CARNOT CYCLE

Reversible cycles cannot be achieved in practice because the irreversibilities associated with each process and cannot be eliminated. Reversible cycles also serve as starting points in the development of actual cycles and are modified as needed.

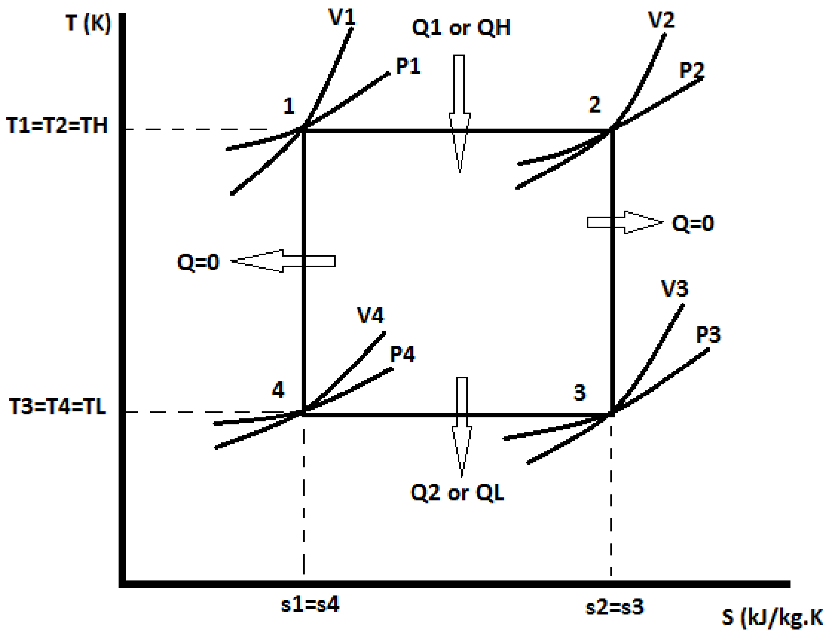
Probably the best known reversible cycle is the **Carnot cycle,** first proposed in 1824 by French engineer Carnot. The theoretical heat engine that operates on the Carnot cycle is called the **Carnot heat engine.**

The Carnot cycle is composed of four reversible processes and it operates in clockwise.

Heat flows in the direction of decreasing temperature from TH to TL.

Two reversible isothermal and **two reversible adiabatic (isentropic)** processes can be executed either in a closed or a steady-flow system.

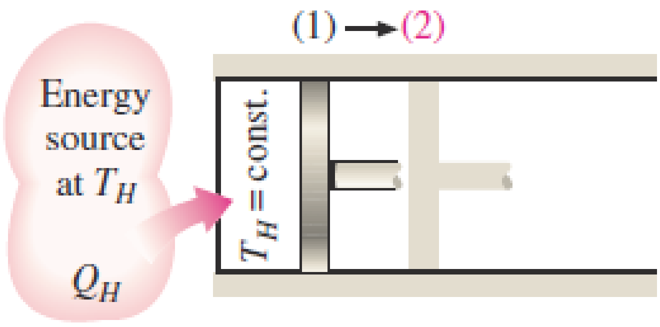
Working Fluid: Ideal gas



Consider a closed system that consists of a gas contained in an adiabatic piston–cylinder device. The insulation of the cylinder head is such that it may be removed to bring the cylinder into contact with reservoirs to provide heat transfer. The four reversible processes that make up the Carnot cycle are as follows:

A- Reversible Isothermal Expansion

(process 1-2, T1=T2=TH=constant). Initially (state 1), the temperature of the gas is TH and the cylinder head is in close contact with a source at temperature TH. The gas is allowed to expand slowly, doing work by the system. As the gas expands, the temperature of the gas tends to decrease. But as soon as the temperature drops by an infinitesimal amount dT, some heat is transferred from the reservoir into the gas, raising the gas temperature to TH. Thus, the gas temperature is kept constant at TH. Since the temperature difference between the gas and the reservoir never exceeds a differential amount dT, this is a reversible heat transfer process. It continues until the piston reaches position 2. The amount of total heat transferred to the gas during this process is QH.



(a) Process 1-2

T = constant: P and V change during process

non-flow energy equation

$Q – W = U_2 - U_1 = cv*(T_2 - T_1)$

$U_2-U_1 = cv*(T_2 - T_1) = 0$

$Q_H = W$

dW = PdV   P*V = R*T P = R*T/V   dW = R*T*dV/V

$Q_H = W = R*T_H*ln(V_2/V_1) = R*T_H*ln(P_1/P_2)$

Only the heat added process of Carnot Cycle is the isothermal expansion process.

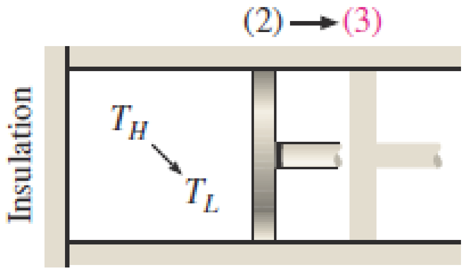
dS = dQ/T $ΔS = (S_2 – S_1) = Q_H/T_H$

$ΔS = (S_2 – S_1) = R*T_H*ln(V_2/V_1)/T_H = R*T_H*ln(P_1/P_2)/T_H$ cancelling T_H terms on equations

$ΔS = (S_2 – S_1) = R*ln(V_2/V_1) = R*ln(P_1/P_2)$

B- Reversible Adiabatic Expansion (isentropic expansion)

process 2-3, temperature drops from TH to TL). At state 2, the reservoir that was in contact with the cylinder head is removed and replaced by insulation so that the system becomes adiabatic. The gas continues to expand slowly, doing work by the system until its temperature drops from TH to TL (state 3). The piston is assumed to be frictionless and the process to be quasi-equilibrium, so the process is reversible as well as adiabatic.



(b) Process 2-3

Q = 0

$T_2/T_3 = (P_2/P_3)^{(\Upsilon-1)/\Upsilon}$ $T_2/T_3 = (V_3/V_2)^{(\Upsilon-1)}$ $P_2/P_3 = (V_3/V_2)^{\Upsilon-1}$

non-flow energy equation

$Q - W = U_3 - U_2 = c_v*(T_3 - T_2)$

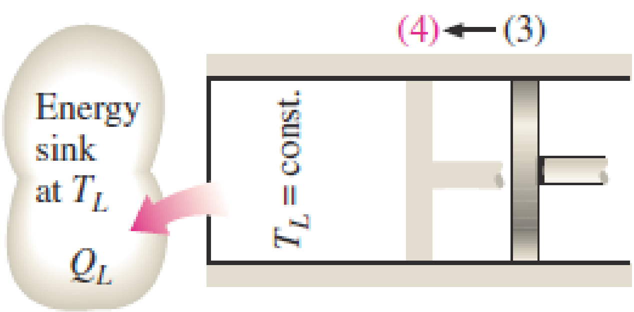
$W = U_2 - U_3 = c_v*(T_2 - T_3)$ or $W = R*(T_2 - T_3)/(\Upsilon-1)$ or $W = (P_2V_2 - P_3V_3)/(\Upsilon-1)$

$dS = dQ/T$ $\Delta S = (S_3 - S_2) = Q/T$

Q = 0 and ΔS = 0 $S_2=S_3$

C- Reversible Isothermal Compression

(process 3-4, T3=T4=TL is constant). At state 3, the insulation at the cylinder head is removed, and the cylinder is brought into contact with a sink at temperature TL. Now the piston is
pushed inward by an external force, doing work on the system. As the gas is compressed, its temperature tends to rise. But as soon as it rises by an infinitesimal amount dT, heat is transferred from the gas to the sink, causing the gas temperature to drop to TL. Thus, the gas temperature remains constant at TL. Since the temperature difference between the gas and the sink never exceeds a differential amount dT, this is a reversible heat transfer process. It continues until the piston reaches state 4. The amount of heat rejected from the gas during this process is QL.



(c) Process 3-4

T = constant: P and V change during process

non-flow energy equation

$Q - W = U_4 - U_3 = cv*(T_4 - T_3)$

$U_4\text{-}U_3 = cv*(T_4 - T_3) = 0$

$Q_L = W$

dW = PdV

P*V = R*T P = R*T/V

dW = R*T*dV/V

$Q_L = W = R*T_L*\ln(V_4/V_3) = R*T_L*\ln(P_3/P_4)$

Only the heat rejected process of Carnot Cycle is the isothermal compression process.

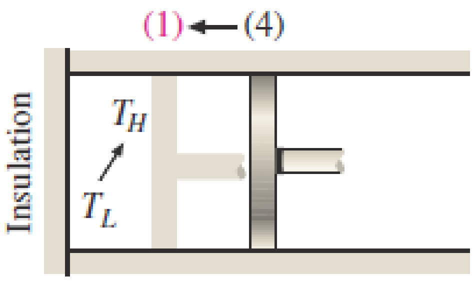
$ds = dQ/T$ $\Delta S = (S_4 - S_3) = Q_L/T_L$

$\Delta S = (S_4 - S_3) = R*T_L*\ln(V_4/V_3)/T_L = R*T_L*\ln(P_3/P_4)/T_L$ cancelling T_L terms on equations

$\Delta S = (S_4 - S_3) = R*\ln(V_4/V_3) = R*\ln(P_3/P_4)$

D- Reversible Adiabatic Compression (isentropic compression)

(process 4-1, temperature rises from TL to TH). State 4 is such that when the low-temperature reservoir is removed, the insulation is put back on the cylinder head, and the gas is compressed in a reversible manner, the gas returns to its initial state (state 1). The temperature rises from TL to TH during this reversible adiabatic compression process, which completes the cycle.



(d) Process 4-1

Q = 0

$T_4/T_1 = (P_4/P_1)^{(\Upsilon-1)/\Upsilon}$ $T_4/T_1 = (V_1/V_4)^{(\Upsilon-1)}$ $P_4/P_1 = (V_4/V_1)^{\Upsilon-1}$

non-flow energy equation

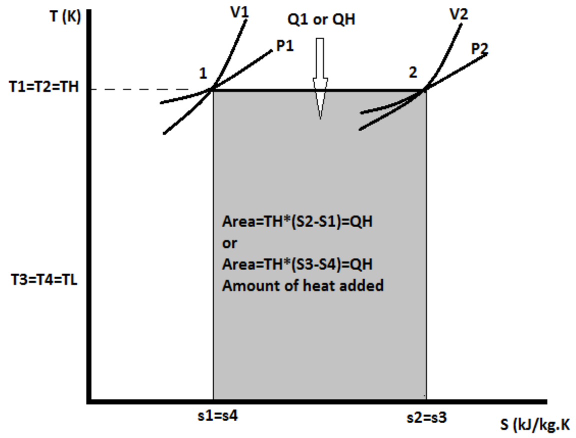
$Q - W = U_1 - U_4 = c_v*(T_1 - T_4)$

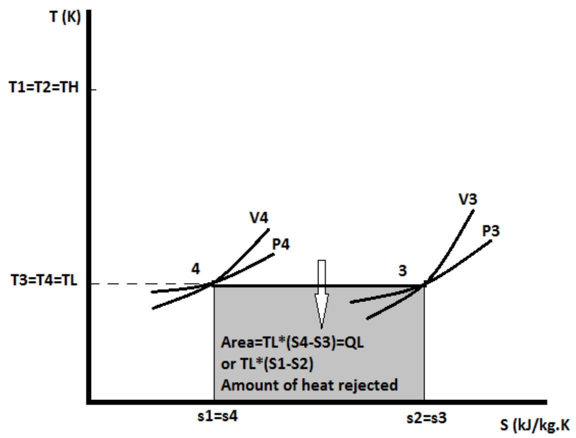
$W = U_4 - U_1 = c_v*(T_4 - T_1)$ or $W = R*(T_4 - T_1)/(\Upsilon-1)$ or $W = (P_4V_4 - P_1V_1)/(\Upsilon-1)$

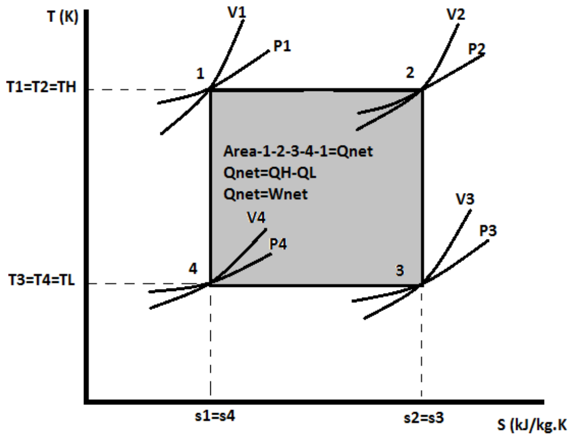
$ds = dQ/T$ $\Delta S = (S_1 - S_4) = Q/T$

Q = 0 and $\Delta S = 0$ $S_1 = S_4$

PROPERTIES OF CARNOT CYCLE







Area-1-2-3-4-1 = Q_{net} = Q_H - Q_L

Since each process occurs in a closed system, non-flow energy equation is written around cycle

For cycle $Q - W = U_1$(initial state) - U_1(final state) = $cv*(T_1 - T_1) = 0$

$Q_{net} = W_{net}$ $Q_H > Q_L$ then Q_{net} or Q_{net} added = $(Q_H - Q_L)$ = + and W_{net} done by the cycle or W_{net} out = +

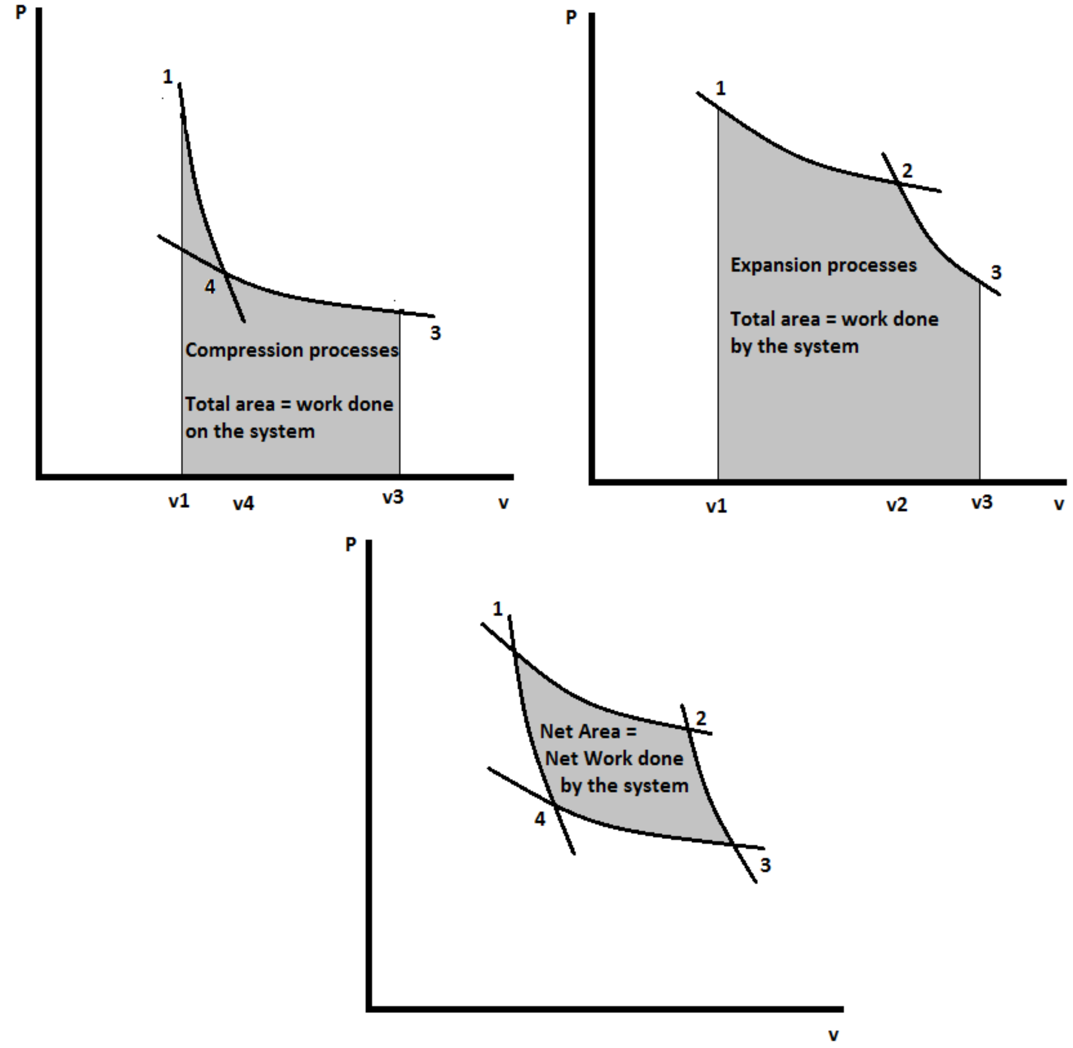
$Q_{net} = W_{net} = (T_H - T_L)*(S_2 - S_1)$ or $=(T_H - T_L)*(S_3 - S_4)$ since $S_2=S_3$ and $S_1=S_4$

Thermal efficiency of Carnot Cycle

Thermal efficiency = $\eta = \frac{TH-TL}{TH}$

Q_H occurs at T_H and Q_L at T_L $\eta = \frac{TH-TL}{TH} = \frac{QH-QL}{QH} = \frac{Qnet}{QH} = \frac{Wnet}{QH}$

Carnot Cycle on P-v diagram

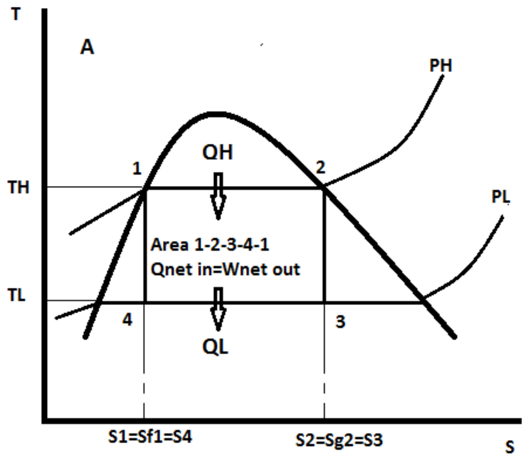


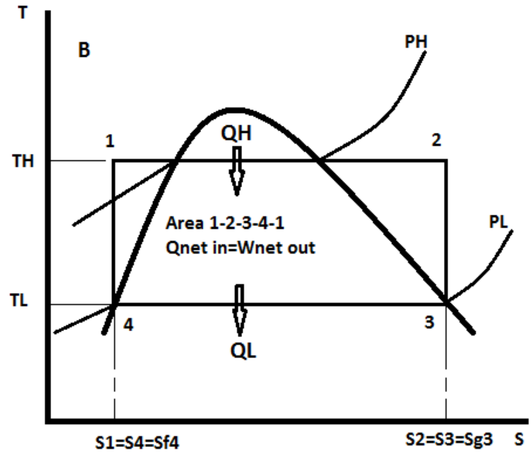
Work done by the system (gain): area under 1-2-3 (expansion processes) = W_{gross}

Work done on the system (loss): area under 3-4-1 (compression processes)

Net work done by the system (area 1-2-3-4-1) = $W_{net\ out}$

Working Fluid: Water





A- process 1-2: isothermal (constant temperature) expansion also constant pressure expansion, non-flow

$Q_H - W = U_2 - U_1$ $W=P_H*(v_2 - v_1)$ $Q_H - P_H*(v_2 - v_1) = U_2 - U_1$ $Q_H = h_2 - h_1 = + h_{fg}$

Or

$dS = dQ/T$ $dQ = T*dS$ $Q_H = T_H*(S_2 - S_1)$ $Q_H = T_H*S_{fg}$

Process 3-4: isothermal (constant temperature) compression also constant pressure compression, non-flow

process needs dryness fractions at 3 and 4

$S_1 = S_{f1} = S_4$ reading $S_1 = S_{f1}$ at T_H or P_H from saturated water table and writing equation

$S_1 = S_{f1} = S_4 = S_{f4} + x_4*S_{fg4}$ S_{f4} and S_{fg4} at T_L or P_L from saturated water table

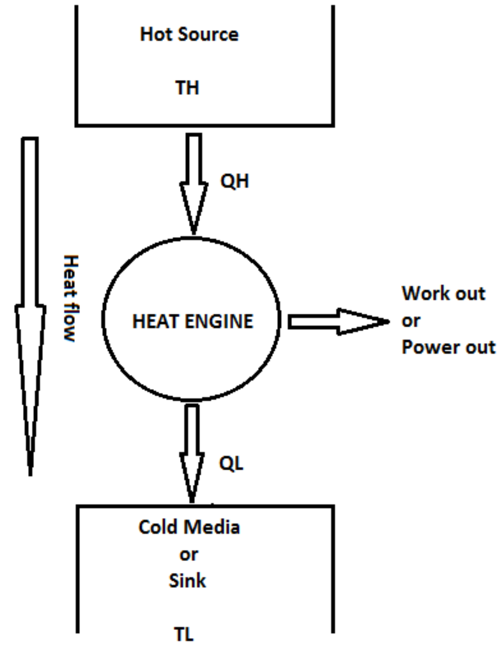
apply same procedure for point 3 to find x_3

$dS = dQ/T$ $dQ = T*dS$ $Q_L = T_L*(S_4 - S_3)$ or $Q_L = T_L*(S_1 - S_2)$ $S_1=S_4$ and $S_2=S_3$

$Q_L - W = U_4 - U_3$ $W=P_L*(v_4 - v_3)$

B- Similar calculations can be used as in case **A**

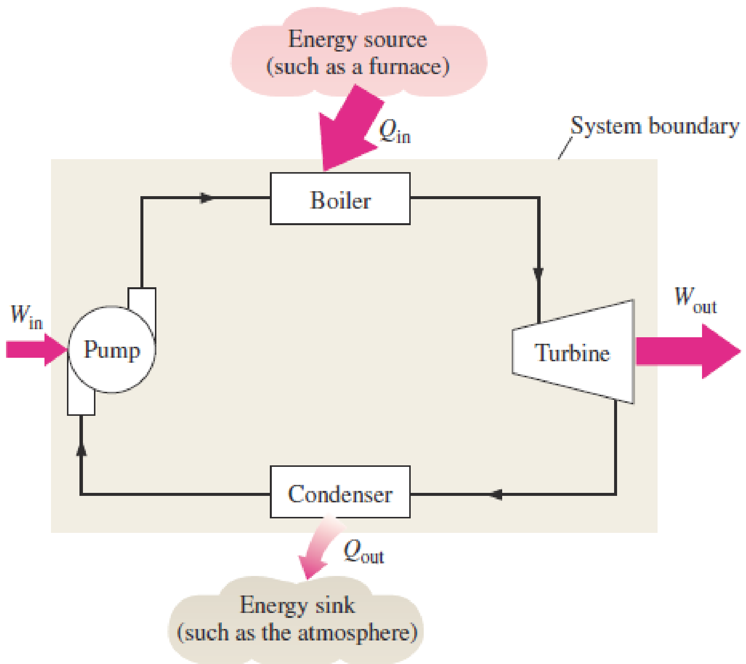
Heat Engines



Thermal efficiency of a Heat Engine

Thermal efficiency = $\eta = \frac{TH - TL}{TH}$

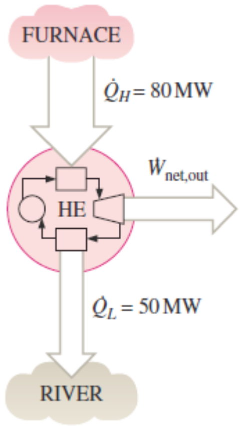
Q_H occurs at T_H and Q_L at T_L $\quad \eta = \frac{TH - TL}{TH} = \frac{QH - QL}{QH} = \frac{Qnet}{QH} = \frac{Wnet}{QH}$



$W_{net\ out} = W_{out} - W_{in}$

$W_{net\ out} = Q_{in} - Q_{out} = Q_H - Q_L$

Example problem



Heat is transferred to a heat engine from a furnace at a rate of 80 MW. If the rate of waste heat rejection to a nearby river is 50 MW, determine the net power output and the thermal efficiency for this heat engine.

Solution The rates of heat transfer to and from a heat engine are given. The net power output and the thermal efficiency are to be determined.
Assumptions Heat losses through the pipes and other components are negligible.
Analysis A schematic of the heat engine is given in Fig. 6–16. The furnace serves as the high-temperature reservoir for this heat engine and the river as the low-temperature reservoir. The given quantities can be expressed as

$$\dot{Q}_H = 80 \text{ MW} \quad \text{and} \quad \dot{Q}_L = 50 \text{ MW}$$

The net power output of this heat engine is

$$\dot{W}_{net,out} = \dot{Q}_H - \dot{Q}_L = (80 - 50) \text{ MW} = 30 \text{ MW}$$

Then the thermal efficiency is easily determined to be

$$\eta_{th} = \frac{\dot{W}_{net,out}}{\dot{Q}_H} = \frac{30 \text{ MW}}{80 \text{ MW}} = 0.375 \text{ (or } 37.5\%\text{)}$$

Discussion Note that the heat engine converts 37.5 percent of the heat it receives to work.