## ENTROPY

The first law of thermodynamics deals with the property energy and the conservation of it.

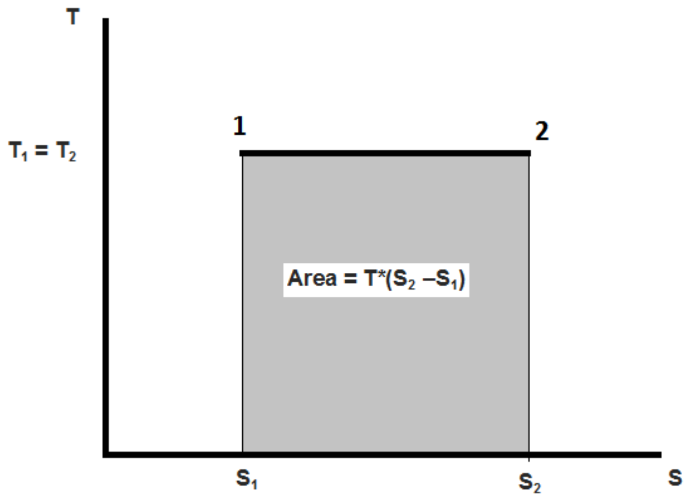
The second law leads to the definition of a new property called **entropy**.

Entropy is the measure of the disorder of a system.

General euation for all working fluids

**$\Delta S = \Delta Q/T$ or $ds = dQ/T$**

**Area = Q = heat transfer = $T*(S_2 - S_1)$**




### PATH FUNCTIONS

General work equation W = F * d = Force * distance
İt is obvious that work is a path function
Similarly, heat transfer is a path function
**Q and W are the path functions**

### STATE FUNCTIONS

Other properties; P, V and T are the state functions
Energy terms; u (internal energy), h (enthalpy) and s (entropy) are the state functions

Initial and final states are important for state functions

Change during a process for ideal gases
$\Delta u = u_2 - u_1 = c_v * (T_2 - T_1)$
**$\Delta h = h_2 - h_1 = c_p * (T_2 - T_1)$**
$\Delta s = s_2 - s_1$ = see above equations in entropy part

## IDEAL GASES

## NON-FLOW PROCESSES

1. Reversible constant volume
2. Reversible constant pressure
3. Reversible constant temperature (isothermal)
4. Reversible adiabatic
5. Polytropic

**1. Reversible constant volume**

V = constant; P and T change during process depending on the amount of heat transfer

$V_1 = V_2$, $m_1 = m_2$, $v_1 = v_2$, $P*v^n$ = constant, **n = ∞**  for v = constant

$P_1/T_1 = P_2/T_2$

non-flow energy equation

$Q – W = U_2-U_1 = c_v*(T_2 - T_1)$

V is constant $W_b = 0$

$Q = U_2-U_1 = c_v*(T_2 - T_1)$  $dQ = c_v*dT$ and the general equation **ds = dQ/T**

**$dS = c_v*dT/T$** taking integral of the equation for a process between initial and final states

$$\int_1^2 dS = c_v * \int_1^2 \frac{dT}{T} \qquad (S_2 - S_1) = \Delta S = c_v * \ln\frac{T_2}{T_1}$$

**2. Reversible constant pressure**

P = constant; V and T change during process depending on the amount of heat transfer

$P_1 = P_2$, $P*v^n$ = constant, **n = 0**  for P = constant

$V_1/T_1 = V_2/T_2$

non-flow energy equation

$Q – W = U_2-U_1 = c_v*(T_2 - T_1)$

P is constant $W_b = P*(V_2 - V_1)$

$Q - P*(V_2 - V_1)= U_2-U_1$

$Q = (U_2 + PV_2) – (U_1 + PV_1)$

$Q = h_2 – h_1 = cp\ (T_2 –T_1)$  $dQ = c_p*dT$ and the general equation **ds = dQ/T**

**$dS = c_p*dT/T$** taking integral of the equation for a process between initial and final states

$$\int_1^2 dS = c_p * \int_1^2 \frac{dT}{T} \qquad (S_2 - S_1) = \Delta S = c_p * \ln\frac{T_2}{T_1}$$

*****Constant volume and constant pressure lines on T-s diagram*****

Slope of the constant volume line higher than the slope of constant pressure line

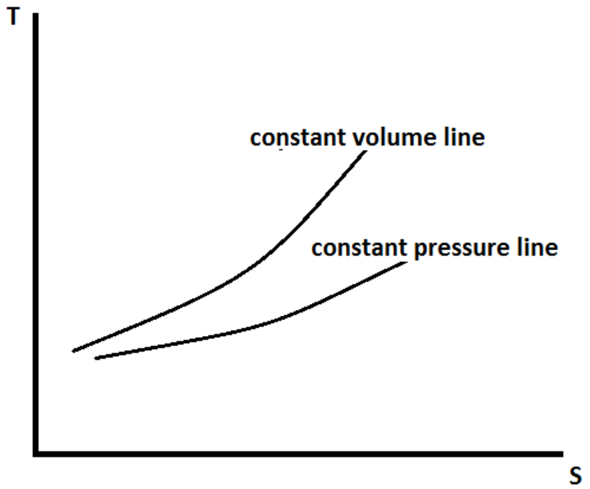
**Slope of constant volume line = dy/dx = dT/ds = $(T_2 - T_1)/(S_2 - S_1) = (T_2 - T_1)/(c_v*\ln(T_2/T_1))$**

**Slope of constant pressure line = dy/dx = dT/ds = $(T_2 - T_1)/(S_2 - S_1) = (T_2 - T_1)/(c_p*\ln(T_2/T_1))$**

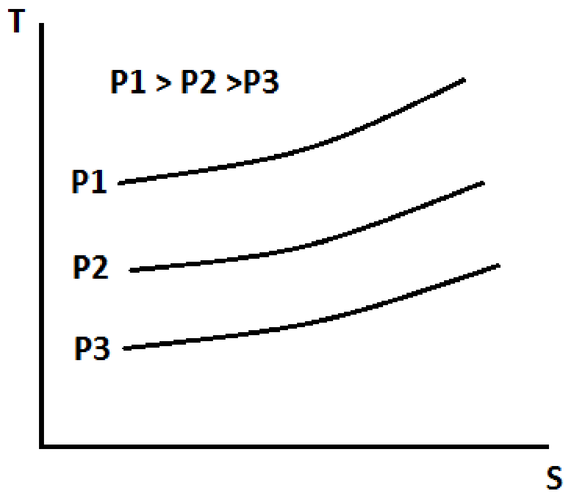
Slope of constant volume line = $1/c_v$

Slope of constant pressure line = $1/c_p$

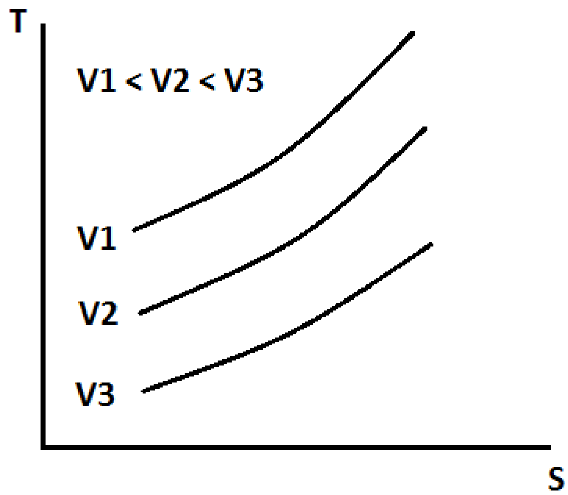
**$C_p > C_v$ then $1/Cv > 1/Cp$**




Constant pressure lines on T-s




Constant volume lines on T-s



**3. Reversible constant temperature**

**(isothermal)**

T = constant: P and V change during process depending on the amount of heat transfer

non-flow energy equation

$T_1 = T_2$, $P*v^n$ = constant, **n = 1**   for T = constant

$P_1*V_1 = P_2*V_2$

non-flow energy equation

$Q – W = U_2 - U_1 = c_v*(T_2 - T_1)$

$U_2 - U_1 = c_v*(T_2 - T_1) = 0$

Q = W

dW = PdV

P*V = R*T   P = R*T/V

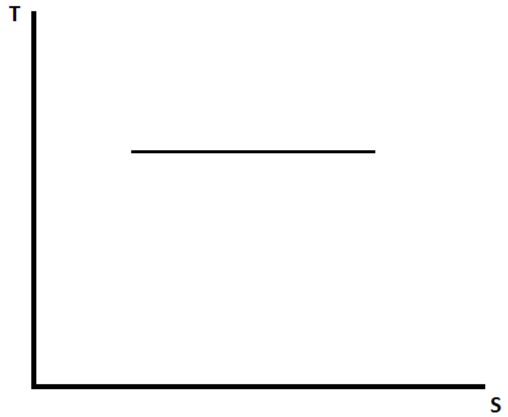
dW = R*T*dV/V

$Q = W = R*T*\ln(V_2/V_1) = R*T*\ln(P_1/P_2)$

general equation **ds = dQ/T**   $\Delta S = (S_2 – S_1) = Q/T$

$\Delta S = (S_2 – S_1) = R*T*\ln(V_2/V_1)/T = R*T*\ln(P_1/P_2)/T$   **cancelling T terms on equation**

$\Delta S = (S_2 – S_1) = R*\ln(V_2/V_1) = R*\ln(P_1/P_2)$




**4. Adiabatic process**

**(i) Reversible adiabatic process (Isentropic)**

**Q = 0**

$P*v^n$ = constant, **n = $\gamma$**   **P, V, T change during process**

$T_1/T_2 = (P_1/P_2)^{(\gamma-1)/\gamma}$

$T_1/T_2 = (V_2/V_1)^{(\gamma-1)}$

$P_1/P_2 = (V_2/V_1)^{\gamma}$

non-flow energy equation

$Q - W = U_2 - U_1 = c_v*(T_2 - T_1)$

$W = U_1 - U_2 = c_v*(T_1 - T_2)$ or

$W = R*(T_1 - T_2)/(\Upsilon-1)$ or

$W = (P_1V_1 - P_2V_2)/(\Upsilon-1)$

general equation **$ds = dQ/T$ $\Delta S = (S_2 - S_1) = Q/T$ $Q = 0$ $\Delta S = 0$**


**(ii) irreversible adiabatic process**

**$Q = 0$ $\Delta S \neq 0$**

**P, V, T change during process**

***** you can not use below equations*****

**$T_1/T_2 = (P_1/P_2)^{(\Upsilon-1)/\Upsilon}$**

**$T_1/T_2 = (V_2/V_1)^{(\Upsilon-1)}$ *** These equations are valid only for reversible adibatic process (isentropic)*****

**$P_1/P_2 = (V_2/V_1)^{\Upsilon}$**

**5. Polytropic process**

$P*v^n$ = constant, **$n \neq 0, 1,$** $\Upsilon$, $\infty$, **n = n**

**P, V, T change during process**

$T_1/T_2 = (P_1/P_2)^{(n-1)/n}$

$T_1/T_2 = (V_2/V_1)^{(n-1)}$

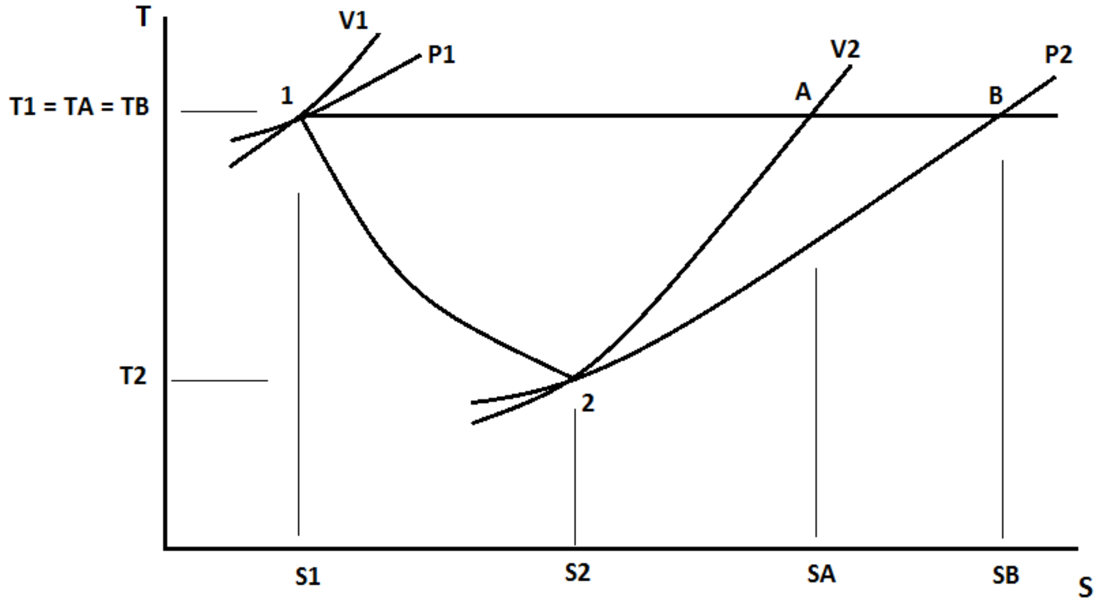
**$P_1/P_2 = (V_2/V_1)^n$**

non-flow energy equation

$Q - W = U_2 - U_1 = c_v*(T_2 - T_1)$

$W = (R/n-1) *(T_1 - T_2)$ or

$W = (P_1V_1 - P_2V_2)/(n-1)$




Process between state 1 and 2 is the polytropic expansion process (P, V and T change)

T decreases, P decreases, V increases

We need **ΔS = ($S_2 – S_1$)**

**If we write a property ($S_2 – S_A$) – ($S_A – S_1$) = ($S_2 – S_A + S_A – S_1$) cancelling $S_A$ terms,**

**Now the equation is ΔS = ($S_2 – S_1$)**

**($S_2 – S_A$) is the entropy change of a constant volume process (initial 2 and final A)**

$$(S_2 - S_A) = \Delta S = c_v * \ln\frac{T_A}{T_2}$$

**$T_A = T_1$** $(S_2 - S_A) = \Delta S = c_v * \ln\frac{T_1}{T_2}$

**($S_A – S_1$) is the entropy change of a constant temperature (isothermal) process (initial 1 and final A)**

$$(S_A - S_1) = \Delta S = R * \ln\frac{V_A}{V_1}$$

$V_A = V_2$ $(S_A - S_1) = \Delta S = R * \ln\frac{V_2}{V_1}$

**ΔS = ($S_2 – S_1$) =** $R * \ln\frac{V_2}{V_1}$ - $c_v * \ln\frac{T_1}{T_2}$ or **ΔS = ($S_2 – S_1$) =** $\boldsymbol{R * \ln\frac{V_2}{V_1} + c_v * \ln\frac{T_2}{T_1}}$

Equation was derived using a constant temperature and a constant volume processes (point A).

Another equation can be derived using a constant temperature and a onstant pressure processes (point B).

**If we write a property ($S_2 – S_B$) – ($S_B – S_1$) = ($S_2 – S_B + S_B – S_1$) cancelling $S_B$ terms,**

**Now the equation is ΔS = ($S_2 – S_1$)**

**($S_2 - S_B$) is the entropy change of a constant pressure process (initial 2 and final B)**

$$(S_2 - S_B) = \Delta S = c_p * \ln\frac{T_B}{T_2}$$

**$T_B = T_1$** $(S_2 - S_B) = \Delta S = c_p * \ln\frac{T_1}{T_2}$

**($S_B - S_1$) is the entropy change of a constant temperature (isothermal) process (initial 1 and final B)**

$$(S_B - S_1) = \Delta S = R * \ln\frac{P_1}{P_B}$$

$P_B = P_2$ $(S_B - S_1) = \Delta S = R * \ln\frac{P_1}{P_2}$

**$\Delta S = (S_2 - S_1) = R * \ln\frac{P_1}{P_2} - c_p * \ln\frac{T_1}{T_2}$ or $\Delta S = (S_2 - S_1) = R * \ln\frac{P_1}{P_2} + c_p * \ln\frac{T_2}{T_1}$**

Now, we have two equations to determine entropy change of a polytropic process

**1- $\Delta S = (S_2 - S_1) = R * \ln\frac{V_2}{V_1} + c_v * \ln\frac{T_2}{T_1}$**

**or**

**2- $\Delta S = (S_2 - S_1) = R * \ln\frac{P_1}{P_2} + c_p * \ln\frac{T_2}{T_1}$**

*****Notice that above equations are general equations when P, V, T change.*****

**Cases**

**(i) Constant volume process: on equation 1, $R * \ln\frac{V_2}{V_1}$ becomes zero and $\Delta S = (S_2 - S_1) = c_v * \ln\frac{T_2}{T_1}$**

**(ii) Constant pressure process: on equation 2, $R * \ln\frac{P_1}{P_2}$ beomes zero and $\Delta S = (S_2 - S_1) = c_p * \ln\frac{T_2}{T_1}$**

**(iii) Constant temperature process: on equation 1 or 2 $c_v * \ln\frac{T_2}{T_1}$ and $c_p * \ln\frac{T_2}{T_1}$ become zero and**

**$\Delta S = (S_2 - S_1) = R * \ln\frac{V_2}{V_1}$ or $\Delta S = (S_2 - S_1) = R * \ln\frac{P_1}{P_2}$**

## IDEAL GASES

## FLOW PROCESSES

**P, V and T change during a flow process, general equations can be used for flow processes.**

**1- $\Delta S = (S_2 - S_1) = R * \ln\frac{V_2}{V_1} + c_v * \ln\frac{T_2}{T_1}$**

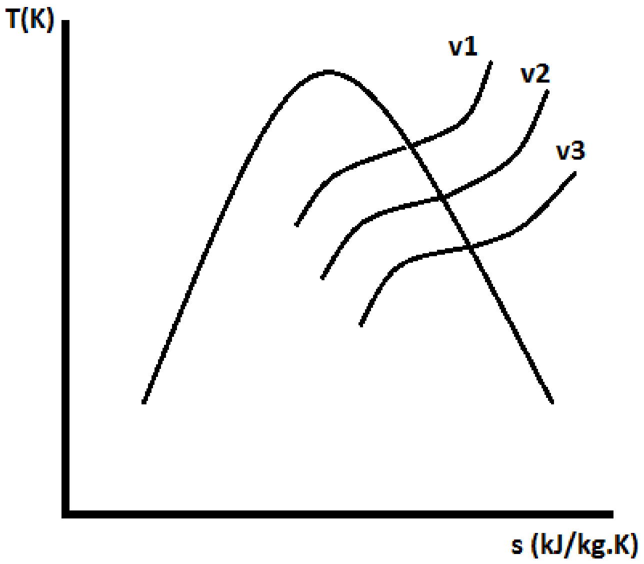
**or**

**2- $\Delta S = (S_2 - S_1) = R * \ln\frac{P_1}{P_2} + c_p * \ln\frac{T_2}{T_1}$**

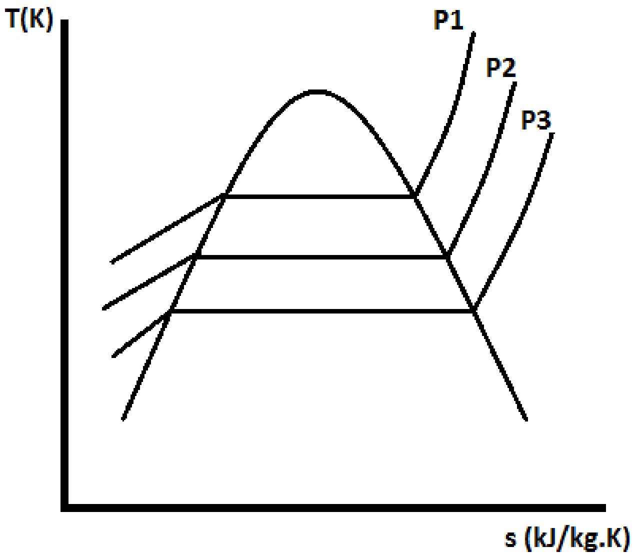
**WATER**
**NON-FLOW PROCESSES**
Entropy change of water during a non-flow process is calculated using entropy values on steam table depending on the initial and final states of water.
**ΔS = (S2 – S1)**

**WATER**
**FLOW PROCESSES**
Entropy change of water during a flow process is calculated using entropy values on steam table depending on the inlet and exit states of water.
**ΔS = (S2 – S1)**

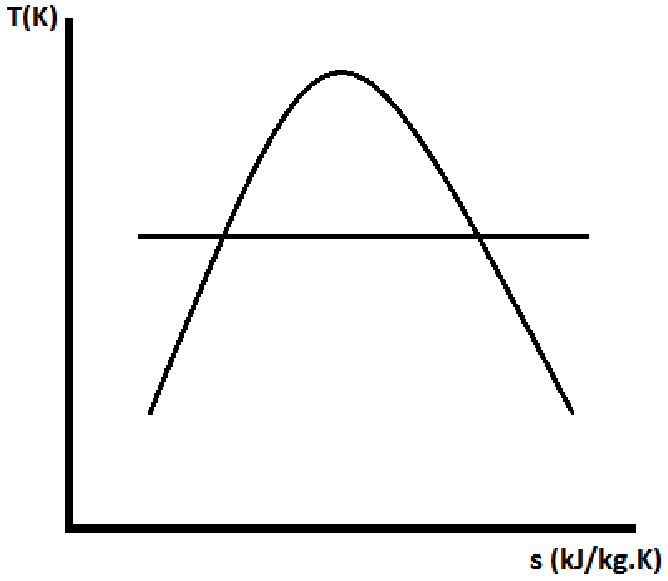
**Mixture region**
**S= Sf + x*Sfg**



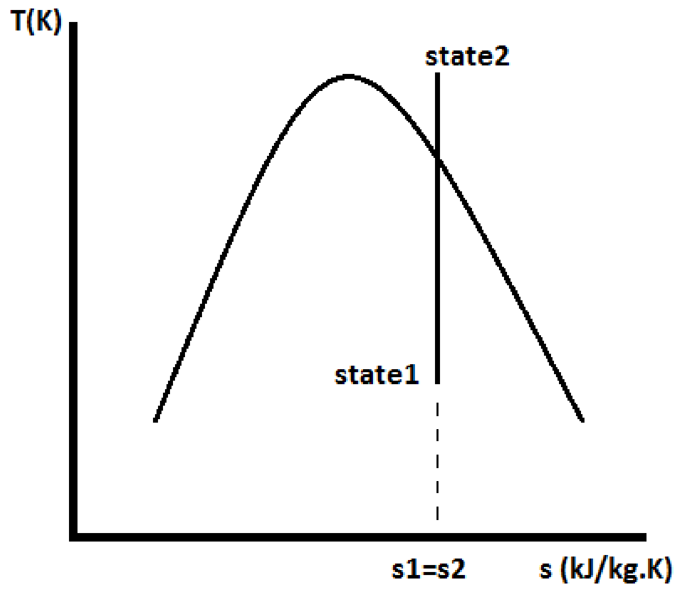
Constant volume lines on T-s diagram



Constant pressure lines on T-s diagram




Constant temperature line on T-s diagram




Reversible adiabatic (isentropic) line on T-s diagram

**Entropy and Irreversibility**

**Example**

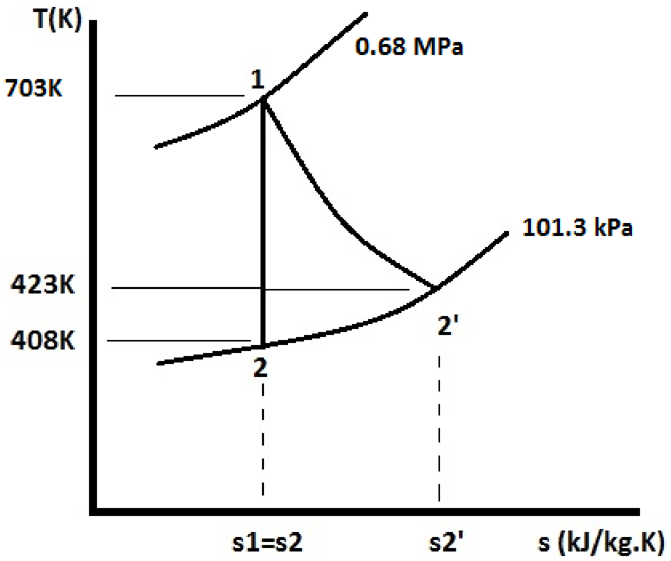
In an air turbine the air expands from 0.68 MPa and 430°C to 101.3 kPa and 150°C. **The heat loss from the turbine can be assumed to be negligible**. Show that the process is irreversible and calculate the entropy change of air.

W.F. : air
System: open, air turbine
Process: flow

Since the heat loss is negligible, process is adiabatic.

(i) For a reversible adiabatic (isentropic)
Q = 0 and ΔS = 0
$T_1/T_2 = (P_1/P_2)^{(\gamma-1)/\gamma}$

$703/T_2 = (680/101.3)^{(1.4-1)/1.4}$
T2 = 408K or 135°C for reversible adiabatic (isentropic) on T-s diagram process line 1-2
but the exit temperature is 150°C, it can be decided that process is not reversible adiabatic.
(ii) irreversible adiabatic
Q = 0 and ΔS ≠ 0 on T-s diagram process line 1-2' (actual process)




The entropy change of the actual process $\Delta S = S_{2'} - S_1$
On T-s diagram $S_1=S_2$ above entropy change equation also can be written as $\Delta S = S_{2'} - S_2$
Point 2 and point 2' lie on the same constant pressure line then
$\Delta S = S_{2'} - S_2 = cp*\ln(T_{2'}/T_2) = 1.00*\ln(423/408) =$ **+ 0.0355 kJ/kg.K**