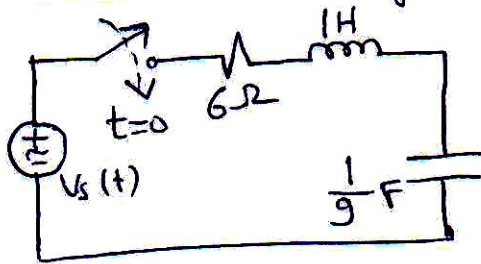


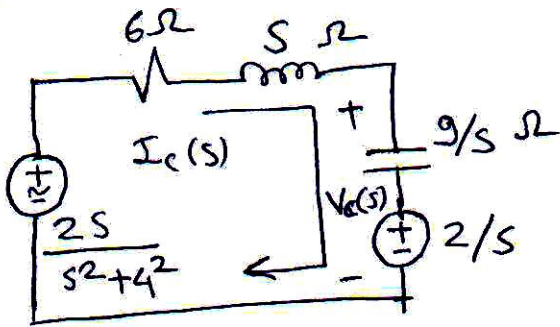
Example: Find voltage across capacitor $V_c(t)$ for $t \geq 0$.



Initial voltage in capacitor is 2V.

$$V_c(0^+) = 2V.$$

$$V_s(t) = 2 \cos 4t$$



$$I_c(s) = \frac{\frac{2s}{s^2+16} - \frac{2}{s}}{6+s+\frac{9}{s}}$$

$$I_c(s) = \frac{2s^2 - 2s^2 - 32}{(s^2+16)s(6s+s^2+9)}$$

$$I_c(s) = \frac{-32s}{(s^2+16)(s+3)^2 s} = \frac{-32}{(s^2+16)(s+3)^2}$$

$$V_c(s) = I_c(s) \frac{9}{s} + \frac{2}{s} = \frac{-32}{(s^2+16)(s+3)^2} \frac{9}{s} + \frac{2}{s}$$

$$V_c(s) = \frac{-288 + 2(s^2+16)(s+3)^2}{s(s^2+16)(s+3)^2}$$

$$V_c(s) = \frac{K_1}{s} + \frac{K_2}{s-j4} + \frac{K_3}{s+j4} + \frac{K_4}{(s+3)^2} + \frac{K_5}{s+3}$$

$$K_1 = V_c(s)s \Big|_{s=0} = \frac{-288 + 288}{16(9)} = 0$$

$$K_2 = V_c(s)(s-j4) \Big|_{s=j4} = \frac{-288 + 2(s^2+16)(s+3)^2}{s(s+j4)(s+3)^2} \Big|_{s=j4}$$

$$K_2 = -0.1008 - j0.3456 = 0.36 \angle -106.2602^\circ$$

$$K_3 = K_2^* = 0.36 \angle 106.2602^\circ$$

$$K_4 = V_c(s)(s+3)^2 \Big|_{s=-3} = \frac{-288 + 2(s^2+16)(s+3)^2}{s(s^2+16)} \Big|_{s=-3}$$

$$K_4 = \frac{-288 + 2(25)(-3+3)^2}{(-3)25} = \frac{288}{75} = 3.84$$

$$K_5 = \left. \frac{d}{ds} \left[V_c(s) (s+3)^2 \right] \right|_{s=-3}$$

$$K_5 = \left. \frac{d}{ds} \left[\frac{-288 + 2(s^2+16)(s+3)^2}{s(s^2+16)} \right] \right|_{s=-3}$$

$$K_5 = \frac{0 + 2(2s)(s+3)^2 + 2(s^2+16)2(s+3)}{s(s^2+16)} + (-288 + 2(s^2+16)(s+3)^2) \underbrace{\frac{d}{ds} \left[s^{-1}(s^2+16)^{-1} \right]}_A$$

$$A = \frac{d}{ds} \left[s^{-1}(s^2+16)^{-1} \right] = (-1)s^{-2}(s^2+16)^{-1} + s^{-1}(-1)(s^2+16)^{-2} 2s$$

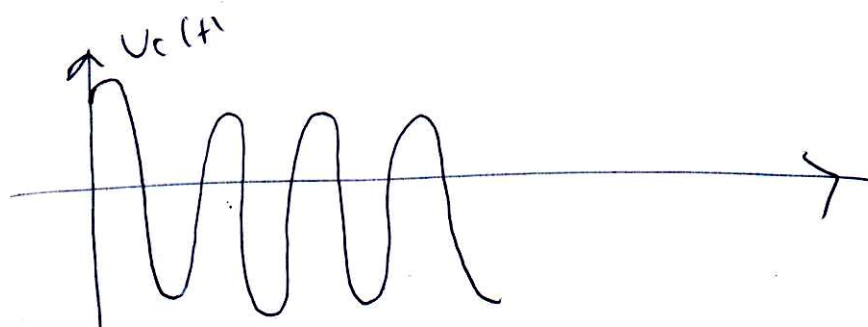
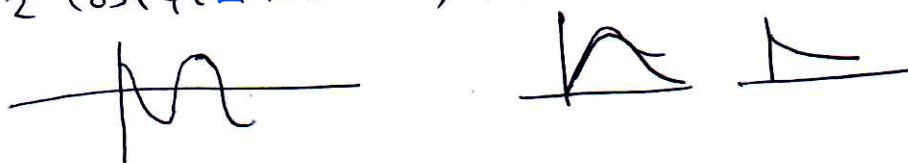
$$K_5 = \frac{4s(s+3)^2 + 2(s^2+16)(s+3)2}{s(s^2+16)} - \frac{(-288 + 2(s^2+16)(s+3)^2)}{s^2(s^2+16)} - \frac{2s(-288 + 2(s^2+16)(s+3)^2)}{s(s^2+16)^2}$$

$$K_5 = 0 + \frac{288}{9(25)} + \frac{6(-288)}{-3 \cdot 25^2} = \frac{288}{225} + \frac{6(288)}{1875} = \boxed{2.051}$$

$$V_c(s) = \frac{0}{s} + \frac{0.36 e^{j106.2602^\circ}}{s-j4} + \frac{0.36 e^{-j106.2602^\circ}}{s+j4} + \frac{3.84}{(s+3)^2} + \frac{2.051}{(s+3)}$$

$$V_c(t) = \left(0.36 e^{-j106.2602^\circ} e^{j4t} + 0.36 e^{+j106.2602^\circ} e^{-j4t} + 3.84 t e^{-3t} + 2.051 e^{-3t} \right) u(t)$$

$$V_c(t) = \left(0.72 \cos(4t - 106.2602^\circ) + 3.84 t e^{-3t} + 2.051 e^{-3t} \right) u(t)$$



$$V_c(t) = (0.72 \cos(4t - 106.2602^\circ) + 3.84t e^{-3t} + 2.051 e^{-3t}) U(t)$$

