EEE 322

Electromechanical Energy Conversion – II

Prepared By

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Given By

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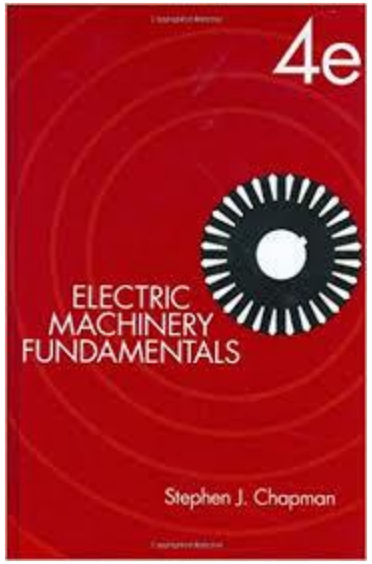
References

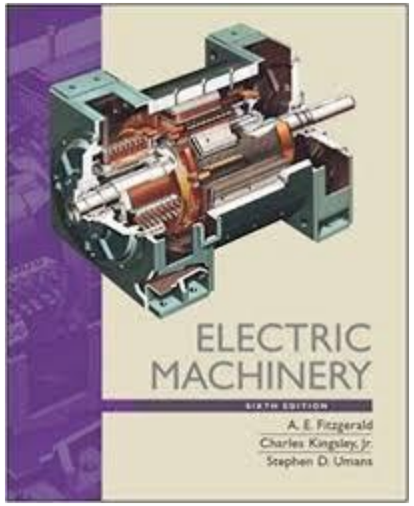
- Electric Machinery Fundamentals, Stephen Chapman, *4th Edition,* McGraw-Hill

(Course Book)



- Electric Machinery, A.E. Fitzgerald, Charles Kingsley, JR., Stephen D. Umans, *6th Edition, McGraw-Hill*

(Supplementary Book)



Contents

Grading Policy

- Midterm-1 : **20%**
- Midterm-2 : **20%**
- Lab : **20%**
- Final : **40%**
- TOTAL : **100%**

✓Exams are closed-book

✓Homework/Project can be assigned during the semester

(The performances of the students can be included into the grading policy)

✓Attendance is minimum 70% to class

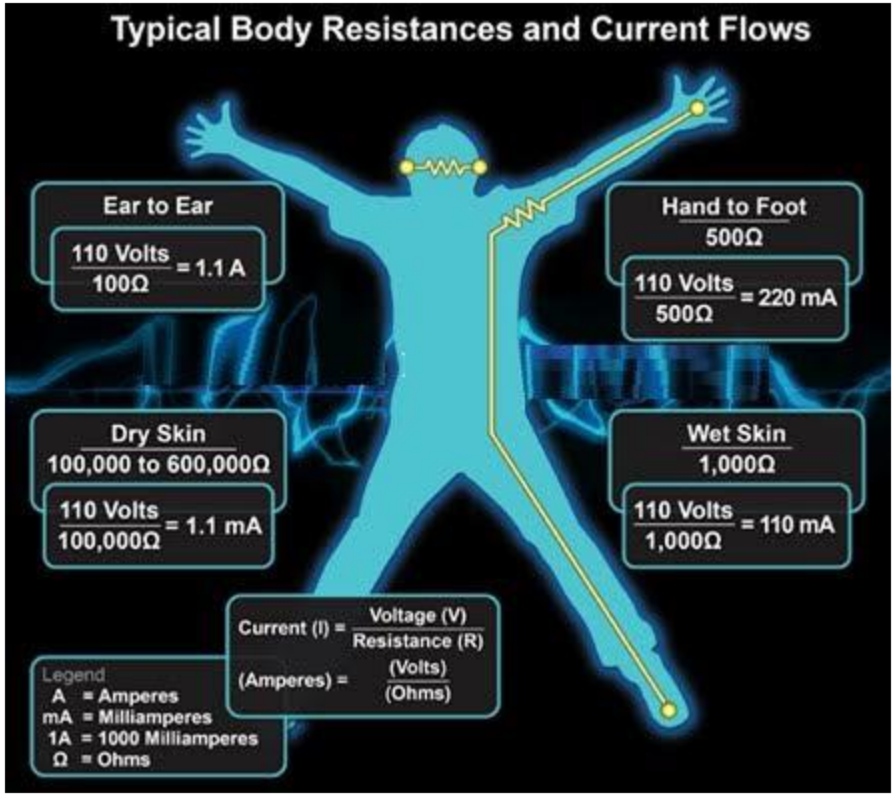
✓Attendance is minimum 80% to laboratory works

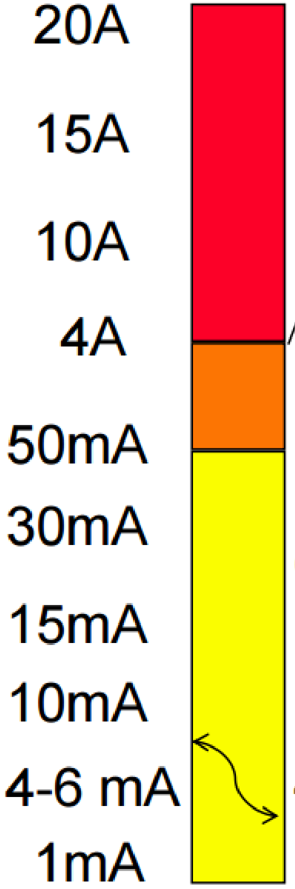
Students who has registered to EEE 322 ***MUST follow*** *the web page of Dr. Ali Osman ARSLAN for all announcements and getting other course related materials.*

Some Advices !

- Keep attendance as much as poosible ! (70% or more)
- Take your own notes
- Practice is good. Do it as much as possible
- Do not try to summarize, try to learn the fundamental idea
- Solve examples, problems as many as possible
- Make your study plan by yourself
- Know yourself ! Study alone or within a group
- Do not postpone anything ! Do it now

Effects of Electrical Shock





4 AMPERES AND OVER

Heart Paralysis, Serious Tissue and Organ Burning

50 mA - 4 Amps Fibrillation

IEC RCD Standards Breathing Difficult, Fibrillation in children

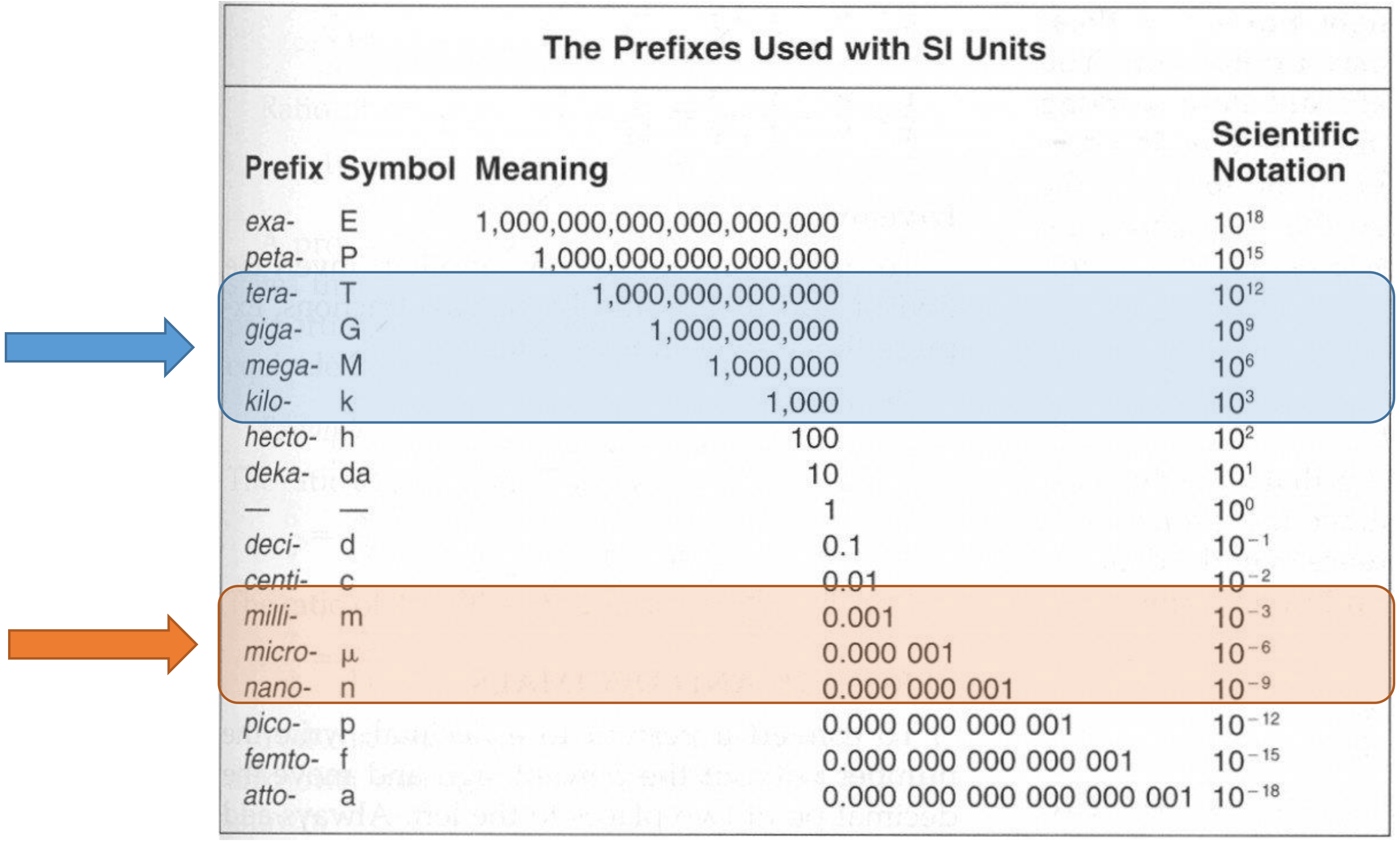
Muscles “Freeze” in 50% of the Population

Let-Go Threshold

4-6 UL 943 Class A GFCI Trip Level

Perception Level

Source: http://www.nema.org



The Prefixes Used with SI Units

Prefix	Symbol	Meaning	Scientific Notation
exa-	E	1,000,000,000,000,000,000	10^{18}
peta-	P	1,000,000,000,000,000	10^{15}
tera-	T	1,000,000,000,000	10^{12}
giga-	G	1,000,000,000	10^{9}
mega-	M	1,000,000	10^{6}
kilo-	k	1,000	10^{3}
hecto-	h	100	10^{2}
deka-	da	10	10^{1}
—	—	1	10^{0}
deci-	d	0.1	10^{-1}
centi-	c	0.01	10^{-2}
milli-	m	0.001	10^{-3}
micro-	μ	0.000 001	10^{-6}
nano-	n	0.000 000 001	10^{-9}
pico-	p	0.000 000 000 001	10^{-12}
femto-	f	0.000 000 000 000 001	10^{-15}
atto-	a	0.000 000 000 000 000 001	10^{-18}

Greek alphabet letters used as symbols in electrical engineering

UPPER CASE

Α α	alpha	Ν ν	nu
Β β	beta	Ξ ξ	ksi
Γ γ	gamma	Ο ο	omicron
Δ δ	delta	Π π	pi
Ε ε	epsilon	Ρ ρ	rho
Ζ ζ	zeta	Σ σς	sigma
Η η	eta	Τ τ	tau
Θ θ	theta	Υ υ	upsilon
Ι ι	iota	Φ φ	phi
Κ κ	kappa	Χ χ	chi
Λ λ	lambda	Ψ ψ	psi
Μ μ	mu	Ω ω	omega

lower case

CHAPTER 1

AC MACHINERY FUNDAMENTALS

What is an AC machine ?

- AC machines are **generators** that convert ***mechanical energy*** to ***ac electrical energy.***

- AC machines are **motors** that convert ***ac electrical energy*** to ***mechanical energy***.

Two major classes of AC machines

- **Synchronous machines**

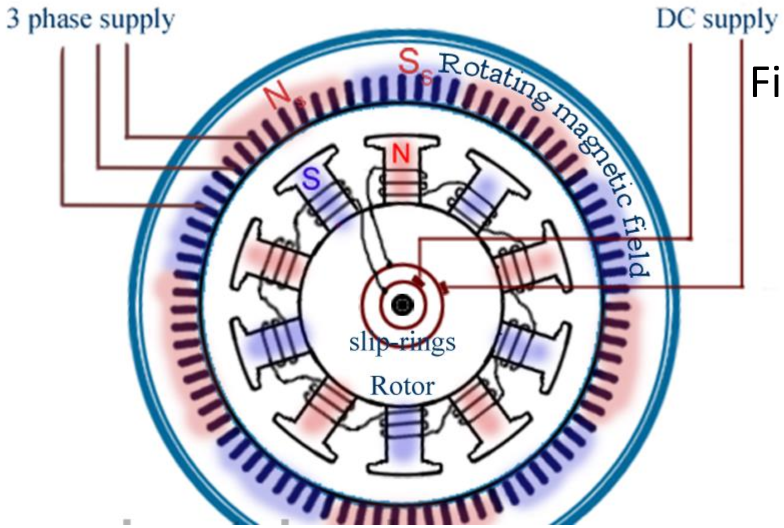
Synchronous machines are motors and generators whose magnetic field current is supplied by a **separate dc power source**

- **Induction machines**

Induction machines are motors and generators whose field current is supplied by **magnetic induction** (transformer action)

Field (excitation) circuit

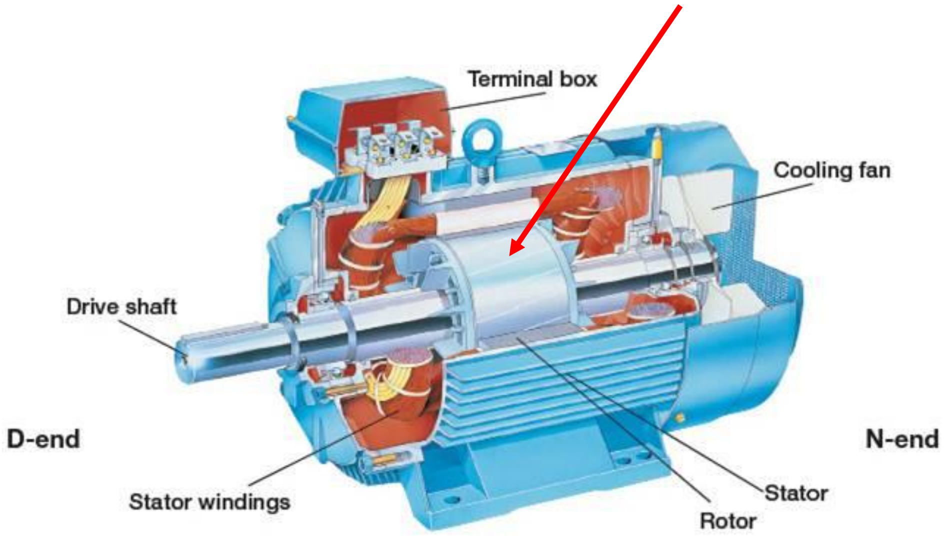
- The **field (excitation) circuits** of most synchronous and induction machines are located **on their rotors**.
- Stator (armature) is the **stationary** part of both machine types
- Rotor is the **rotating** part of both machine types



Field circuit

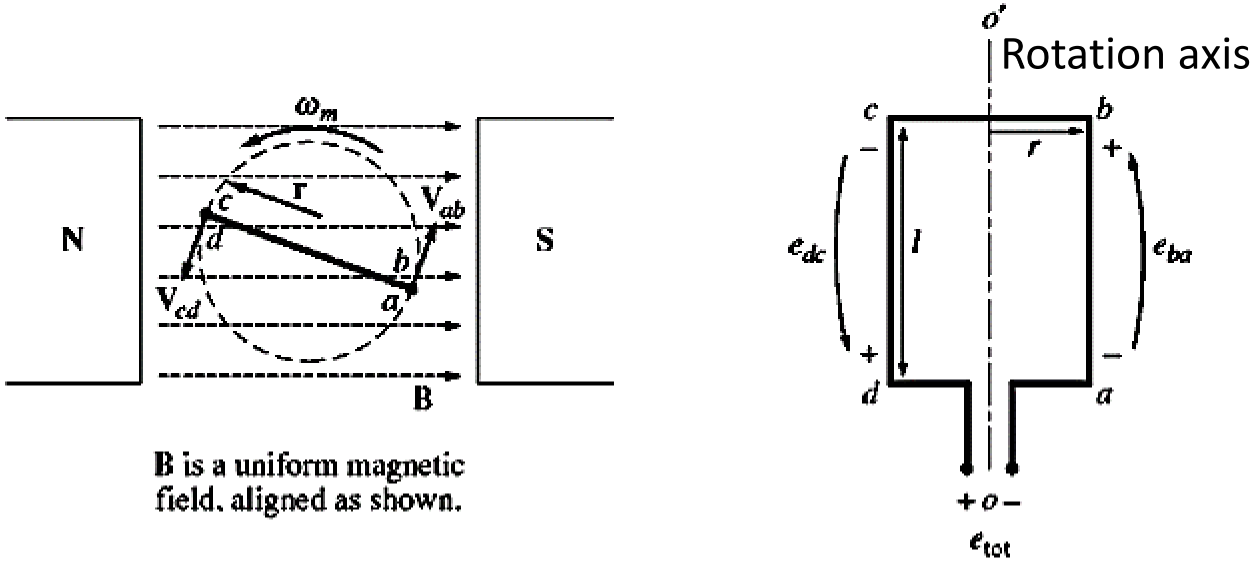
Three-phase synchronous machine

Field circuit



Three-phase induction machine

Voltage induced in a simple rotating loop



(a) (b)

FIGURE 4–1
A simple rotating loop in a uniform magnetic field. (a) Front view; (b) view of coil.

$$e = (v\ x\ B)\ .\ l$$

e is the induced voltage at the terminals of the segment

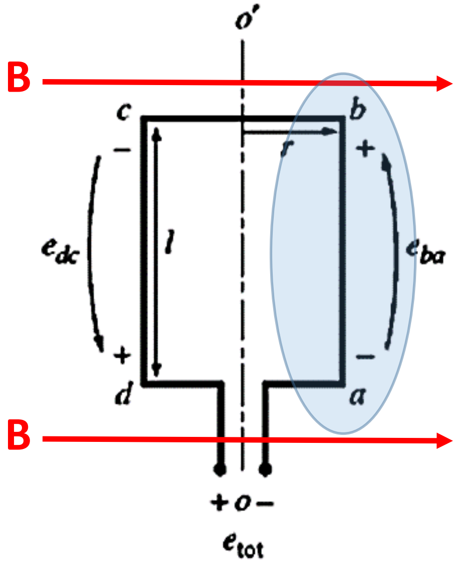
v is the linear velocity of the segment

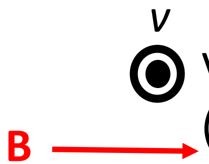
B is the magnetic field generated by the *N-S* poles

l is the length of the segment

- Two poles (*N-S poles*) produce a **constant (uniform) magnetic field** oriented from *N* to *S-pole*
- If a rectangular conductor is rotated in **counter-clock wise** direction in this magnetic field **by applying a mechanical force**, **a voltage is induced at the terminals of the rectangular conductor** **(e_{tot})**

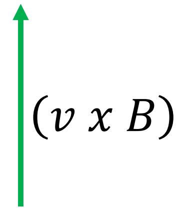
Voltage induced in segment *ab*

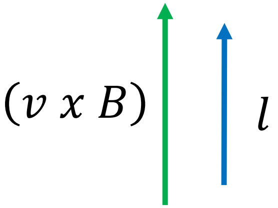




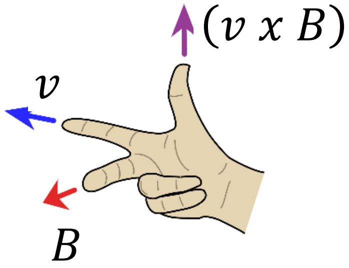
Velocity of the segment *ab*, *v* **(out of page)**

Cross product (vxB) is





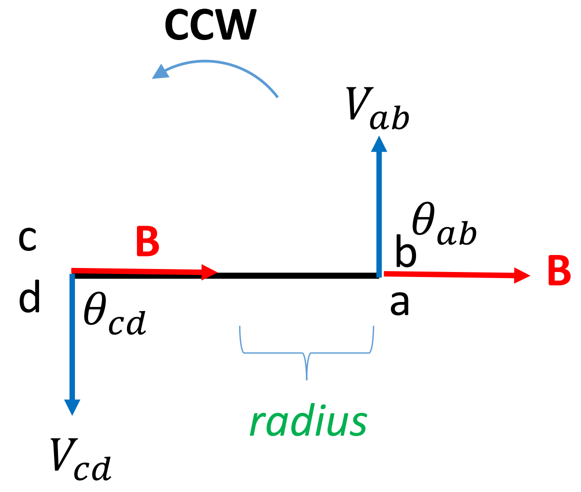
Dot product is (vBl) (angle between (***vxB***) and l is **α=0**)



$$e_{ba} = (v\ x\ B)\ .\ l = vBl\underbrace{(\cos\alpha)}_{\mathbf{1}}\underbrace{sin\theta_{ab}}_{\textbf{(angle between } \boldsymbol{v} \textbf{ and } \boldsymbol{B}\textbf{, due to cross-product)}} \Rightarrow e_{ba} = (v\ x\ B)\ .\ l = vBlsin\theta_{ab}$$

Voltage induced in segment *ab*

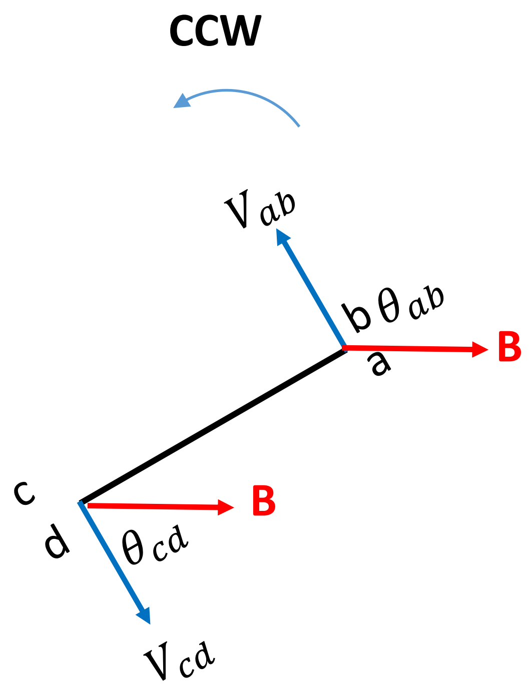
- Determining the angles θ_{ab} and θ_{cd}



Case when $\theta_{ab} = 90°$

$\theta_{cd} = 90°$

$\theta_{ab} + \theta_{cd} = 180°$



Case when $\theta_{ab} = 120°$

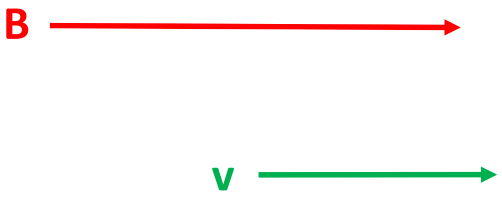
$\theta_{cd} = 60°$

$\theta_{ab} + \theta_{cd} = 180°$

Voltage induced in segment *ab*

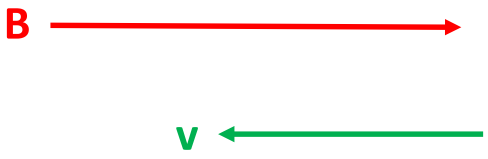
$$e_{ba} = (v \; x \; B) \, . \, l = vBlsin\theta_{ab}$$

$\theta_{ab} = 0$



$e_{ba} = 0$
(segment *ab* is at the **bottom**)

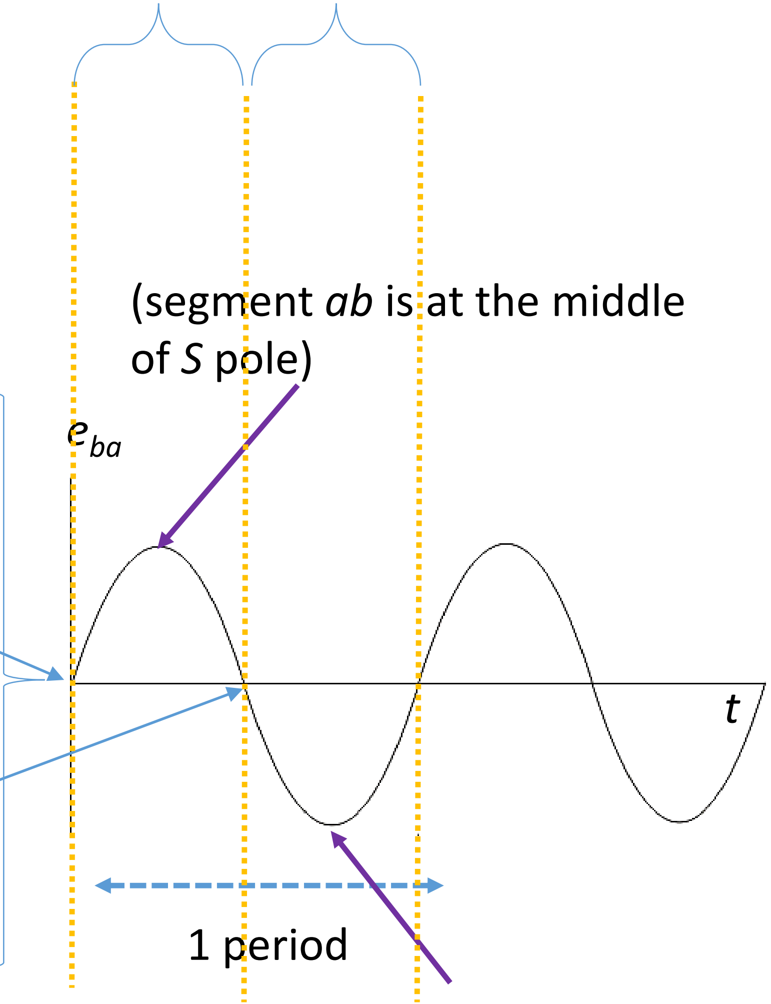
$\theta_{ab} = 180$



$e_{ba} = 0$
(segment *ab* is at the **top**)

segment *ab* is
at the **right** side of
rotation axis

segment *ab* is
at the **left** side of
rotation axis

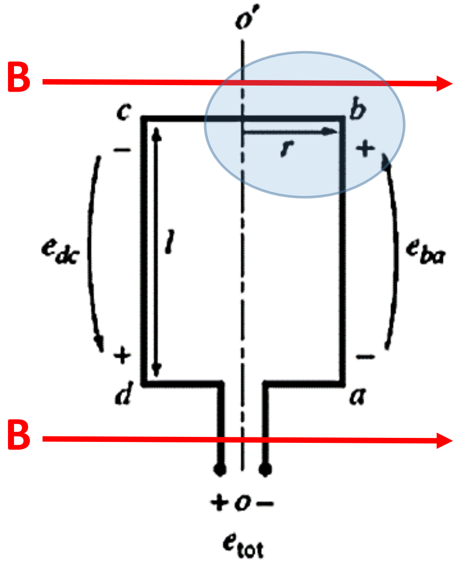


(segment *ab* is at the middle of *S* pole)

1 period

(segment *ab* is at the middle of *N* pole)

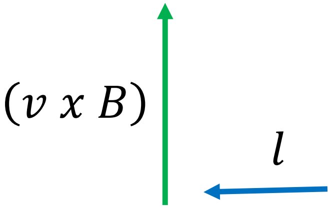
Voltage induced in segment *bc (first half)*



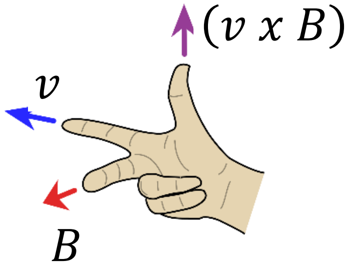
v ⊙ Velocity of the segment *bc*, *v* (**out of page**)

B →

Cross product (vxB) is ➡ $(v\ x\ B)$

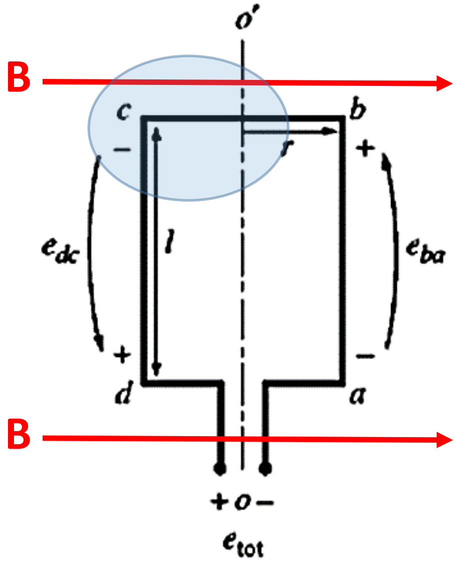


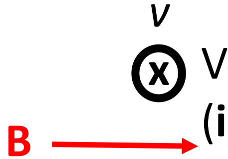
Dot product is **0** (angle between (vxB) and l is **α=90°**)



$$e_{bc1} = (v\ x\ B)\,.\,l = vBl\underbrace{(\cos\alpha)}_{\mathbf{0}}sin\theta_{bc1} \Rightarrow e_{bc1} = 0$$

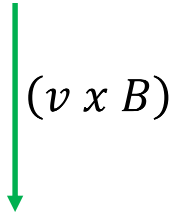
Voltage induced in segment *bc (second half)*

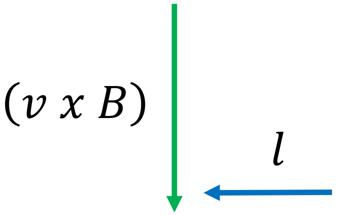




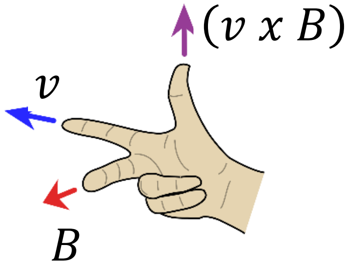
Velocity of the segment *ab*, *v* **(into the page)**

Cross product (vxB) is ⇒



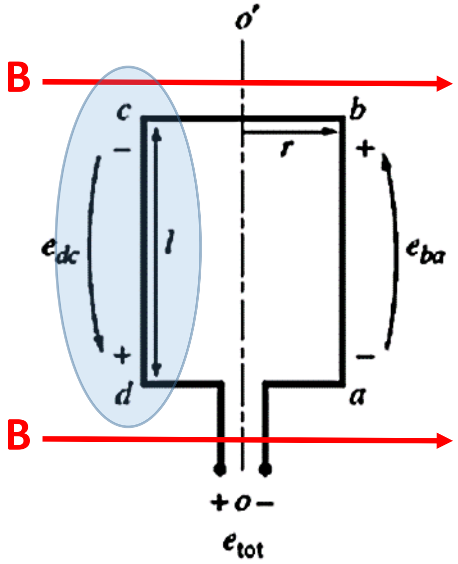


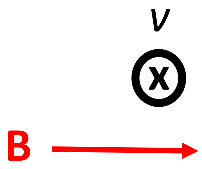
Dot product is **0** (angle between (vxB) and l is **α=90°**)



$$e_{bc2} = (v\ x\ B)\,.\,l = vBl\underbrace{(\cos\alpha)}_{\mathbf{0}}sin\theta_{bc2} \Rightarrow e_{bc2} = 0$$

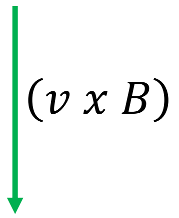
Voltage induced in segment *dc*

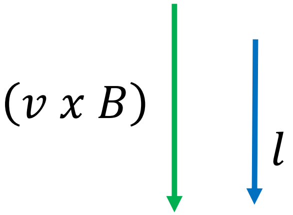




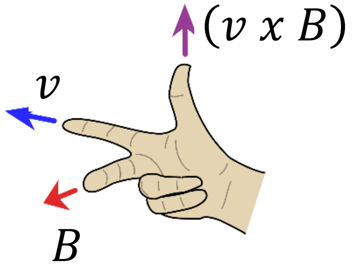
Velocity of the segment *ab*, *v* **(into the page)**

Cross product (vxB) is ➡





Dot product is (vBl) (angle between (vxB) and l is **α=0**)



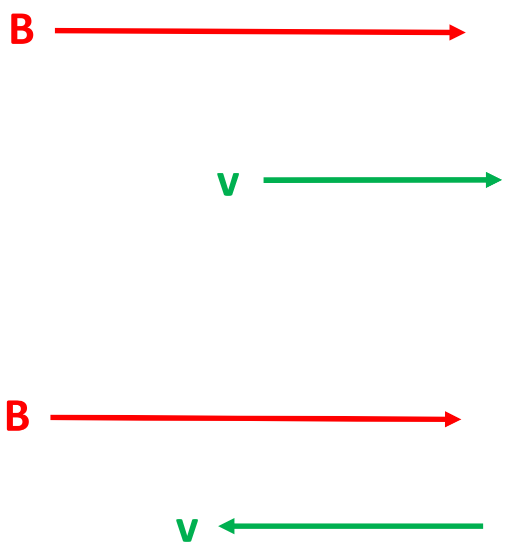
$$e_{dc} = (v\ x\ B)\ .\ l = vBl\underbrace{(\cos\alpha)}_{\mathbf{1}}sin\theta_{cd} \quad ➡ \quad e_{dc} = (v\ x\ B)\ .\ l = vBlsin\theta_{cd}$$

Voltage induced in segment *dc*

$$e_{dc} = (v\ x\ B)\ .\ l = vBlsin\theta_{cd}$$

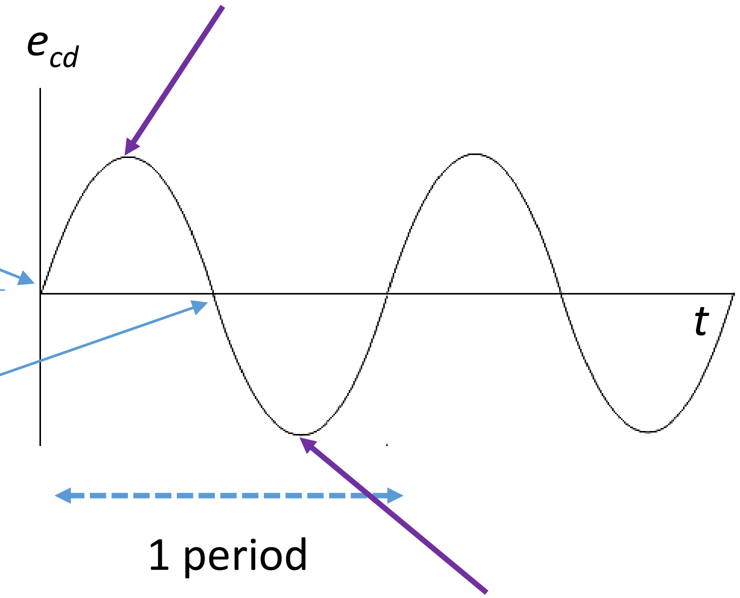
$\theta_{cd} = 0$ ⇨

$\theta_{cd} = 180$ ⇨



$e_{dc} = 0$
(segment cd is at the bottom)

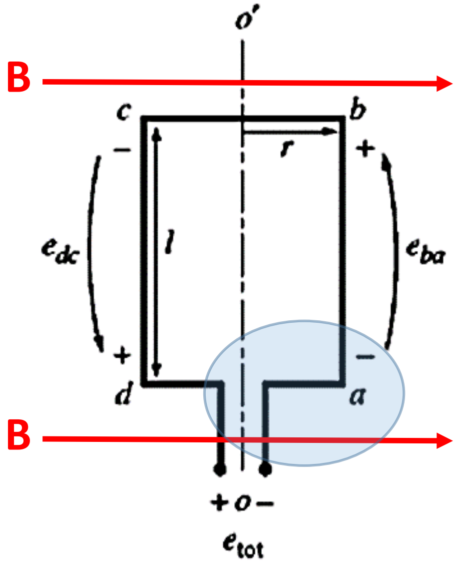
$e_{dc} = 0$
(segment *cd* is at the top)

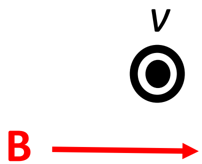


(segment *cd* is at the middle of *N* pole)

(segment *cd* is at the middle of *S* pole)

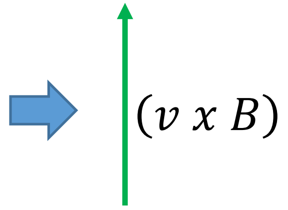
Voltage induced in segment *ad (first half)*

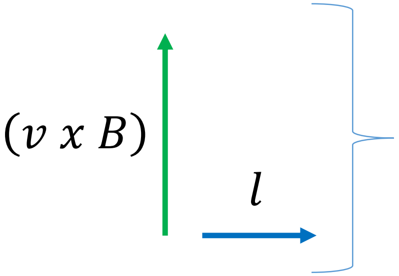




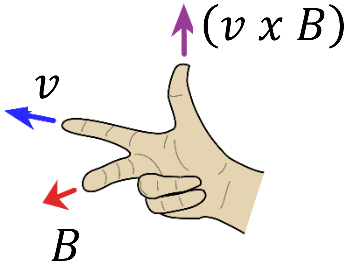
Velocity of the segment *bc*, *v* (**out of page**)

Cross product (vxB) is



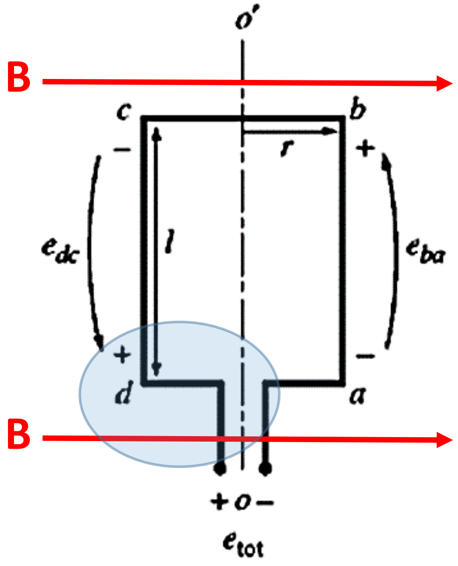


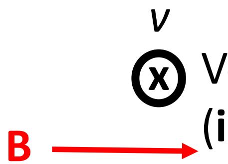
Dot product is **0** (angle between (vxB) and l is **α=90°**)



$$e_{ad1} = (v\ x\ B)\,.\,l = vBl\underbrace{(\cos\alpha)}_{\mathbf{0}}sin\theta_{ad1} \Rightarrow e_{ad1} = 0$$

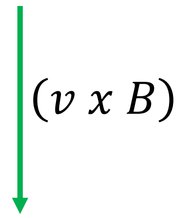
Voltage induced in segment *ad (second half)*

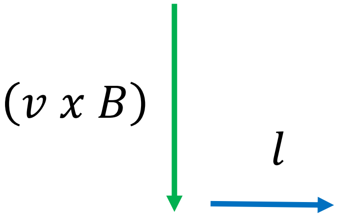




Velocity of the segment *ab*, *v* **(into the page)**

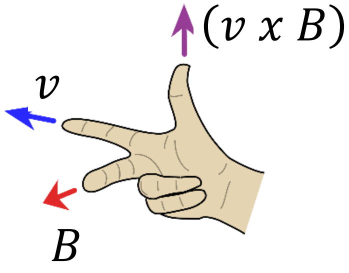
Cross product (vxB) is





Dot product is **0**

(angle between (vxB) and l is **α=90°**)



$$e_{ad2} = (v\ x\ B)\ .\ l = vBl\underbrace{(\cos\alpha)}_{\mathbf{0}}sin\theta_{ad2} \Rightarrow e_{ad2} = 0$$

Total induced voltage in the rectangular conductor

The ***total induced voltage*** in the rectangular conductor is the sum of individual induced voltages on each segment:

$$e_{ba} = vBlsin\theta_{ab}$$

$$e_{dc} = vBlsin\theta_{cd}$$

$$e_{bc1} = 0$$

$$e_{bc2} = 0$$

$$e_{ad1} = 0$$

$$e_{ad2} = 0$$

$+$

$$e_{tot} = vBlsin\theta_{ab} + vBlsin\theta_{cd}$$

since

$$\theta_{ab} + \theta_{cd} = 180° \Rightarrow \theta_{cd} = 180° - \theta_{ab}$$

$$e_{tot} = vBlsin\theta_{ab} + vBlsin(180° - \theta_{ab})$$

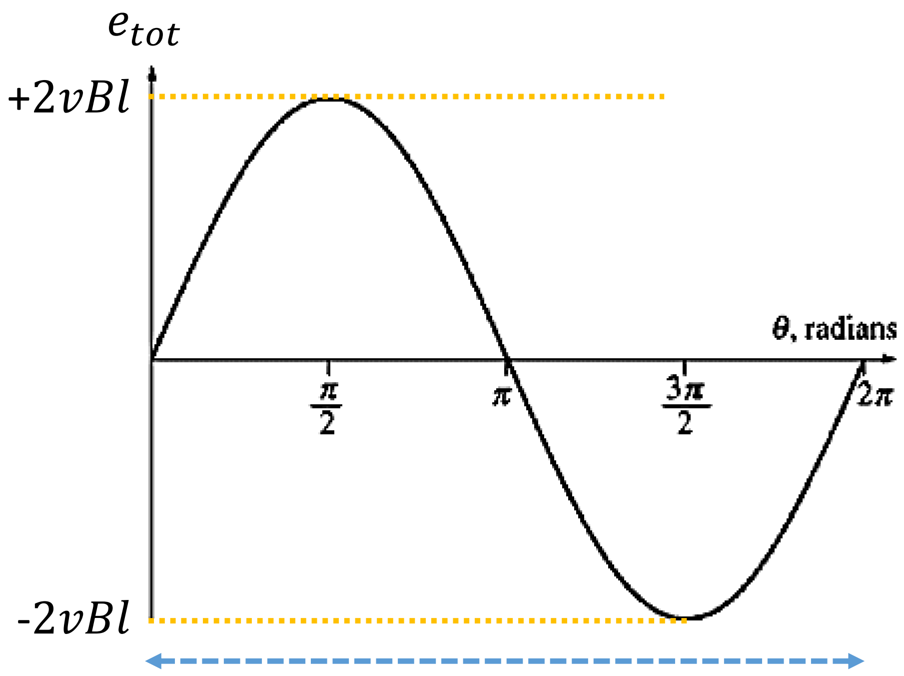
$$e_{tot} = vBlsin\theta_{ab} + vBl[\underbrace{\sin(180)}_{\mathbf{0}} . cos\theta_{ab} - \underbrace{\cos(180)}_{\mathbf{-1}} . sin\theta_{ab}]$$

$$e_{tot} = vBlsin\theta_{ab} + vBlsin\theta_{ab} = 2vBlsin\theta_{ab}$$

Let $\theta = \theta_{ab}$

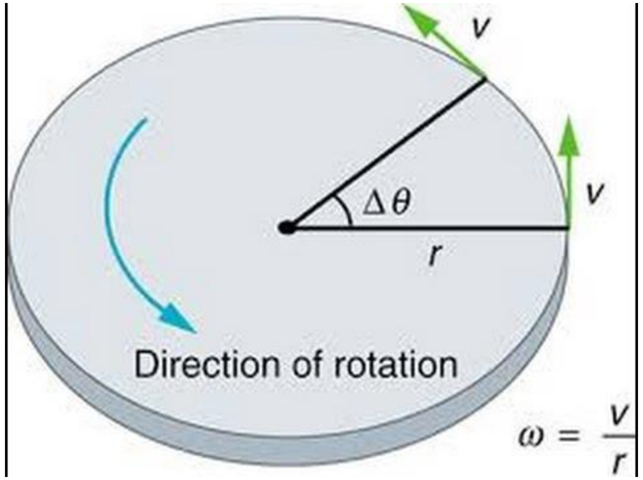
$$\boxed{e_{tot} = 2vBlsin\theta}$$

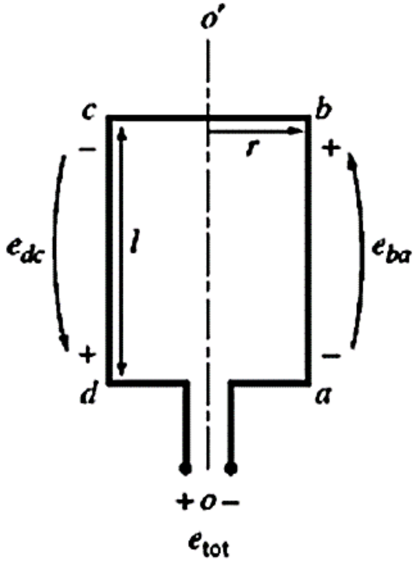
Total induced voltage in the rectangular conductor



1 period (*1 full rotation of the rectangular conductor*)

$$e_{tot} = 2vBlsin\theta$$





w ➡ Angular velocity (rad/s)

v ➡ Linear velocity (m/s)

r ➡ Radius of rotation (m)

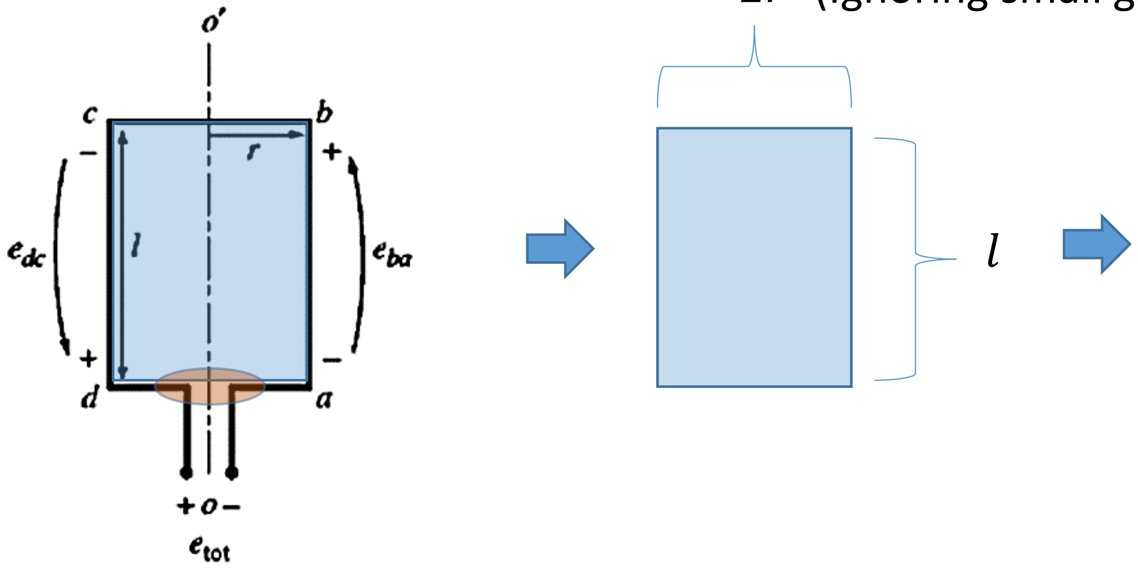
$$v = wr$$

$$\theta = wt$$

$$e_{tot} = 2vBlsin\theta$$

$$e_{tot} = 2wrBlsin(wt)$$

$\sim 2r$ (Ignoring small gap at the bottom)



Area of the rectangular conductor (loop) is approximately

$$A = 2rl$$

$$e_{tot} = 2wrBlsin(wt)$$

$$e_{tot} = AwBsin(wt)$$

Since maximum flux occurs when loop is perpendicular to B lines

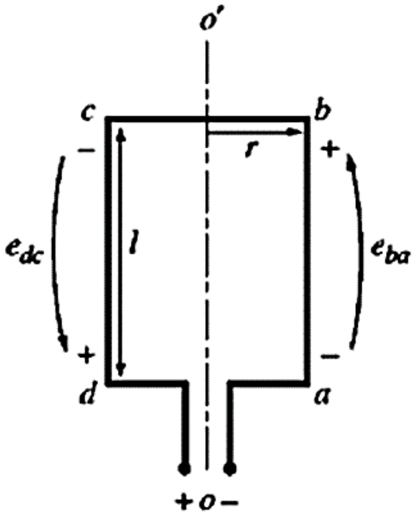
$$\emptyset_{max} = AB$$

$$e_{tot} = \emptyset_{max}wsin(wt)$$

Final form of the induced voltage in the rectangular conductor

$e_{tot} = \emptyset_{max} w sin(wt)$

- The generated voltage in a loop is sinusoidal with a period of $T = \frac{2\pi}{w}$
- The voltage has a peak value depending on the following parameters:
  - ➢ The flux in the machine, $\emptyset$
  - ➢ The speed (angular or linear) of rotation, $w\ or\ v$
  - ➢ A constant representing the construction of the machine (*which is not shown in the above equation*)

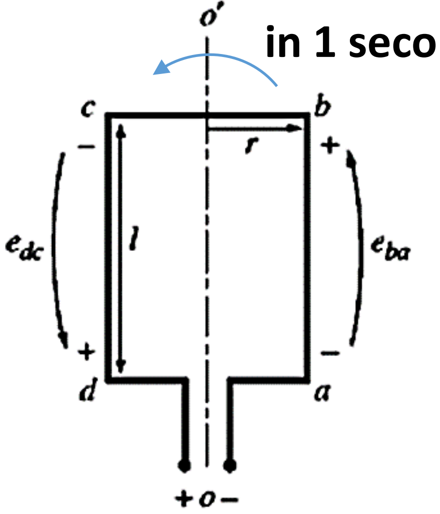


e_{tot}

Examples for the induced voltage in the rectangular conductor

Case 1:

1 rotation in 1 second



$$T = \frac{2\pi}{w} = \frac{2\pi}{2\pi} = 1\ s$$

Assume ➔ $\emptyset_{max} = 1\ Wb$

Since;

$$e_{tot} = \emptyset_{max} w sin(wt)$$

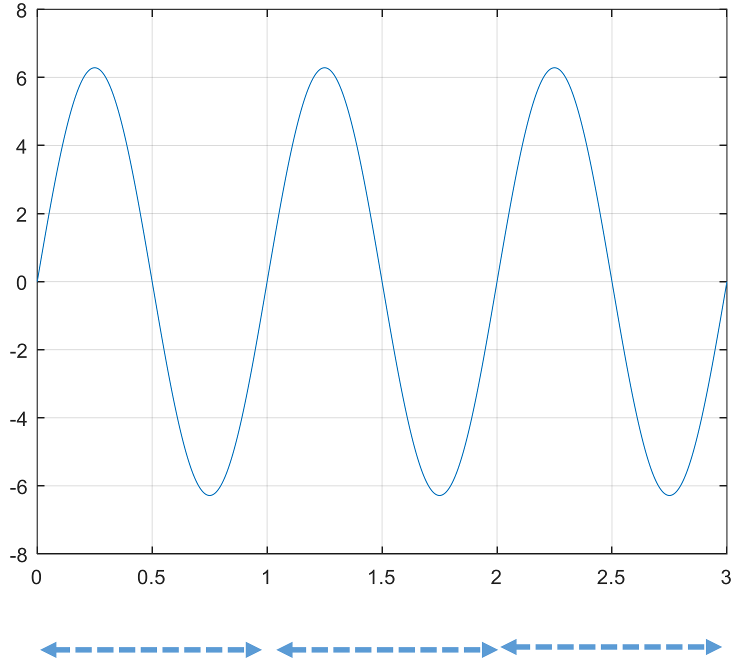
$$e_{tot} = 1 w sin(wt)$$

$$w = (1)(2\pi)\ rad/s$$

$$w = 6.28318531\ rad/s$$

$$e_{tot} = (1)(6.28318531) sin(6.28318531t)$$

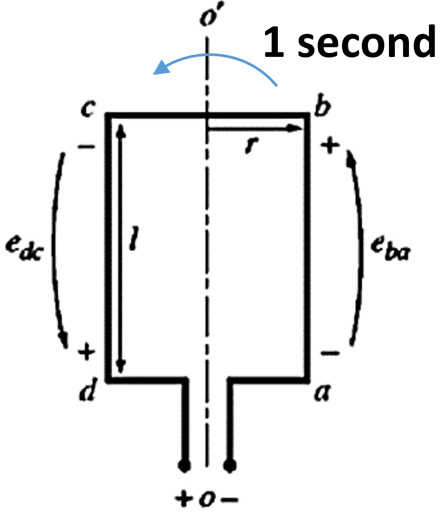
Plot of e_{tot} for 3 periods (*cycles*)



1st period **2nd period** **3rd period**

Examples for the induced voltage in the rectangular conductor

Case 2:



66.6 rotations in 1 second

Assume ➔ $\emptyset_{max} = 1\,Wb$

Since;

$$e_{tot} = \emptyset_{max} w \sin(wt)$$

$$e_{tot} = 1 w \sin(wt)$$

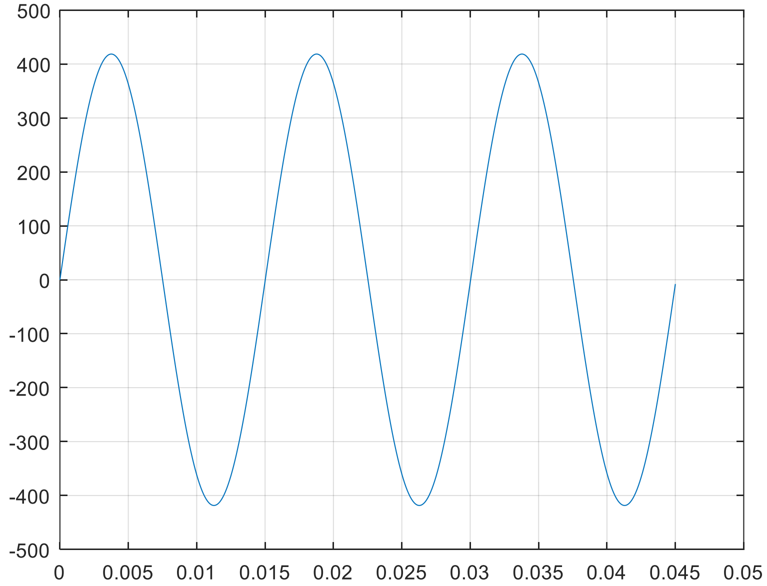
$$w = (66.6)(2\pi)\ rad/s$$

$$w = 418.46\ rad/s$$

$$e_{tot} = (1)(418.46)\sin(418.46t)$$

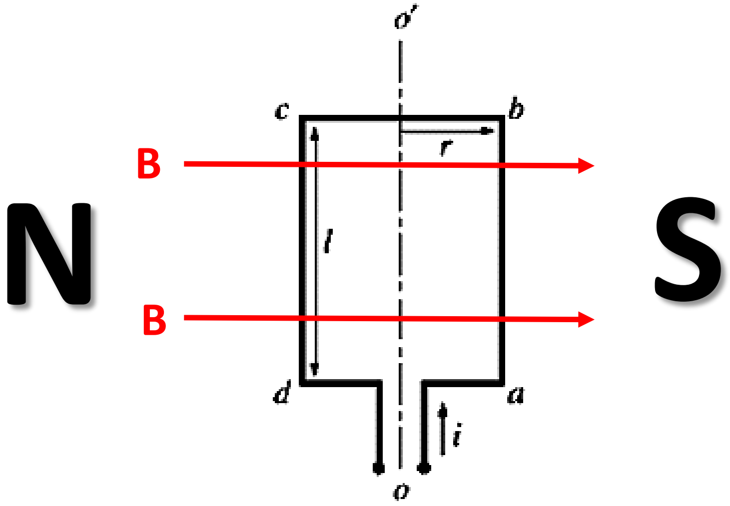
$$T = \frac{2\pi}{(66.6)(2\pi)} = \frac{1}{66.6} = 0.01501\ s$$

Plot of e_{tot} for 3 periods (*cycles*)



1st period **2nd period** **3rd period**

Torque induced in a current-carrying loop



$$F = i.(l\ x\ B)$$

F is the induced force on the segment

i is the loop current

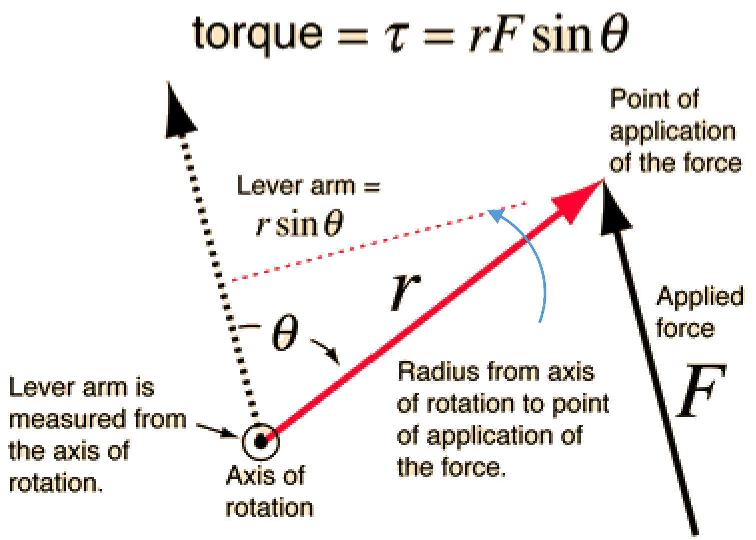
l is the length of the segment

B is the magnetic field generated by the *N-S* poles

- Two poles (*N-S poles*) produce a **constant (uniform) magnetic field** oriented from *N* to *S-pole*
- If a current ***i*** is applied to the rectangular conductor as shown in the figure, **a force is induced on the rectangular conductor**
- Direction of length vector ***l*** is accepted as direction of current ***i***

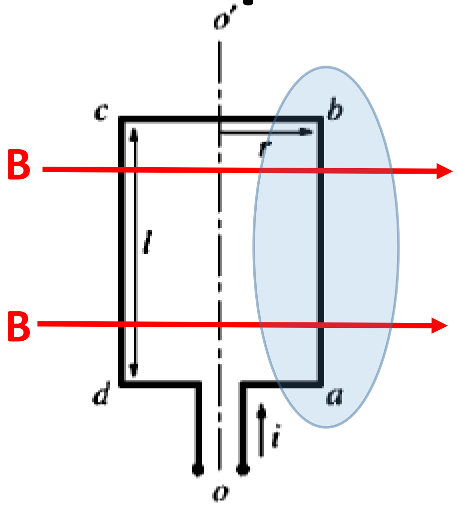
Torque

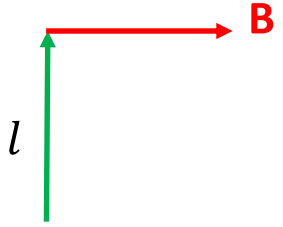
Torque = (force applied)(perpendicular distance)



Counter-clock wise (**CCW**) direction

Torque induced in segment *ab*





(Direction of *l* is the direction of current *i*)

⊗ $(l \; x \; B)$

Directed into the page

scalar

$$F_{ab} = i.(l \; x \; B)$$

⊗ (directed into the page)

$F_{ab} = ilB$ *(Magnitude is constant, independent of rotation)*

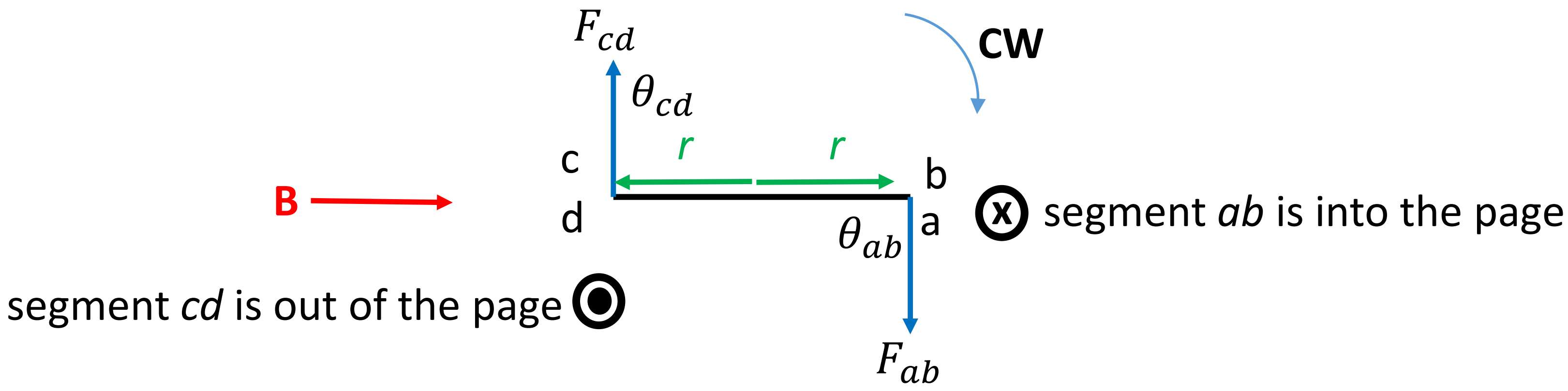
Since Fab is directed into the page, induced *Tab* on segment *ab* is in the clock-wise **(CW)** direction

$$T_{ab} = rF_{ab}sin\underbrace{(\theta_{ab})}$$ (*clockwise*)

(angle between *r* and Fab, due to definition of torque)

Torque induced in segment *ab*

- Determining the angles θ_{ab} and θ_{cd}



segment *ab* is into the page

segment *cd* is out of the page

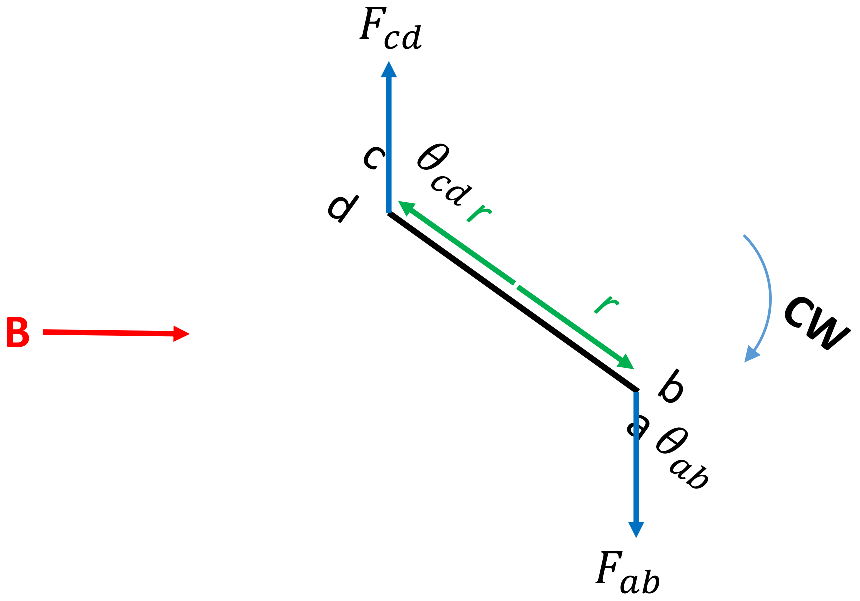
Case when $\theta_{ab} = 90°$

$$\theta_{cd} = 90°$$

$$\theta_{ab} + \theta_{cd} = 180°$$

Torque induced in segment *ab*

- Determining the angles θ_{ab} and θ_{cd}

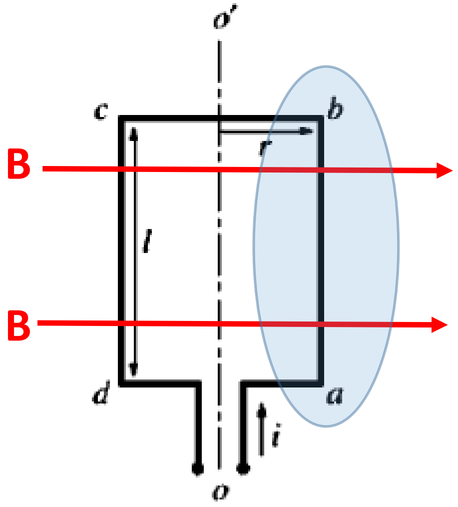


Case when $\theta_{ab} = 30°$

$$\theta_{cd} = 150°$$

$$\theta_{ab} + \theta_{cd} = 180°$$

Torque induced in segment *ab*



$$T_{ab} = rF_{ab}\sin(\theta_{ab})$$

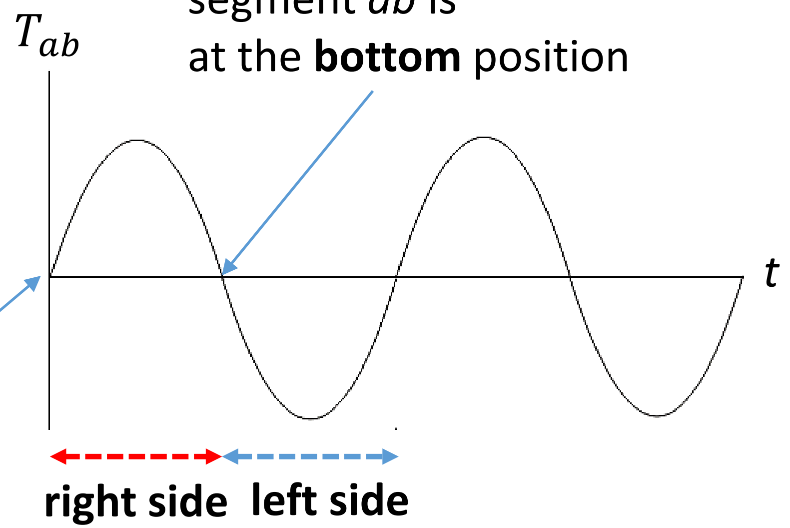
(*clockwise*)

- When segment *ab* is at the **top** position between *N-S* poles

$$\left.\begin{matrix} \otimes\ F_{ab} \\ \odot\ r \end{matrix}\right\} \quad \theta_{ab} = 180° \Rightarrow \sin(\theta_{ab}) = 0 \Rightarrow T_{ab} = 0$$

- When segment *ab* is at the **bottom** position between *N-S* poles

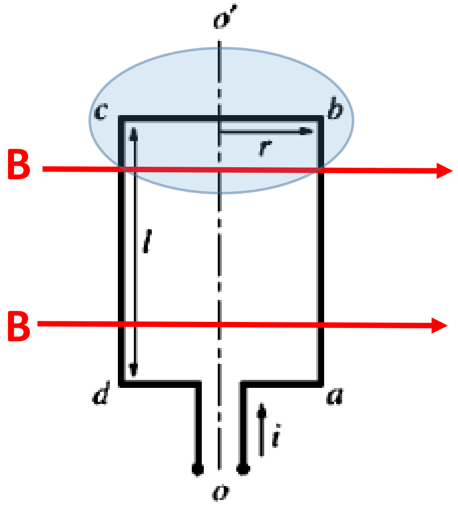
$$\left.\begin{matrix} \otimes\ F_{ab} \\ \otimes\ r \end{matrix}\right\} \quad \theta_{ab} = 0° \Rightarrow \sin(\theta_{ab}) = 0 \Rightarrow T_{ab} = 0$$

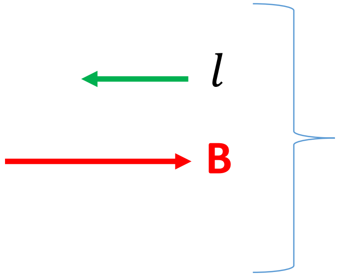


segment *ab* is
at the **bottom** position

segment *ab* is
at the **top** position

Torque induced in segment *bc*

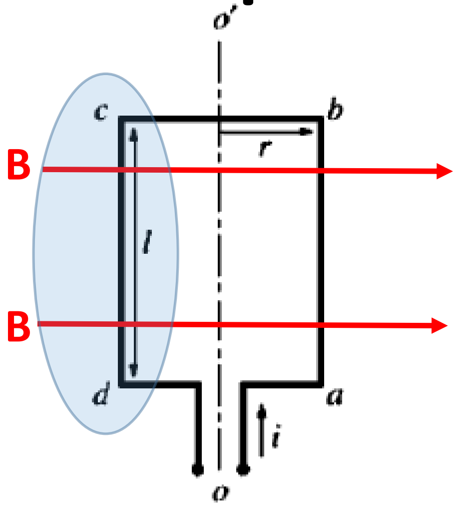


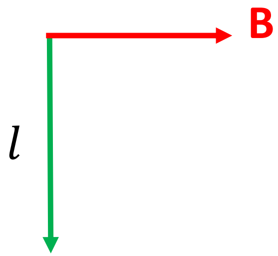


$$(l\ x\ B) = 0 \Rightarrow F_{bc} = i.(l\ x\ B) = 0 \Rightarrow T_{bc} = 0$$

There is **no induced force** (*hence no torque*) on segment *bc*

Torque induced in segment *cd*





(Direction of *l* is the direction of current *i*)

⊙ $(l\ x\ B)$

Directed out of page

scalar

$$F_{cd} = i.(l\ x\ B)$$

⊙ directed out of page

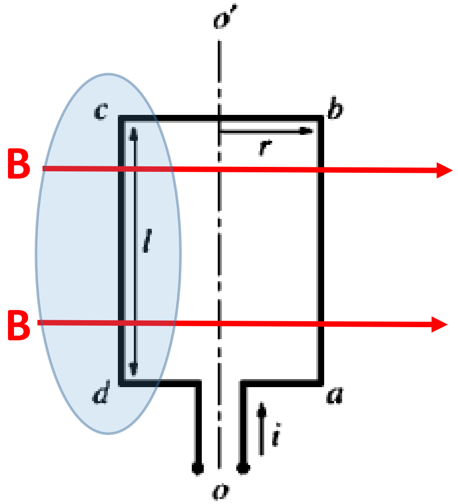
$F_{cd} = ilB$ *(Magnitude is constant, independent of rotation)*

Since Fcd is directed out of the page, induced *Tcd* on segment *cd* is in the clock-wise **(CW)** direction

$$T_{cd} = rF_{cd}sin\underbrace{(\theta_{cd})}$$ (*clockwise*)

(angle between *r* and Fcd, due to definition of torque)

Torque induced in segment *cd*



$$T_{cd} = rF_{cd}sin(\theta_{cd})$$

(*clockwise*)

- When segment *cd* is at the **top** position between *N-S* poles

$\odot$ F_{cd}

$\odot$ r

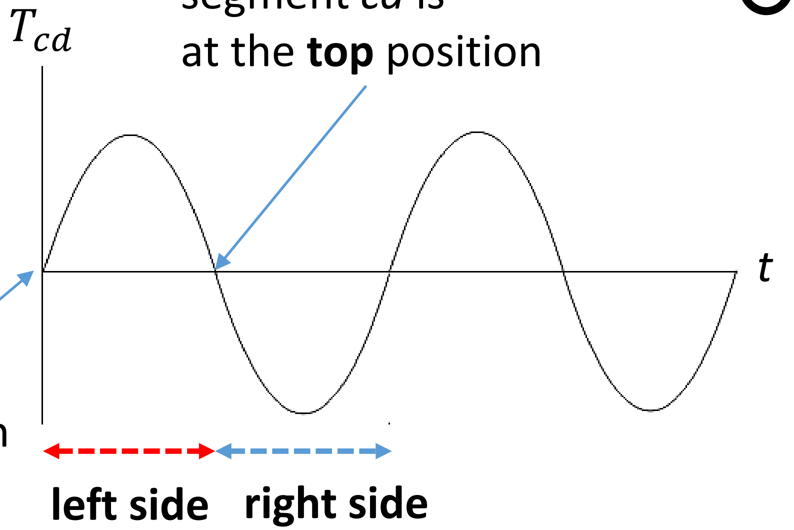
$\theta_{cd} = 0° \Rightarrow sin(\theta_{cd}) = 0 \Rightarrow T_{cd} = 0$

- When segment *ab* is at the **bottom** position between *N-S* poles

$\odot$ F_{cd}

$\otimes$ r

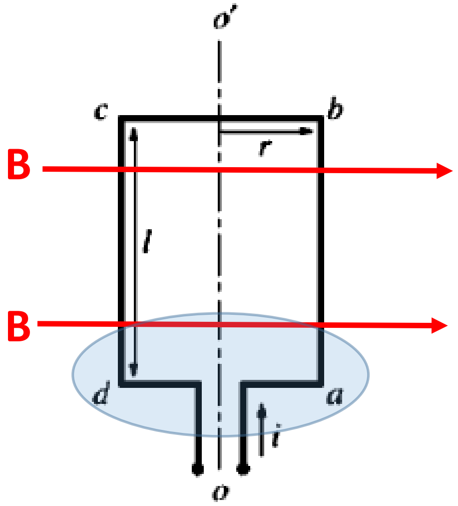
$\theta_{cd} = 180° \Rightarrow sin(\theta_{cd}) = 0 \Rightarrow T_{cd} = 0$

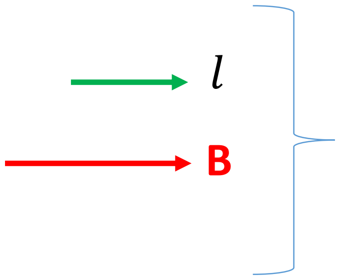


segment *cd* is
at the **top** position

segment *cd* is
at the **bottom** position

Torque induced in segment *ad*





$$(l\ x\ B) = 0 \Rightarrow F_{ad} = i.(l\ x\ B) = 0 \Rightarrow T_{ad} = 0$$

There is **no induced force** (*hence no torque*) on segment *ad*

Total induced torque in the rectangular conductor

The ***total induced torque*** in the rectangular conductor is the sum of individual induced torques on each segment:

$$T_{ab} = rF_{ab}sin(\theta_{ab})$$

$$T_{bc} = 0$$

$$T_{cd} = rF_{cd}sin(\theta_{cd})$$

$$T_{ad} = 0$$

+

$$T_{tot} = rF_{ab}sin\theta_{ab} + rF_{cd}sin\theta_{cd}$$

Let $F = F_{ab} = F_{cd} = ilB$

since

$$\theta_{ab} + \theta_{cd} = 180° \Rightarrow \theta_{cd} = 180° - \theta_{ab}$$

$$T_{tot} = rFsin\theta_{ab} + rFsin(180° - \theta_{ab})$$

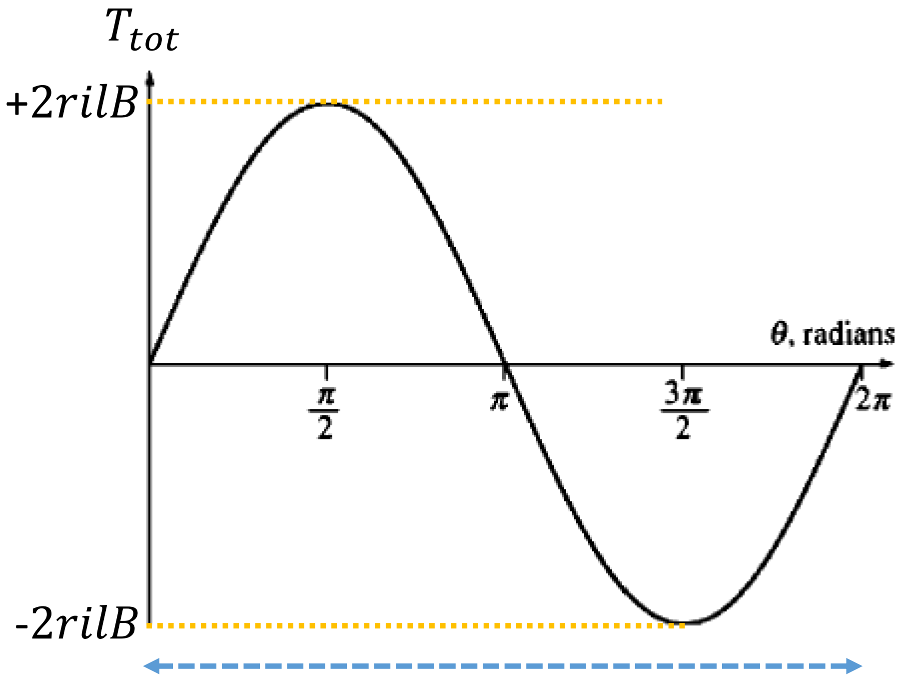
$$T_{tot} = rFsin\theta_{ab} + rF[\underbrace{\sin(180)}_{\mathbf{0}}\,.\,cos\theta_{ab} - \underbrace{\cos(180)}_{\mathbf{-1}}\,.\,sin\theta_{ab}]$$

$$T_{tot} = rFsin\theta_{ab} + rFsin\theta_{ab} = 2rFsin\theta_{ab}$$

Let $\theta = \theta_{ab}$

$T_{tot} = 2rFsin\theta$ ⇒ since $F = ilB$ ⇒ $\boxed{T_{tot} = 2rilBsin\theta}$

Total induced torque in the rectangular conductor



1 period (*1 full rotation of the rectangular conductor*)

$$T_{tot} = 2rilBsin\theta$$

Final form of the induced torque in the rectangular conductor

Since $\theta = wt$

$$T_{tot} = 2rilBsin\theta \Rightarrow T_{tot} = 2rilBsin(wt)$$

- The induced torque in a loop is sinusoidal with a period of $T = \frac{2\pi}{w}$

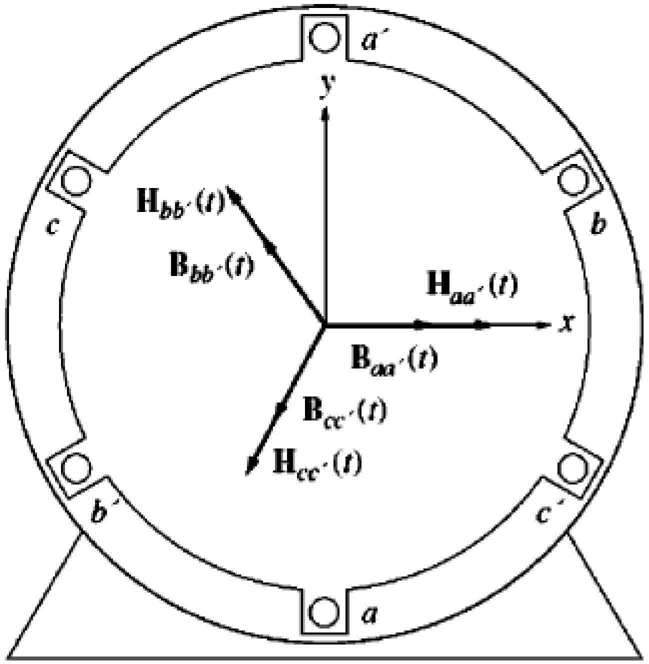
- The torque has a peak value depending on the following parameters:

  - *r* and *l* parameters (*physical dimensions of the loop*)
  - The magnitude of the current *i* in the loop
  - The magnetic field *B* generated by pole pair
  - A constant representing the construction of the machine

  (*which is not shown in the above equation*)

IMPORTANT NOTE: Since *i* of the loop generates its own magnetic field and B is the magnetic field generated by pole pair, **there must be two types of magnetic field in a machine to induce a torque**

Stator magnetic field



- If a three-phase set of currents, each of equal magnitude and differing in phase by 120° flows in a three-phase winding, then it produces a **rotating magnetic field** of **constant magnitude**.
- The three-phase winding consists of three separate windings spaced 120 electrical degrees apart around the surface of the machine.

$i_{aa'}(t) = I_M sin(wt)$ ⇨ generates ⇨ $H_{aa'}(t) = H_M sin(wt)/\_\_0°$ (A.turns/m)

$i_{bb'}(t) = I_M sin(wt - 120°)$ ⇨ generates ⇨ $H_{bb'}(t) = H_M sin(wt - 120°)/\_\_ + 120°$ (A.turns/m)

$i_{cc'}(t) = I_M sin(wt + 120°)$ ⇨ generates ⇨ $H_{cc'}(t) = H_M sin(wt + 120°)/\_\_ - 120°$ (A.turns/m)

Three-phase currents (*positive sequence*)

Magnetic field intensities generated by each phase

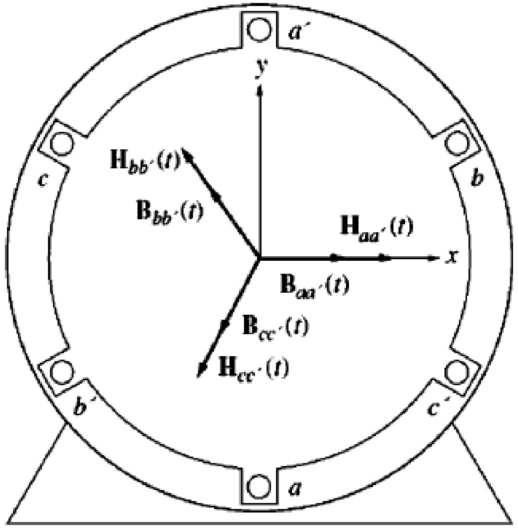
Stator magnetic field

Since **B=μH**

$H_{aa'}(t) = H_M sin(wt) \angle 0°$ → $B_{aa'}(t) = B_M sin(wt) \angle 0°$ (Wb/m^2) or (T)

$H_{bb'}(t) = H_M sin(wt - 120°) \angle + 120°$ → $B_{bb'}(t) = B_M sin(wt - 120°) \angle + 120°$ (T)

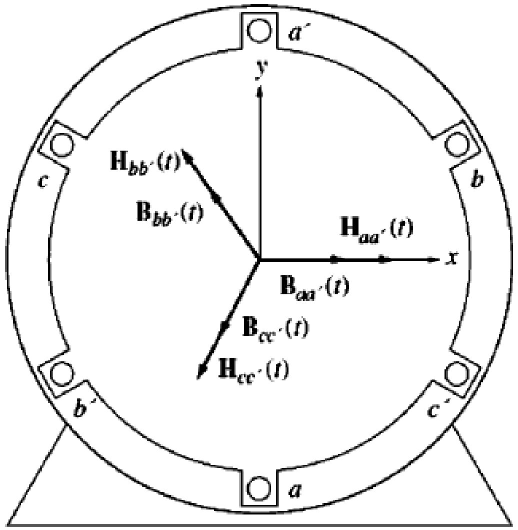
$H_{cc'}(t) = H_M sin(wt + 120°) \angle - 120°$ → $B_{cc'}(t) = B_M sin(wt + 120°) \angle - 120°$ (T)



Net stator magnetic field

Since there are **three magnetic fields simultaneously** at a given time, the net stator magnetic field $\boldsymbol{B_{net}}$ can be calculated:

$$B_{net}(t) = B_{aa'}(t) + B_{bb'}(t) + B_{cc'}(t)$$



Net stator magnetic field

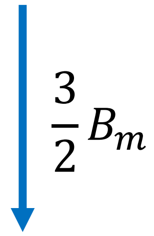
For example at *t*=0 ➔ ***wt* = 0**

$$B_{aa'}(t=0) = B_M sin(wt) = 0$$

$$B_{bb'}(t=0) = B_M sin(wt - 120°)\angle +120° = B_M \underbrace{sin(-120°)}_{-\sqrt{3}/2}\angle +120° = \frac{-\sqrt{3}}{2} B_M \angle +120°$$

$$B_{cc'}(t=0) = B_M sin(wt + 120°)\angle -120° = B_M \underbrace{sin(+120°)}_{\sqrt{3}/2}\angle -120° = \frac{\sqrt{3}}{2} B_M \angle -120°$$

$$B_{net}(t=0) = \frac{-\sqrt{3}}{2} B_M \angle +120° + \frac{\sqrt{3}}{2} B_M \angle -120° = \frac{3}{2} B_M(-j) = \frac{3}{2} B_M \angle -90°$$



Net stator magnetic field

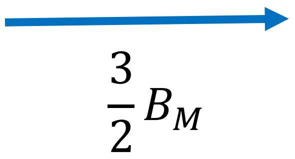
For example at ***wt* = 90°**

$$B_{aa'}(wt=90) = B_M sin(90)/\underline{\quad}0^\circ \ = B_M/\underline{\quad}0^\circ$$

$$B_{bb'}(wt=90) = B_M \underbrace{sin(90-120^\circ)}_{-1/2}/\underline{\quad}+120^\circ \quad = -\frac{1}{2}B_M/\underline{\quad}+120^\circ$$

$$B_{cc'}(wt=90) = B_M \underbrace{sin(90+120^\circ)}_{-1/2}/\underline{\quad}-120^\circ \ = -\frac{1}{2}B_M/\underline{\quad}-120^\circ$$

$$B_{net}(wt=90) = \ B_M/\underline{\quad}0^\circ \ -\frac{1}{2}B_M/\underline{\quad}+120^\circ \ -\frac{1}{2}B_M/\underline{\quad}-120^\circ \ \ = \frac{3}{2}B_M/\underline{\quad}0^\circ$$



Net stator magnetic field

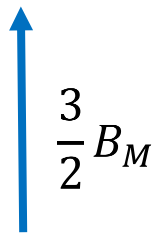
For example at ***wt* = 180°**

$$B_{aa'}(wt = 180) = B_M sin(180)\angle 0° = 0$$

$$B_{bb'}(wt = 180) = B_M \underbrace{sin(180 - 120°)}_{\sqrt{3}/2}\angle + 120° \quad = \frac{\sqrt{3}}{2}B_M\angle + 120°$$

$$B_{cc'}(wt = 180) = B_M \underbrace{sin(180 + 120°)}_{-\sqrt{3}/2}\angle - 120° \quad = -\frac{\sqrt{3}}{2}B_M\angle - 120°$$

$$B_{net}(wt = 180) = \frac{\sqrt{3}}{2}B_M\angle + 120° - \frac{\sqrt{3}}{2}B_M\angle - 120° = \frac{3}{2}B_M(j) = \frac{3}{2}B_M\angle 90°$$



Net stator magnetic field

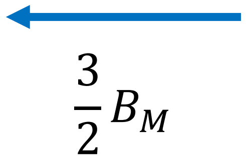
For example at ***wt* = 270°**

$$B_{aa'}(wt = 270) = B_M sin(270)\angle 0^\circ = -B_M\angle 0^\circ$$

$$B_{bb'}(wt = 270) = B_M \underbrace{sin(270 - 120^\circ)}_{1/2}\angle + 120^\circ \quad = \frac{1}{2}B_M\angle + 120^\circ$$

$$B_{cc'}(wt = 270) = B_M \underbrace{sin(270 + 120^\circ)}_{1/2}\angle - 120^\circ = \frac{1}{2}B_M\angle - 120^\circ$$

$$B_{net}(wt = 270) = \ -B_M\angle 0^\circ \ + \frac{1}{2}B_M\angle + 120^\circ + \frac{1}{2}B_M\angle - 120^\circ = -\frac{3}{2}B_M$$



Net stator magnetic field

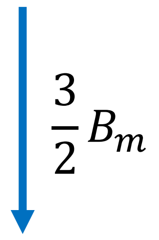
For example at *t*=0 ➔ ***wt* = 360=0°**

$$B_{aa'}(t=0)=B_M sin(wt)=0$$

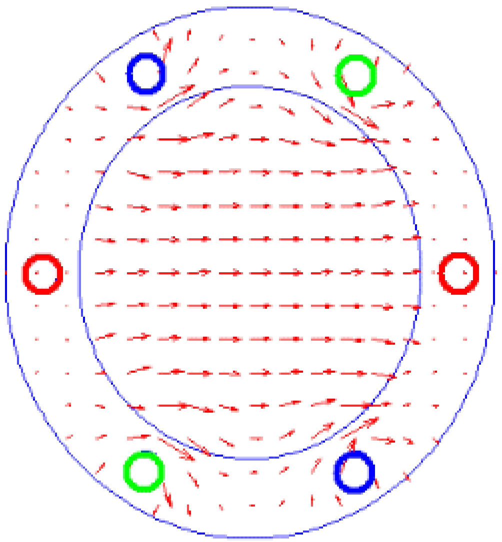
$$B_{bb'}(t=0)=B_M sin(wt-120°)/\underline{\quad}+120° = B_M \underbrace{sin(-120°)}_{-\sqrt{3}/2}/\underline{\quad}+120° \ = \frac{-\sqrt{3}}{2}B_M/\underline{\quad}+120°$$

$$B_{cc'}(t=0)=B_M sin(wt+120°)/\underline{\quad}-120° \ = B_M \underbrace{sin(+120°)}_{\sqrt{3}/2}/\underline{\quad}-120° \ = \frac{\sqrt{3}}{2}B_M/\underline{\quad}-120°$$

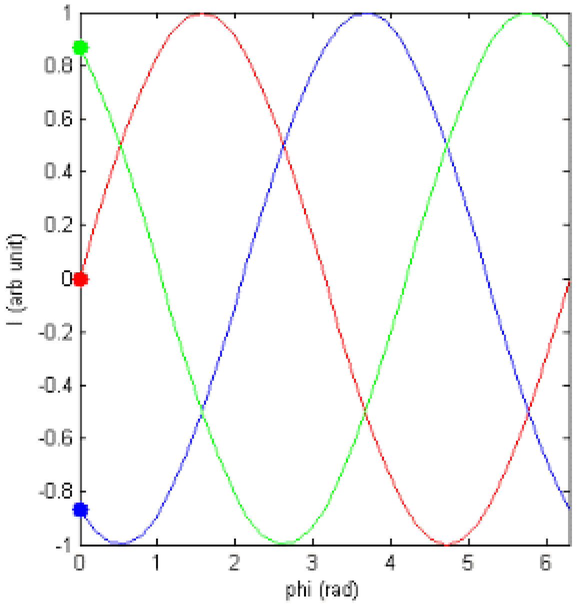
$$B_{net}(t=0)=\frac{-\sqrt{3}}{2}B_M/\underline{\quad}+120°+\frac{\sqrt{3}}{2}B_M/\underline{\quad}-120° \ = \frac{3}{2}B_M(-j)=\frac{3}{2}B_M/\underline{\quad}-90°$$



Rotating stator magnetic field



Rotation in **CCW** direction



Three-phase currents (*positive sequence*)

Rotating stator magnetic field

- Rotating stator magnetic field can be represented as moving **North** and **South** poles

- **North pole** is where the flux leaves the stator
- **South pole** is where the flux enters the stator

- These magnetic poles complete **one mechanical rotation** around the stator surface **for one electrical cycle** of the three-phase applied current to the stator

- Therefore, the mechanical speed of rotation of the magnetic field in revolutions per second **(rev/s)** is equal to the electric frequency in hertz (**Hz)**:

$$f_e = f_m$$

$$w_e = w_m$$

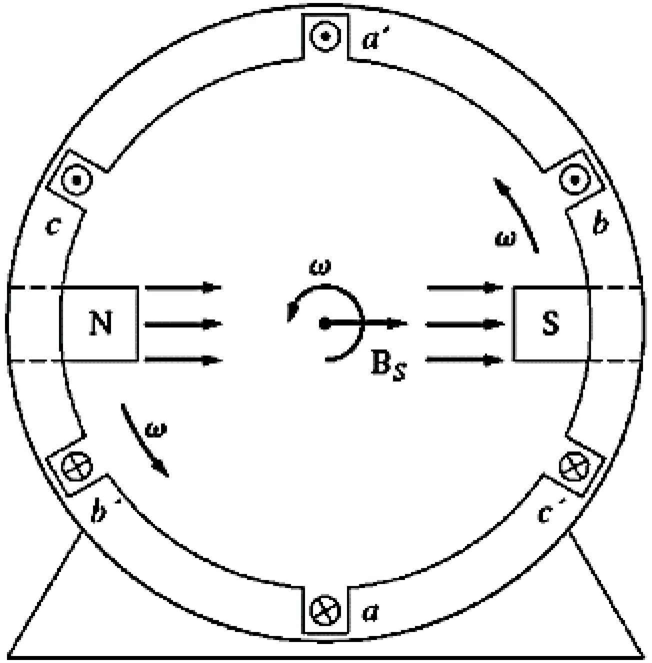
f_e ➡ Electrical frequency (Hz)

f_m ➡ Mechanical speed (rev/s)

w_e ➡ Electrical frequency (rad/s)

w_m ➡ Mechanical speed (rad/s)

For **2-pole** stator



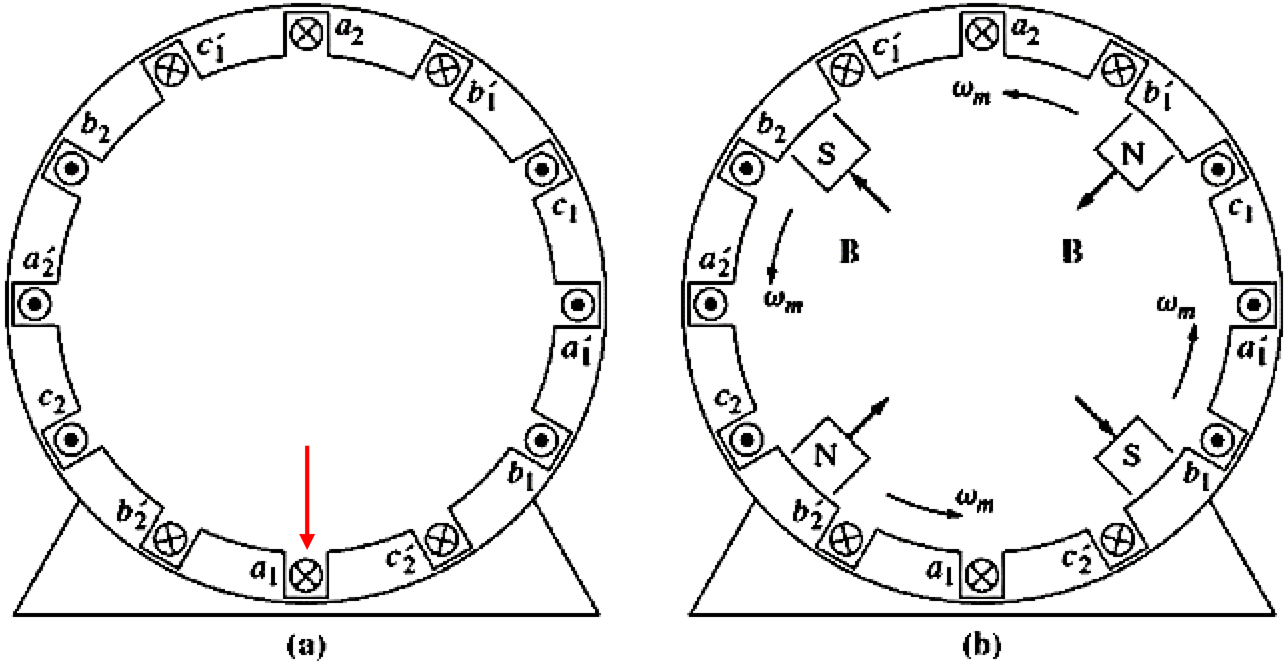
a-c'-b-a '-c-b'

Rotation in **CCW** direction

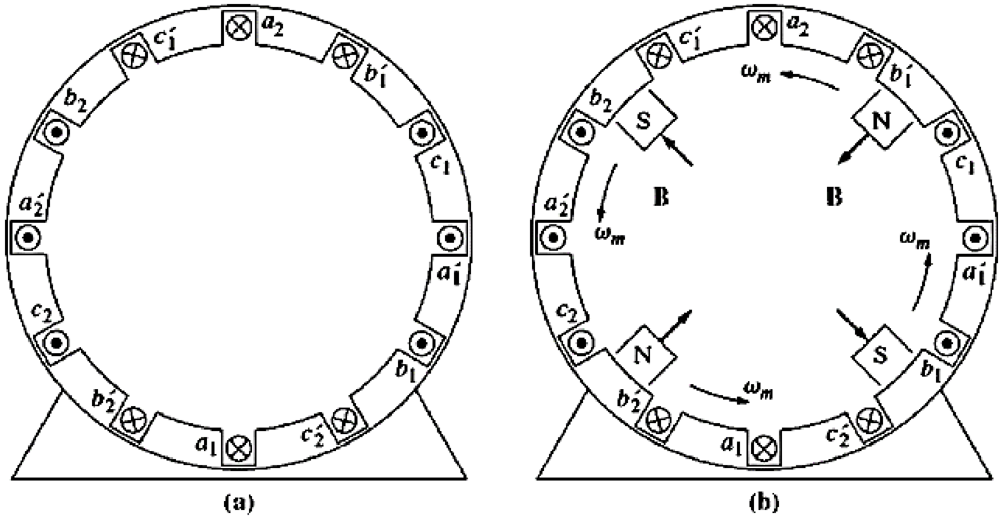
How pole number is increased in stator ?

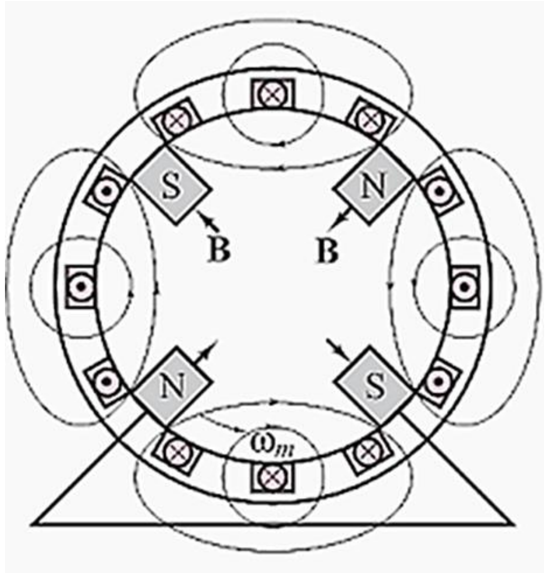
- Let’s repeat twice the phase pattern of the stator as follows:

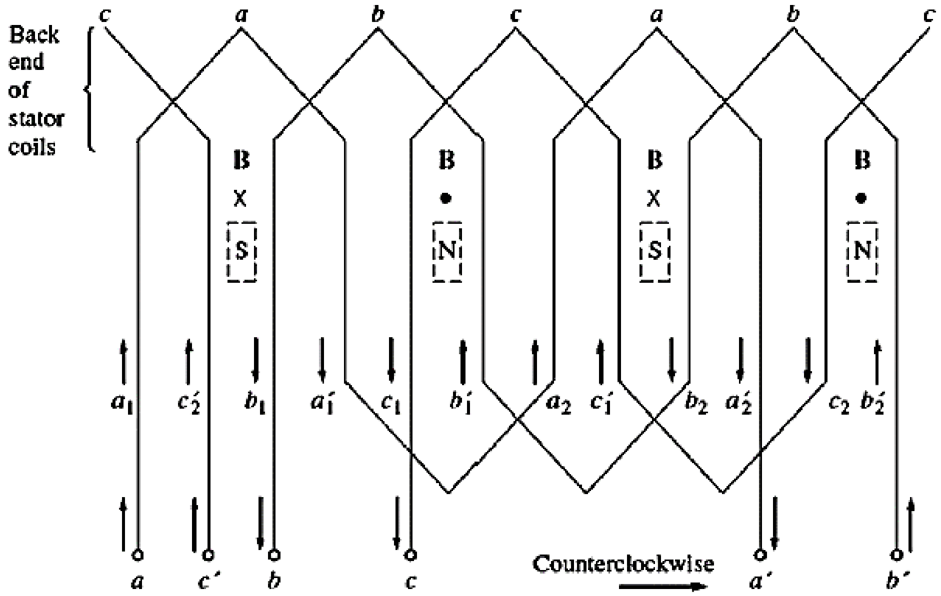
a-c '-b-a' -c-b '-a-c '-b-a '-c-b'



(a) A simple **four-pole** stator winding. (b) The resulting stator magnetic poles. Notice that there are moving poles of alternating polarity every 90° around the stator surface.



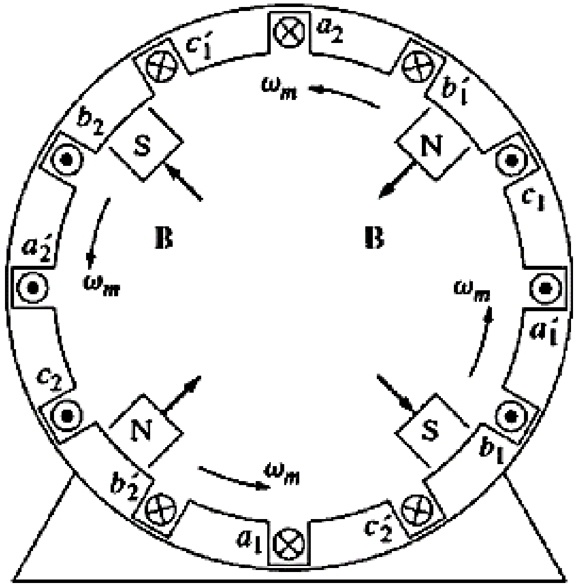




A winding diagram of the stator as seen from its inner surface, showing how the stator currents produce north and south magnetic poles

How pole number is increased in stator ?

- When a three-phase set of currents is applied to the stator, **two north poles** and **two south poles** are produced in the stator winding
- A pole moves only **halfway** around the stator surface in **one electrical cycle**
- Since one electrical cycle is 360 electrical degrees, and since the mechanical motion is 180 mechanical degrees, the relationship between the electrical angle and the mechanical angle is given as;



four-pole stator winding

$$\theta_e = 2\theta_m$$

θ_e → Electrical angle (*rad* or *degree*)

θ_m → Mechanical angle (*rad* or *degree*)

How pole number is increased in stator ?

Since;

$$\theta_e = 2\theta_m$$

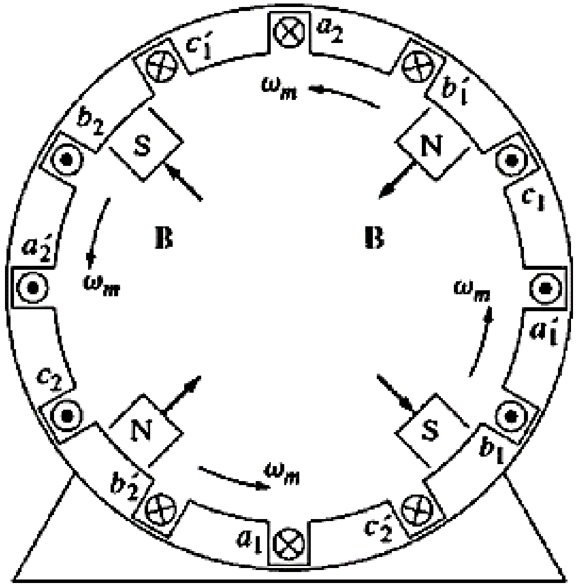
$$\left.\begin{array}{l}\theta_e = w_e.t\\ \theta_m = w_m.t\end{array}\right\} \quad w_e = 2w_m$$

Since in general;

$$w = 2\pi f$$

$$\left.\begin{array}{l}w_e = 2\pi f_e\\ w_m = 2\pi f_m\end{array}\right\} \quad f_e = 2f_m$$

For a 4-pole stator



four-pole stator winding

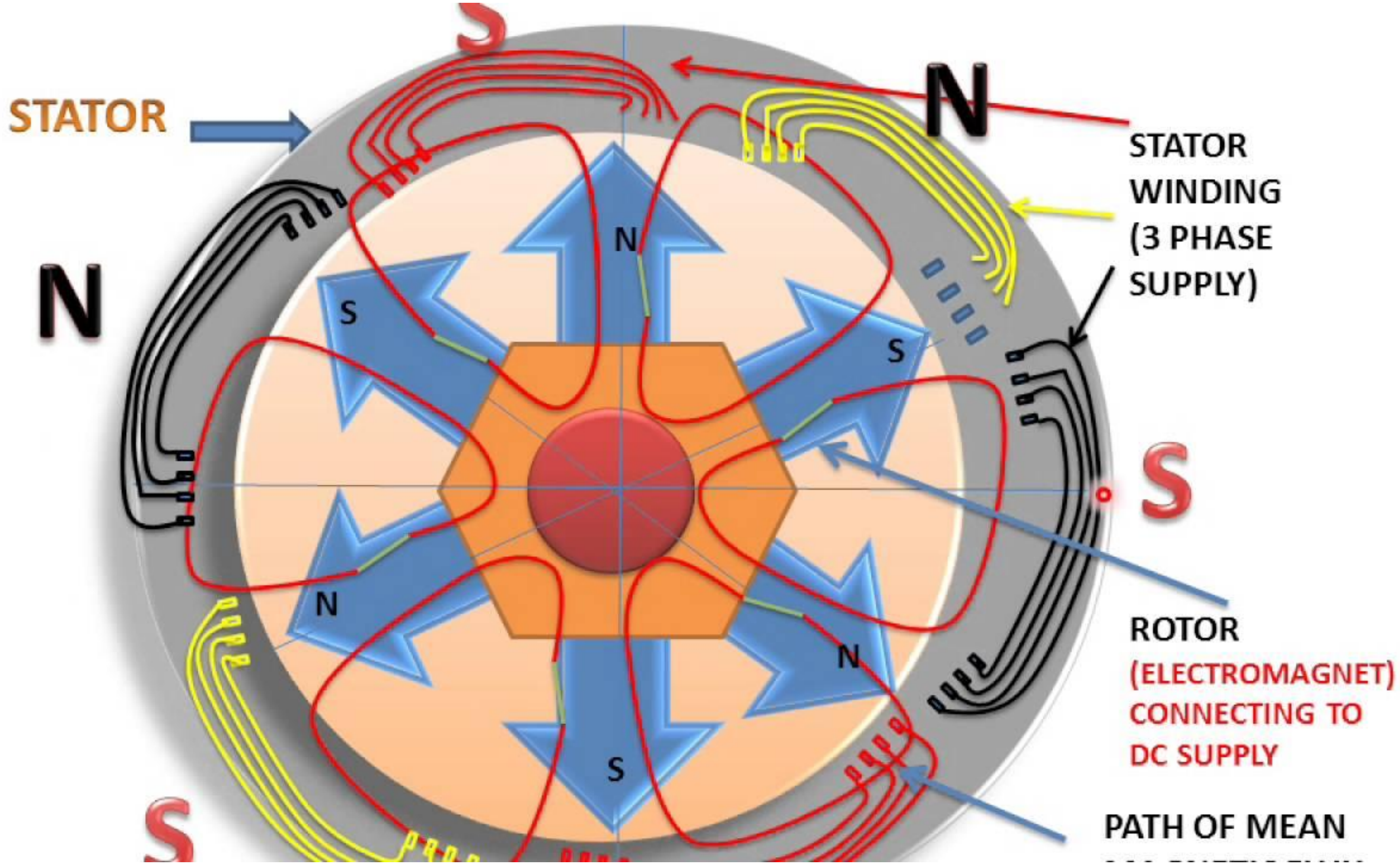
How pole number is increased in stator ?

- In general, if the number of magnetic poles on the stator is **P**, then there are **P/2 repetitions** of the winding sequence **a-c '-b-a '-c-b'** around the inner surface of the stator

- The following equations then can be derived:

$$\left.\begin{aligned} \theta_e &= \frac{P}{2}\theta_m \\ w_e &= \frac{P}{2}w_m \\ f_e &= \frac{P}{2}f_m \end{aligned}\right\} P$$

→ Pole number on the stator
(always an *even number*)



Six-pole stator winding with a rotor inside

Relationship between electrical frequency and rotation speed of stator magnetic field

- This relationship is very important for the analysis of AC machines (synchronous/induction)

$$n_m = \frac{120 f_e}{P}$$

n_m ➡ The speed of rotating stator magnetic field (**rev/min**) or (**rpm**)

f_e ➡ Electrical frequency of the three-phase supply connected at stator terminals (**Hz**) (50 Hz or 60 Hz)

P ➡ Pole number of the stator (always an *even number*)

- Let’s proof this equation:

$$n_m = \frac{120 f_e}{P}$$

n_m ⇨ Rotation number per minute

$\frac{n_m}{60}$ ⇨ Rotation number per second

$\frac{(n_m).\,2\pi\ rad}{60\ second}$ ⇨ Travelled mechanical rotational angle per second

So;

$w_m = \frac{2\pi.\,(n_m)}{60}$ ⇨ Mechanical speed (rad/s)

Since;

$$w_e = \frac{P}{2} w_m$$

$$w_e = \frac{P}{2} . \frac{2\pi . (n_m)}{60}$$

Since;

$$w_e = 2\pi f_e$$

$$f_e = \frac{w_e}{2\pi} = \frac{P}{2} . \frac{2\pi . (n_m)}{60 . (2\pi)}$$

$$f_e = \frac{P . (n_m)}{120}$$

or;

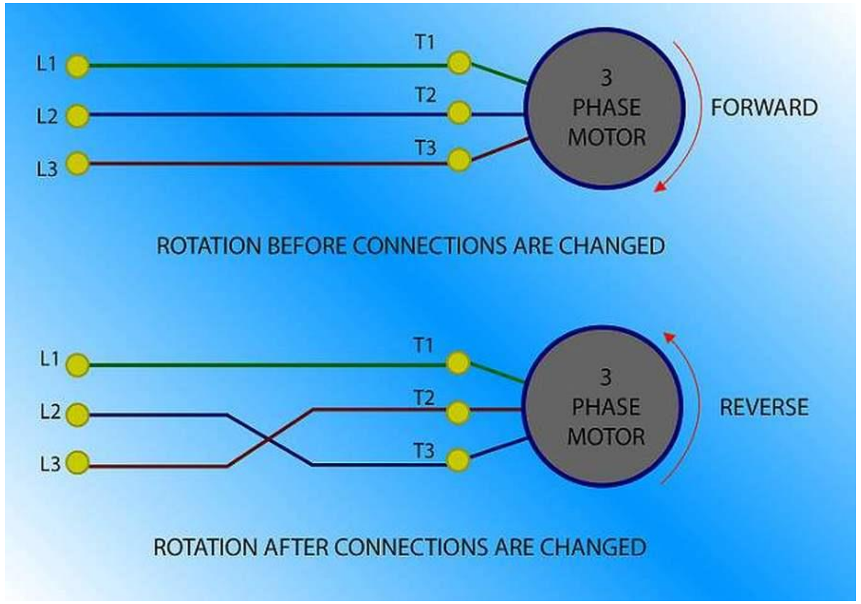
$$\boxed{n_m = \frac{120 f_e}{P}}$$

Reversing the Direction of Magnetic Field Rotation

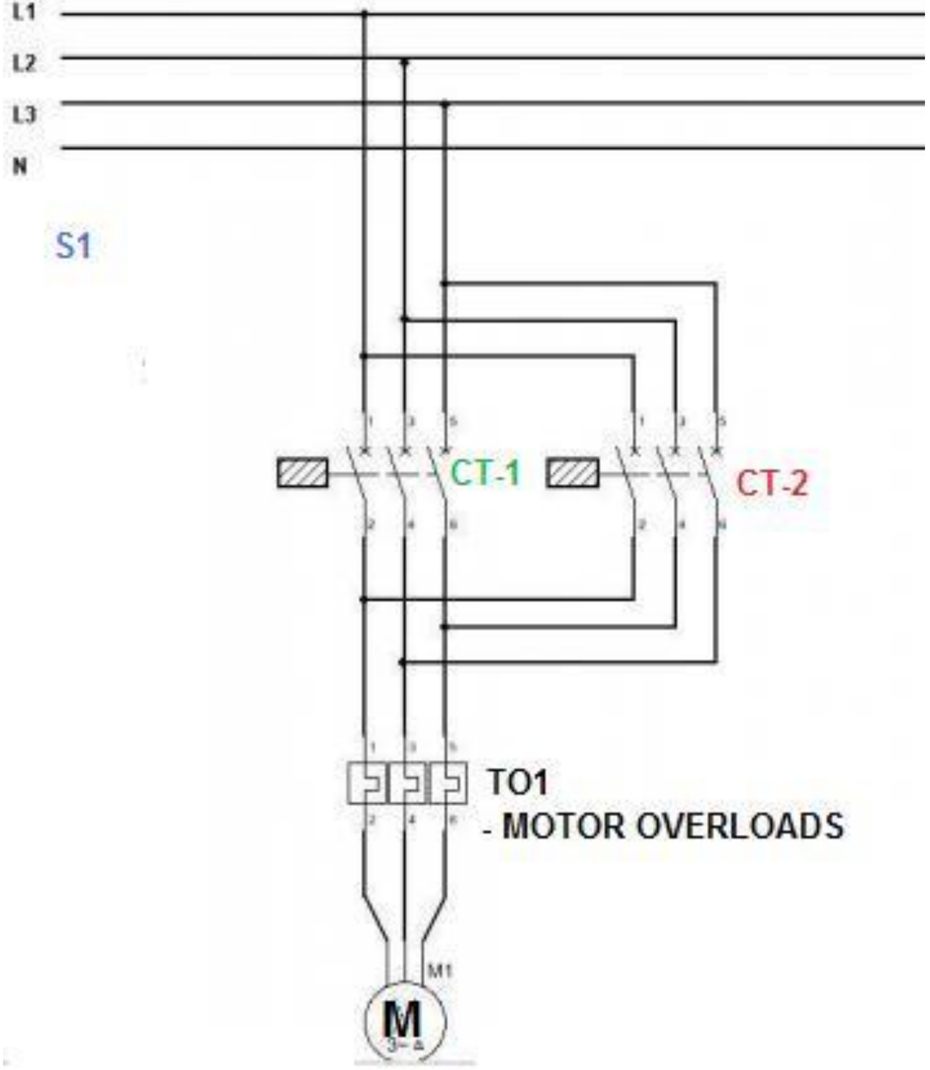
- If the **two phases** of the stator connections is **exchanged** (**swapped**), then the direction of the rotating stator magnetic field will be **reversed**

- This means that it is possible to reverse the direction of rotation of an AC motor just by switching the connections on any two phases of the three-phase supply

- Since mechanical rotation direction is directly related with the rotation direction of the stator magnetic field



Reversing the Direction of Magnetic Field Rotation



Reversing a 3-phase asynchronous motor rotation using two contactors

Reversing the Direction of Magnetic Field Rotation

- Let’s exchange **phase-B** and **C (phase-A remains unchanged)**

$$B_{aa'}(t) = B_M sin(wt)/\underline{\quad}0^\circ$$

$$B_{bb'}(t) = B_M sin(wt - 120^\circ)/\underline{\quad} + 120^\circ$$

$$B_{cc'}(t) = B_M sin(wt + 120^\circ)/\underline{\quad} - 120^\circ$$

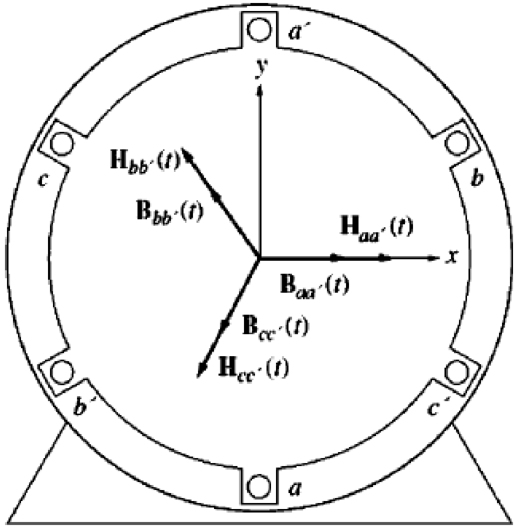
Before exchanged

$$B_{aa'}(t) = B_M sin(wt)/\underline{\quad}0^\circ$$

$$B_{bb'}(t) = B_M sin(wt - 120^\circ)/\underline{\quad} - 120^\circ$$

$$B_{cc'}(t) = B_M sin(wt + 120^\circ)/\underline{\quad} + 120^\circ$$

After exchanged



Net stator magnetic field

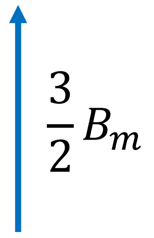
For example at *t*=0 ➔ ***wt* = 0**

$$B_{aa'}(t=0) = B_M sin(wt) = 0$$

$$B_{bb'}(t=0) = B_M sin(wt - 120°)/\underline{\quad} - 120° = B_M \underbrace{sin(-120°)}_{-\sqrt{3}/2}/\underline{\quad} - 120° = \frac{-\sqrt{3}}{2} B_M/\underline{\quad} - 120°$$

$$B_{cc'}(t=0) = B_M sin(wt + 120°)/\underline{\quad} + 120° = B_M \underbrace{sin(+120°)}_{\sqrt{3}/2}/\underline{\quad} + 120° = \frac{\sqrt{3}}{2} B_M/\underline{\quad} + 120°$$

$$B_{net}(t=0) = \frac{-\sqrt{3}}{2} B_M/\underline{\quad} - 120° + \frac{\sqrt{3}}{2} B_M/\underline{\quad} + 120° = \frac{3}{2} B_M(+j) = \frac{3}{2} B_M/\underline{\quad} + 90°$$



Net stator magnetic field

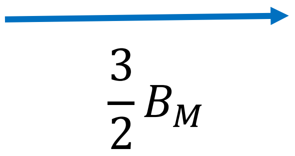
For example at ***wt* = 90°**

$$B_{aa'}(wt = 90) = B_M sin(90)\angle 0^\circ \ = B_M \angle 0^\circ$$

$$B_{bb'}(wt = 90) = B_M \underbrace{sin(90 - 120^\circ)}_{-1/2} \angle -120^\circ \ = -\frac{1}{2} B_M \angle -120^\circ$$

$$B_{cc'}(wt = 90) = B_M \underbrace{sin(90 + 120^\circ)}_{-1/2} \angle +120^\circ \ = -\frac{1}{2} B_M \angle +120^\circ$$

$$B_{net}(wt = 90) = \ B_M \angle 0^\circ \ -\frac{1}{2} B_M \angle -120^\circ \ -\frac{1}{2} B_M \angle +120^\circ \ = \frac{3}{2} B_M \angle 0^\circ$$



Net stator magnetic field

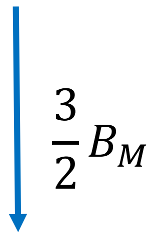
For example at ***wt* = 180°**

$$B_{aa'}(wt = 180) = B_M sin(180)\angle 0° = 0$$

$$B_{bb'}(wt = 180) = B_M \underbrace{sin(180 - 120°)}_{\sqrt{3}/2}\angle + 120° \quad = \frac{\sqrt{3}}{2} B_M \angle - 120°$$

$$B_{cc'}(wt = 180) = B_M \underbrace{sin(180 + 120°)}_{-\sqrt{3}/2}\angle - 120° \;= -\frac{\sqrt{3}}{2} B_M \angle + 120°$$

$$B_{net}(wt = 180) = \;\frac{\sqrt{3}}{2} B_M \angle - 120° - \frac{\sqrt{3}}{2} B_M \angle + 120° = \frac{3}{2} B_M(-j) = \frac{3}{2} B_M \angle - 90°$$



Net stator magnetic field

For example at ***wt* = 270°**

$$B_{aa'}(wt = 270) = B_M sin(270)\angle 0° = -B_M\angle 0°$$

$$B_{bb'}(wt = 270) = B_M \underbrace{sin(270 - 120°)}_{1/2}\angle + 120° \quad = \frac{1}{2}B_M\angle - 120°$$

$$B_{cc'}(wt = 270) = B_M \underbrace{sin(270 + 120°)}_{1/2}\angle - 120° \ = \frac{1}{2}B_M\angle + 120°$$

$$B_{net}(wt = 180) = \ -B_M\angle 0° \ +\frac{1}{2}B_M\angle - 120° + \frac{1}{2}B_M\angle + 120° = -\frac{3}{2}B_M$$

Net stator magnetic field

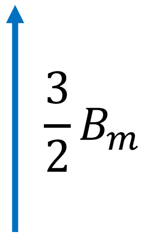
For example at *t*=0 ➔ ***wt* = 360=0°**

$$B_{aa'}(t=0) = B_M sin(wt) = 0$$

$$B_{bb'}(t=0) = B_M sin(wt - 120°)\angle -120° = B_M \underbrace{sin(-120°)}_{-\sqrt{3}/2}\angle +120° = \frac{-\sqrt{3}}{2} B_M \angle -120°$$

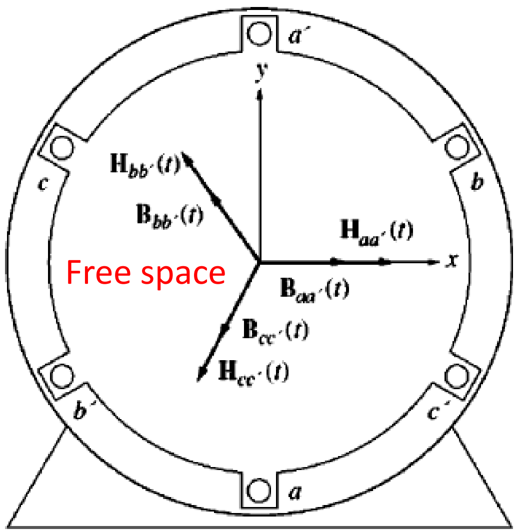
$$B_{cc'}(t=0) = B_M sin(wt + 120°)\angle +120° = B_M \underbrace{sin(+120°)}_{\sqrt{3}/2}\angle -120° = \frac{\sqrt{3}}{2} B_M \angle +120°$$

$$B_{net}(t=0) = \frac{-\sqrt{3}}{2} B_M \angle -120° + \frac{\sqrt{3}}{2} B_M \angle +120° = \frac{3}{2} B_M(+j) = \frac{3}{2} B_M \angle +90°$$



MAGNETOMOTIVE FORCE AND FLUX DISTRIBUTION ON AC MACHINES

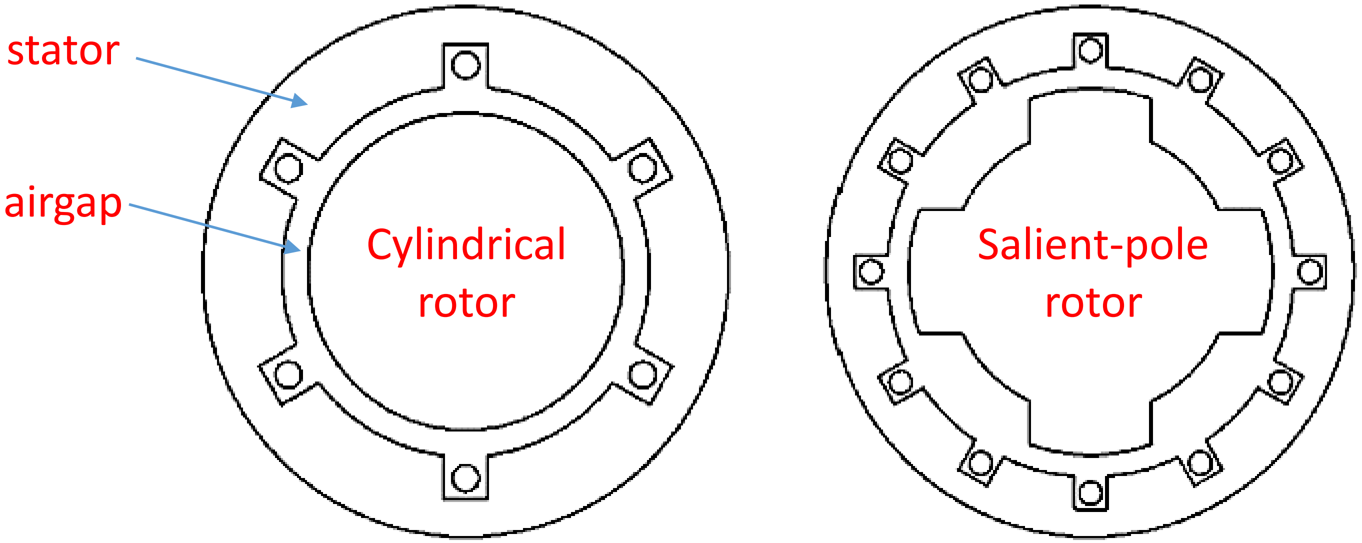
- The flux produced inside an ac machine was treated as if it were in **free space**
- The direction of the flux density (**B**) produced by a coil of wire was assumed to be perpendicular to the coil plane
- Flux direction can be shown by right-hand rule



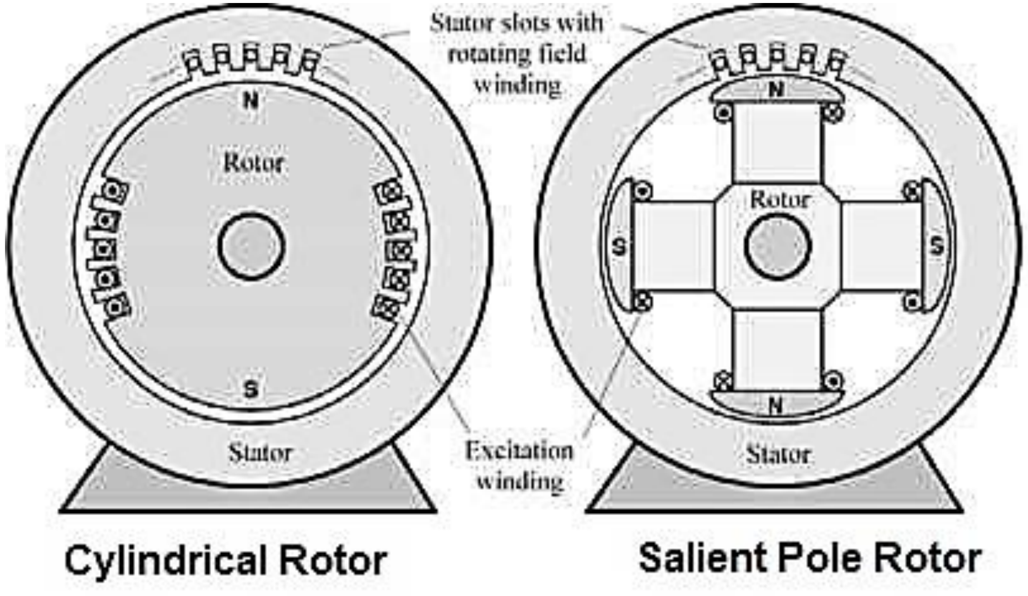
MAGNETOMOTIVE FORCE AND FLUX DISTRIBUTION ON AC MACHINES

- The flux in a real machine does not behave as simple as mentioned before
- Because there is a **ferromagnetic rotor** in the center of the machine and
- There is as a **small air gap** between the rotor and the stator

- The rotor can be cylindrical (non-salient pole) or non-cylindrical (salient-pole)



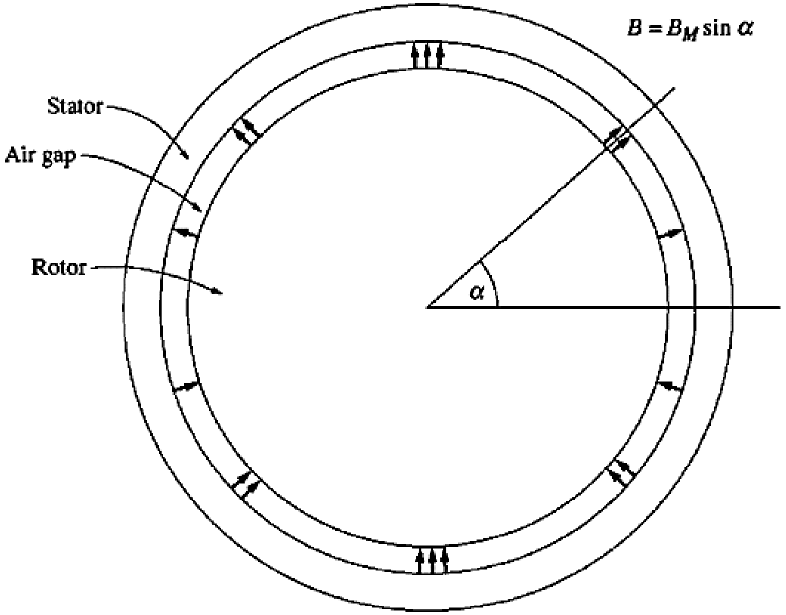
MAGNETOMOTIVE FORCE AND FLUX DISTRIBUTION ON AC MACHINES



Reference: https://etrical.blogspot.com.tr/2016/10/cylindrical-salient-pole-rotor.html

MAGNETOMOTIVE FORCE AND FLUX DISTRIBUTION ON AC MACHINES

- Consider **cylindrical-rotor** AC machine (*much easier to analyze*)



$$R_{stator} = \frac{stator\ length}{\mu_s(Area\ of\ stator)}$$

$$R_{air} = \frac{air\ gap\ length}{\mu_0(Area\ of\ airgap)}$$

$$R_{rotor} = \frac{rotor\ length}{\mu_r(Area\ or\ rotor)}$$

Since $\mu_s\ , \mu_r \gg \mu_0$ ➡ $R_{air} \gg R_{stator}, R_{rotor}$

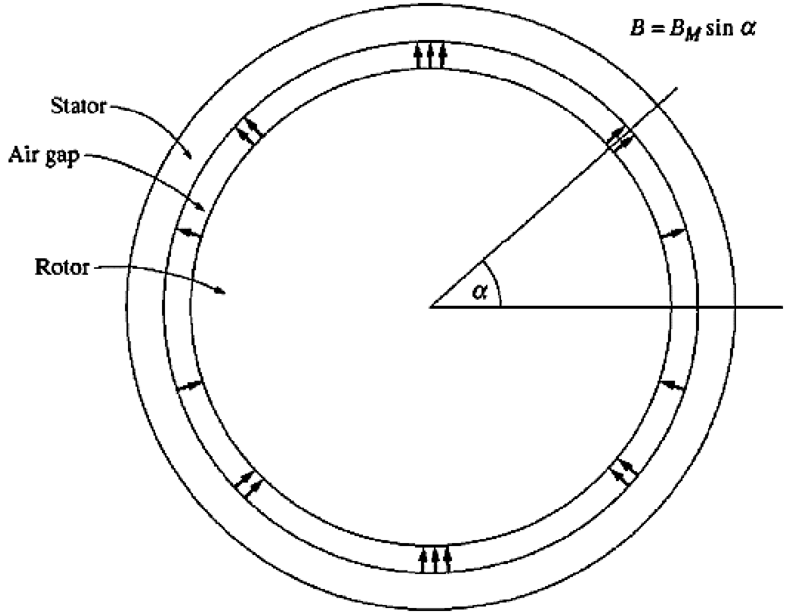
- Flux density vector **B** takes the **shortest possible path across the air gap** and jumps **perpendicularly** between the rotor and the stator
- Because the reluctance of air is much greater than that of stator and rotor, hence **B** wants to travel the airgap as fast as possible

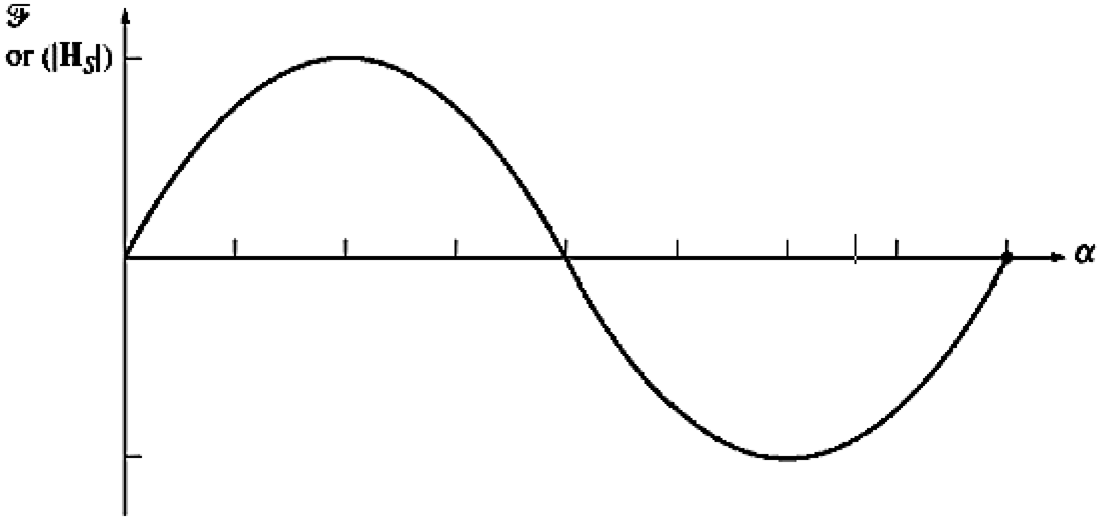
MAGNETOMOTIVE FORCE AND FLUX DISTRIBUTION ON AC MACHINES

- To produce a **sinusoidal voltage** in the machine, the magnitude of the flux density vector **B** must vary in a sinusoidal manner along the surface of the air gap

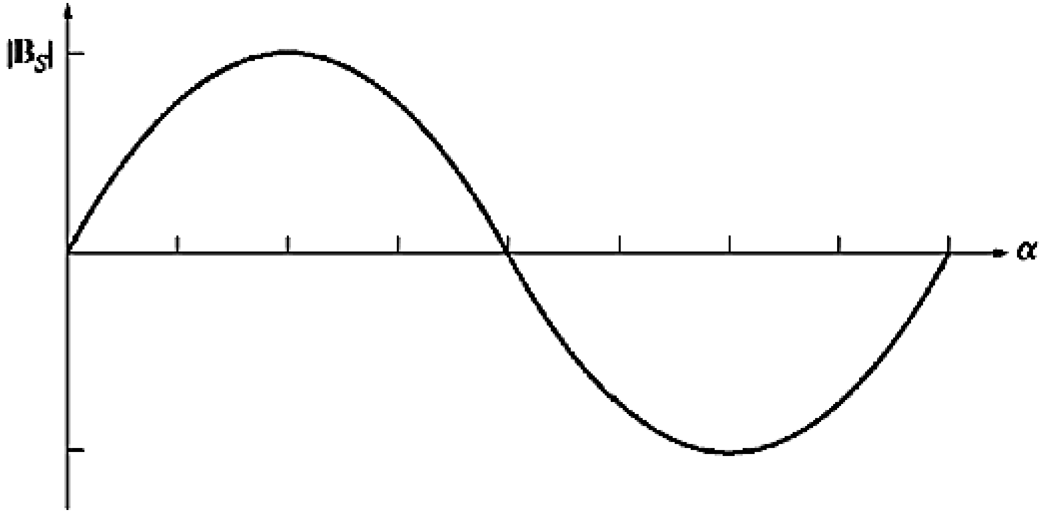
- **B** varies sinusoidally only if the magnetizing intensity **H** and the magnetomotive force **F** varies also in sinusoidal manner along the surface of the air gap

- But how is this possible in a real machine ?





The magnetomotive force **F** or magnetizing intensity **H** as a function of angle **α** in the air gap



The flux density **B** as a function of angle **α** in the air gap

MAGNETOMOTIVE FORCE AND FLUX DISTRIBUTION ON AC MACHINES

- The most straightforward way to achieve (*approach*) a sinusoidal variation of magnetomotive force **F** along the surface of the air gap is;

  - ❑ Distribute the turns of the winding that produces **F** in closely spaced slots around the surface of the machine (***increase total number of slots***)

  - ❑ And vary the number of conductors in each slot in a sinusoidal manner according to the equation given below:

$$n_c = N_c \cos(\alpha)$$

n_c ⇨ The number of conductors in the slot at angle, α

N_c ⇨ The number of conductors at the position α=0°

MAGNETOMOTIVE FORCE AND FLUX DISTRIBUTION ON AC MACHINES

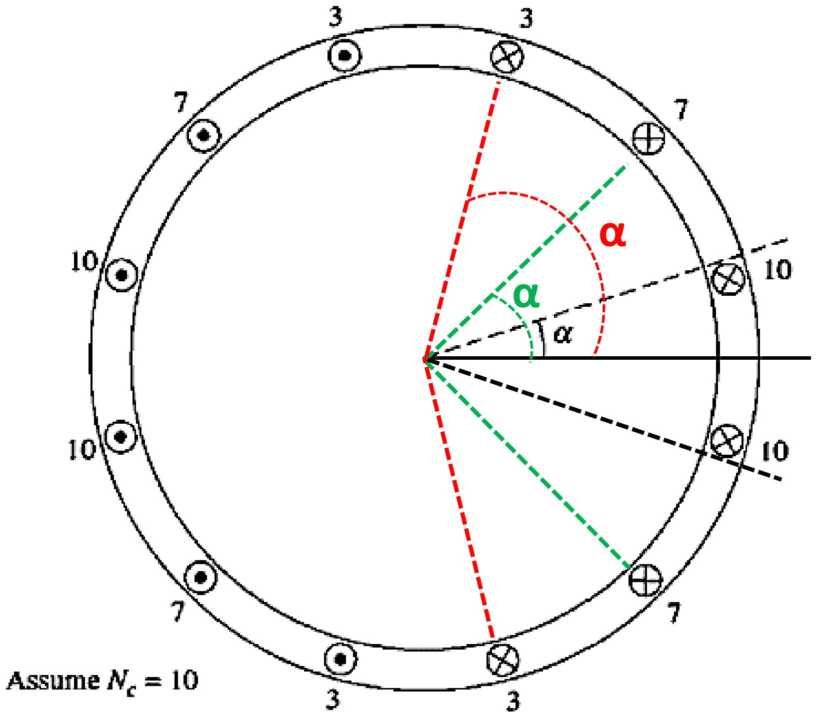
$N_c = 12$ (total number of slots in the figure)

$n_c = N_c \cos(\alpha)$

For α = 15° ➔ n_c = 12xcos(33.5) ≅ 10

For α = 45° ➔ n_c = 12xcos(54.3) = 7.07 ≅ 7

For α = 75° ➔ n_c = 12xcos(75.5) = 2.588 ≅ 3



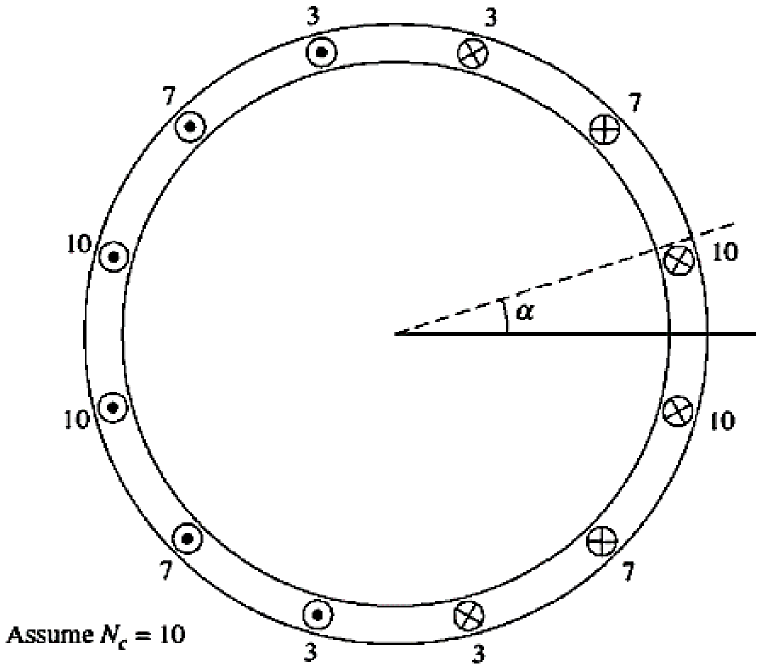
α=33.5°

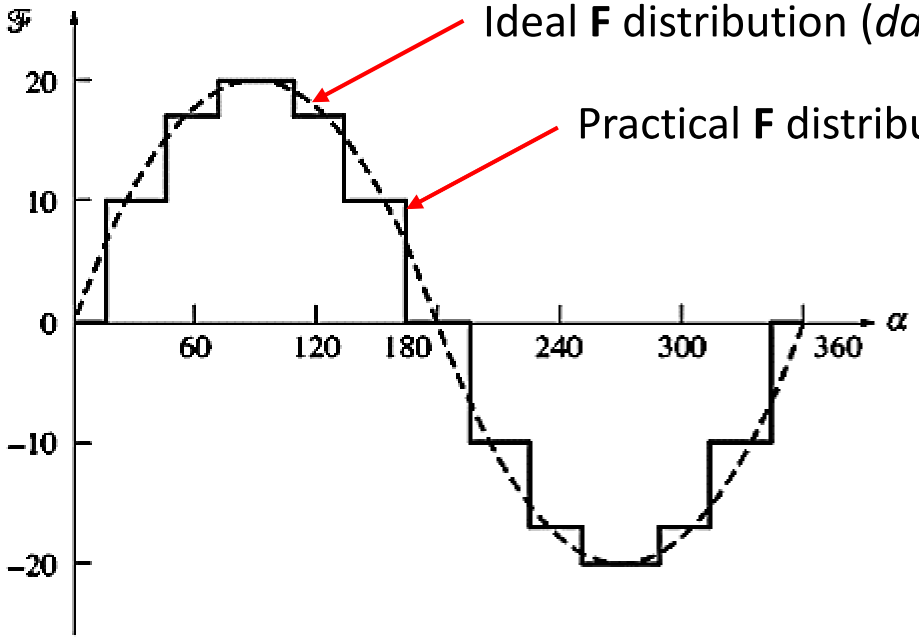
α=54.3°

α=75.5°

Note: If total number of slots are increased, a better sinusoidal approximation is achieved

MAGNETOMOTIVE FORCE AND FLUX DISTRIBUTION ON AC MACHINES





Ideal **F** distribution (*dashed line*)

Practical **F** distribution (*line*)

The magnetomotive force **F** distribution resulting from the stator winding distribution (*left side*) as compared to an ideal distribution

MAGNETOMOTIVE FORCE AND FLUX DISTRIBUTION ON AC MACHINES

- Stator voltages will have the **same shape** as that of the air-gap flux density distribution
- So, if the air-gap flux density distribution is sinusoidal, the output voltages in the stator coils are also sinusoidal (*AC generator action*)
- If the air-gap flux density distribution is not sinusoidal , the output voltages in the stator coils are also not sinusoidal
- The reason for being **non-sinusoidal shape** is the **inclusion of the harmonics**

Fractional-Pitch Windings

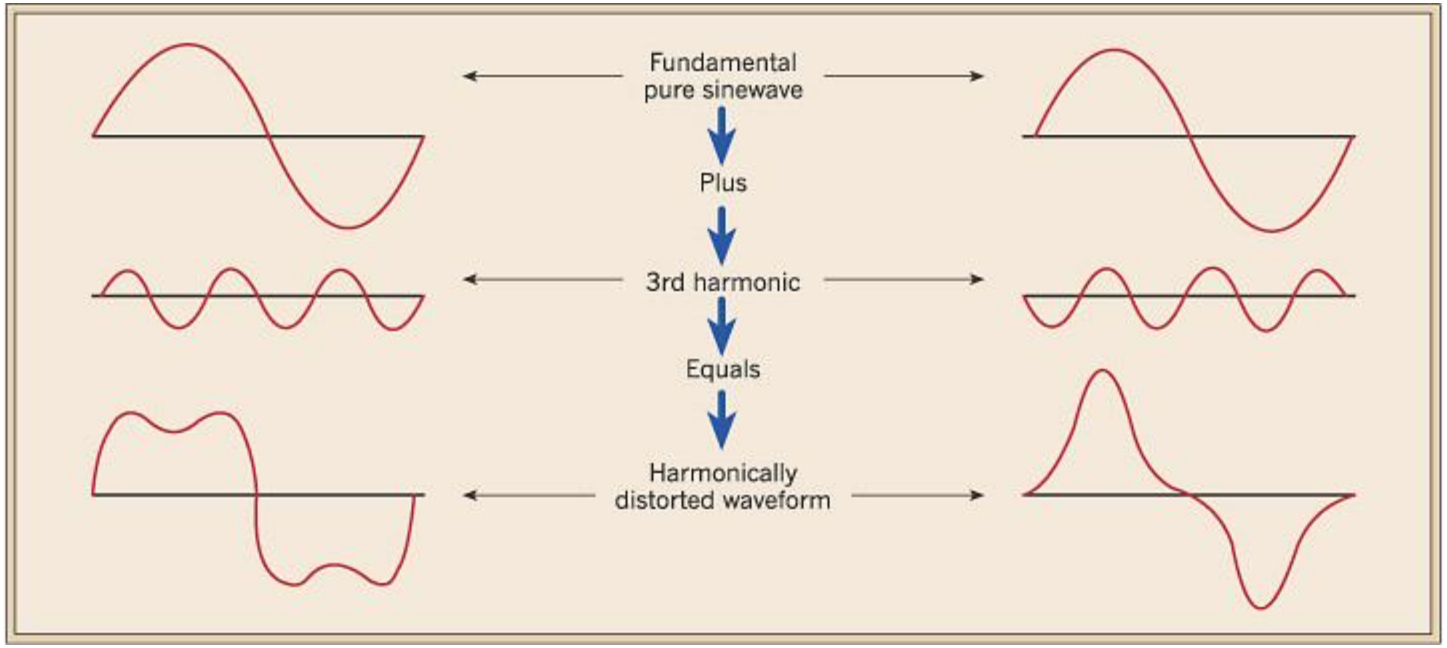
Why harmonics are present in AC machines ?

- There are only a **finite number of slots** in a real machine
- Only **approximated number of turns** can be considered for the windings in each slot
- Usually, it is often convenient for the machine designer to use **equal numbers of turns** in each slot instead of varying the number of turns

Because of the **above three reasons**, there will be **higher-order harmonic components** in **F** and **H**

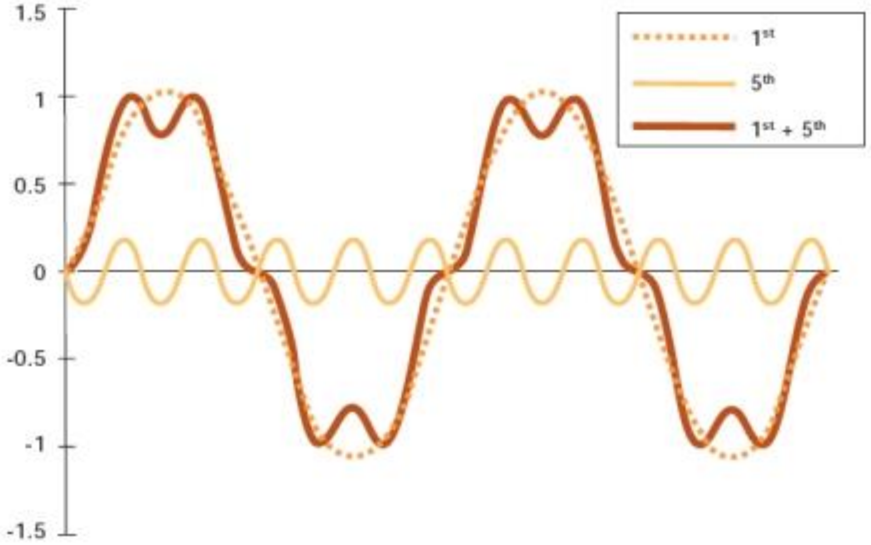
Using **fractional-pitch windings** is one of the techniques to **suppress the harmonics** in AC machines

Effect of 3rd harmonic



Reference: https://etrical.blogspot.com.tr/2016/07/advantages-disadvantages-of-harmonics.html

Effect of 5th harmonic



Reference: https://joliettech.com/information/reducing-harmonics-caused-by-variable-speed-drives/

Pole pitch

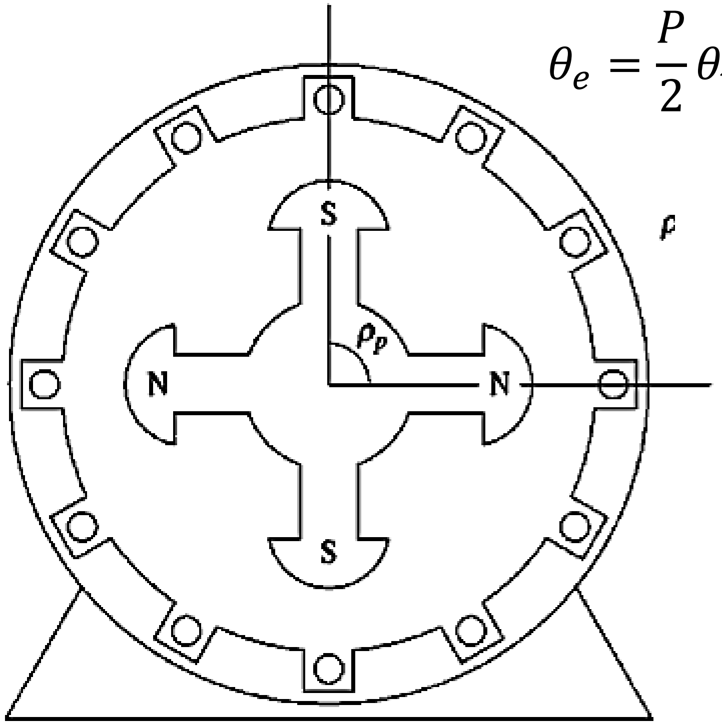
- Pole pitch is defined as the angle distance between two neighboring poles in an AC machine
- Pole pitch is defined in terms of **mechanical degrees** as follows:

$$P_p = \frac{360°}{P}$$ → Pole pitch in mechanical degrees

P → Pole number

- Regardless of the number of pole numbers, **pole pitch is always 180 electrical degrees**

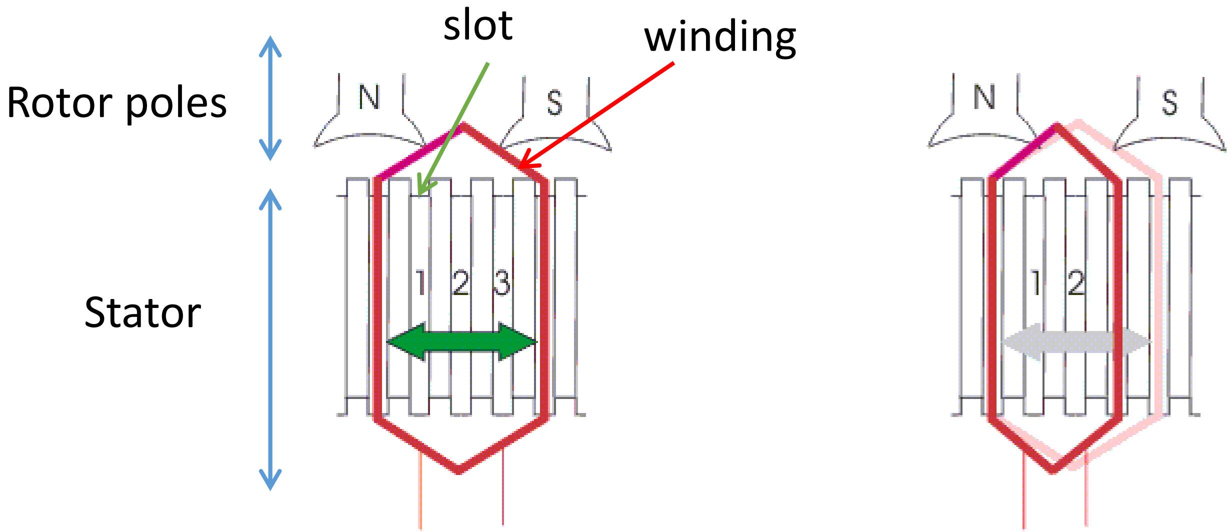
$$\theta_e = \frac{P}{2}\theta_m$$



The pole pitch of a four-pole machine **is 90 mechanical** or **180 electrical degrees**

Full-pitch *vs* fractional-pitch coil

- If the stator coil stretches across the same angle as the pole pitch, it is called ***full-pitch coil***
- If the stator coil stretches across an angle smaller than the pole pitch, it is called ***fractional-pitch coil***



Rotor poles

Stator

slot

winding

full-pitch coil

fractional-pitch coil

(*short-pitch*)

Coil pitch

- Coil pitch is expressed as a fraction indicating the portion of the pole pitch it spans
- Coil pitch is given as follows:

$$\rho = \frac{\theta_m}{P_p}.180^\circ$$

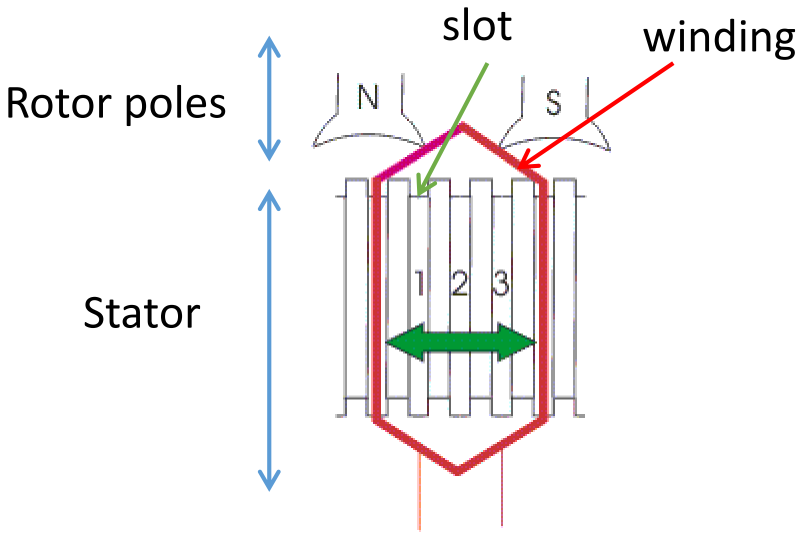
ρ ⇨ Coil pitch in electrical degrees

θ_m ⇨ Mechanical angle covered by the coil in degrees

P_p ⇨ Pole pitch in mechanical degrees

- For fractional-pitch coils, ρ is **always less** than 180°
- For full-pitch coils, ρ is **always equal** to 180°

Coil pitch: An example



Rotor poles

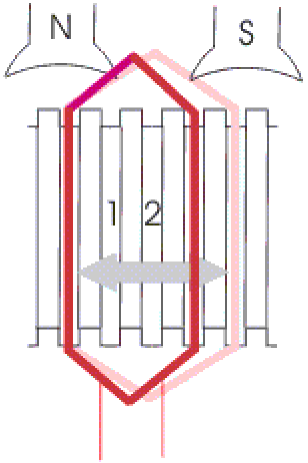
Stator

slot

winding

full-pitch coil

$$\rho = 180°$$



fractional-pitch coil

$$\rho = \left(\frac{3}{4}\right).180° = 135°$$

Induced Voltage of a Fractional-Pitch Coil

- The **induced voltage of a fractional-pitch coil** (*without giving the proof*) is expressed as:

$$e_{ind} = \emptyset w k_p \cos(wt)$$

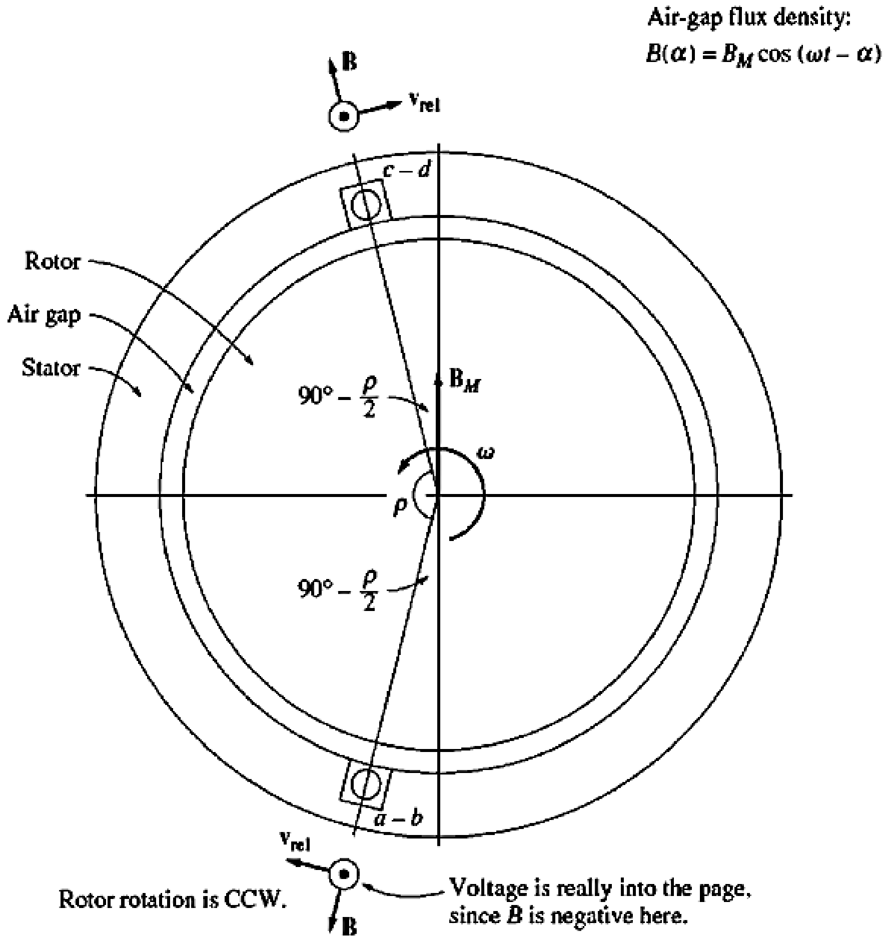
where k_p is the "**pitch factor", d**efined as;

$$k_p = sin(\frac{\rho}{2})$$

ρ ➡ Coil pitch in electrical degrees

Since $0 \leq \sin\left(\frac{\rho}{2}\right) \leq 1$

e_{ind} is reduced by a factor less than 1 due to fractional-pitch coil



A fractional-pitch winding of pitch ***p***

Induced Voltage of a Fractional-Pitch Coil

- For ***N*-turn**, the **induced voltage of a fractional-pitch coil** is given as:

$$e_{ind} = N\emptyset w k_p \cos(wt)$$

- The peak voltage is

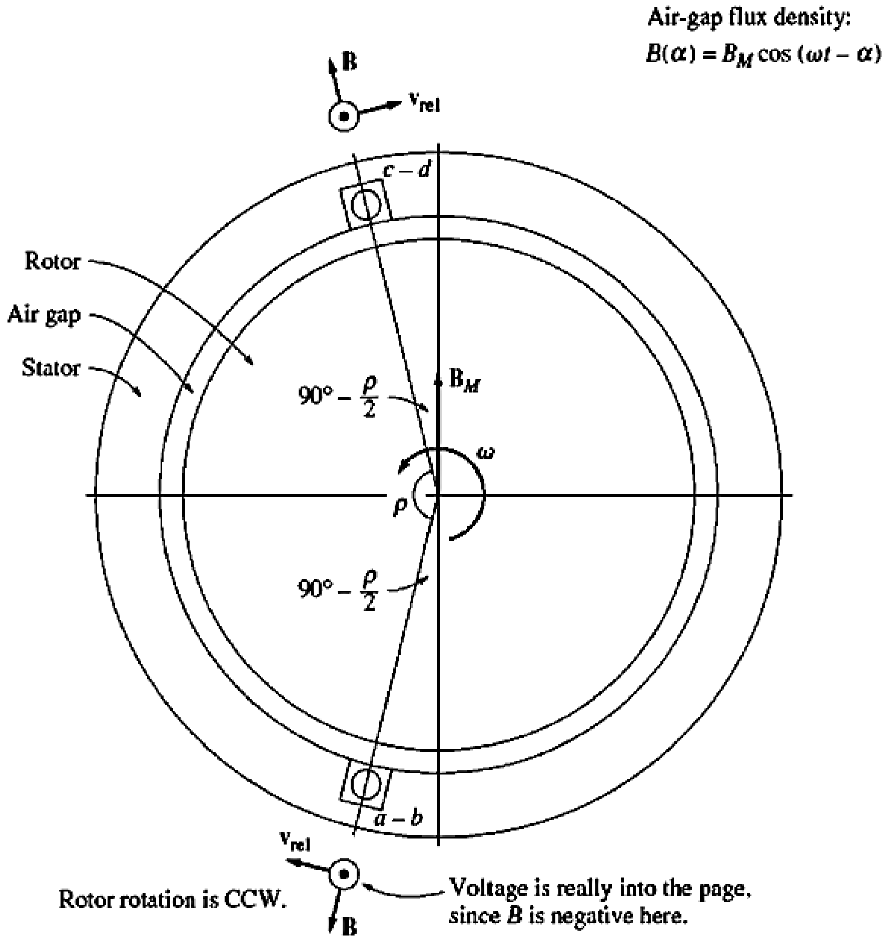
$$E_{max} = N\emptyset w k_p = N\emptyset 2\pi f k_p$$

- The rms voltage is

$$E_A = \frac{N\emptyset w k_p}{\sqrt{2}} = \frac{N\emptyset 2\pi f k_p}{\sqrt{2}}$$

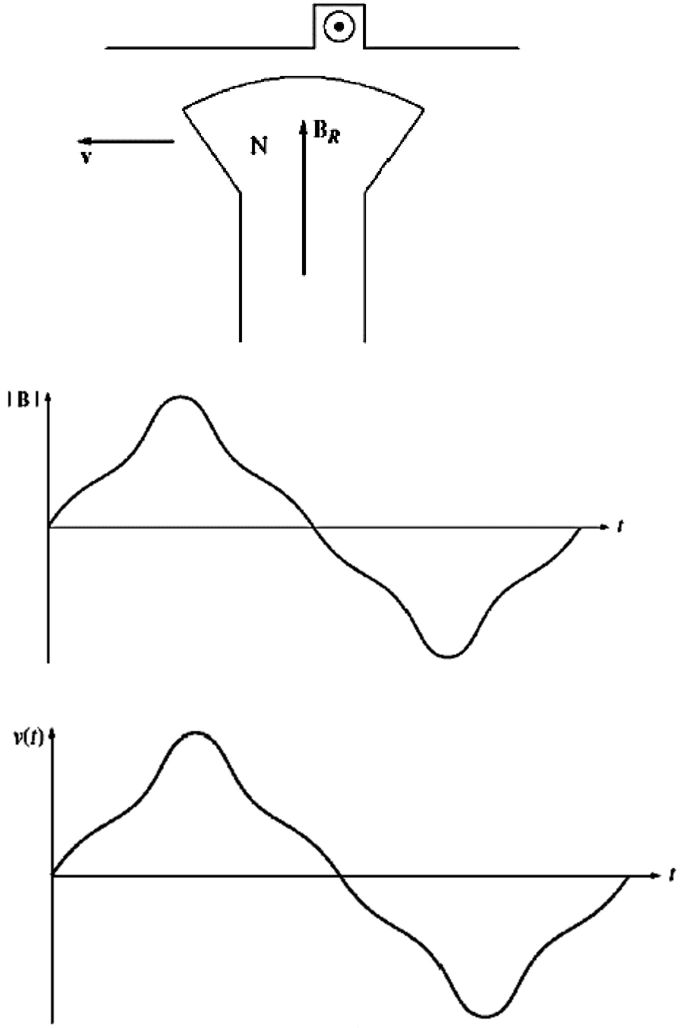
$$E_A = \sqrt{2}\pi N k_p \emptyset f$$

If $k_p = 1$ $(\rho = 180°)$ ➡ Full-pitch winding



A fractional-pitch winding of pitch ***p***

Harmonic suppression and fractional-pitch windings



- This figure shows a **salient-pole synchronous machine** whose rotor is sweeping across the stator surface.

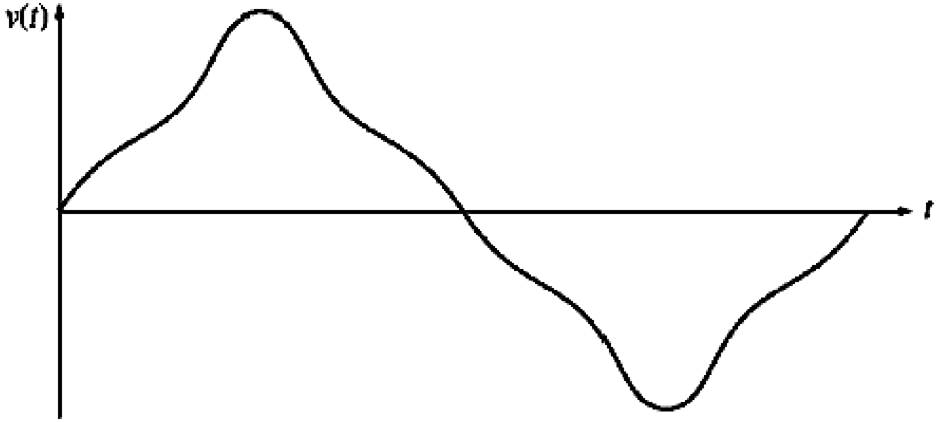
- Because the reluctance of the magnetic field path is much lower under the center of the rotor than it is toward the sides (*smaller air gap*), the flux is strongly concentrated at that point and the flux density is very high there.

- The **magnetic field density** and the **induced voltage** in the winding is shown in the figure.

- As seen, **induced voltage** is not sinusoidal and it contains **many harmonic frequency components**.

Harmonic suppression and fractional-pitch windings

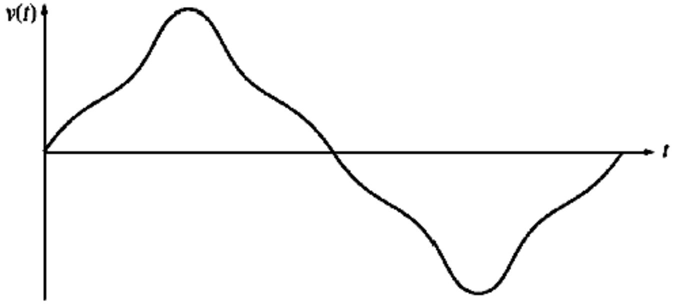
- Since the resulting voltage waveform is **symmetric** about the time axis, **no even harmonics** are present in the phase voltage
- However, the **odd harmonics** (*third, fifth, seventh, ninth, etc*.) *are* present in the phase voltage



Harmonic suppression and fractional-pitch windings

- **Third-harmonic component** is disappeared from the output voltage of the machine for **three-phase (Y or Δ)** connection.

- For example if the AC machine is **Y-connected**:



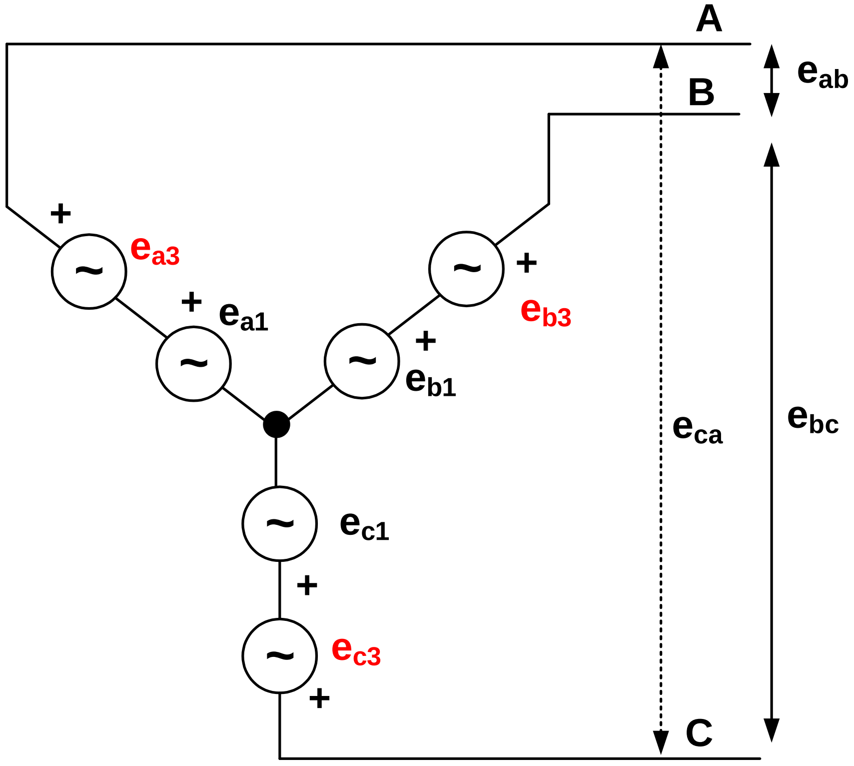
$$\left.\begin{aligned} e_{a1}(t) &= E_{M1}\sin(wt) \\ e_{b1}(t) &= E_{M1}\sin(wt-120°) \\ e_{c1}(t) &= E_{M1}\sin(wt-240°) \end{aligned}\right\}\ \text{Fundamental components}$$

$$\left.\begin{aligned} e_{a3}(t) &= E_{M3}\sin(3wt) \\ e_{b3}(t) &= E_{M3}\sin(3wt-3x120°) \\ e_{c3}(t) &= E_{M3}\sin(3wt-3x240°) \end{aligned}\right\}\ \text{Third-harmonic components}\ \Rightarrow\ e_{a3}(t)=e_{b3}(t)=e_{c3}(t)$$

$$E_{M3}=\frac{E_{M1}}{3}$$

(They are equal !)

Harmonic suppression and fractional-pitch windings



Y-connected stator of AC machine

- **Third-harmonic component** is disappeared from the **Y-connected** output of the machine:

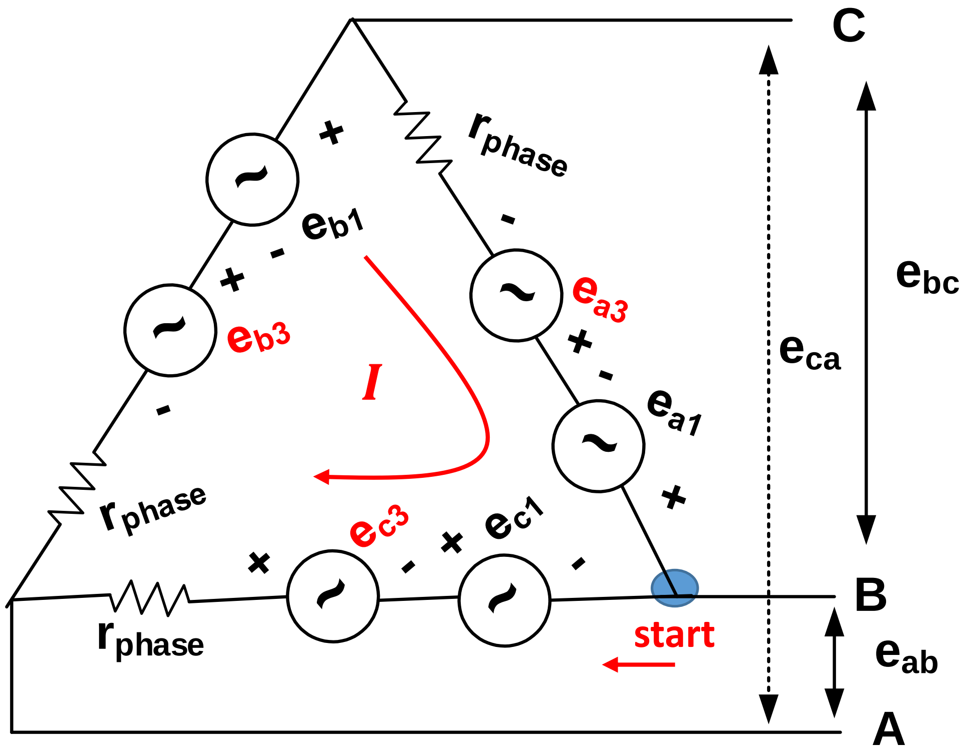
$$e_{ab} = e_{a1} + \cancel{e_{a3}} - e_{b1} - \cancel{e_{b3}}$$

$$e_{ab} = e_{a1} - e_{b1}$$

- Third harmonic components **disappear** from the **line-to-line voltage of the AC machine**

- Third harmonic components also **disappear** from other **line-to-line voltages** (e_{bc} and e_{ca})

Harmonic suppression and fractional-pitch windings



Δ-connected stator

- **Third-harmonic component** is disappeared from the **Δ-connected** output of the machine:

  I ⇒ circulating current (*loop current*) in delta

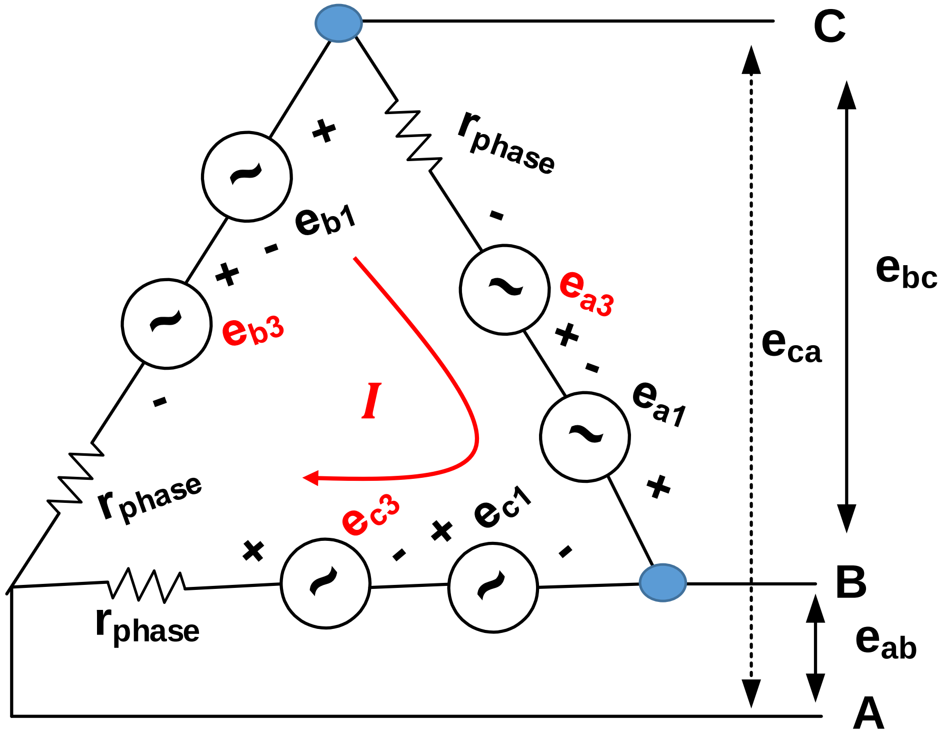
- Applying **KVL** to delta:

$$e_{c1} + e_{c3} - r_{phase}.I - r_{phase}.I + e_{b3} + e_{b1} - r_{phase}.I + e_{a3} + e_{a1} = 0$$

- Since $e_{a1} + e_{b1} + e_{c1} = 0$, (*balanced*), the following equation is obtained for loop current:

$$I = \frac{e_{a3} + e_{b3} + e_{c3}}{3r_{phase}} = \frac{3e_{a3}}{3r_{phase}} = \frac{e_{a3}}{r_{phase}}$$

Harmonic suppression and fractional-pitch windings



Δ-connected stator

$$e_B + e_{c1} + e_{c3} - 2r_{phase}.I + e_{b3} + e_{b1} = e_C$$

$$e_B - e_C = -e_{c1} \underbrace{- e_{c3} + 2r_{phase}.I - e_{b3}}_{\mathbf{0}} - e_{b1}$$

$$\underbrace{e_B - e_C}_{\textbf{LL voltage}} = -e_{c1} - e_{b1}$$

- Third harmonic components **disappear** from the **line-to-line voltage of the AC machine**
- Third harmonic components also **disappear** from other **line-to-line voltages**

Harmonic suppression and fractional-pitch windings

- We see that the **third-harmonic components** are removed from the **line-to-line voltages**.

- Also any **multiple** of the **third-harmonic components** (**such as the 9th, 12th, etc…**) are also eliminated from the **line-to-line voltages**.

- These special harmonic frequencies are called "***triplen harmonics***" and are **automatically eliminated** in three-phase machines (*either **Y** or **Δ** connected*)

- So, the remaining harmonic frequencies at machine terminals are **5th, 7th, 11th, 13th, etc…**

- The **most strong harmonics** are **5th and 7th**. Because their **magnitude** is **greater** then other harmonics (**11th, 13th , etc…**)

- **Fractional-pitch windings** on the stator can be used **to suppress these harmonics** at the terminals of AC machines.

- If they are effectively eliminated, then the machine's output voltage waveform would be essentially a **pure sinusoid** at the fundamental frequency (***50** or **60 Hz***).

Harmonic suppression and fractional-pitch windings

*How can some of the **harmonic content** of the winding's terminal voltage be **eliminated** using **fractional windings** ?*

- Assume a coil spans $\boldsymbol{\rho}$ **electrical degrees** at its **fundamental frequency (50 or 60 Hz)**;
- It will span $\mathbf{2}\boldsymbol{\rho}$ **electrical degrees** at its **second-harmonic frequency**,
- It will span $\mathbf{3}\boldsymbol{\rho}$ **electrical degrees** at its **third-harmonic frequency**,
- So the **pitch factor** at the **harmonic order** $\boldsymbol{\vartheta}$ can be written as follows:

$$k_p = sin\frac{\vartheta\rho}{2}$$

- The **pitch factor** of a winding is different for each harmonic frequency
- By a ***proper choice of coil pitch*** it is possible to almost ***eliminate harmonic frequency components*** at the output

Harmonic suppression and fractional-pitch windings: An Example

Question: A three-phase, two-pole stator has coils with a 5/6 pitch. What are the pitch factors for the harmonics present in this machine's coils? Does this pitch help suppress the harmonic content of the generated voltage?

$$\rho_p = \frac{360°}{P} = \frac{360°}{2} = 180°$$ (pole pitch)

$$\rho = \frac{\theta_m}{P_p}.180° = \frac{\frac{5}{6} * 180°}{180°}.180° = 150°$$ (coil pitch)

Harmonic suppression and fractional-pitch windings: An Example

Pitch factor at **fundamental** frequency: $k_p = \sin\left(\frac{150°}{2}\right) = 0.9659$

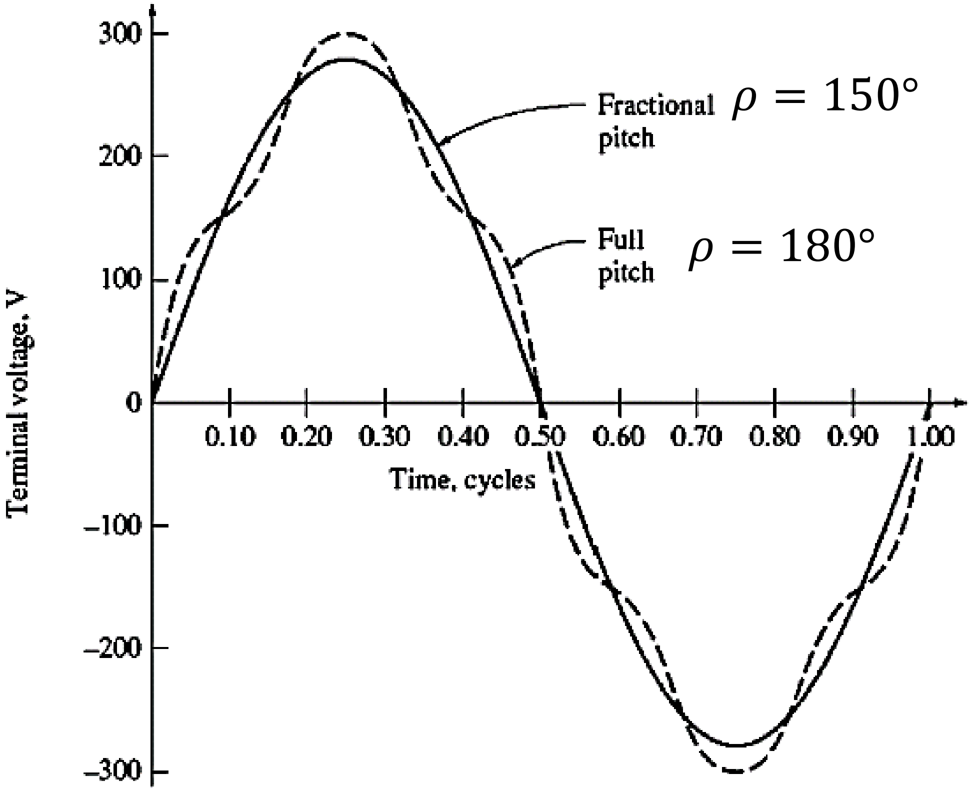
Pitch factor at **third harmonic** frequency: $$k_p = \sin\left(\frac{3 * 150°}{2}\right) = -0.7071$$

Pitch factor at **fifth harmonic** frequency: $$k_p = \sin\left(\frac{5 * 150°}{2}\right) = 0.2588$$

Pitch factor at **seventh harmonic** frequency: $$k_p = \sin\left(\frac{7 * 150°}{2}\right) = 0.2588$$

Pitch factor at **ninth harmonic** frequency: $$k_p = \sin\left(\frac{9 * 150°}{2}\right) = -0.7071$$

Harmonic suppression and fractional-pitch windings: An Example



$\rho = 150°$

$\rho = 180°$

The line-to-line output voltage of a three-phase generator with **full-pitch** and **fractional-pitch windings**. Although the peak voltage of the fractional-pitch winding is **slightly smaller** than that of the full-pitch winding its output voltage is much **purer**

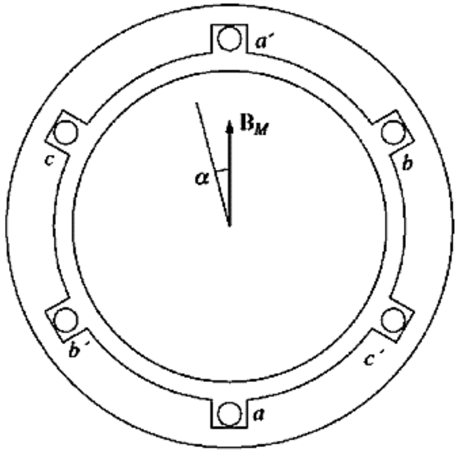
Induced Voltage in a Three-Phase Set of Coils

- Three coils **(each has N_c turns)** are placed in **stator** around the **rotor magnetic field** as shown in the Figure.
- The induced voltages in stator at each phase are given as follows:

$$e_{aa'}(t) = N_c \emptyset w sin(wt)$$

$$e_{bb'}(t) = N_c \emptyset w sin(wt - 120°)$$

$$e_{cc'}(t) = N_c \emptyset w sin(wt - 240°)$$



- A **three-phase set of voltages** (hence currents) generates a **uniform rotating magnetic field** in the AC machine.
- A **uniform rotating magnetic field** generates a **three-phase set of voltages** in stator of the AC machine.

The RMS Voltage in a Three-Phase Stator

- Phase-A induced instantaneous voltage in stator:

$$e_{aa'}(t) = N_c\emptyset w sin(wt)$$

- Peak value of this voltage:

$$e_{aa'(peak)} = N_c\emptyset w$$

- RMS value of this voltage:

$$e_{aa'(RMS)} = \frac{N_c\emptyset w}{\sqrt{2}} = \frac{N_c\emptyset 2\pi f}{\sqrt{2}} = \sqrt{2}N_c\emptyset\pi f$$

- Since stator is **balanced**:

$$e_{aa'(RMS)} = e_{bb'(RMS)} = e_{cc'(RMS)}$$

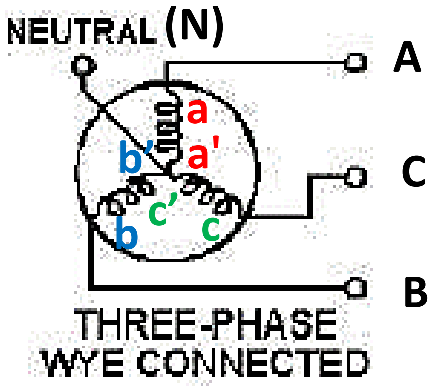
Stator connection as Wye (Y)

- If stator is **Wye (Y)** connected, **phase voltages** **(line-to-neutral [LN])** of stator:

$$\left.\begin{array}{l} e_{AN}(t) = N_c \emptyset w sin(wt) \\ e_{BN}(t) = N_c \emptyset w sin(wt - 120°) \\ e_{CN}(t) = N_c \emptyset w sin(wt - 240°) \end{array}\right\} \quad e_{AN(RMS)} = e_{BN(RMS)} = e_{CN(RMS)} = e_{LN(RMS)}$$

- The **line voltages** **(line-to-line [LL])** of stator:

$$e_{AB(RMS)} = e_{BC(RMS)} = e_{CA(RMS)} = \sqrt{3}.\, e_{LN(RMS)}$$



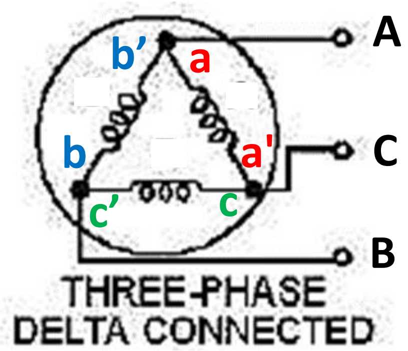
Stator connection as Delta (Δ)

- If stator is **Delta (Δ)** connected, **phase voltages** of stator:

$$\left.\begin{aligned} e_{aa'}(t) &= N_c \emptyset w sin(wt) \\ e_{bb'}(t) &= N_c \emptyset w sin(wt - 120°) \\ e_{cc'}(t) &= N_c \emptyset w sin(wt - 240°) \end{aligned}\right\} \quad e_{aa'(RMS)} = e_{bb'(RMS)} = e_{cc'(RMS)}$$

- The **line voltages** **(line-to-line [LL])** of stator:

$$e_{AB(RMS)} = e_{BC(RMS)} = e_{CA(RMS)} = e_{aa'(RMS)}$$



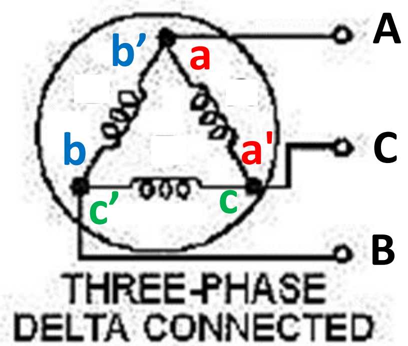
Stator connection as Delta (Δ)

- If stator is **Delta (Δ)** connected, **phase voltages** of stator:

$$\left.\begin{aligned} e_{aa'}(t) &= N_c \emptyset w sin(wt) \\ e_{bb'}(t) &= N_c \emptyset w sin(wt - 120°) \\ e_{cc'}(t) &= N_c \emptyset w sin(wt - 240°) \end{aligned}\right\} \; e_{aa'(RMS)} = e_{bb'(RMS)} = e_{cc'(RMS)}$$

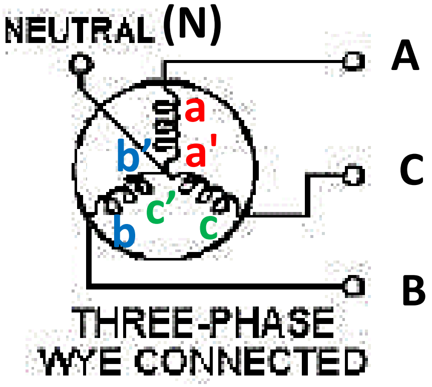
- The **line voltages** **(line-to-line [LL])** of stator:

$$e_{AB(RMS)} = e_{BC(RMS)} = e_{CA(RMS)} = e_{aa'(RMS)}$$



Example:

Question: A three-phase stator is Y-connected as shown in the figure. The stator diameter and coil length are 0.5 m and 0.3 m, respectively. There are 15 turns per coil. The stator has two poles and the peak value of the flux density of the machine is 0.2 T. If the speed of the rotor (*not shown in the figure*) is adjusted to 3600 r/min (rpm), answer the following questions:



a) Express the phase voltages (*LN-voltages*) of the stator in time domain and plot them as a function of time for two cycles.

Solution:

$$n_m = \frac{120 f_e}{P} \Rightarrow f_e = \frac{n_m P}{120} = \frac{3600x2}{120} = 60\ Hz \Rightarrow w_e = 2\pi f_e = 2\pi 60 = 376.9911\ rad/s$$

$$\emptyset = B.A = 0.2T * 0.5\ * 0.3m = 0.03\ Wb$$

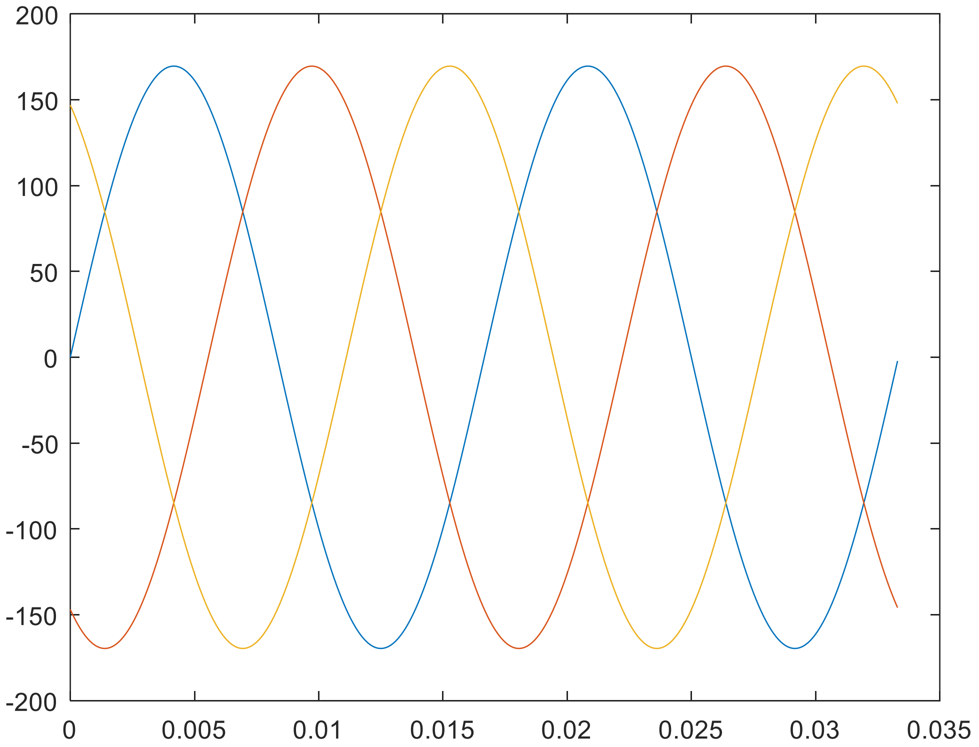
Example:

$e_{aN}(t) = N_c\emptyset w sin(wt) = 15\text{x}0.03\text{x}376.9911\text{sin}(376.9911\text{t}) = 169.65\text{sin}(376.9911\text{t})$ (volts)

$e_{bN}(t) = N_c\emptyset w sin(wt - 120°) = 169.65\text{sin}(376.9911\text{t} - 2\pi/3)$ (volts)

$e_{cN}(t) = N_c\emptyset w sin(wt - 240°) = 169.65\text{sin}(376.9911\text{t} - 4\pi/3)$ (volts)

(we assume positive sequence)

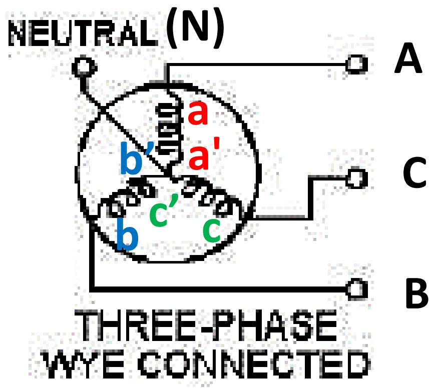


Example:

(b) Calculate the rms value of the phase voltages of stator

Solution:

$$e_{AN(RMS)} = e_{BN(RMS)} = e_{CN(RMS)} = \frac{169.65}{\sqrt{2}} = 119.96 \text{ Volts}$$



c) Calculate the rms value of the stator terminal voltage.

Solution:

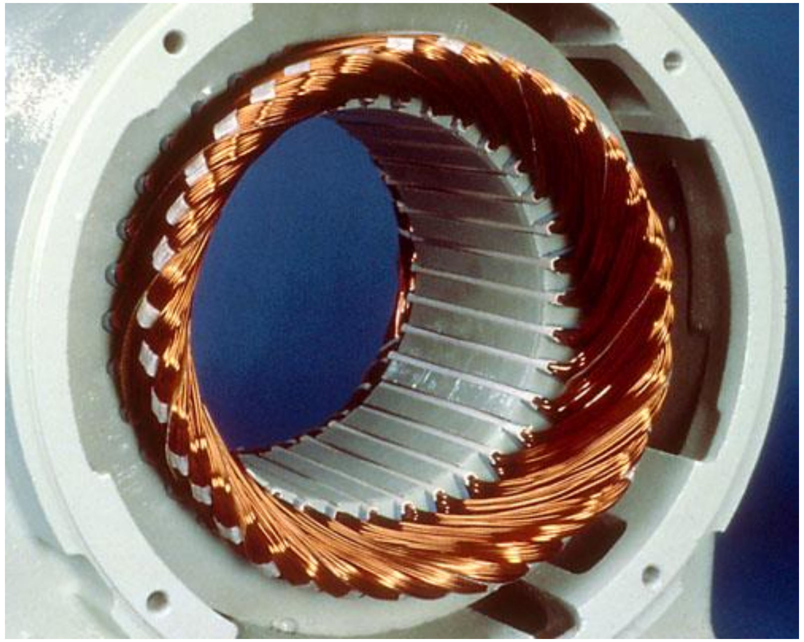
Terminal voltage = Line-to-line voltage ⇐ ***IMPORTANT***

$$e_{AB(RMS)} = e_{BC(RMS)} = e_{CA(RMS)} = 119.96 * \sqrt{3} = 207.77 \text{ Volts}$$

Winding Insulation of AC Machines

- **Winding insulation** is **one of the most important** part of AC machine design.
- If the insulation of an AC motor/generator **breaks down**, the machine **shorts out**.
- The **repair** of the machine with shorted insulation is **very expensive**.
- **Overheating** (*overloading the machine*) is the **cause** of the **winding insulation** from **breaking down .**
- It is necessary to **limit the temperature of the windings**.
- This can be partially done by providing a **cooling air circulation** over them.
- The **maximum winding temperature** limits the **maximum power** that can be supplied continuously by the machine.

Healthy stator →



Electrical winding is necessary
- ✓ To isolate the conductors in each winding
- ✓ To isolate the windings from each other
- ✓ To isolate all windings from stator

Reference: www.easa.com

Winding Insulation of AC Machines

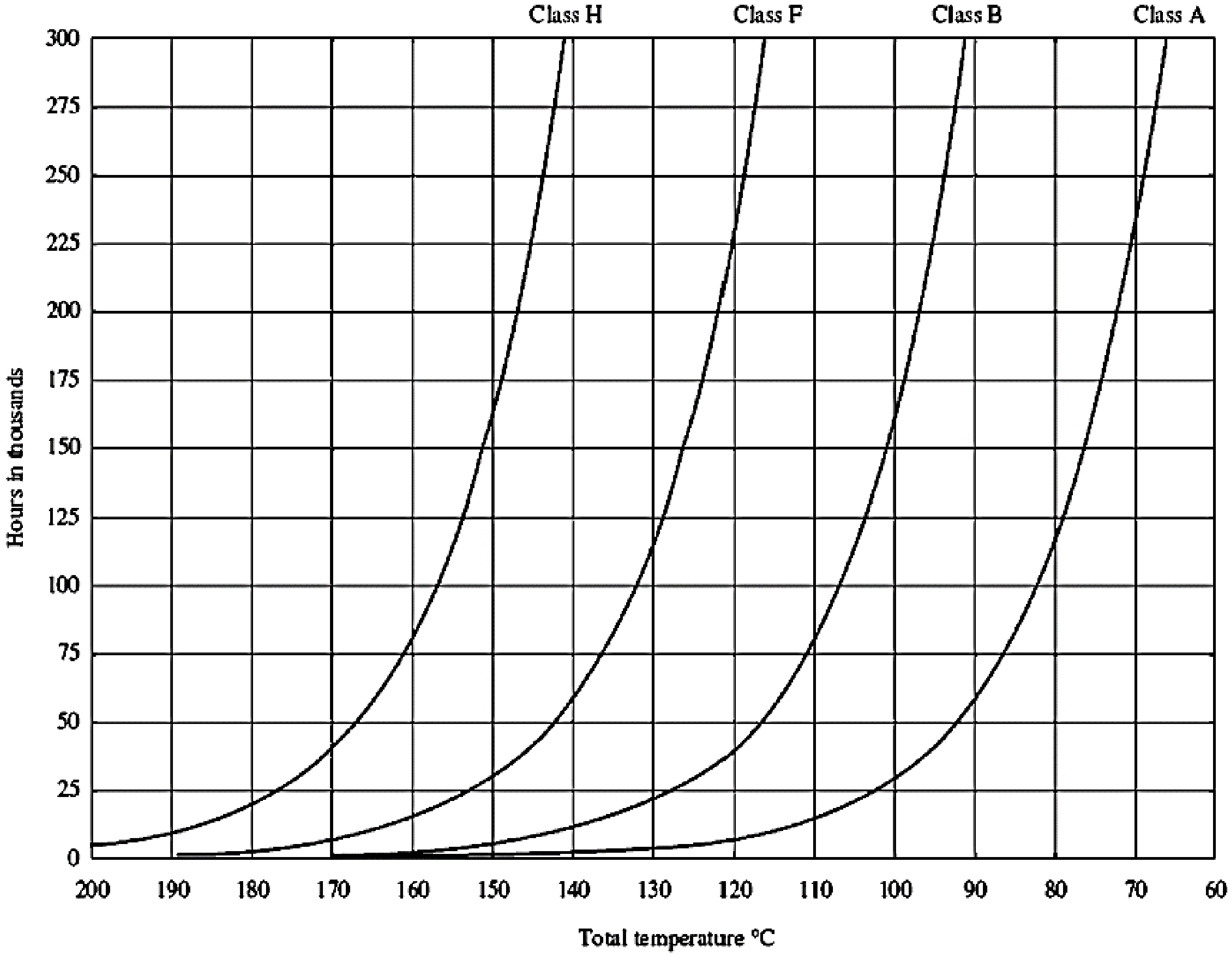
- The increase in temperature produces a **gradual degradation (*slow declination*)** of the insulation.

- **Mechanical shock/vibration**, or **electrical stress (*electric field*)** are the other factors for the insulation degradation.

- The life expectancy of a motor is generally **halved** for each **10 percent rise in temperature** above the rated temperature of the winding.

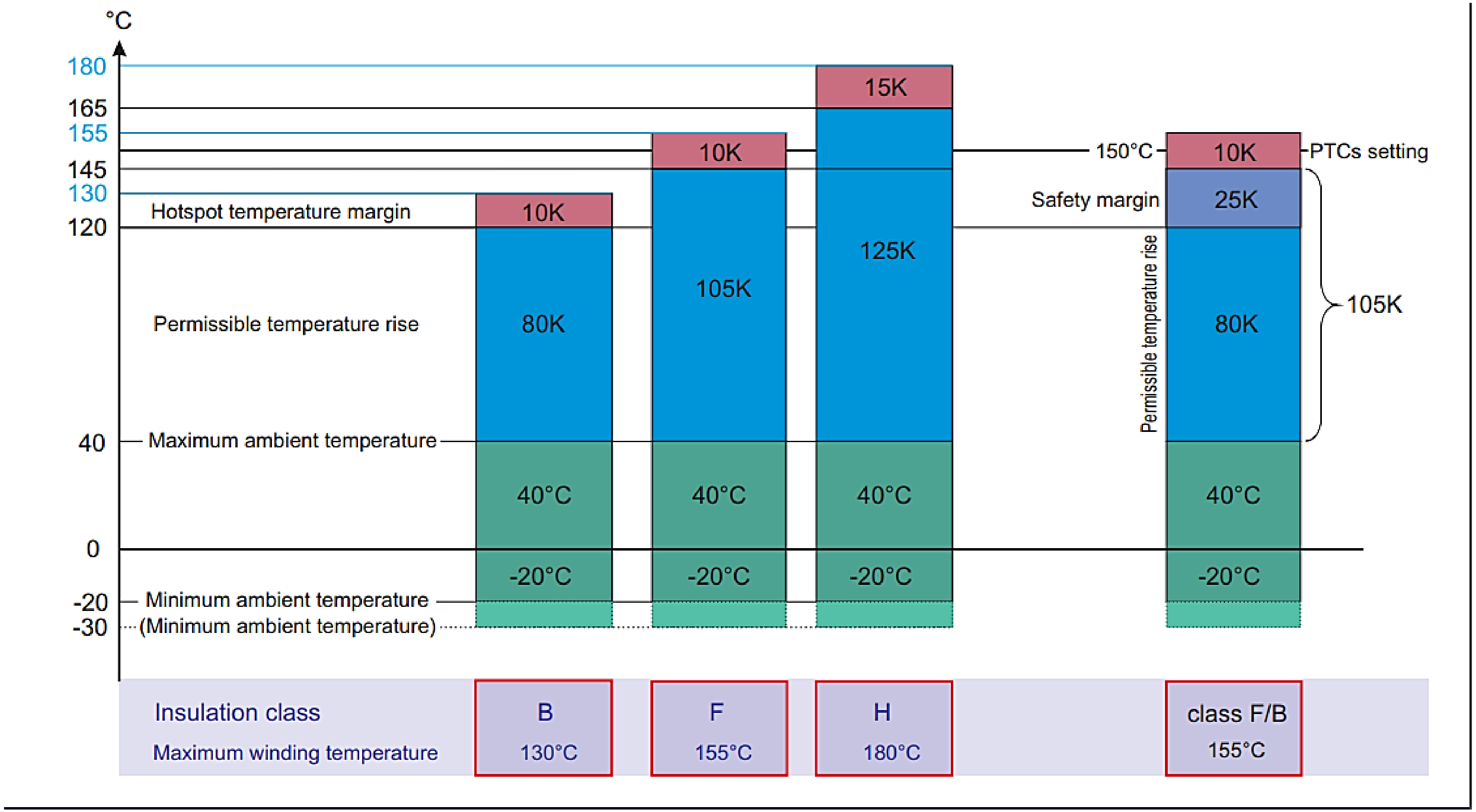
- To standardize the temperature limits of machine insulation, the National Electrical Manufacturers Association (**NEMA**) in the United States has defined a series of insulation system classes: **Class A, B, F, H**

- Similar standards have been defined by the International Electrotechnical Commission (**IEC**) and by various national standards organizations in other countries.



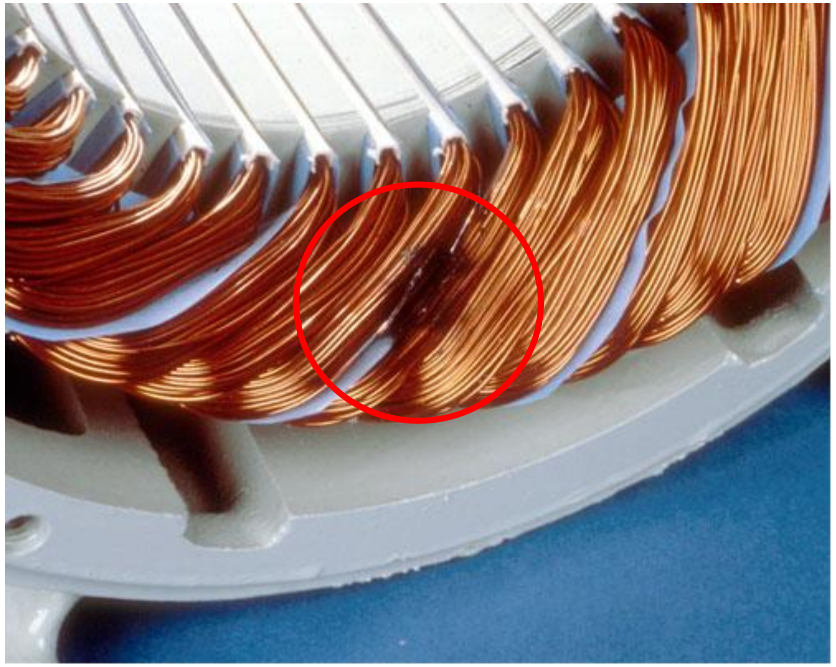
(*NEMA Standard MG 1-1993*)

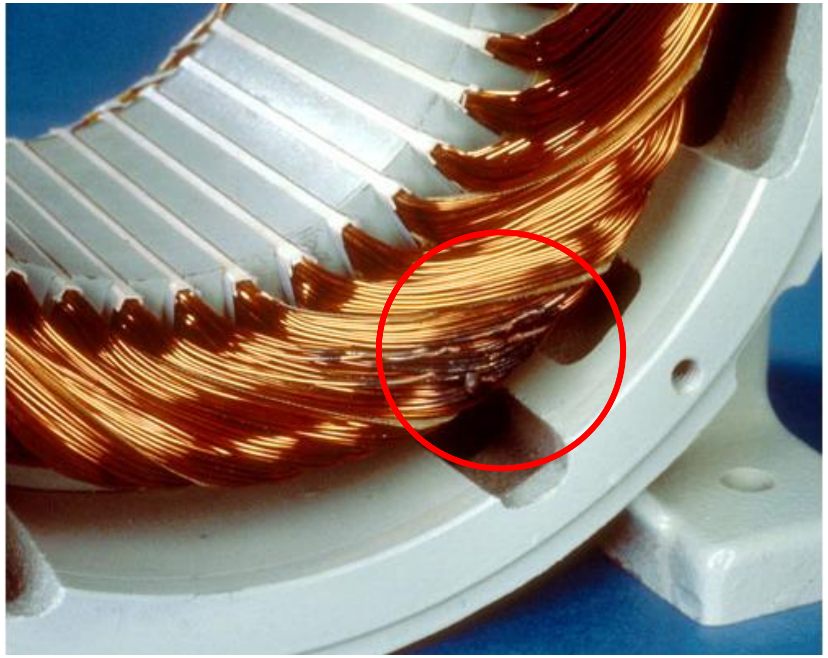
Mean life of a machine in thousands of hours versus the temperature of the windings, for several different insulation classes



IEC 60034-1 Insulation Class

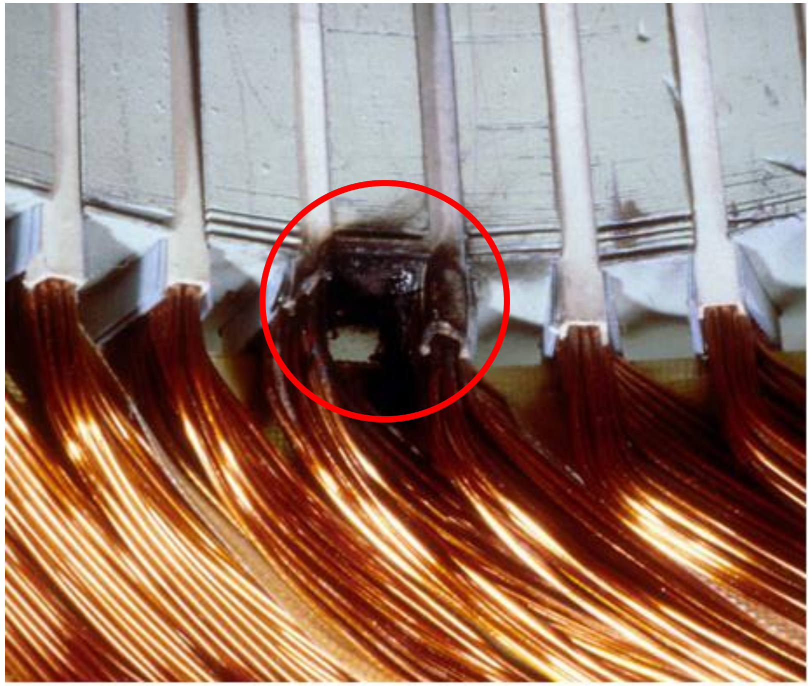
Some examples to winding insulation failure

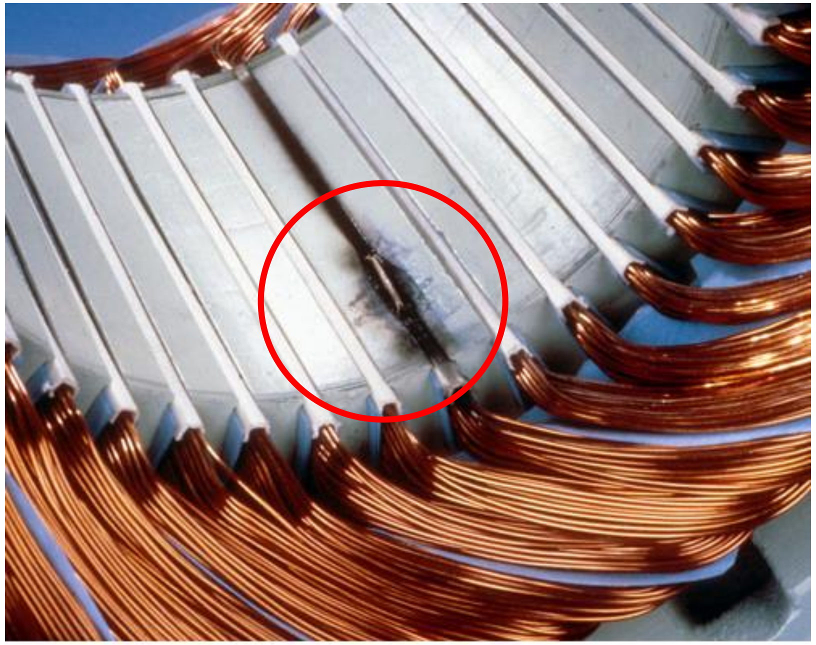




Reference: www.easa.com

Some examples to winding insulation failure





Reference: www.easa.com

Power Flow in AC Machines

- AC generators convert **mechanical power** into **electrical power**.
- AC motors convert **electrical power** into **mechanical power**.
- There is always **some losses** (*electrical/mechanical*) in either AC motors and generators, given by the equation:

$$P_{losses} = P_{in} - P_{out} \quad (P_{in} > P_{out})$$

- The **efficiency** of an AC machine is defined by the following equation:

$$\eta = \frac{P_{out}}{P_{in}} x100\%$$ (Efficiency is usually represented by Greek symbol “**eta**”)

P_{out} is the output power of the AC machine (W, kW, MW)

P_{in} is the output power of the AC machine (W, kW, MW)

$$\left.\begin{array}{l} 0 \leq \eta \leq 100\% \\ 0 \leq \eta \leq 1 \end{array}\right\}$$ (Efficiency can not be **greater than 100%** or **negative**)

Losses in AC Machines

- There are generally **four types of losses** that occur in AC machines:

  - ✓ Electrical or copper losses (I^2R losses)
  - ✓ Core losses
  - ✓ Mechanical losses
  - ✓ Stray losses

They all **decreases efficiency**. Hence machine design stage is very important in order to reduce/minimize these losses

Electrical Losses

- Electrical or copper losses are the **resistive heating losses** that occur in the stator (*armature*) and rotor (*field*) windings of the machine.

- The **stator copper losses** (**SCL**) in a three-phase AC machine are given by the equation:

$$P_{SCL} = 3{I_A}^2 R_A$$

I_A is the **armature current** of each phase of the stator
R_A is the **per-phase armature resistance** of the (stator)

- The **rotor copper losses (RCL)** of a synchronous AC machine are given by the following equation:

$$P_{RCL} = {I_F}^2 R_F$$

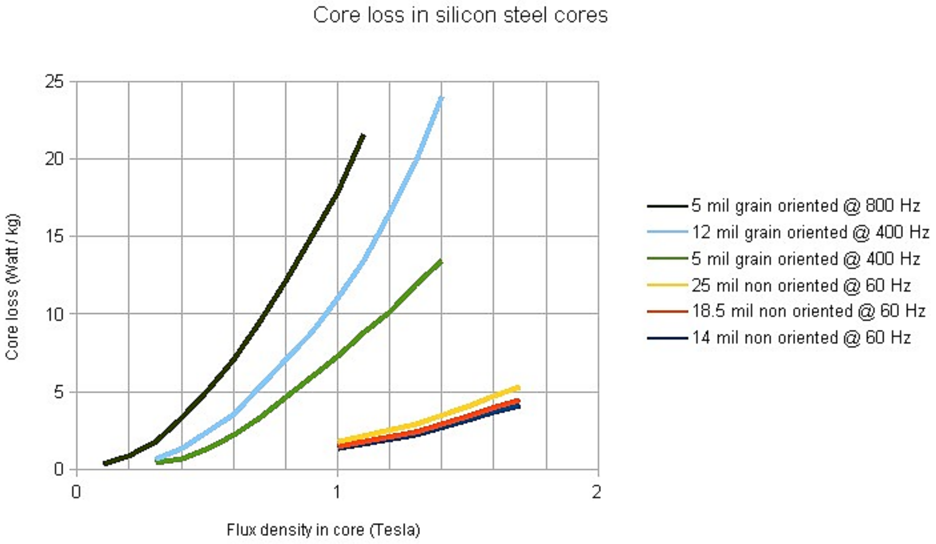
I_F is the **field current** flowing through the **field circuit** on the **rotor** of the AC synchronous machine
R_F is the **resistance of the field winding** on the **rotor**

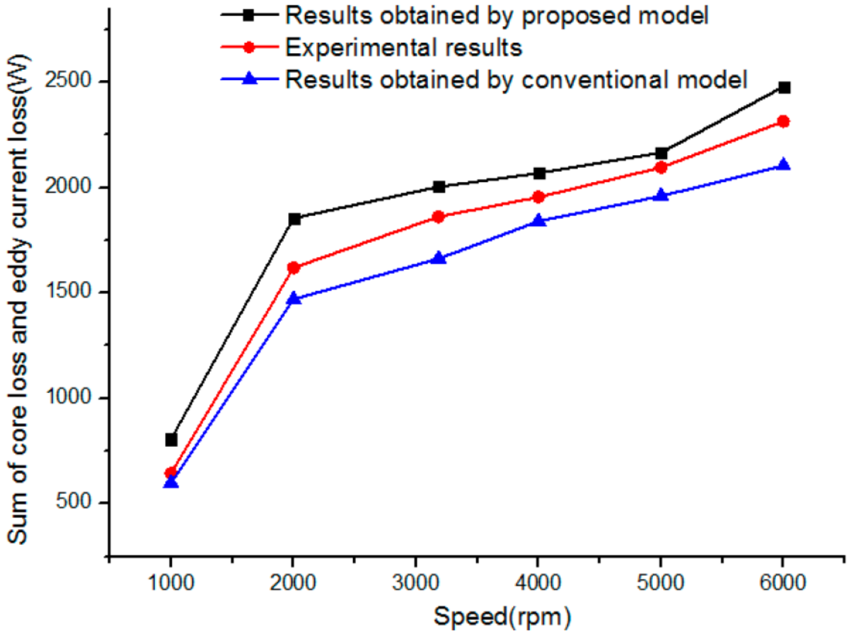
Core Losses

- The core losses are the summation of **hysteresis losses** and **eddy current losses** occurring in the metal structure (both for *rotor* and *stator*) of the AC machine.

- Core losses vary as the **square of the flux density (B^2)**.

- For the stator, core losses are the 1.5 power of the speed of rotating magnetic field (**$n^{1.5}$**).



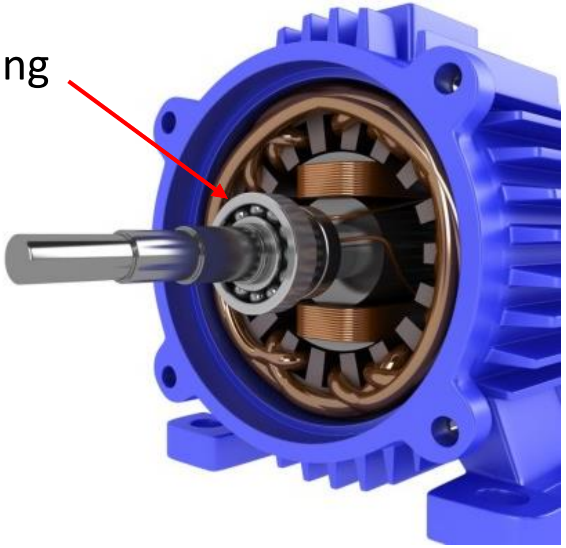


Mechanical and Stray Losses

- The mechanical losses in an AC machine are the losses associated with mechanical rotation.
- There are two basic types of mechanical losses:
  - ✓ Friction
  - ✓ Windage
- **Friction losses** are caused by the **friction of the bearings** in the AC machine.
- **Windage losses** are caused by the **friction** between the **moving parts of the machine** and the **air inside** the motor's casing. Windage losses vary as the **cube of the speed of rotation of the machine**.

Bearings

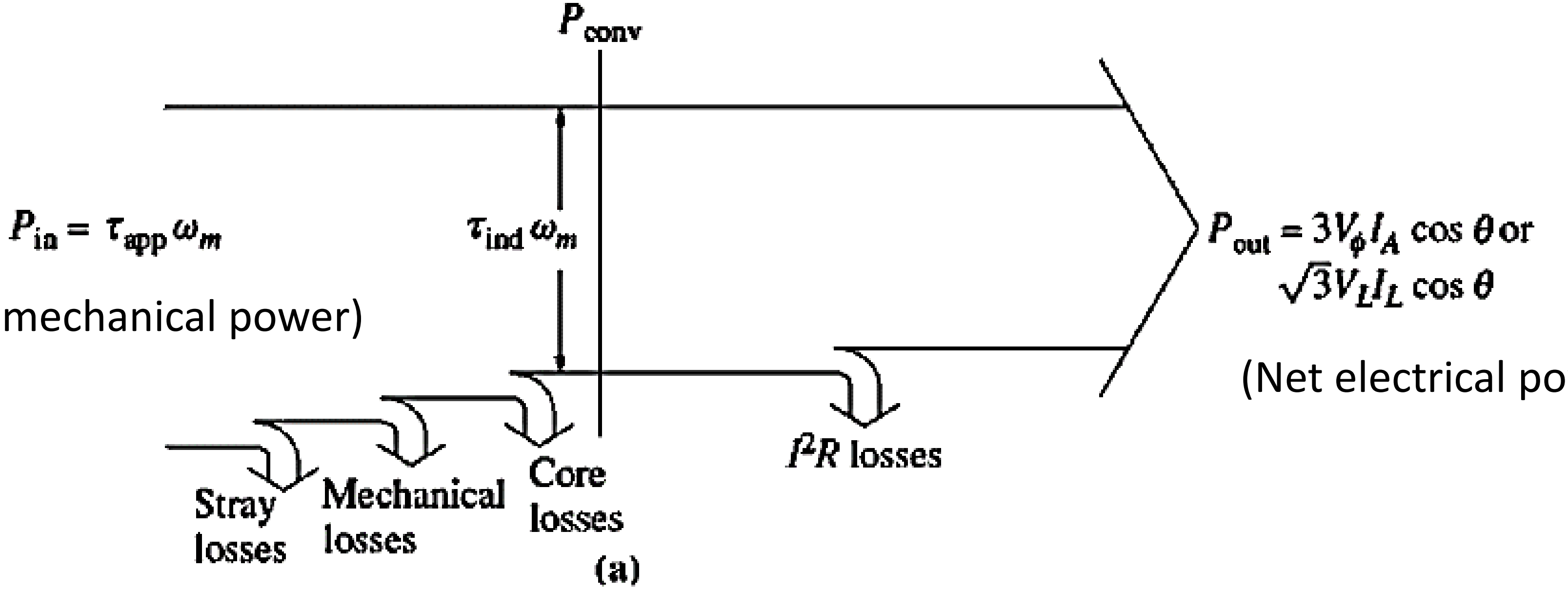
Motor bearing



Stray losses (*miscellaneous losses*) are the losses that cannot be placed in one of the previous categories.

For most AC machines, **stray losses** are taken by convention to be **1 percent of full load**.

Power-Flow Diagram of Three-Phase AC Generator



(Converted electrical power)

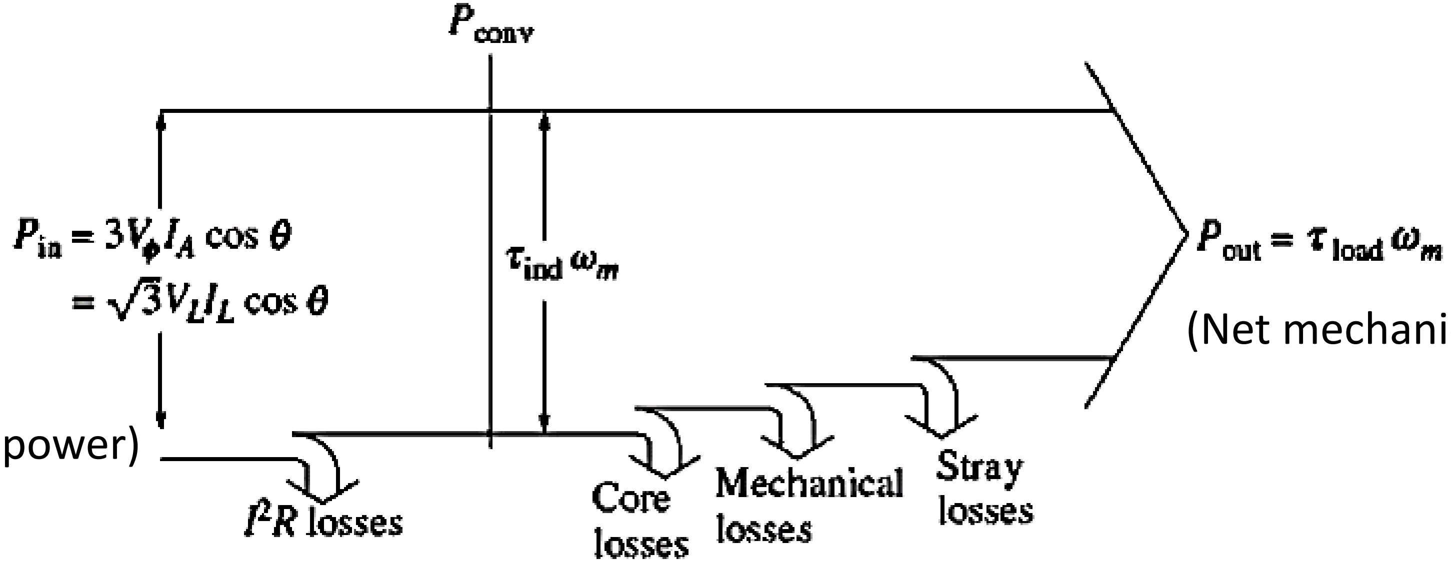
(input mechanical power)

(Net electrical power output)

$$P_{conv} = \tau_{ind}.w_m$$

Power-Flow Diagram of Three-Phase AC Motor

$$P_{conv} = \tau_{ind}.w_m$$



(Converted mechanical power)

(input
electrical power)

(Net mechanical power output)

Voltage Regulation of the Generator

- **Voltage regulation** (**VR**) is a measure of the ability of a generator (AC/DC) to keep a constant voltage at its terminals as the load varies.

- **Voltage regulation** is a performance indicator that is used to compare the generators.

- **Voltage regulation** is calculated by the following equation:

$$VR = \frac{V_{NL} - V_{FL}}{V_{FL}} x100\%$$

V_{NL} is the **No-Load** voltage (*LL or LN*) of the generator
V_{FL} is the **Full-Load** (*rated-load*) voltage (*LL or LN*) of the generator

- A **small voltage regulation** is "***better***" in the sense that the voltage at the terminals of the generator is **more constant** with **variations in load**.

Speed Regulation of the Motor

- **Speed regulation (SR)** is a measure of the ability of a motor to keep a constant shaft speed as load varies.

- **Speed regulation** is a performance indicator that is used to compare the motors.

- **Speed regulation** is calculated by the following equation:

$$SR = \frac{n_{NL} - n_{FL}}{n_{FL}} x100\%$$

n_{NL} is the **No-Load** speed of the motor
n_{FL} is the **Full-Load** (*rated-load*) speed of the motor

- **SR** is a rough measure of the shape of a **torque-speed characteristics** of the motor.
- A positive **SR** means that a motor's speed drops with increasing load. (*common behavior*)
- A negative **SR** means that a motor 's speed increases with increasing load. (not *common behavior*)

END OF CHAPTER 1

AC MACHINERY FUNDAMENTALS