EEE 322

Electromechanical Energy Conversion – II

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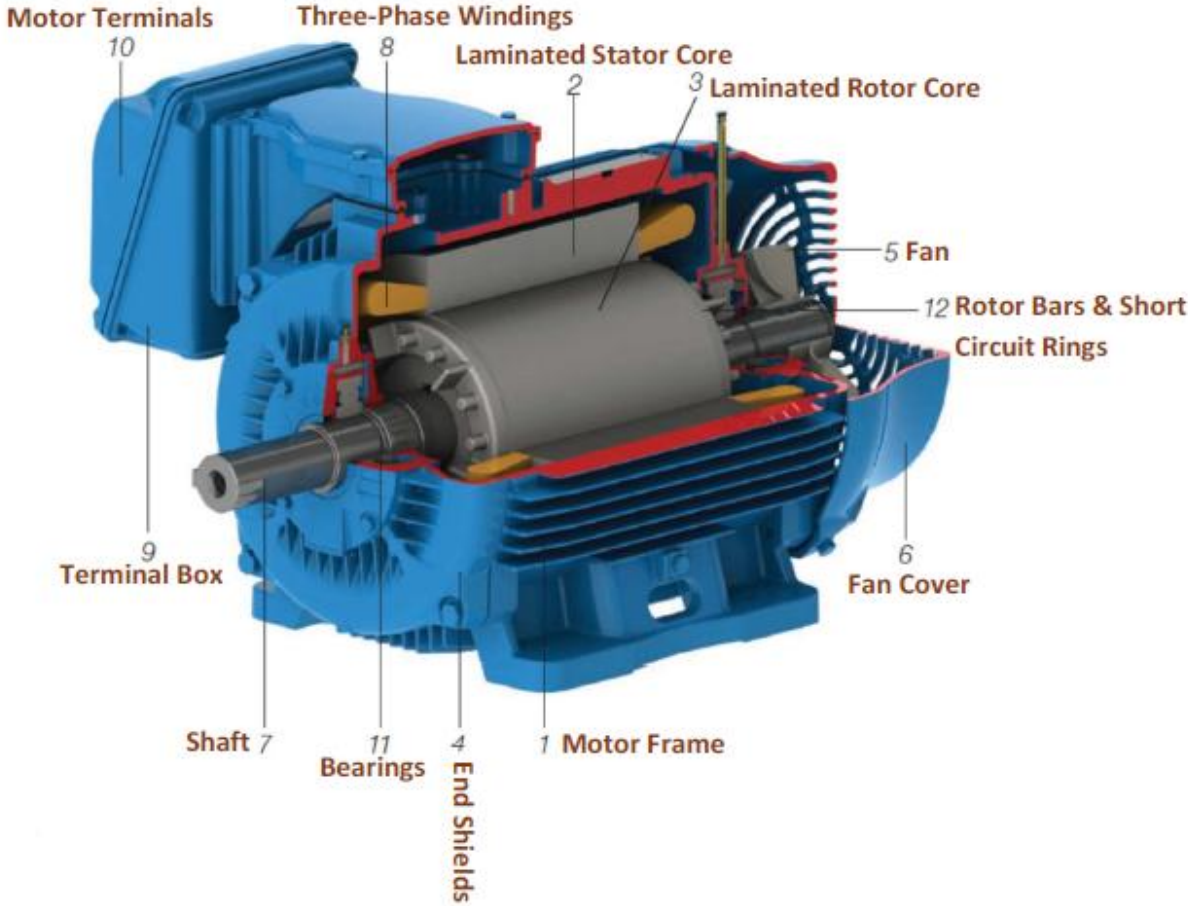
CHAPTER 4

INDUCTION MOTORS

Induction motors



Source: https://www.electricalengineeringtoolbox.com

Induction motors

- So far we have seen that **damping** (***amortisseur***) windings on a **synchronous motor** could develop a **starting torque** **without** the necessity of supplying an **external DC field current**.
- So we can define "**induction machine**" as a machine **with only damping (*amortisseur*) windings** without supplying an external DC field current field.
- In induction machines, **rotor voltage** (which produces the rotor current and the rotor magnetic field B_R) **is induced in the rotor windings**.
- So, the **difference** between an ***induction machine*** and ***synchronous machine*** is that no DC field current is required to run the induction machine.
- The **induction machine** can be either used as a ***motor*** or a ***generator***.
- **Induction machines** are also called "**asynchronous machines**".

Different size induction motors

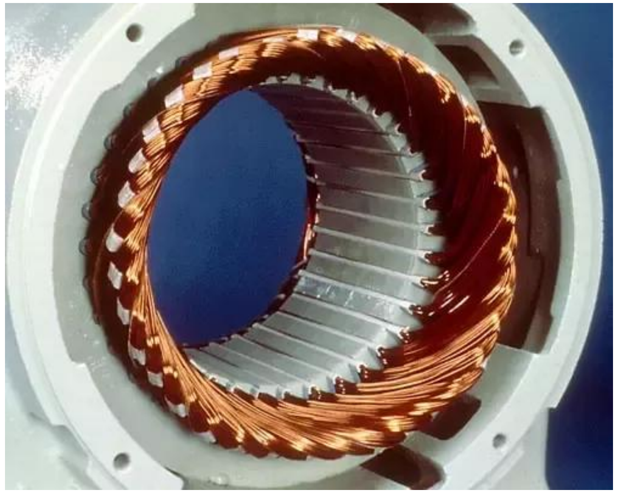
0.5 Hp induction motor

2 Hp, 400V induction motor

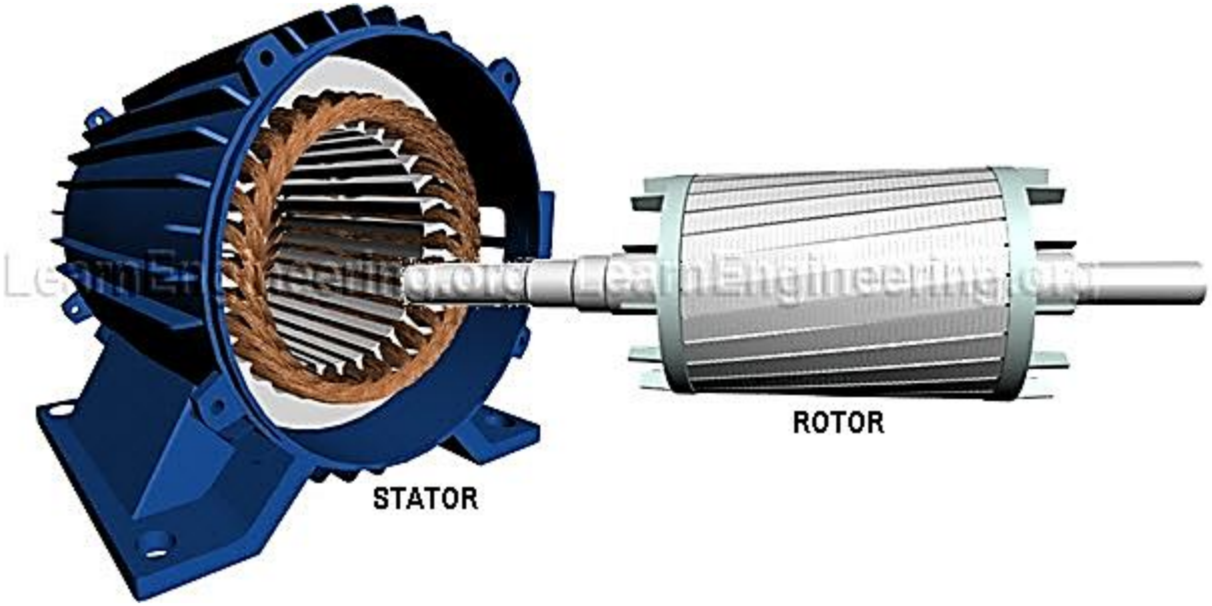
300 Hp, 460V induction motor

Induction motors

- An **induction motor** has the **same physical stator** as a **synchronous machine**.
- But, **induction motor** has a **different rotor structure** than a **synchronous machine**.

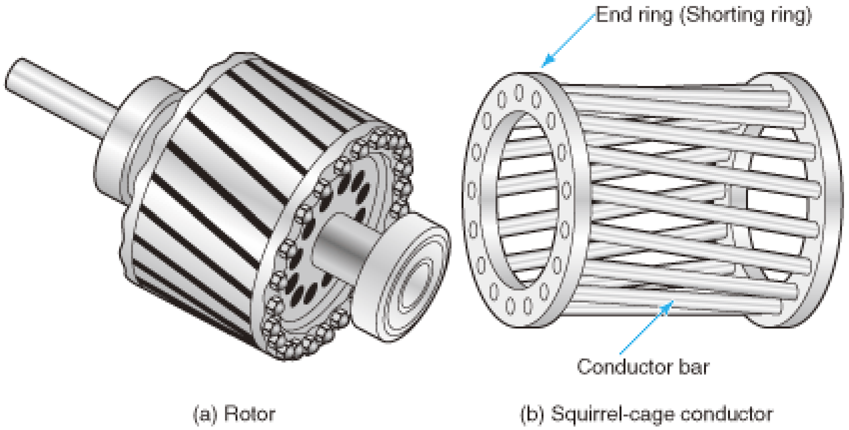


The stator of an induction motor

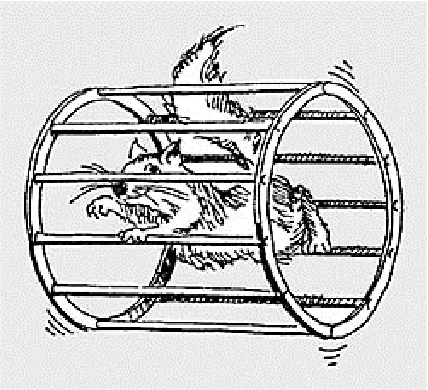


Rotor types of induction motors

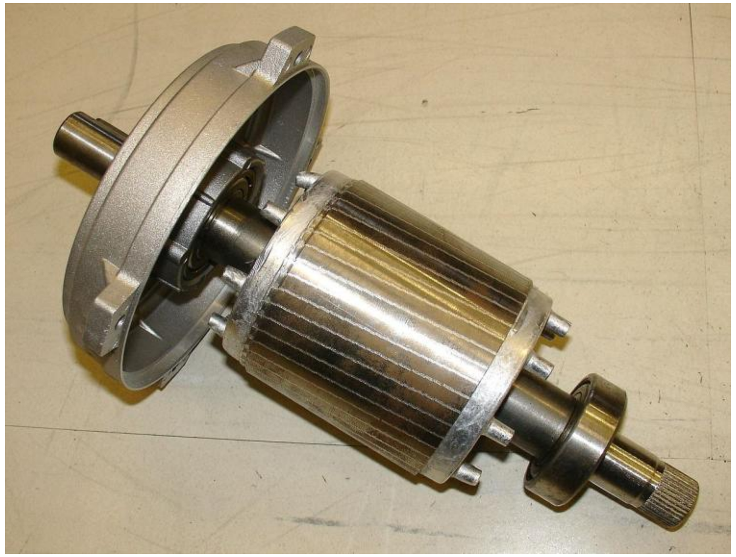
- There are **two different types of rotors** which can be placed inside the stator of an induction motor:
  - Squirrel cage rotor
  - Wound rotor



Schematic diagram of squirrel cage rotor



A squirrel running inside of a rotating cage



A real photo of a squirrel cage rotor

Rotor types of induction motors

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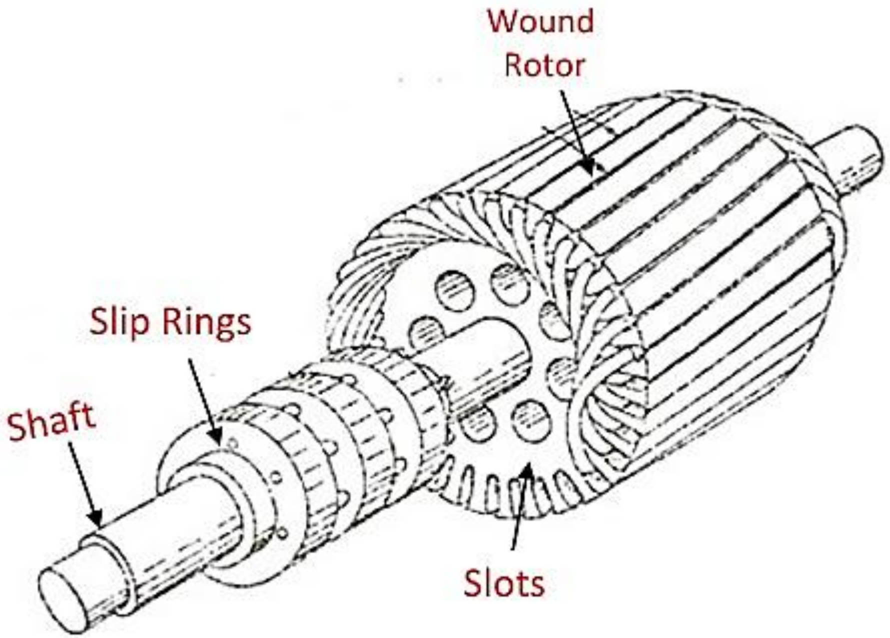
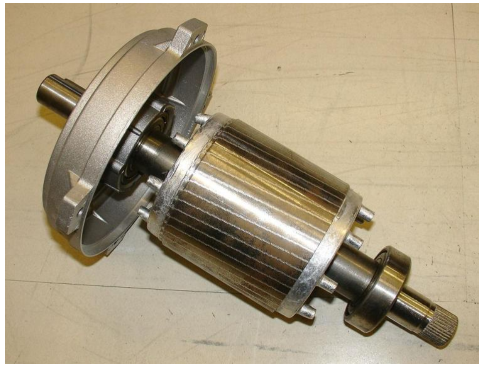


Schematic diagram of wound rotor

A real photo of a wound rotor

Rotor types of induction motors



Squirrel Cage

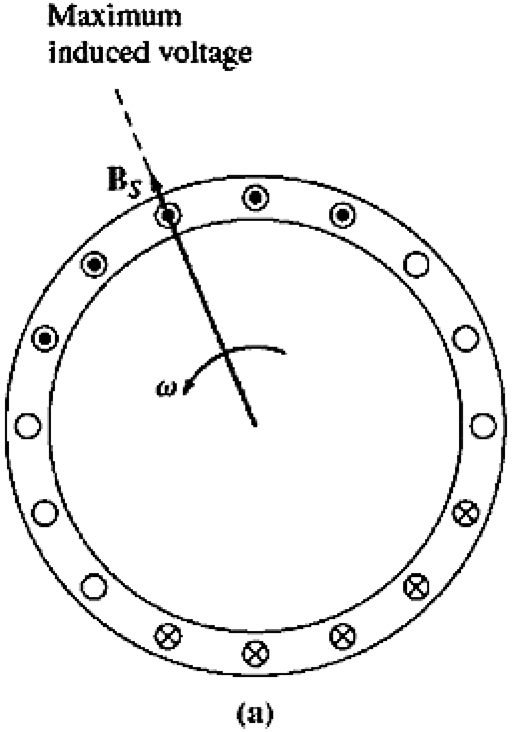
- Cheap
- Requires less maintenance than wound rotor
- Used only in induction motors
- Can not be used in induction generators

Wound

- Expensive
- Usually Y-connected
- Extra resistance can be added to rotor circuit to modify torque-speed characteristics
- requires much more maintenance than squirrel cage rotor
- Can be used in wind turbines (**D**oubly **F**ed **I**nduction **G**enerators)

Development of induced torque

- A three-phase set of voltages has been applied to the stator.
- Then a three-phase set of stator currents is flowing in stator windings.
- These currents produce a magnetic field B_S, which is rotating in counterclockwise direction as shown in the figure.
- The rotating speed of B_S is given by:



$$n_{syn} = \frac{120 f_e}{P}$$

where

- f_e is the frequency of the applied voltage to the stator windings (*Source frequency, 50 or 60 Hz*)
- P is the number of poles of the machine

Development of induced torque

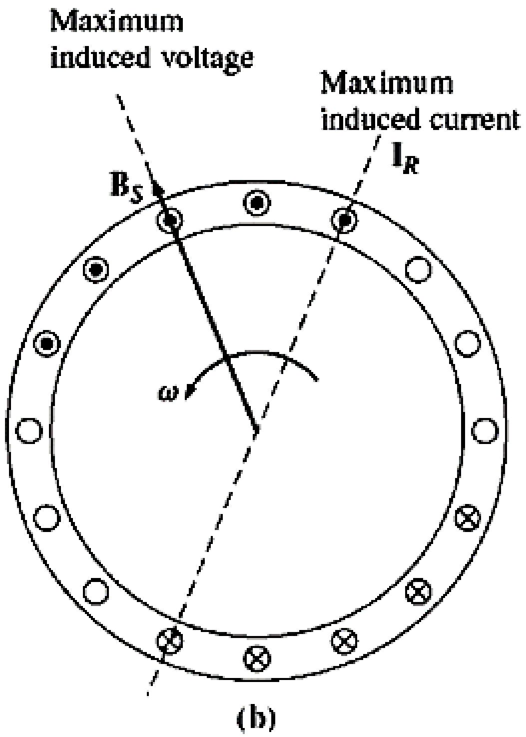
- This rotating magnetic field B_S passes over the rotor bars (*assuming squirrel cage rotor*) and induces a voltage on rotor bars.
- The induced voltage on a rotor bar is given by the equation:

$$e_{ind} = (v \ x \ B_S).l$$

where

- v is the velocity of the bar relative to the magnetic field
- l is length of conductor in the magnetic field

- Since the rotor is mostly **inductive ($X \gg R$)**, the peak rotor current I_R **lags** behind the peak rotor voltage as shown in the figure.

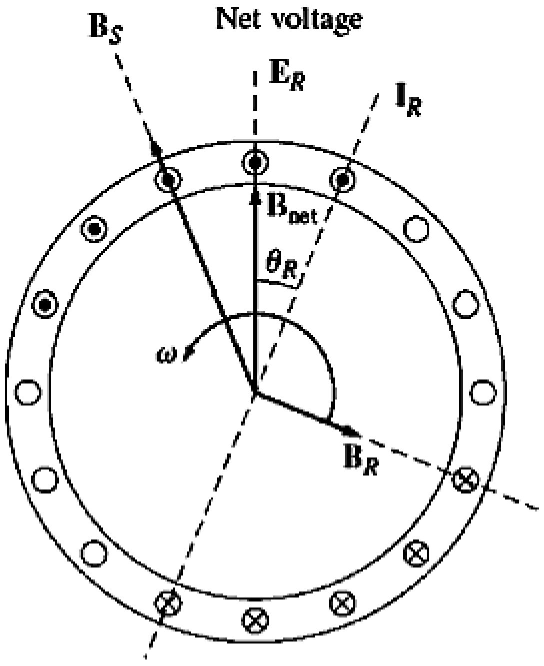


Development of induced torque

- The rotor current I_R flowing through the rotor bars produces a rotor magnetic field B_R.
- Rotor magnetic field B_R lags 90° behind itself as shown in the figure.
- B_R and B_S *interacts* to produce a counterclockwise torque induced in the machine, given by the following equation:

$$\tau_{ind} = kB_R x B_S$$

- Since the induced torque is in the counterclockwise direction, the rotor starts to move in that direction.



(c)

Upper limit of rotor’s speed

- There is a **finite upper limit** to the motor's speed. This can be explained as follows:

  - If the rotor were turning at synchronous speed n_{syn}, then the rotor bars would be stationary relative to the rotating magnetic field B_S and hence $v = 0$
  - Then there would be no induced voltage, $e_{ind} = 0$
  - If $e_{ind} = 0$ then there would be no rotor current and no rotor magnetic field, $B_R = 0$.
  - With no rotor magnetic field, the induced torque $\tau_{ind} = 0$ would be zero.
  - Since there is no induced torque on the rotor, the rotor starts to slow down as a result of friction losses.
  - Thereby, an induction motor’s speed is very close to the synchronous speed, but the speed of the rotor can never exactly reach synchronous speed.

Rotor slip

- So far, we have understood that there is always a **speed difference** between **rotating stator magnetic field** and the **rotor** itself.
- The **slip speed** is defined as this speed difference, and given by the following equation:

$$n_{slip} = n_{syn} - n_m$$

where

n_{slip} is the slip speed of the induction machine
n_{sync} is the speed of the stator rotating magnetic field B_S
n_m is the mechanical shaft (rotor) speed of the induction machine

$$w_m = \frac{2\pi n_m}{60}$$

- **Slip** is defined as follows:

$$s = \frac{n_{slip}}{n_{syn}} x100\% = \frac{n_{syn} - n_m}{n_{syn}} x100\%$$

$$s = \frac{w_{slip}}{w_{syn}} x100\% = \frac{w_{syn} - w_m}{w_{syn}} x100\%$$

- **0 ≤ s ≤ 100%**
- if the rotor turns at synchronous speed, **s = 0**
- If the rotor is stationary (not rotating), **s = 1**

Rotor slip

- Since;

$$s = \frac{n_{syn} - n_m}{n_{syn}} x100\%$$

- Solving the above equation for n_m yields;

$n_m = (1 - s)n_{syn}$ → The unit is **rev/min** or **rpm**

- Or;

$w_m = (1 - s)w_{syn}$ → The unit is **rad/sec**

Electrical frequency on the rotor

- An induction motor works by inducing voltages and currents in the rotor of the machine, and for that reason it has sometimes been called a "***rotating transformer***".

- Like a transformer, the **primary** (***stator***) induces a voltage in the **secondary** (***rotor***).

- But unlike a transformer, the **secondary frequency** is **not necessarily the same** as the **primary frequency**.

- If the **rotor** is **locked (*blocked*)** so that it cannot move, then the rotor will have **the same frequency as the stator**.

- If the **rotor** turns at **synchronous speed**, **the frequency on the rotor will be zero**.

- What will the rotor frequency be for any arbitrary rate of rotor rotation?

  - If $n_m = 0$ ➔ **s = 1** ➔ frequency of the voltage/current of the rotor (f_r) will be **equal** to the frequency of the three-phase applied voltage of the stator (f_e), $f_r = f_e$
  - If $n_m = n_{syn}$ ➔ **s = 0** ➔ $f_r = 0$ (*since no induced voltage*)
  - By combining the above two conditions we can derive an equation as follows: $f_r = sf_e$

Electrical frequency on the rotor

- Since;

$f_r = sf_e$ and $s = \frac{n_{syn} - n_m}{n_{syn}}$

$$f_r = \frac{n_{syn} - n_m}{n_{syn}} f_e$$

- Moreover since;

$$n_{syn} = \frac{120 f_e}{P}$$

$$f_r = \left(n_{syn} - n_m\right) \frac{P}{120 f_e} f_e$$

$$f_r = \frac{P}{120} \left(n_{syn} - n_m\right)$$

Example:

A 208-V, 10-hp, four-pole, 60-Hz, Y-connected induction motor has a full-load slip of 5 percent. Answer the following questions:

(a) What is the synchronous speed of this motor?
(b) What is the rotor speed of this motor at the rated load?
(c) What is the rotor frequency of this motor at the rated load?
(d) What is the shaft torque of this motor at the rated load?

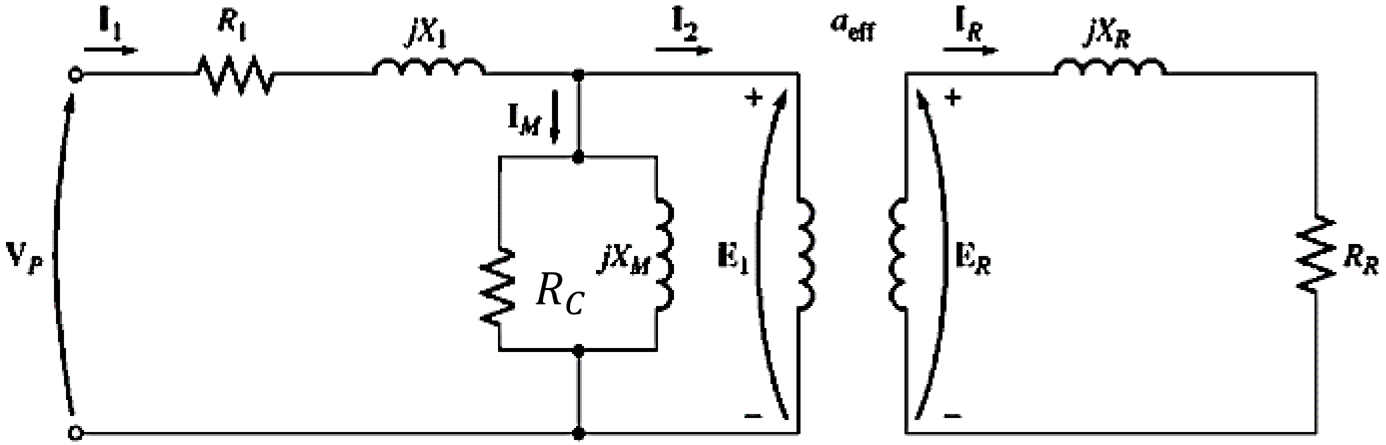
Equivalent circuit of an induction motor

- Since the voltages and currents in the rotor circuit of an induction motor is essentially generated like a transformer operation, the equivalent circuit of an induction motor can be accepted to be **very similar** to the equivalent circuit of a **transformer**.

- An induction motor is called a **singly excited machine** (*as opposed to a doubly excited synchronous machine*), since power is supplied to **only the stator circuit**.

- Because an induction motor does not have **an independent field circuit**, its model will not contain an internal voltage source such as the internal generated voltage E_A in a synchronous machine .

Transformer model of an induction motor



V_P is the **per-phase** stator voltage (Amper-*rms*)

I_1 is the **phase current** of the stator (Amper-*rms*)

R_1 is the **per-phase** stator **resistance** (*ohm*)

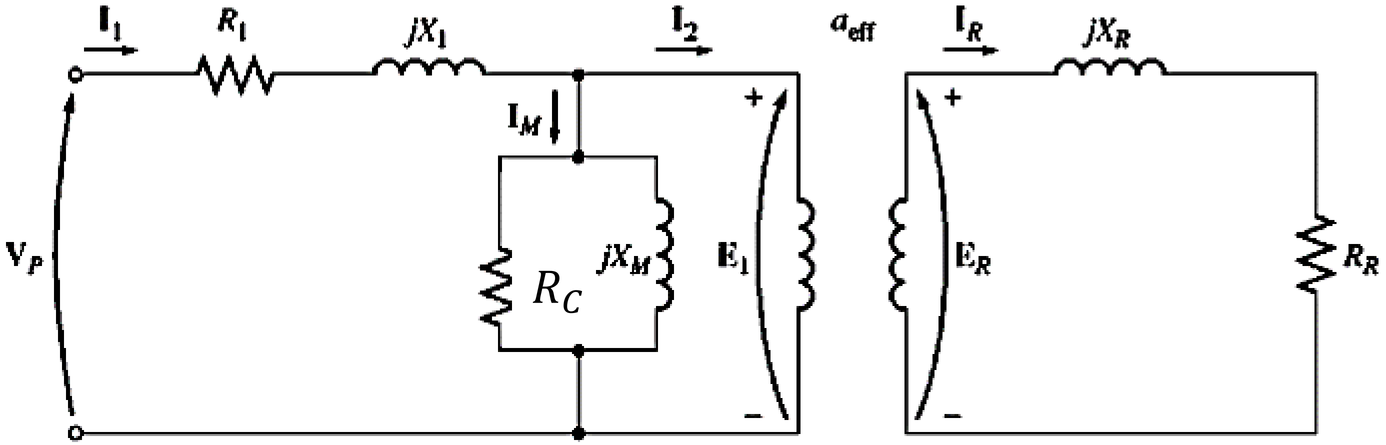
X_1 is the **per-phase** stator **leakage reactance** (*ohm*)

R_C is the **resistance** used to model **core losses** of the induction motor (*hysteresis + eddy current losses*) (*ohm*)

X_M is the **magnetizing reactance** of the stator (*ohm*)

I_M is the **magnetizing current** of the stator (*ohm*)

Transformer model of an induction motor



I_R is the equivalent rotor **current** (Amper-*rms*)

R_R is the **per-phase** rotor equivalent **resistance** (*ohm*)

X_R is the **per-phase** rotor **leakage reactance** (*ohm*)

E_R is the **induced per-phase equivalent rotor voltage** (Volt-*rms*)

a_{eff} is the effective turns ratio between stator and rotor

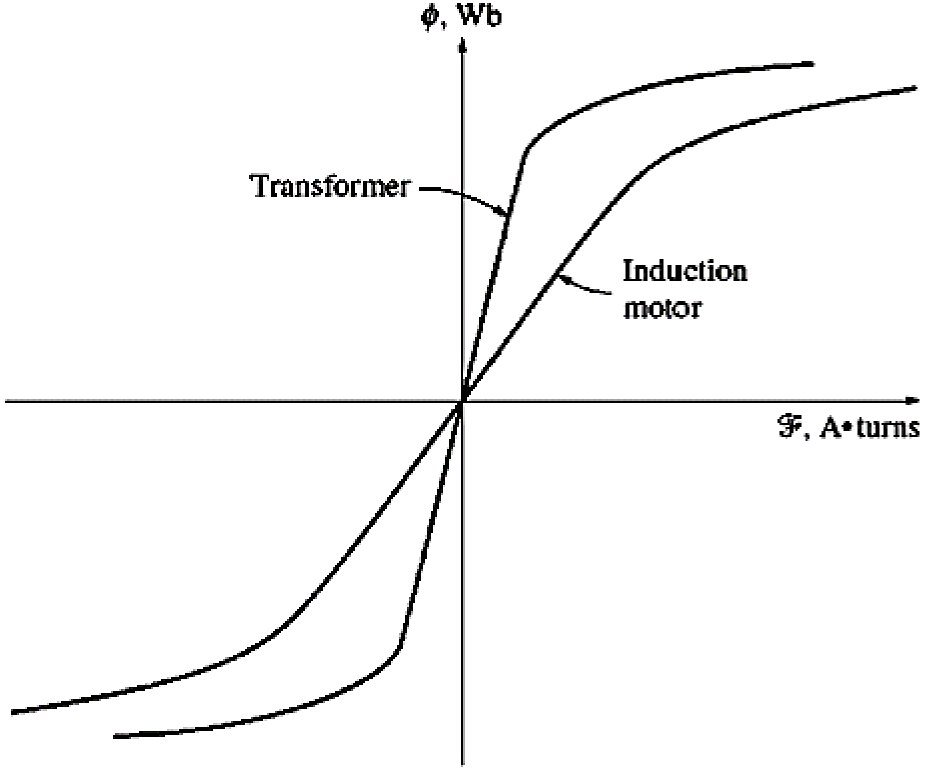
Comparison of the magnetization curve of an induction motor with a transformer

- The difference shown in figure is due to the **air gap** between rotor and stator in the induction motor.

- Because, **air gap** greatly **increases** the **reluctance** of the flux path and therefore reduces the coupling between primary and secondary windings.

- The **higher reluctance** means that a **higher magnetizing current** is required to obtain a given flux level.

- Therefore, the magnetizing reactance X_M in the equivalent circuit is much smaller than that of an ordinary transformer.



The magnetization curve of an induction motor compared to that of a transformer

Rotor circuit model

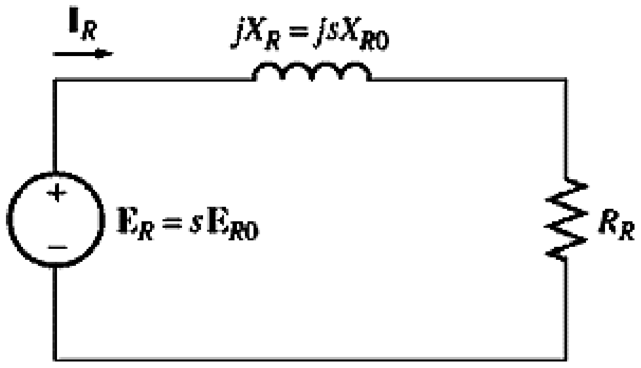
- When a voltage is applied to the stator windings, a voltage is induced in the rotor windings of an induction motor.

- The greater the relative motion between the rotor and the stator magnetic fields, the greater the resulting rotor voltage and rotor frequency.

- The **largest relative motion** occurs when the rotor is **stationary** **(s=1)**, called the "**locked-rotor**" or "**blocked-rotor**" condition, so the **largest voltage** and **rotor frequency** are induced in the rotor at that condition.

- The **smallest voltage** (0V) and **frequency** (0Hz) occur when the rotor moves at the **same speed** as the stator magnetic field, resulting in no relative motion **(s=0)**.

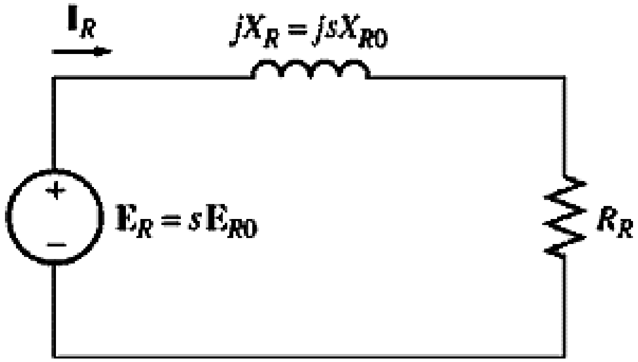
- The **magnitude** and **frequency** of the **voltage** induced in the **rotor** at any speed **($0 \le s \le 1$)** is **directly proportional** to the **slip of the rotor, s**:



The rotor circuit model of an induction motor

Rotor circuit model

- Therefore, **induced per-phase equivalent rotor voltage** (E_R) can be written as follows:



The rotor circuit model of an induction motor

$$E_R = s.E_{R0}$$

where

E_{R0} is the maximum possible value of rotor voltage obtained at **s=1** (*rotor is stationary*)

- So;

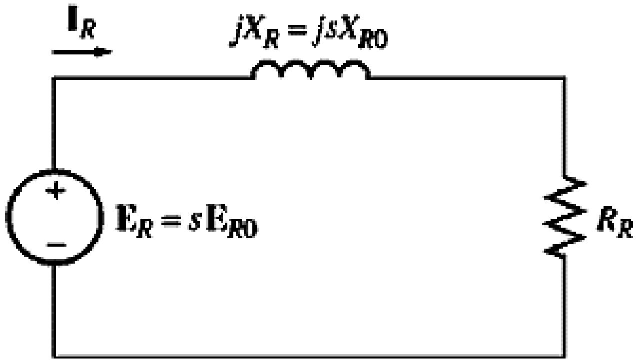
    - At s=1 ➔ $E_R = E_{R0}$
    - At s=0 ➔ $E_R = 0$

Rotor circuit model

- Similary, **the frequency of the induced voltage/current in the rotor** (f_R) can be written as follows:

$$f_R = s . f_e$$

- So;
  - At s=1 ➔ $f_R = f_e$
  - At s=0 ➔ $f_R = 0$



The rotor circuit model of an induction motor

- The **rotor reactance** depends on the **inductance** of the rotor and the **frequency** of the voltage/current in the rotor.
- The **rotor reactance** can be written as follows:

$$X_R = w_r L_R = 2\pi f_r L_R \implies X_R = 2\pi s f_e L_R = s(\underbrace{2\pi f_e L_R}_{X_{R0}}) = sX_{R0}$$

where

- L_R is the inductance of the rotor
- X_{R0} is the locked rotor reactance

Rotor circuit model

- The rotor current can be found as

$$\mathbf{I}_R = \frac{\mathbf{E}_R}{R_R + jX_R}$$

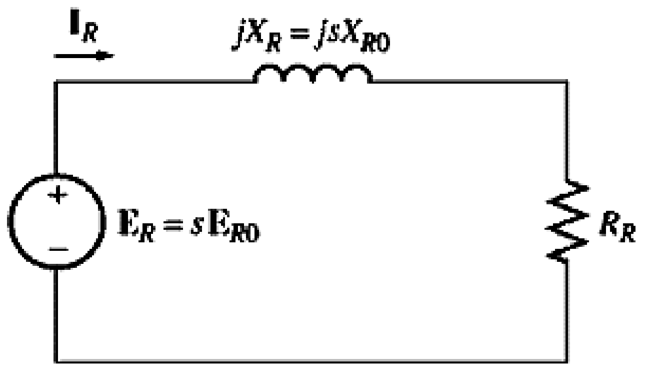
$$\boxed{\mathbf{I}_R = \frac{\mathbf{E}_R}{R_R + jsX_{R0}}}$$

$$\boxed{\mathbf{I}_R = \frac{\mathbf{E}_{R0}}{R_R/s + jX_{R0}}}$$

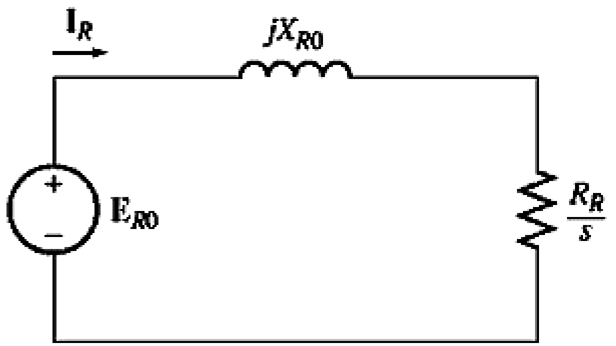
- Now the equivalent rotor impedance from this point of view can be written as:

$$Z_{R,\mathrm{eq}} = R_R/s + jX_{R0}$$

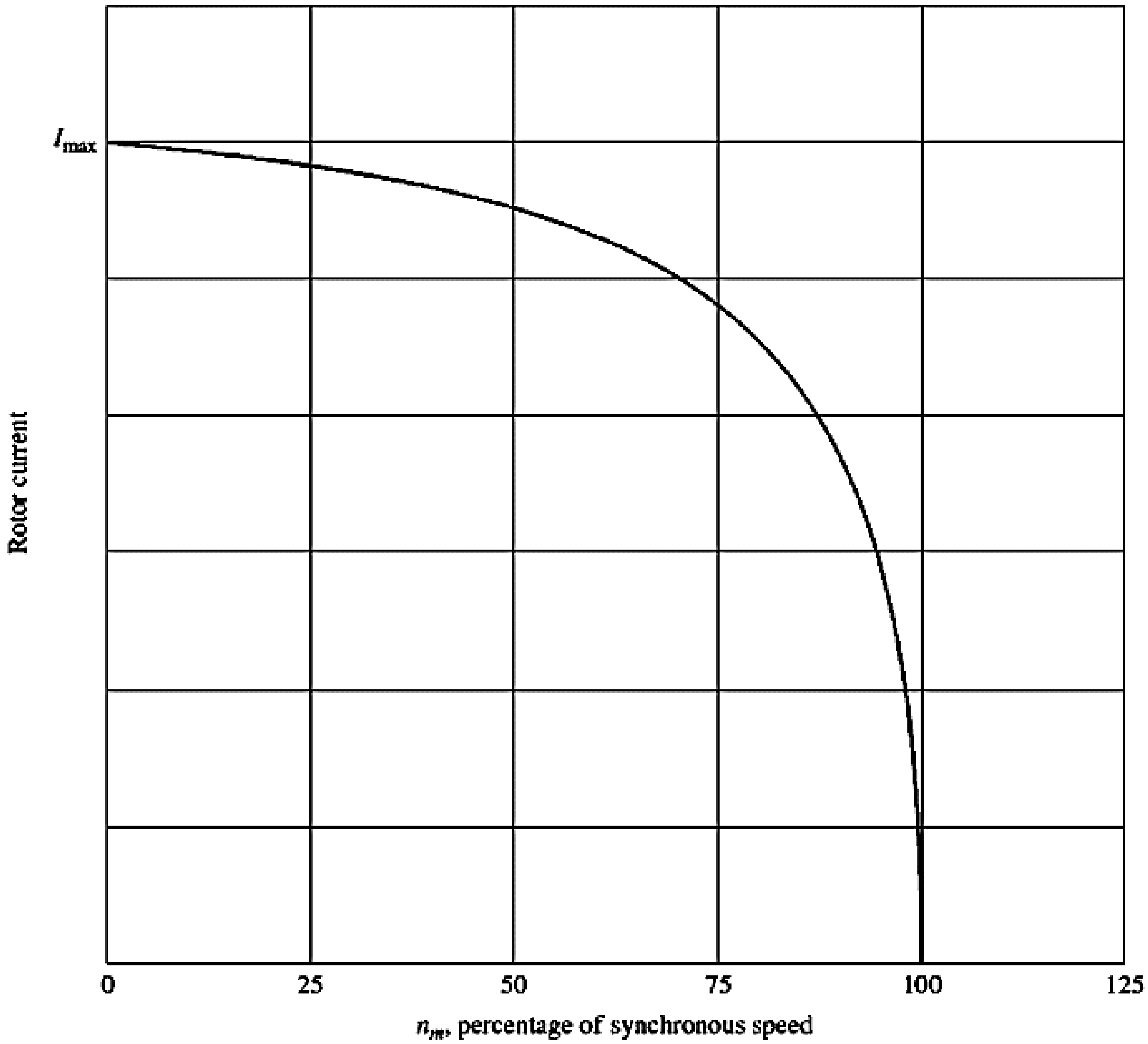
- The final modified equivalent circuit of the rotor can be redrawn as shown in the figure



The rotor circuit model of an induction motor



The rotor circuit model with all the frequency (slip) effects concentrated in the resistor



Rotor current as a function of rotor speed

Final equivalent circuit of induction motor

- To produce the **final per-phase equivalent circuit** for an **induction motor**, **rotor circuit is referred to stator side**.
- In a **transformer**, the *voltages*, *currents*, and *impedances* on the **secondary side** can be **referred to the primary side** by means of the **turns ratio of the transformer**.
- These referring actions are given by the following equations:

$$\mathbf{V}_P = \mathbf{V}'_S = a\mathbf{V}_S$$

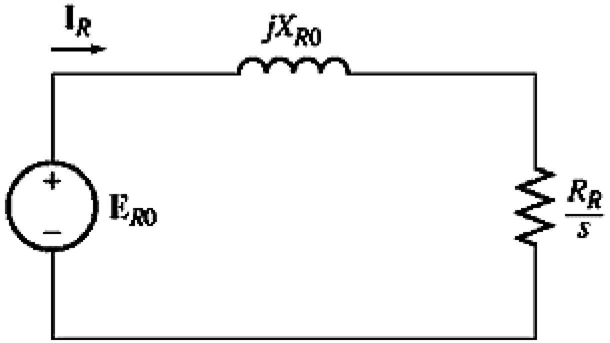
$$\mathbf{I}_P = \mathbf{I}'_S = \frac{\mathbf{I}_S}{a}$$

$$Z'_S = a^2 Z_S$$

- The **same sort of transformation** can also be done for the **induction motor's rotor circuit**:

Final equivalent circuit of induction motor

- The **same sort of transformation** can also be done for the **induction motor's rotor circuit**:



$$\mathbf{E}_1 = \mathbf{E}'_R = a_{\text{eff}}\mathbf{E}_{R0}$$ (The **rotor voltage** referred to **stator**)

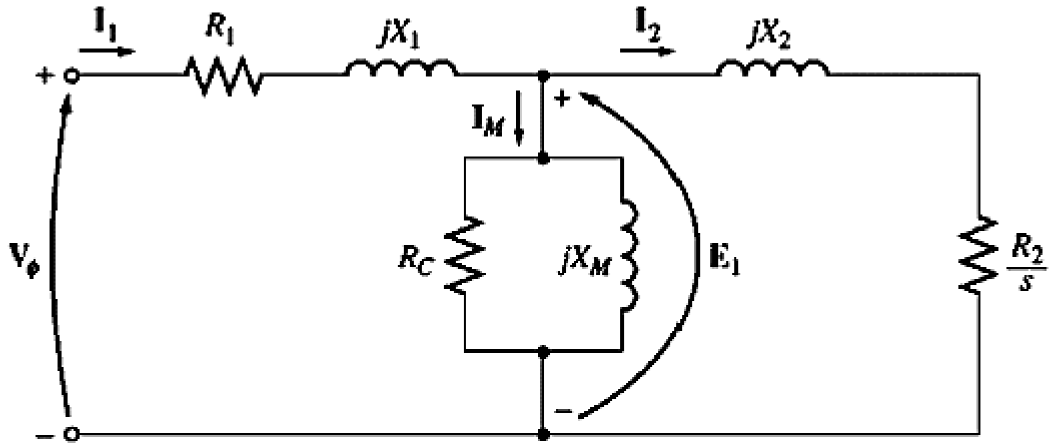
$$\mathbf{I}_2 = \frac{\mathbf{I}_R}{a_{\text{eff}}}$$ (The **rotor current** referred to **stator**)

$$Z_2 = a_{\text{eff}}^2\left(\frac{R_R}{s} + jX_{R0}\right)$$ (The **rotor impedance** referred to **stator**)

$$\left.\begin{aligned} R_2 &= a_{\text{eff}}^2 R_R \\ X_2 &= a_{\text{eff}}^2 X_{R0} \end{aligned}\right\}$$ (The **rotor resistance and reactance** referred to **stator**)

Final equivalent circuit of induction motor

- The **final per-phase equivalent circuit** of the **induction motor** is shown in the figure.



The per-phase equivalent circuit of an induction motor

- **Rotor resistance** (R_R) and the **locked-rotor rotor reactance** (X_{R0}) are **very difficult** (*or impossible*) to determine directly on **cage rotors**.

- **Effective turns ratio** (a_{eff}) is **also difficult** to obtain for **cage rotors**.

- But we can estimate the values of **referred rotor resistance** (R_2) and **reactance** (X_2) with some tests.

Input power of an induction motor

- **The input power** (P_{in}) of induction motor is in the form of **three-phase electrical power**:

$$\left.\begin{aligned} P_{in} &= \sqrt{3}V_T I_L\cos(\theta) \\ P_{in} &= 3V_{\emptyset} I_{ph}\cos(\theta) \end{aligned}\right\} \textit{(Either of them can be used)}$$

where;

V_T is the **terminal voltage** (or ***line voltage*** or ***line-to-line voltage***) of the stator

I_L is the **line current** of the stator

$V_{\emptyset}$ is the **phase voltage** (or ***line-to-neutral voltage***) of the stator

$I_{\emptyset}$ is the **phase current** of the stator

θ is the **phase angle** between $V_{\emptyset}$ and $I_{\emptyset}$

$\cos(\theta)$ is the **power factor** of the induction motor (it is **always lagging** for an **induction motor**)

Output power of an induction motor

- **The output power** (P_{out}) of induction motor is in the form of **mechanical power**:

$$P_{out} = \tau_{load} w_m$$

where;

τ_{load} is the **load torque on the shaft (*or rotor*)** (Nm)

w_m is the **angular speed of the rotor** (rad/sec)

- The **efficiency** of an **induction motor** can be calculated as follows:

$$\eta = \frac{P_{out}}{P_{in}} \, x \, 100\% \quad \left. \begin{array}{l} (0 < \eta < 100\%) \\ (P_{out} < P_{in}, \text{ because of } \textbf{losses}) \end{array} \right.$$

Losses of an induction motor

- There are **five different losses** in an induction motor:
- **Stator copper losses (P_{SCL}):** These losses occur as a result of **heating of the stator windings** because of **current flow**:
- **Core losses (P_{CORE}):** These losses occur as a result of **hysteresis losses** and **eddy current losses** in the stator.
- **Rotor copper losses (P_{RCL}):** These losses occur as a result of **heating of the rotor bars/windings** because of **current flow**:
- **Friction and windage losses ($P_{F\&W}$):** These losses are mechanical losses due to rotation of the rotor side.
- **Stray losses (P_{stray}):** These losses are the losses occurring in the induction motor that **can not be identified exactly**. Stray losses can be sometimes assumed to be **zero**.

Losses of an induction motor

- Generally, the **higher the speed** of an induction motor, the **higher** its **friction/windage, and stray losses**.

$n_m \nearrow \qquad P_{F\&W} \nearrow \qquad P_{stray} \nearrow$

- On the other hand, the **higher the speed** of the motor (*up to synchronous speed*), the **lower its core losses**.

$n_m \nearrow \qquad P_{core} \swarrow$

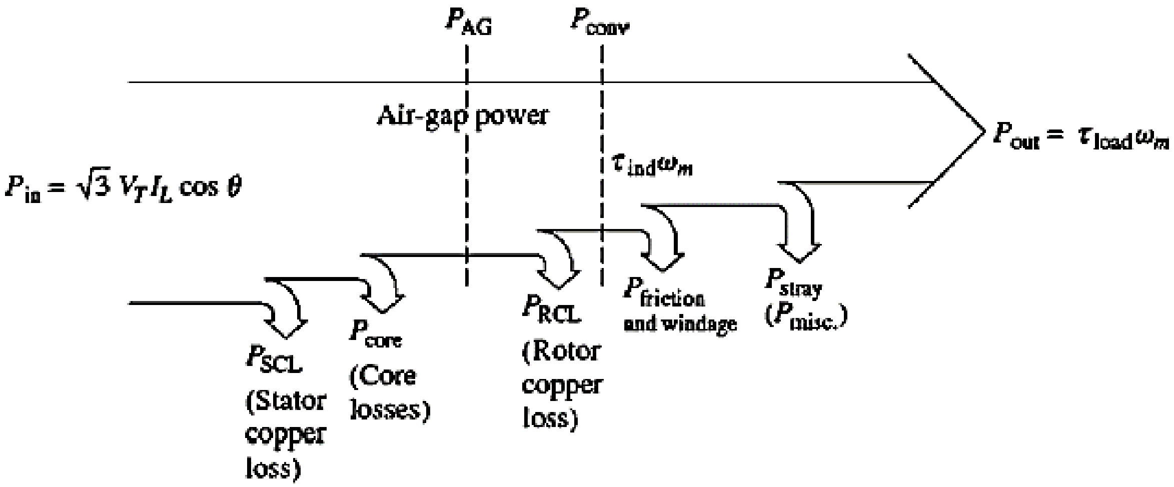
- These **<u>three losses</u>** can be together encountered as "**rotational losses**"

- Since they change **oppositely as speed changes,** their sum is **assumed to be constant (*independent of speed*)**

$$P_{rotational\ losses} = P_{F\&W} + P_{stray} + P_{core} \cong constant$$

Power flow diagram of an induction motor

- Finally, the power flow diagram of an induction motor is shown in the figure:



$$\tau_{ind} = \frac{P_{conv}}{\omega_m}$$ (induced torque=developed torque)

$\tau_{ind} > \tau_{load}$ (Because of the $P_{F\&W}$ and P_{styray} losses)

$P_{AG} = P_{in} - (P_{SCL} + P_{core})$ (Air-gap power)

$$P_{conv} = P_{in} - (P_{SCL} + P_{core} + P_{RCL})$$
$$P_{conv} = P_{AG} - P_{RCL}$$

Converted power (P_{conv}) is sometimes called as "***developed mechanical power***" and it is the power when **stator copper losses**, **core losses**, and **rotor copper losses** are **subtracted** from the **input power.**

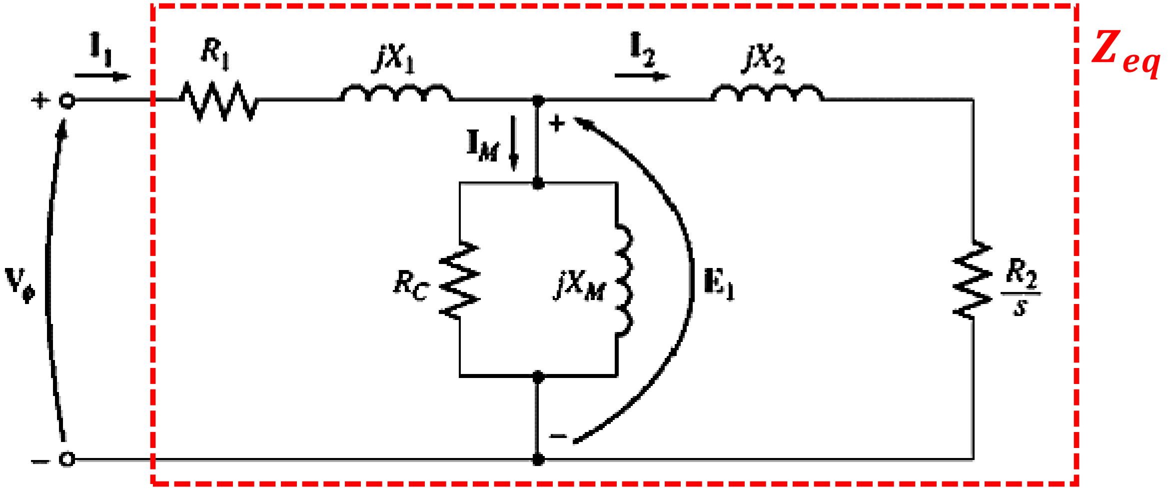
$P_{out} = P_{conv} - (P_{F\&W} + P_{stray})$

Example:

A **480-V**, **60-Hz**, **50-hp**, **three-phase** induction motor is drawing **60 A** at **0.85 PF lagging**. The stator copper losses are **2 kW**, and the rotor copper losses are **700 W**. The friction and windage losses are **600 W**, the core losses are **I800 W**, and the **stray losses are negligible**. Find the following quantities:

(a) The air-gap power
(b) The power converted
(c) The output power
(d) The efficiency of the motor

Power and torque in an induction motor



The per-phase equivalent circuit of an induction motor

$$\mathbf{I}_1 = \frac{\mathbf{V}_\phi}{Z_{eq}}$$ (Per-phase stator current)

$$Z_{eq} = R_1 + jX_1 + \cfrac{1}{G_C - jB_M + \cfrac{1}{\cfrac{R_2}{s} + jX_2}}$$ (Per-phase equivalent impedance)

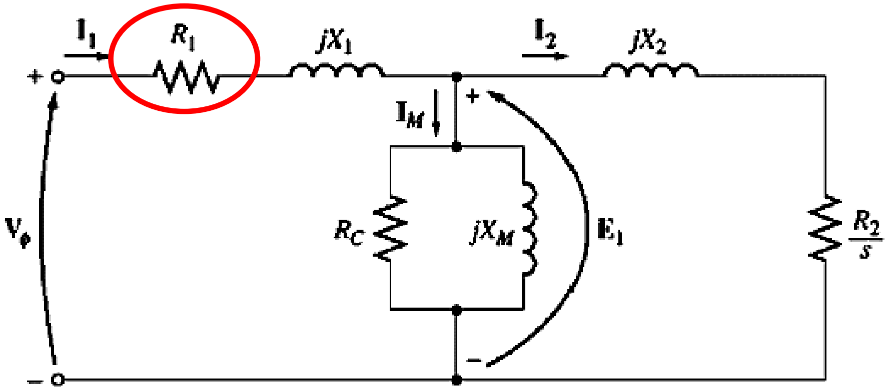
$$G_C = 1/R_C$$

$$B_M = 1/jX_M$$

Power and torque in an induction motor

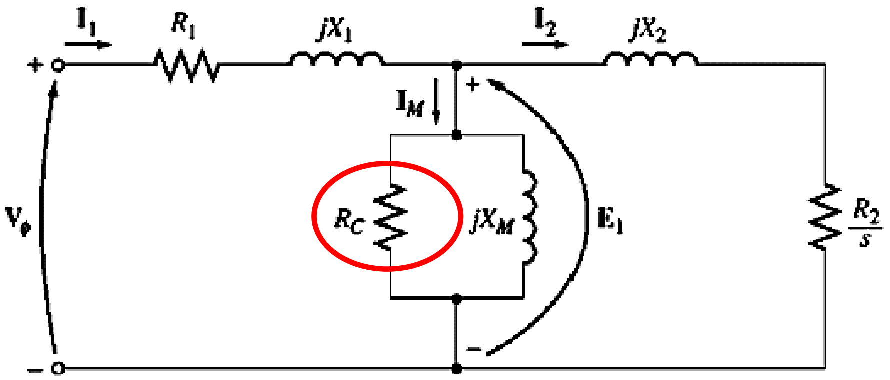
- **Stator copper losses** can be calculated as:

$$P_{SCL} = 3R_1 {I_1}^2 \quad \text{(Watts)}$$



- **Core losses** can be calculated as:

$$P_{core} = \frac{3{E_1}^2}{R_C} = 3G_C{E_1}^2 \quad \text{(Watts)}$$



Power and torque in an induction motor

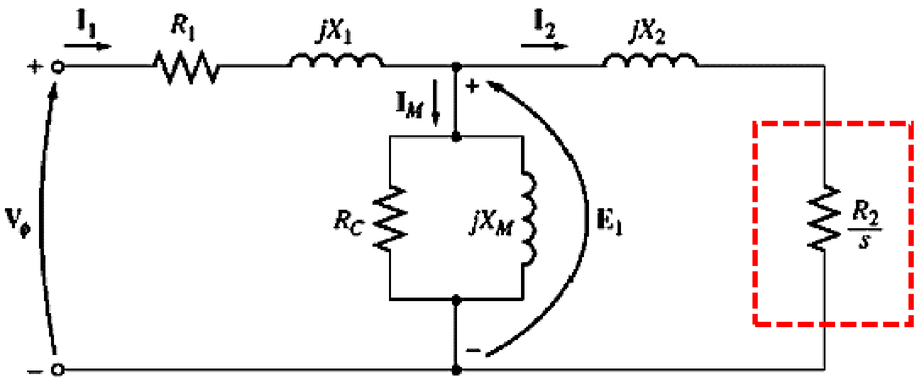
- **Air-gap power** was given previously as:

$$P_{AG} = P_{in} - (P_{SCL} + P_{core})$$

- An **alternative equation** for the **air-gap power** can also be written as:

$$P_{AG} = 3{I_2}^2 \frac{R_2}{s}$$

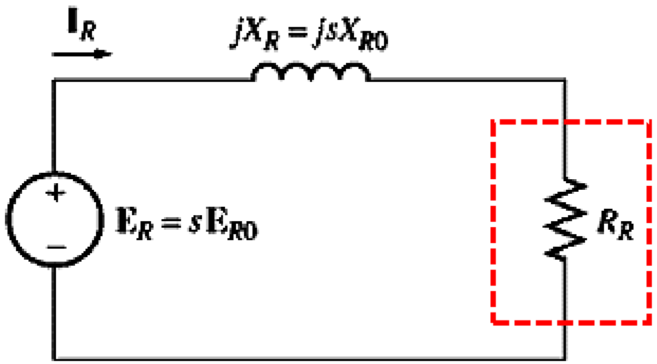
Because the circuit element that can **consume** only the **real power** on the rotor side (P_{AG}) is the **resistive element** ($\frac{R_2}{s}$)



Power and torque in an induction motor

- The **actual resistive losses** in the **rotor circuit** are given by:

$$P_{RCL} = 3R_R {I_R}^2$$



The actual rotor circuit model

- After **referring the rotor circuit to the stator**, the above equation is modified as follows:

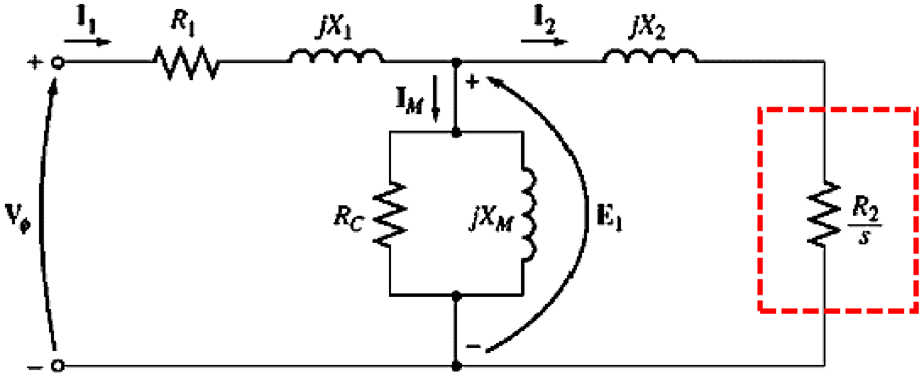
$$P_{RCL} = 3R_R {I_R}^2$$

$$P_{RCL} = 3\frac{R_2}{\cancel{{a_{eff}}^2}} {I_2}^2 \cancel{{a_{eff}}^2}$$

$$P_{RCL} = 3R_2 {I_2}^2$$

$$\boxed{\mathbf{I}_2 = \frac{\mathbf{I}_R}{a_{\text{eff}}}}$$

$$\boxed{R_2 = a_{\text{eff}}^2 R_R}$$



The rotor circuit is referred to stator

Power and torque in an induction motor

- Converted power can be formulated as:

$$
\begin{aligned}
P_{\text{conv}} &= P_{\text{AG}} - P_{\text{RCL}} \\
&= 3I_2^2 \frac{R_2}{s} - 3I_2^2 R_2 \\
&= 3I_2^2 R_2\left(\frac{1}{s} - 1\right)
\end{aligned}
$$

$$\boxed{P_{\text{conv}} = 3I_2^2 R_2\left(\frac{1-s}{s}\right)}$$

- Since we already found that:

$$P_{RCL} = 3R_2{I_2}^2$$

$$P_{AG} = 3{I_2}^2\frac{R_2}{s}$$

$$\left.\begin{matrix} P_{RCL} = 3R_2{I_2}^2 \\ P_{AG} = 3{I_2}^2\frac{R_2}{s} \end{matrix}\right\} \quad P_{\text{RCL}} = sP_{\text{AG}} \quad \Rightarrow$$

$$
\begin{aligned}
P_{\text{conv}} &= P_{\text{AG}} - P_{\text{RCL}} \\
&= P_{\text{AG}} - sP_{\text{AG}}
\end{aligned}
$$

$$\boxed{P_{\text{conv}} = (1 - s)P_{\text{AG}}}$$

- ➢ The **lower** the **slip** of the motor, the **lower** the **rotor losses**.
- ➢ if the rotor is not turning (**s=1**), the **air-gap power** is entirely consumed in the rotor ($P_{AG} = P_{RCL}$).

Power and torque in an induction motor

- Since;

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m}$$

$$\boxed{\boxed{P_{\text{conv}} = (1 - s)P_{\text{AG}}}}$$

$$w_m = (1 - s)w_{syn}$$

(previously defined)

- We can also write an **alternative formula** for the **induced torque** as follows:

$$\tau_{\text{ind}} = \frac{(1 - s)P_{\text{AG}}}{(1 - s)\omega_{\text{sync}}}$$

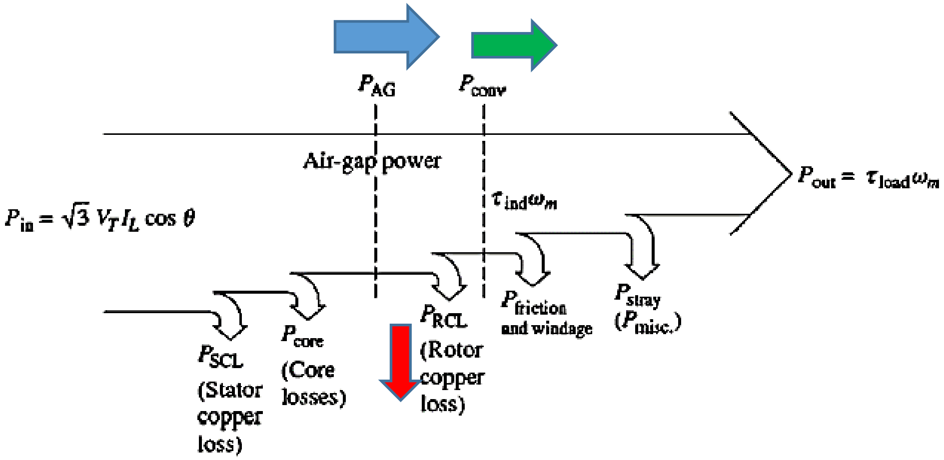
$$\boxed{\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}}}$$

Separating rotor copper losses and converted power in an induction motor's equivalent circuit

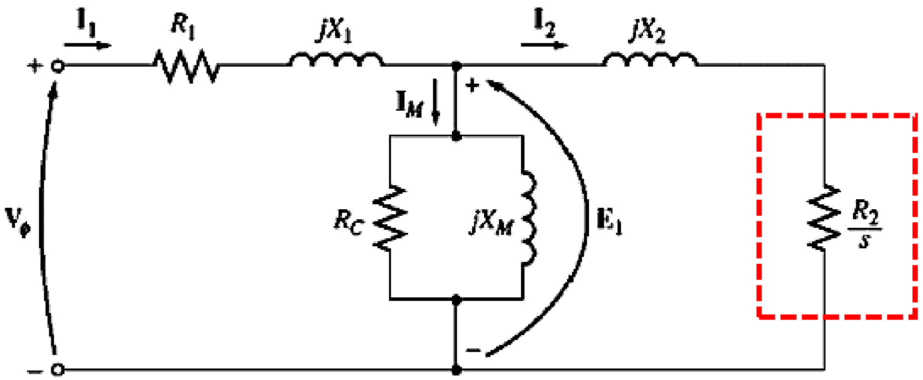
- Major part of the **air-gap power** is the **converted power**, while the **smaller part** is the **rotor copper losses**:

$$P_{AG} = P_{conv} + P_{RCL}$$

- So, it is possible to express the **air-gap power** as **two different circuit elements** in the equivalent circuit of the induction motor.

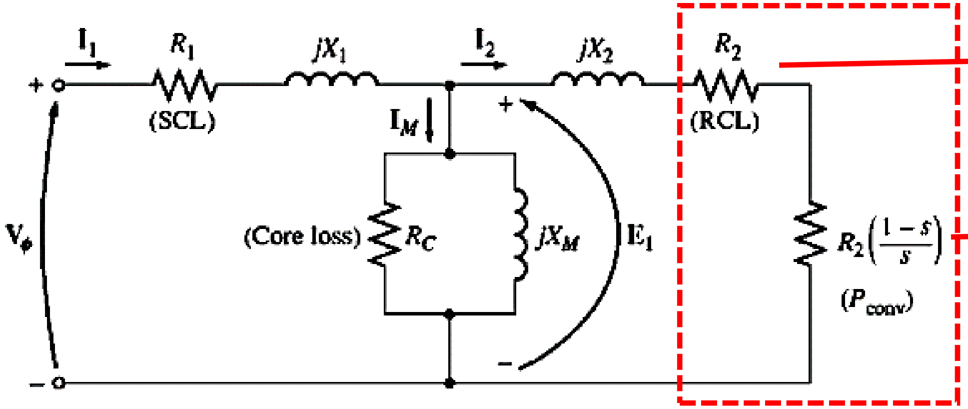


Separating rotor copper losses and converted power in an induction motor's equivalent circuit



$$\frac{R_2}{s} = R_2 + \underbrace{R_2\left(\frac{1-s}{s}\right)}_{R_{conv}}$$

(After separation)



$$\left.\begin{array}{l} P_{RCL} = 3R_2{I_2}^2 \\ \boxed{P_{\text{conv}} = 3I_2^2 R_2\left(\frac{1-s}{s}\right)} \end{array}\right\} \quad P_{AG} = P_{conv} + P_{RCL}$$

Example:

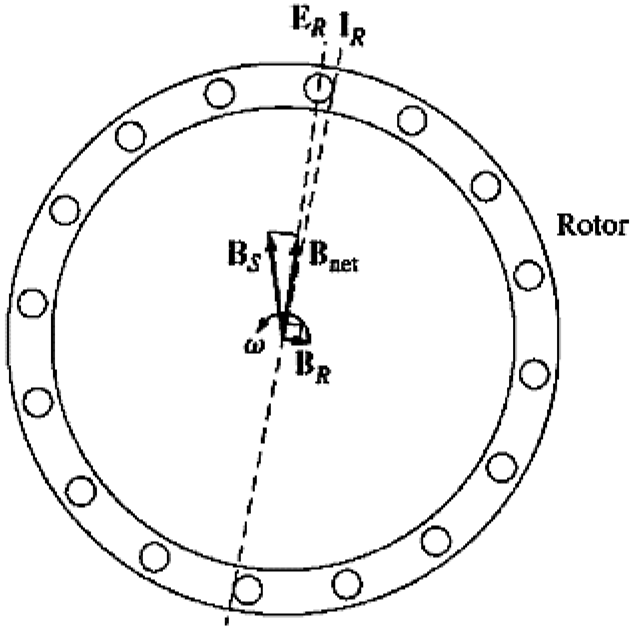
A **460-V**, **25-hp**, **60-Hz**, **four-pole**, **Y-connected** induction motor has the following impedances in ohms per phase referred to the stator circuit:

R1 = 0.641 ohm
X1 = 1.106 ohm
R2 = 0.332 ohm
X2 = 0.464 ohm
XM = 26.3 ohm

The total **rotational losses** are **1100 W** and are assumed to be **constant**. The core loss is lumped in with the rotational losses. For a **rotor slip of 2.2 percent** at the **rated voltage and rated frequency**. Find the motor's
(a) Speed
(b) Stator current
(c) Power factor
(d) Converted power and output power
(e) Induced torque and load torque
(f) Efficiency

Induction motor torque-speed characteristics

- The figure shows a **cage-rotor induction motor** that is initially operating at **no load**.
- Because of **no-load condition**, the **rotor speed** is very close to **synchronous speed (s≈0)**.
- **Bnet** is mainly produced by the **magnetization current** $\boldsymbol{I_M}$.
- The magnitude of the **magnetization current** $\boldsymbol{I_M}$ is proportional to $\boldsymbol{E_1}$**.**
- The **voltage drop** on the elements $(\boldsymbol{R_1 + jX_1})$ is relatively **very small** and hence **Bnet** is approximately **constant** as **load changes**.
- At **no-load**, the **relative motion** between the **rotor** and the **stator magnetic field** is **very small** and hence the **voltage induced on rotor bars (**$\boldsymbol{E_R}$**)** and the **rotor frequency (**$\boldsymbol{f_r}$**)** are also **very small**.
- Since $\boldsymbol{E_R}$ is **very small**, the rotor current $(\boldsymbol{I_R})$ is also **very small** and $\boldsymbol{B_R}$ is also **very small** (as seen in the figure)



Induction motor torque-speed characteristics

- Since at **no-load** **(s≈0)**, the **rotor frequency (f_r) is very small,** the **rotor reactance** becomes also **very small** ($X_R = 2\pi f_R L_R$).

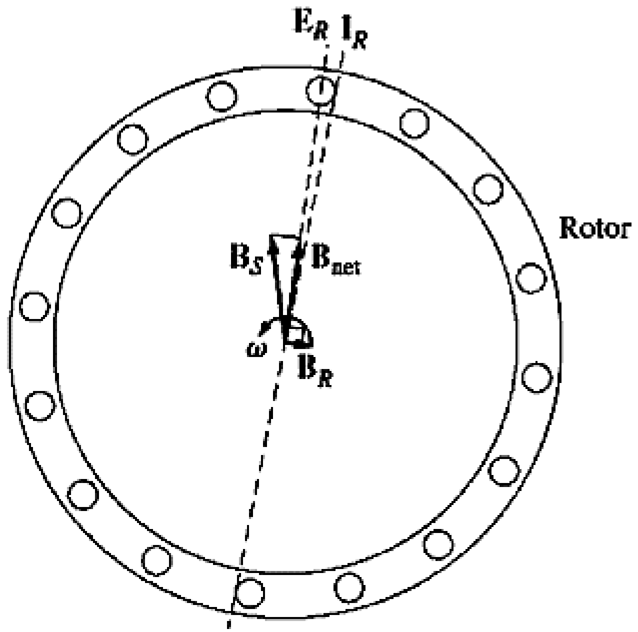
- Since X_R is **too small**, the **rotor circuit** becomes **resistive** **($R_R + jX_R$)** and E_R and I_R are almost **in-phase** at **no-load**.

- At **no-load**, the **induced torque** just keeps the rotor turning, and is given by the following equation:

$$\tau_{\text{ind}} = k\mathbf{B}_R \times \mathbf{B}_{\text{net}}$$

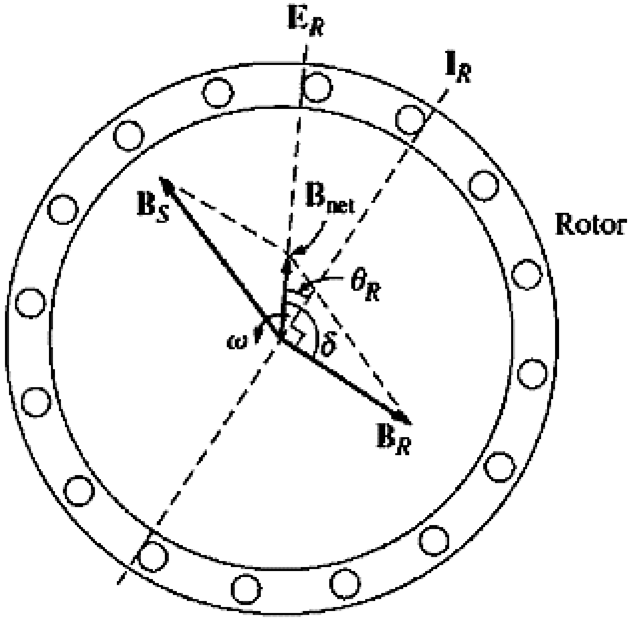
$$\tau_{\text{ind}} = k B_R B_{\text{net}} \sin \delta$$

- Since $\boldsymbol{B_R}$ is too small, the **induced torque** is also **quite small**, just large enough to overcome the motor's rotational losses.



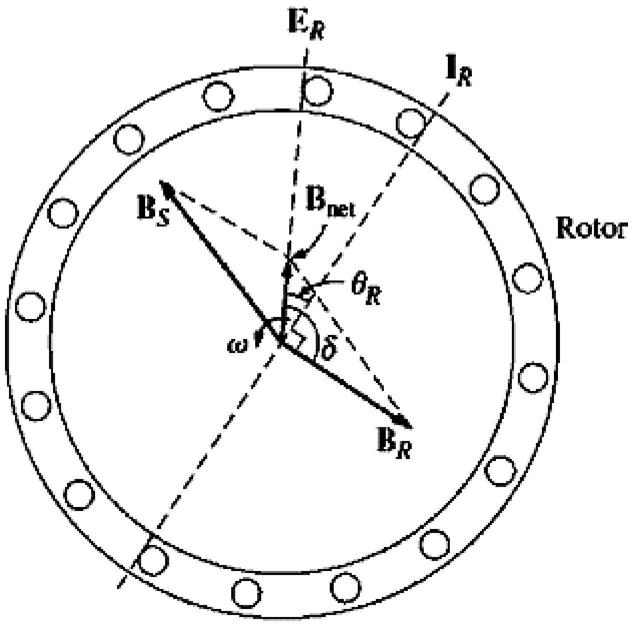
Induction motor torque-speed characteristics

- Now suppose that a **load is attached** to the induction motor.
- As the **motor's load increases**, the **speed drops** and hence **slip increases**.
- Since the **rotor speed is slower**, there is now more relative motion between the **rotor** and the **stator magnetic fields**.
- This greater relative motion produces a **stronger rotor voltage ($\boldsymbol{E_R}$)** which in turn produces a **larger rotor current**.
- With a **larger rotor current ($\boldsymbol{I_R}$)** the **rotor magnetic field ($\boldsymbol{B_R}$)** also **increases**.
- Since the **rotor slip is larger** now, the **rotor frequency rises**. $(f_R = sf_e)$
- As a result, the **rotor's reactance increases** ($X_R = 2\pi f_R L_R$).



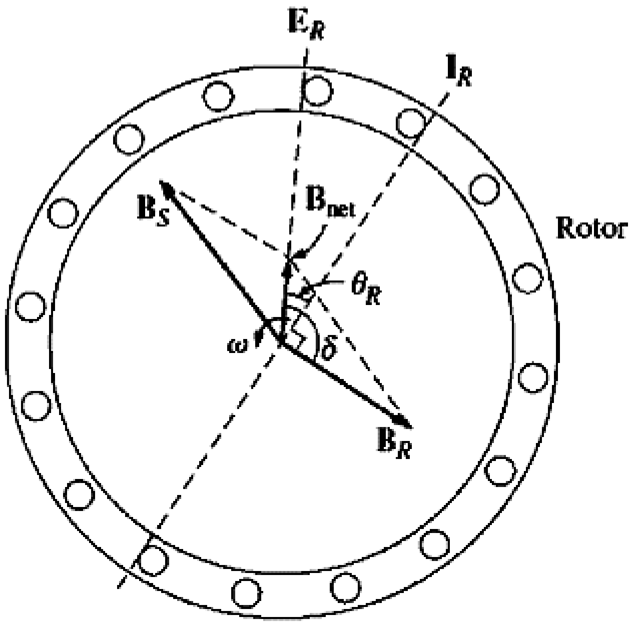
Induction motor torque-speed characteristics

- The **rotor current** now lags further behind the **rotor voltage**.
- Also the **angle** of the rotor magnetic **increases**. (Becomes much larger than 90 degrees).
- The **increase** in ($\boldsymbol{B_R}$) tends to increase the torque, while the **increase** in the **angle** tends to decrease the torque ($\delta > 90$)
- Since the **first effect is larger than the second one**, the overall induced torque increases as the load of the motor is increased.



Induction motor torque-speed characteristics

- What is the **upper limit** of this torque increase ?
- The **maximum induced torque** is defined as "**pullout torque**"
- This happens when the point is reached where, as the **load on the shaft** is **increased**, the **sin($\boldsymbol{\delta}$)** term **decreases** more than the $\boldsymbol{B_R}$ term **increases**.
- At that point, a further increase in load decreases the induced torque and the motor stops.
- If we do not reduce stator voltage immediately at this point, the stator windings will be damaged because of very large current flow

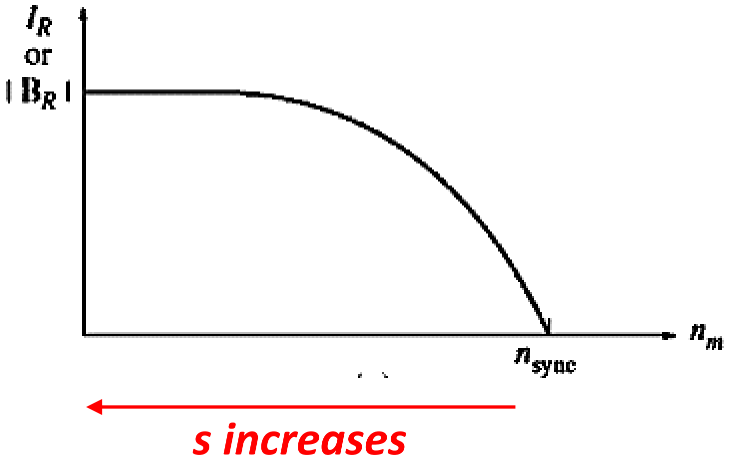


Induction motor torque-speed characteristics

- The **rotor magnetic field** is **directly proportional** to the **current flowing in the rotor**, as long as the rotor is **unsaturated**.

- The **current flow in the rotor** increases with **increasing slip** (*decreasing speed*) according to the following equation:

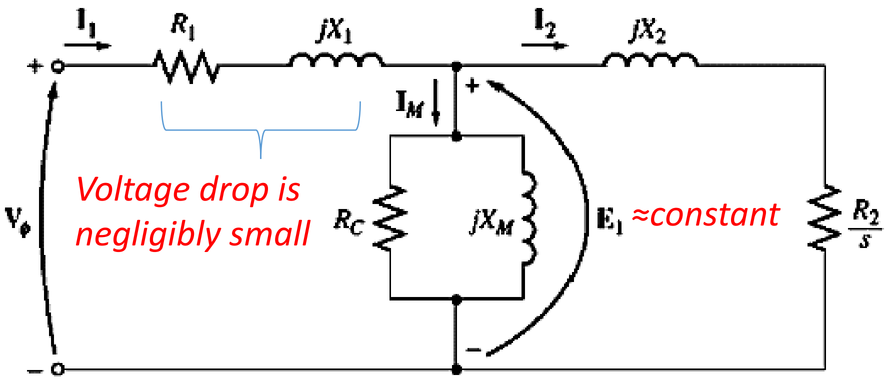
$$\mathbf{I}_R = \frac{\mathbf{E}_{R0}}{R_R/s + jX_{R0}}$$



The plot of rotor current versus speed for an induction motor.

Induction motor torque-speed characteristics

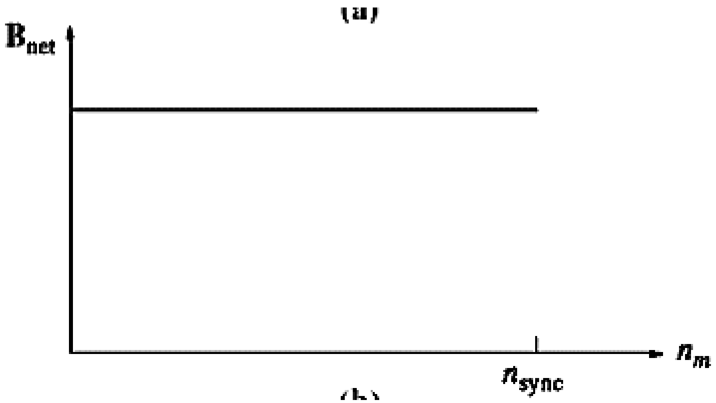
- As explained before, the Bnet of the motor is **approximately constant** and **independent** of the **load size**. (*refer to the eq. circuit of induction motor*)



constant

Voltage drop is negligibly small

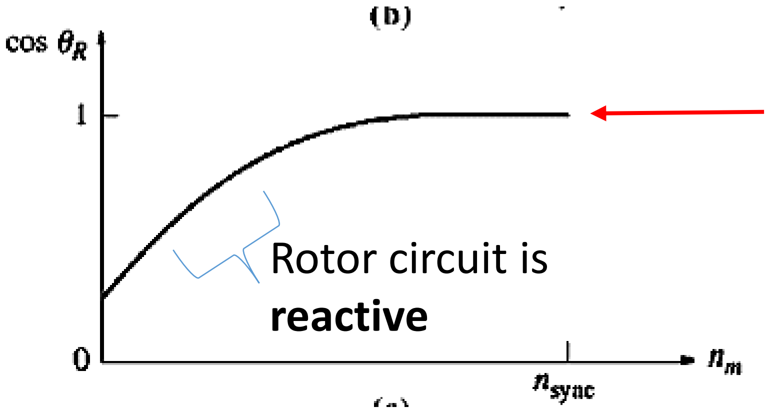
≈constant



Plot of net magnetic field versus speed for the motor

I_M is **almost constant** and hence Bnet is **almost constant.**

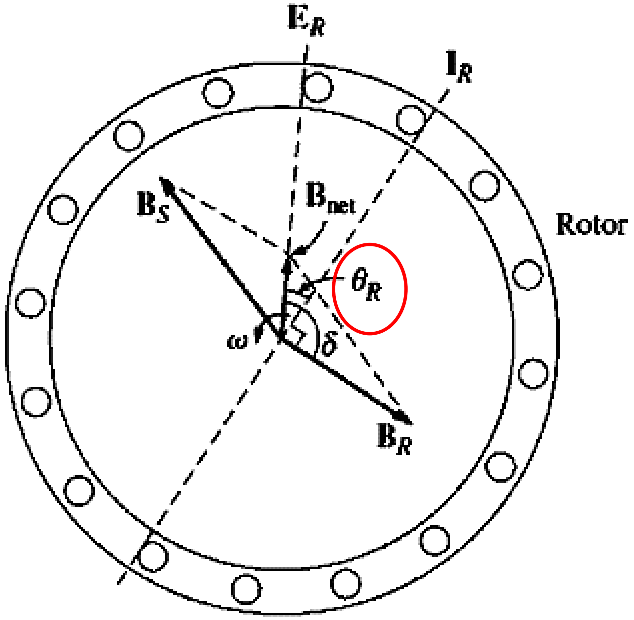
Induction motor torque-speed characteristics



Rotor circuit is **almost resistive**

Rotor circuit is **reactive**

s increases



$$\delta = \theta_R + 90°$$

$$\sin\delta = \sin(\theta_R + 90°) = \cos\theta_R.$$

Induction motor torque-speed characteristics

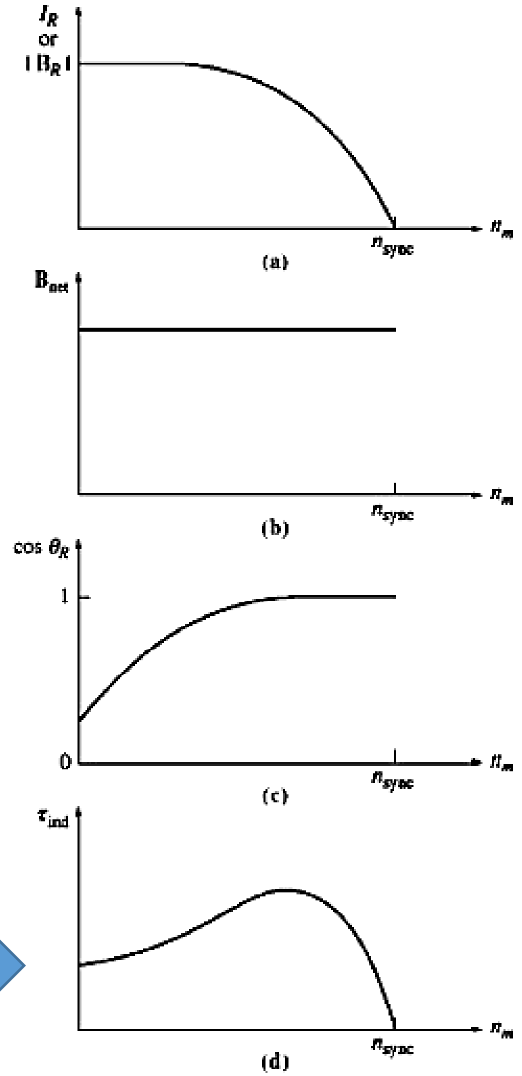
- Since the induced torque equation is given as:

$$\tau_{\text{ind}} = k\mathbf{B}_R \times \mathbf{B}_{\text{net}}$$

$$\tau_{\text{ind}} = k\underbrace{\mathbf{B}_R \mathbf{B}_{\text{net}} \sin\delta}$$

Three parameters** can **be graphically multiplied** to obtain **torque-speed chararacteristics

*The resulting **torque-speed characteristic** of an **induction motor***



Derivation of induction motor induced-torque equation

- It is possible to use the **equivalent circuit** of an induction motor and **the power flow diagram** for the motor to derive a **general expression for induced torque as a function of speed**.

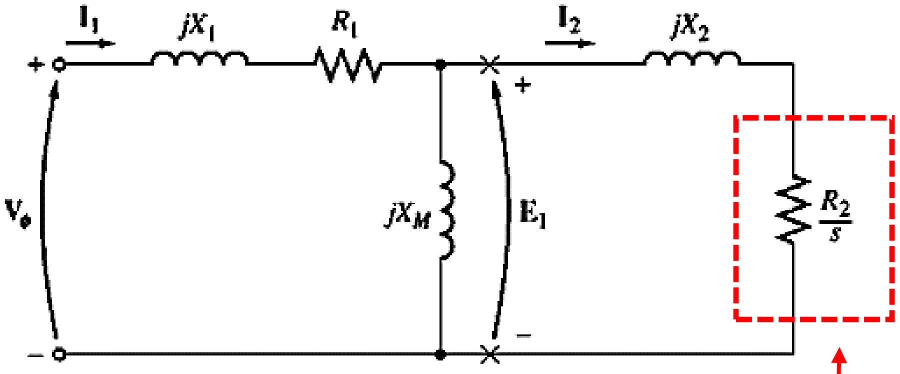
- So far, the **induced torque** equation is given as:

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m}$$

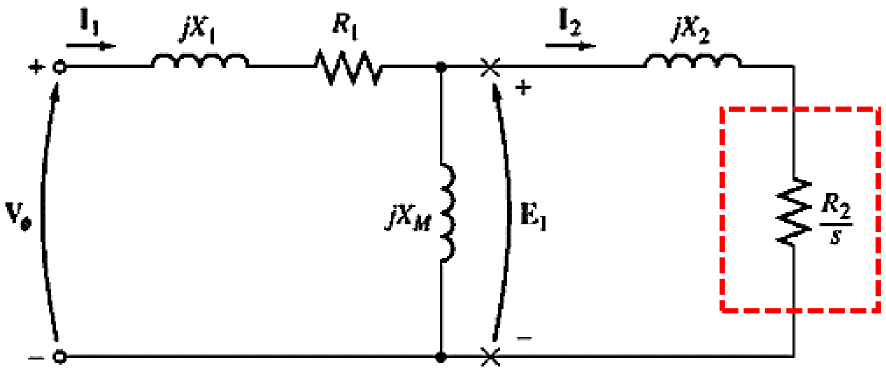
or

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}}$$

The **air-gap power** is the power **crossing the gap** from the stator circuit to the rotor circuit. It is equal to the power absorbed in the resistance ***R2/s***.



Derivation of induction motor induced-torque equation



- The **airgap power** of **one phase** of the motor:

$$P_{\mathrm{AG},1\phi} = I_2^2 \frac{R_2}{s}$$

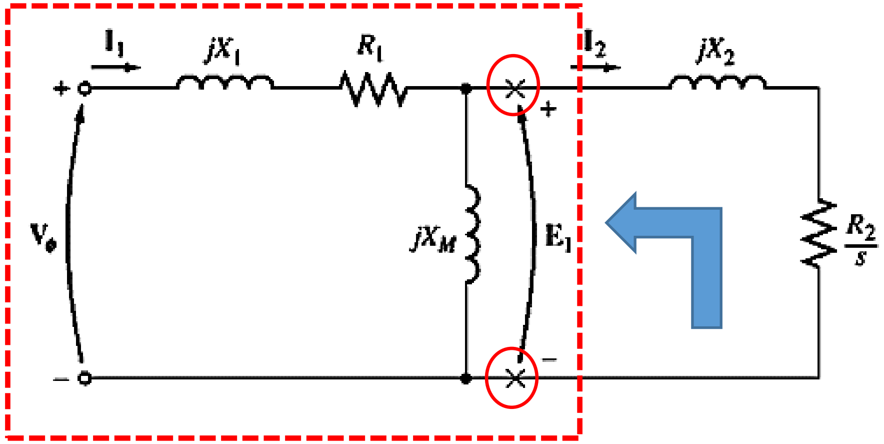
- The **total** (**three-phase**) **airgap power** of the motor:

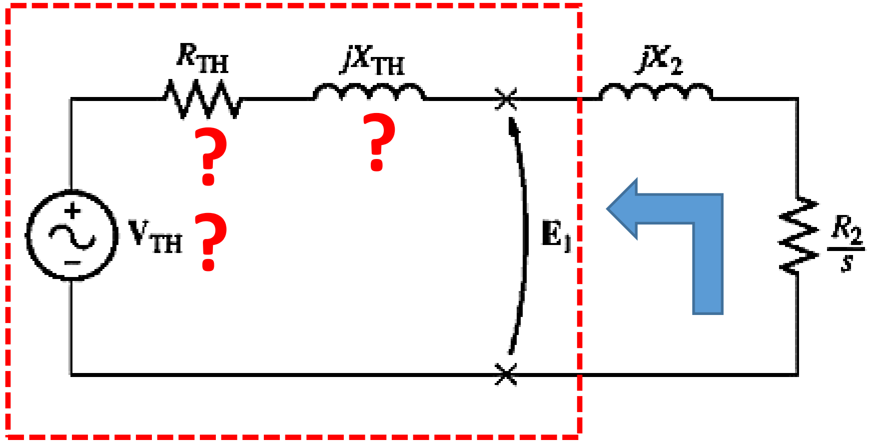
$$P_{\mathrm{AG}} = 3 I_2^2 \frac{R_2}{s}$$

- If we are able to calculate I_2, we can find P_{AG}

Derivation of induction motor induced-torque equation

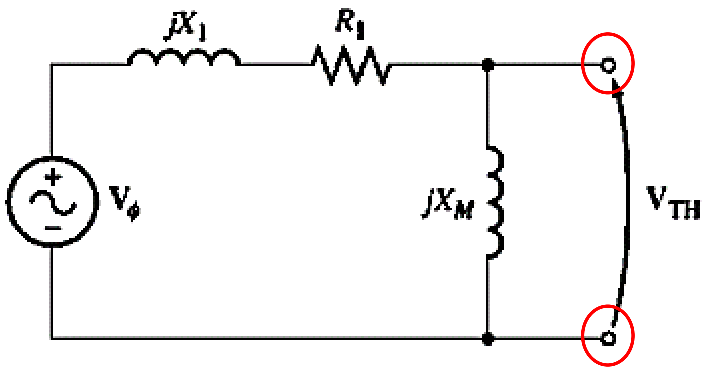
- One efficient way to find I_2 is to use **Thevenin's theorem**:





Derivation of induction motor induced-torque equation

- **Thevenin's voltage** can be calculated by leaving the terminals **open-circuit**:



$$\mathbf{V}_{TH} = \frac{jX_M}{R_1 + jX_1 + jX_M}\mathbf{V}_\phi$$

- The **magnitude** (*rms*) of Thevenin's voltage is written as:

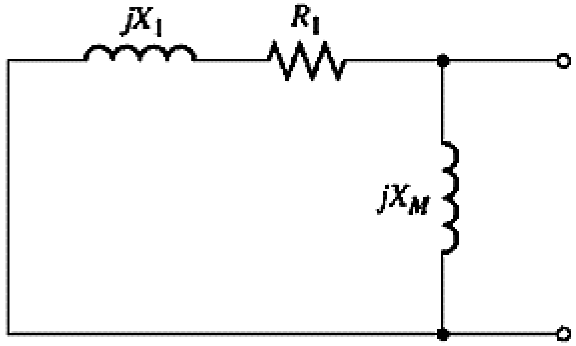
$$V_{TH} = \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}V_\phi$$

Since **magnetization reactance** $X_M \gg X_1$ and $X_M \gg R_1$, Thevenin's voltage can be approximated as:

$$\boxed{V_{TH} \approx V_\phi \frac{X_M}{X_1 + X_M}}$$

Derivation of induction motor induced-torque equation

- **Thevenin's equivalent impedance** can be calculated by **killing the voltage source**:



$$Z_{TH} = \frac{jX_M(R_1 + jX_1)}{R_1 + j(X_1 + X_M)}$$

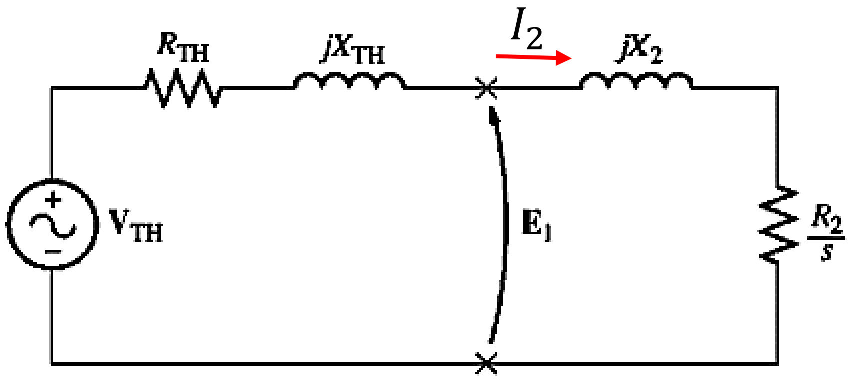
$$Z_{TH} = R_{TH} + jX_{TH}$$

Since **magnetization reactance** $X_M \gg X_1$ and $X_M + X_1 \gg R_1$, Thevenin's resistance and reactance can be approximated as:

$$\boxed{R_{TH} \approx R_1\left(\frac{X_M}{X_1 + X_M}\right)^2}$$

$$\boxed{X_{TH} \approx X_1}$$

Derivation of induction motor induced-torque equation



$$\boxed{V_{TH} \approx V_\phi \frac{X_M}{X_1 + X_M}}$$

$$\boxed{R_{TH} \approx R_1 \left(\frac{X_M}{X_1 + X_M}\right)^2}$$

$$\boxed{X_{TH} \approx X_1}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_{TH}}{R_{TH} + R_2/s + jX_{TH} + jX_2}$$

- The magnitude of this current is

$$I_2 = \frac{V_{TH}}{\sqrt{(R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2}}$$

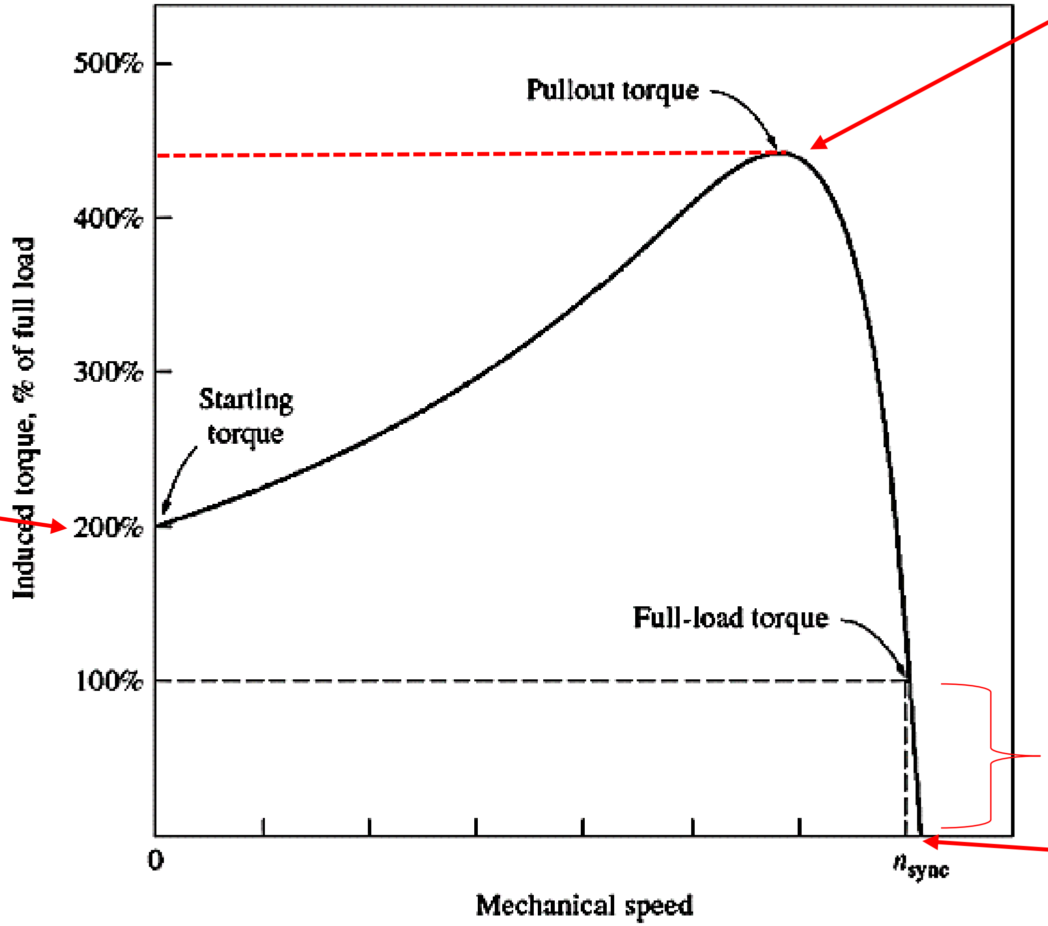
- The **air-gap power** is therefore given by

$$P_{AG} = 3I_2^2 \frac{R_2}{s} = \frac{3V_{TH}^2 R_2/s}{(R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2}$$

- And the **induced torque** is given by

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} \boxed{= \frac{3V_{TH}^2 R_2/s}{\omega_{sync}[(R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2]}}$$

Derivation of induction motor induced-torque equation



There is a **maximum possible torque** that **cannot be exceeded**, called the **pullout torque** or **breakdown torque**

The **starting torque** on the motor is **slightly larger** than its full -load torq ue, so this motor will start carrying any load that it can supply at full power

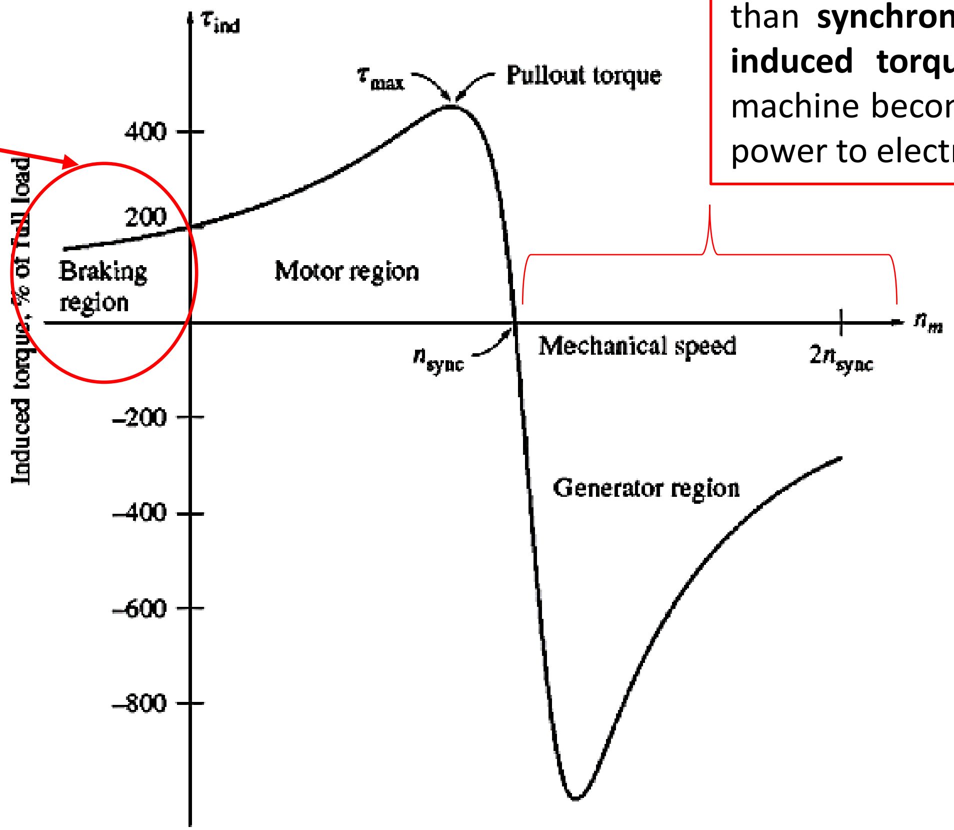
The torque-speed curve is **nearly linear** between no-load and full-load. In this range, the **rotor resistance** is **much larger** than the **rotor reactance**, so the rotor current , the rotor magnetic fi eld, and the induced torque increase linearly with increasing slip.

The **induced torque** of the motor is **zero** at **synchronous speed**

A typical induction motor ***torque-speed characteristics***

Derivation of induction motor induced-torque equation

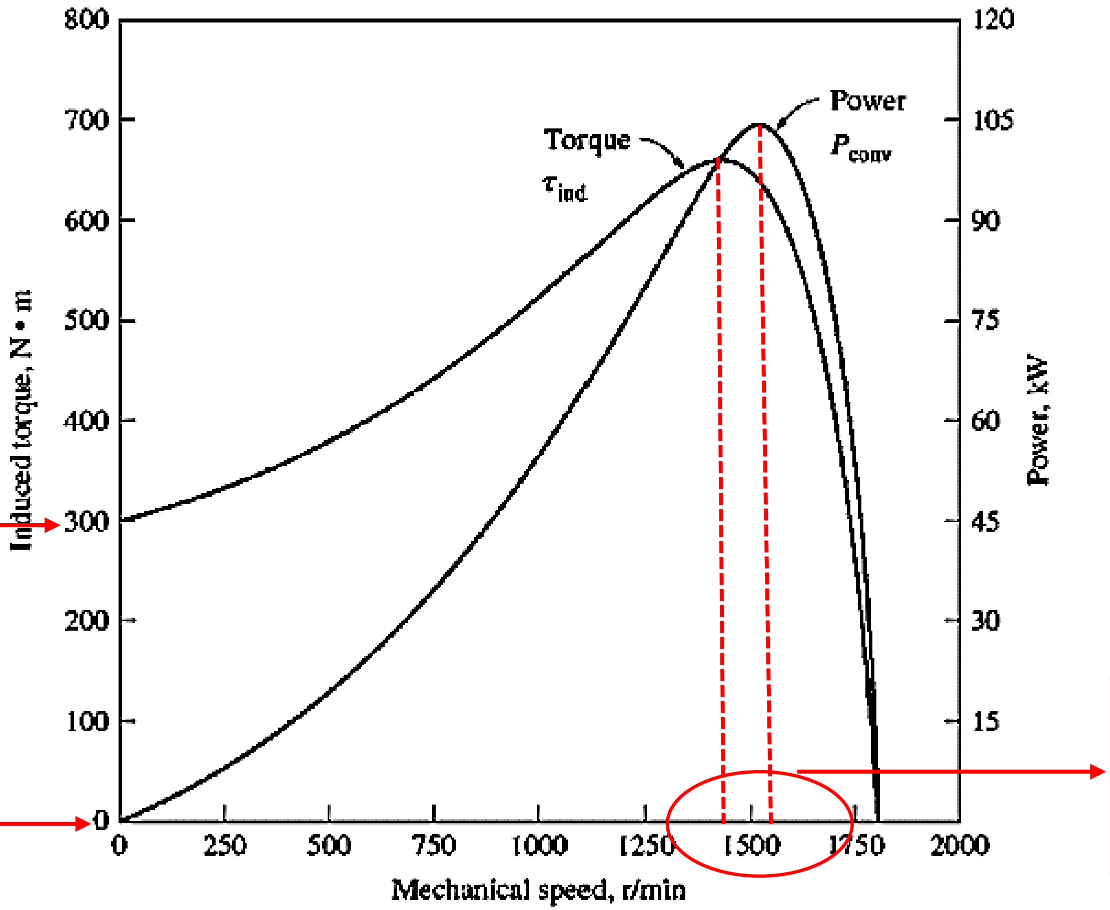
If the motor is **turning backward relative to the direction of the magnetic fields**, the induced torque in the machine will stop the machine very rapidly and will try to rotate it in the other direction. Since reversing the direction of magnetic field rotation is simply a matter of **switching any two stator phases, this fact can be used as a way to very rapidly stop an induction motor.** The act of switching two phases in order to stop the motor very rapidly is called “***plugging***”



If the rotor of the induction motor is driven **faster** than **synchronous speed**, then the **direction of the induced torque** in the machine **reverses** and the machine becomes a **generator**, converting mechanical power to electric power (***induction generator***).

Derivation of induction motor induced-torque equation

$$P_{conv} = \tau_{ind}\,\omega_m$$



There is an induced torque at start-up even if the motor's speed is zero. This torque is called "***start-up torque***"

No power is converted to mechanical form when the rotor is **at zero speed**

The **peak power** supplied by the induction motor occurs at a **different speed** than the **maximum torque**

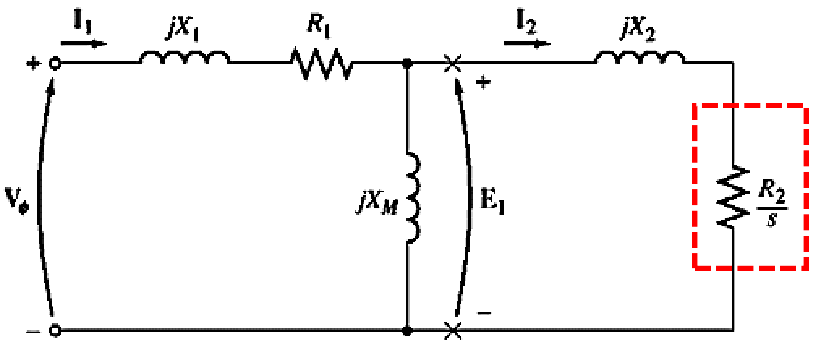
Maximum (pullout) torque in an induction motor

- The **induced torque** is given as:

$$\tau_{ind} = \frac{P_{AG}}{w_{syn}}$$

- The **maximum possible torque** occurs when the **air-gap power is maximum**. (*Synchronous speed is* ***constant***)

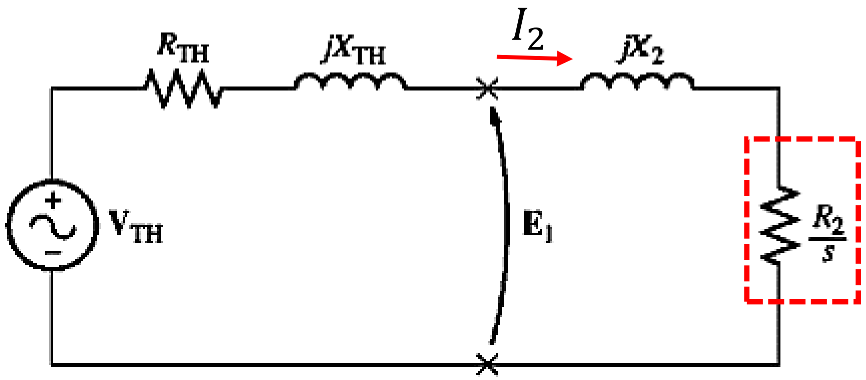
- Since the **air-gap power** is equal to the **power consumed in the resistor R2/s**, the **maximum induced torque** will occur when the **power consumed by that resistor is maximum**.



$$\Rightarrow \underbrace{P_{AG} = 3I_2^2\frac{R_2}{s}}_{\textbf{maximum}} \Rightarrow \tau_{ind(max)}$$

Maximum (pullout) torque in an induction motor

- So when is the **power supplied** to **R2/s** becomes **maximum** ?
- The **maximum power transfer theorem** states that maximum power transfer to the load resistor **R2/s** will occur **when the magnitude of this resistance is equal to the magnitude of the source impedance**:



$$\boxed{R_{TH} \approx R_1 \left(\frac{X_M}{X_1 + X_M} \right)^2}$$

$$\boxed{X_{TH} \approx X_1}$$

$$Z_{source} = R_{TH} + jX_{TH} + jX_2$$

$$\boxed{\frac{R_2}{s} = \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}} \Rightarrow \boxed{s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}}$$

(*Slip at pullout torque*)

Maximum (pullout) torque in an induction motor

- Since **induced torque** is given by:

$$\boxed{\tau_{\text{ind}} = \frac{3V_{\text{TH}}^2 R_2/s}{\omega_{\text{sync}}[(R_{\text{TH}} + R_2/s)^2 + (X_{\text{TH}} + X_2)^2]}}$$

- We can find the **pullout torque** by inserting $s = s_{max}$ in the above equation:

$$\boxed{s_{\max} = \frac{R_2}{\sqrt{R_{\text{TH}}^2 + (X_{\text{TH}} + X_2)^2}}}$$

$$\boxed{\tau_{\max} = \frac{3V_{\text{TH}}^2}{2\omega_{\text{sync}}[R_{\text{TH}} + \sqrt{R_{\text{TH}}^2 + (X_{\text{TH}} + X_2)^2}]}}$$

Maximum (pullout) torque in an induction motor

- Since the **referred rotor resistance R2** appears only in the **numerator**, so **the slip of the rotor at maximum torque** becomes **directly proportional** to the **rotor resistance**:

$$\boxed{s_{\max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}}$$

→ **Denominator is constant**

$R_2 \nearrow$ $s_{max} \nearrow$

This feature can be used to **change** the **torque-speed characteristics of an induction motor**

Maximum (pullout) torque in an induction motor

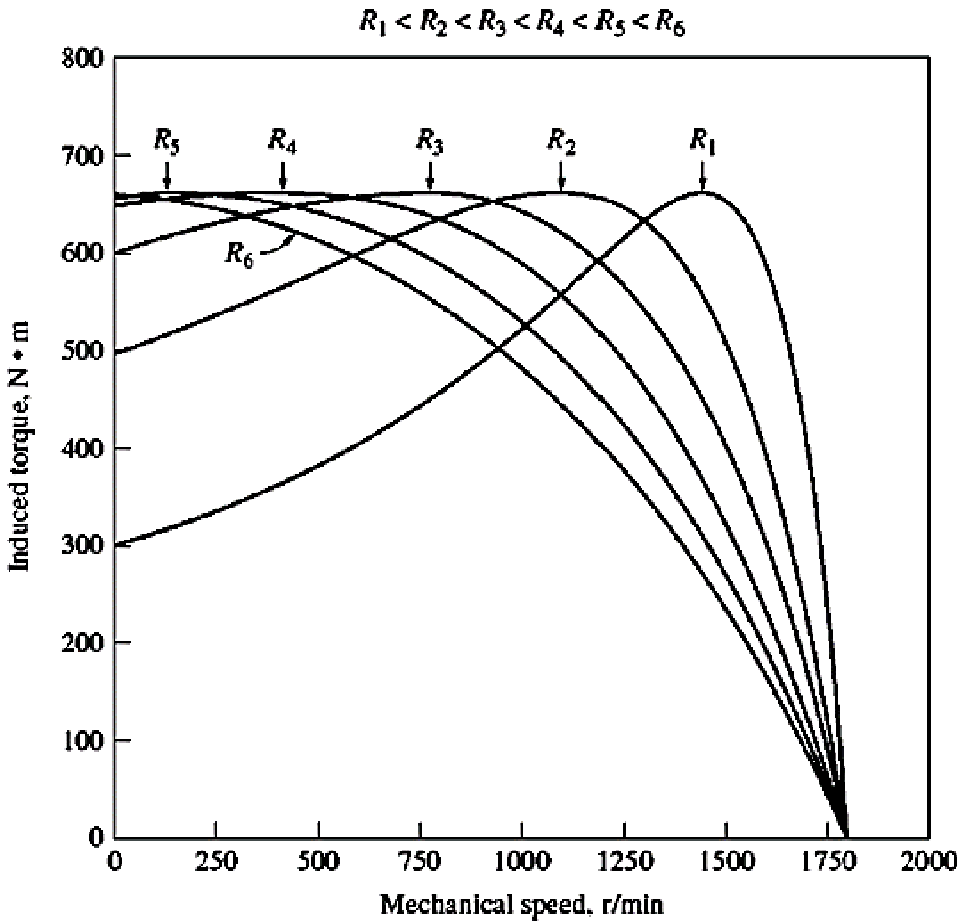
- **Inserting a resistance into the rotor circuit** of a **wound rotor** induction motor **changes** the **torque-speed characteristics**.

- This can be done using **slip rings**.

- The figure shows that **as the rotor resistance is increased**, the **pullout speed of the motor decreases.** (*Pullout speed is the speed at which the maximum torque is induced in the induction motor*)

- However, **as the rotor resistance is increased, the maximum torque remains constant.**

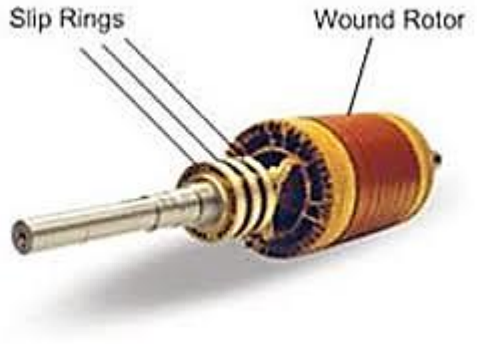
$$\boxed{\tau_{\max} = \frac{3V_{TH}^2}{2\omega_{sync}[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}]}}$$

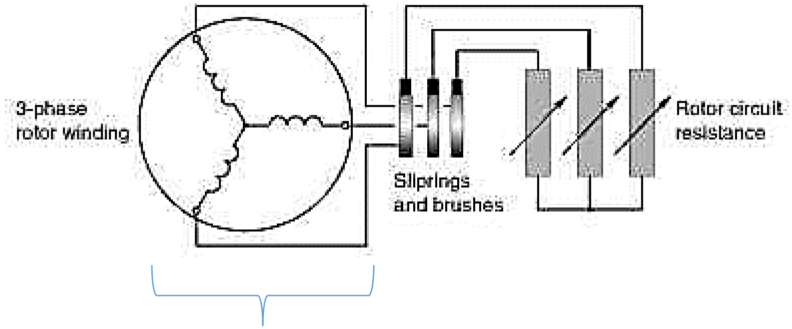


The effect of varying rotor resistance on the torque-speed characteristic of a wound-rotor induction motor.

Maximum (pullout) torque in an induction motor

- It is possible to take advantage of this characteristic of **wound-rotor** induction motors **to start very heavy loads**.
- If a resistance is inserted into the rotor circuit, **the maximum torque can be adjusted to occur at starting conditions**.
- Therefore, the **maximum possible torque** would be available **to start heavy loads**.
- On the other hand, once the load is turning, the extra resistance can be removed from the circuit, and **the maximum torque** will move up to **near-synchronous speed** for regular operation.





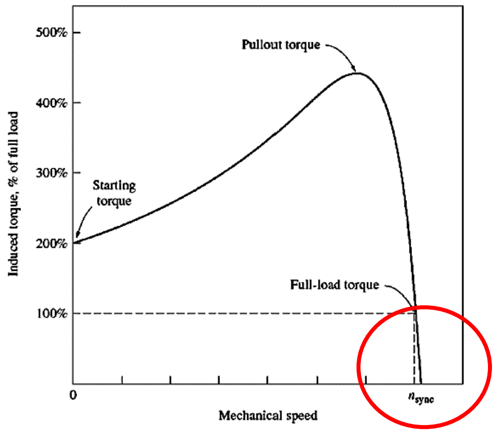
wound-rotor induction motor

Example:

A **two-pole**, **50-Hz** induction motor supplies **15 kW** to a load at a speed of **2950 r/min.** Answer the following questions:

(a) What is the motor's slip?
(b) What is the induced torque in the motor in Nm under these conditions? (*assume efficiency is 100%*)
(c) What will the operating speed of the motor be if its torque is doubled?

In the **low-slip region**, the torque-speed curve is **linear**, so **induced torque is directly proportional to slip**.



s ↗ τ_{ind} ↗

(d) How much power will be supplied by the motor when the torque is doubled?

Rotor design due to desired torque-speed characteristics

- if a **rotor is designed with high resistance**, then **pullout torque** (*maximum torque*) is obtained at **lower speeds** or **higher slip**. (*refer to the figure*)

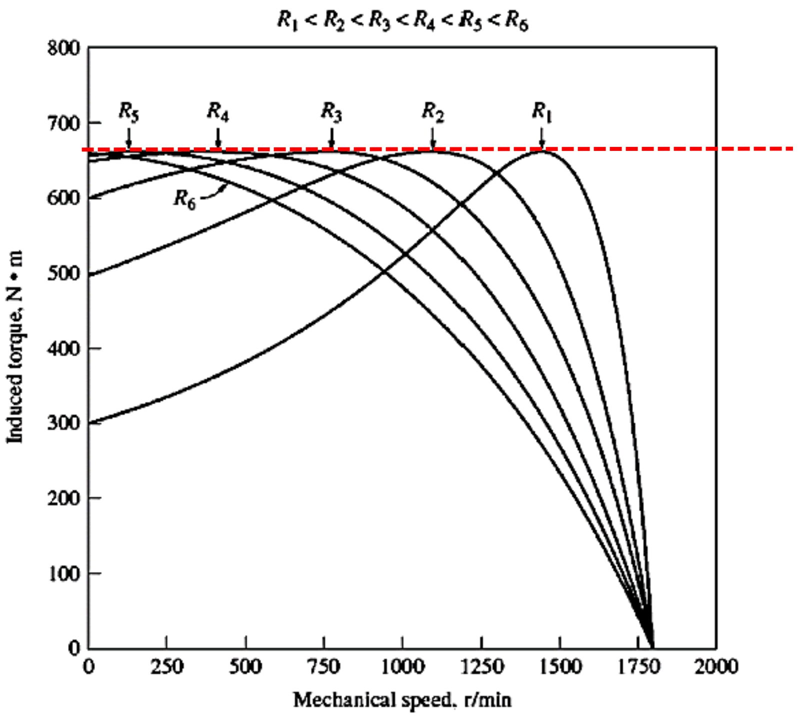
- So **pullout torque** is approached to **starting conditions of the motor if high resistance rotor is used**.

- Consequently, **starting torque** is **increased if the rotor is designed with high resistance**.

- On the other hand, for **higher slip values**, **Pconv** is **reduced:**

$$\boxed{P_{\text{conv}} = (1 - s)P_{\text{AG}}}$$

- So, **the overall efficiency of the motor reduces** if the **rotor is designed with high resistance.**



pullout torque

speed reduces or slip increases

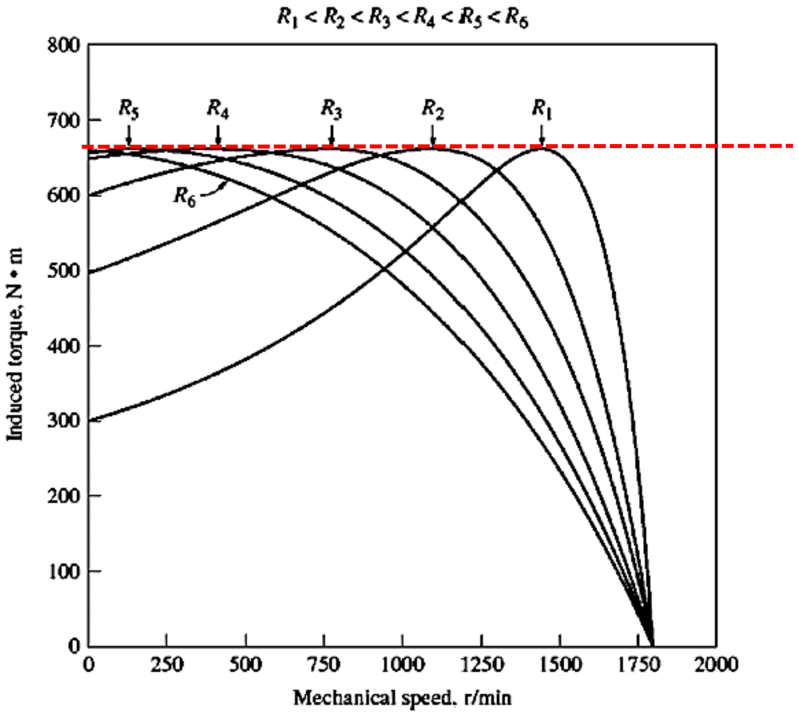
Rotor design due to desired torque-speed characteristics

- If a **low resistance rotor** is used, the induction motor will have a **low starting torque** (*refer to the figure*).

- Since, **the slip is small** at rated conditions, converted power will be high:

$$\boxed{P_{\text{conv}} = (1 - s)P_{\text{AG}}}$$

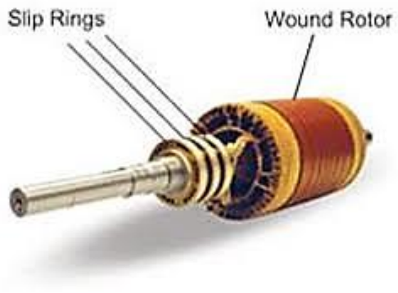
- As a result, **the overall efficiency of the motor** becomes **high** if a **low resistance rotor** is used.



pullout torque

speed reduces or slip increases

Rotor design due to desired torque-speed characteristics



- So the solution is **to use a wound-rotor induction motor**.

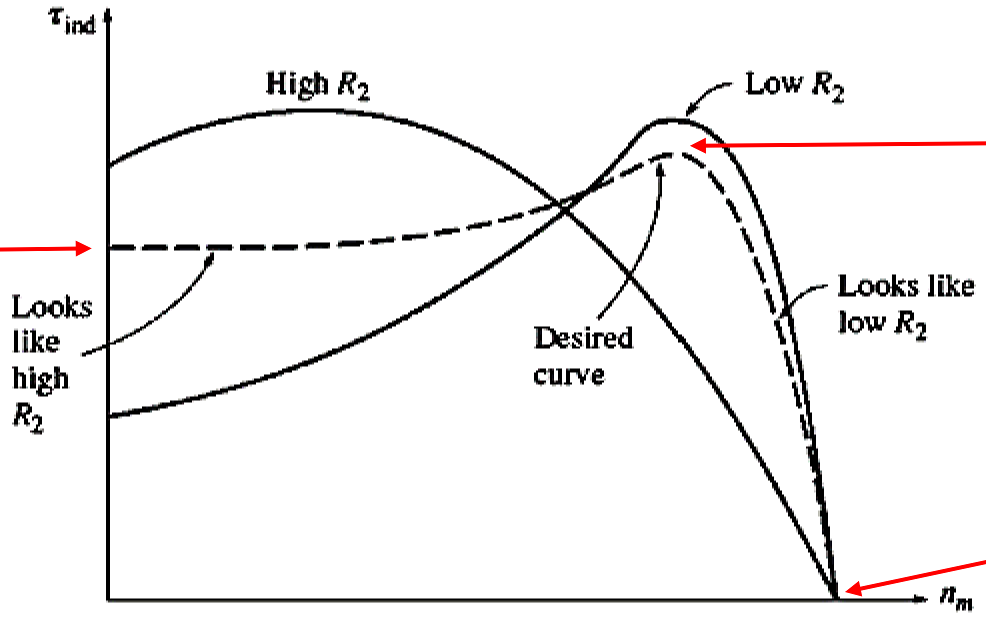
- **Add an extra resistance** to the rotor circuit during starting **to increase starting torque**.

- When the motor reaches at its **steady-state speed**, **remove this resistance** from the rotor circuit **to increase the overall efficiency of the induction motor**.

- Unfortunately, **wound-rotor** motors are
  - ❑ more expensive
  - ❑ need more maintenance
  - ❑ require a more complex automatic control circuit than **squirrel-cage rotor** motors.

- So how can we get a solution with a **squirrel-cage rotor** to generate
  - ❑ **high starting torque**
  - ❑ **high efficiency during normal operation**

Rotor design due to desired torque-speed characteristics



Pullout torque is obtained near synchronous speed

High starting torque

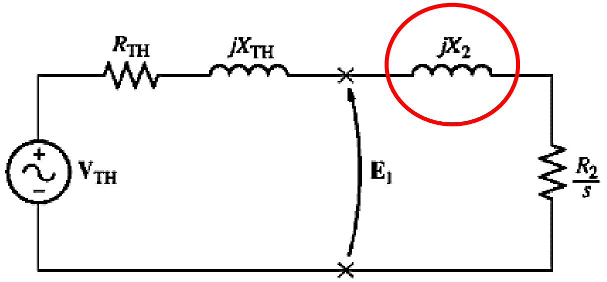
Synchronous speed

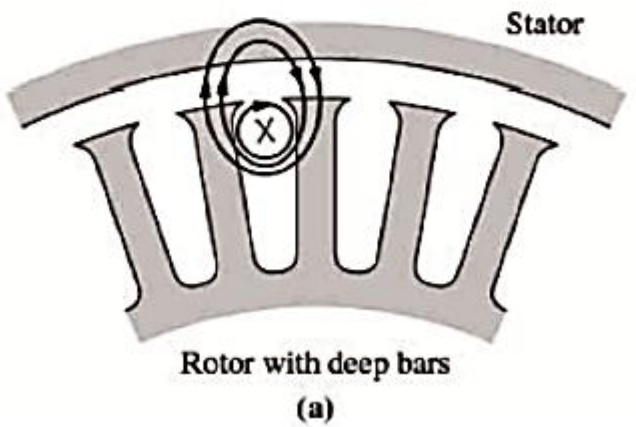
The desired torque-speed characteristics of an induction motor.

Control of motor characteristics by cage rotor design

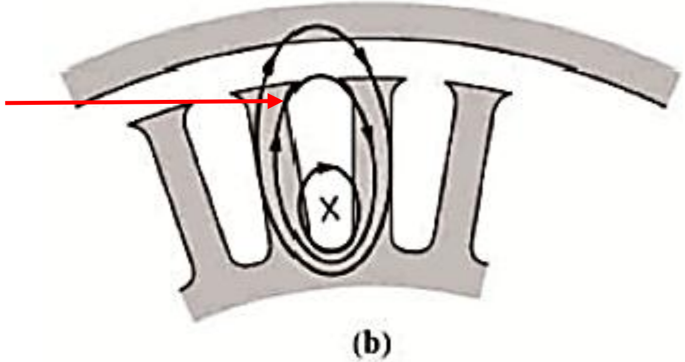
Definition of Rotor Leakage Reactance:

Rotor leakage reactance is the reactance due to the rotor flux lines that **do not couple with the stator windings**.





Rotor leakage reactance is small



Rotor leakage reactance is high

- So if the rotor current passes **at the top of the bar**, the flux is **tightly linked** to the stator and **rotor leakage reactance** becomes **small**. (*See upper figure*)

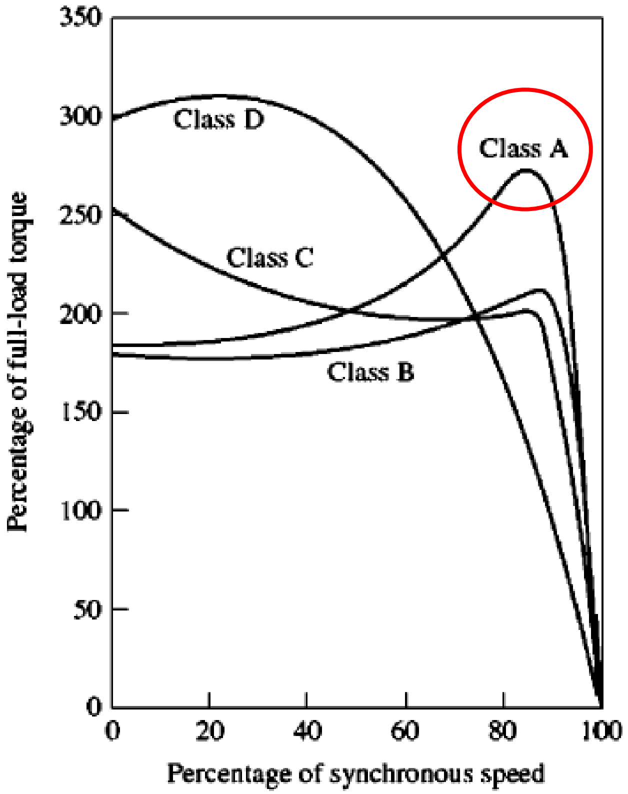
- So if the rotor current passes **in the bottom of the bar**, the flux is **loosely linked** to the stator and rotor leakage reactance becomes **large**. (*See bottom figure*)

Control of motor characteristics by cage rotor design

<u>NEMA Design Class A: Large bars near the surface:</u>

- As seen in the figure, the rotor bars are quite large and placed near the surface of the rotor.
- Such a design will have a **low rotor resistance** (*due to its large cross section*) and a **low leakage reactance** (*due to the bar's location near the stator*).
- Because of the **low rotor resistance**, the **pullout torque** will be **quite near synchronous speed** and the motor will be **quite efficient under normal operation**.
- Since **rotor resistance is small**, the **motor's starting torque will be small** and **its starting current is high**.

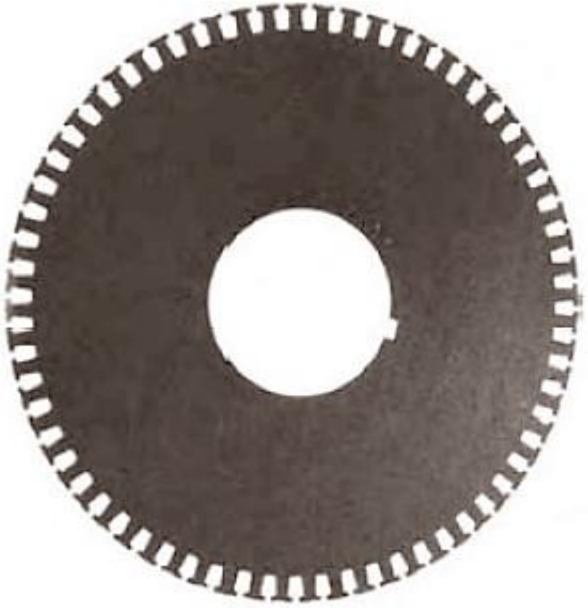
(a)



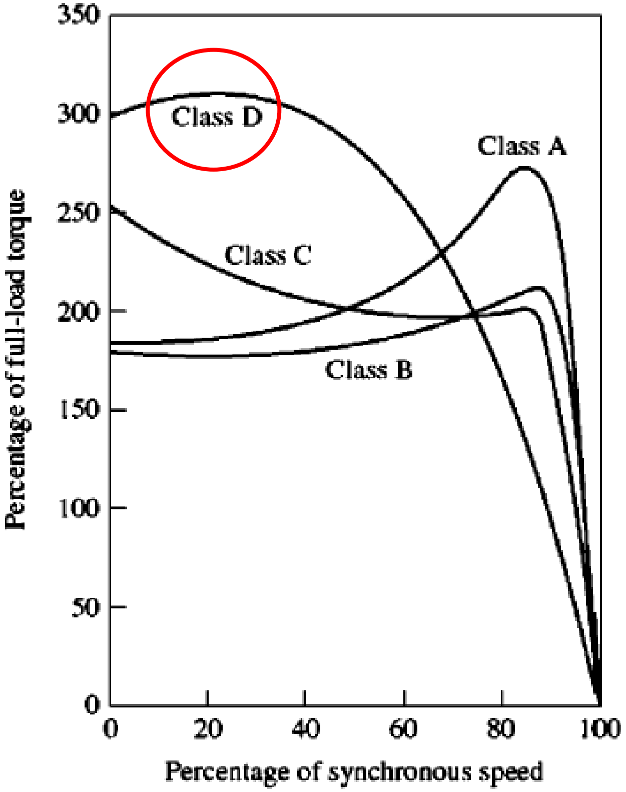
Control of motor characteristics by cage rotor design

NEMA Design Class D: small bars near the surface:

- Since the **cross-sectional area of the bars is small**, the **rotor resistance is relatively high**.
- Since the **bars are located near the stator**, the **rotor leakage reactance is still small**.
- Since **the rotor resistance is high**, this motor has a **high starting torque** and **the pullout torque occurs at low speeds (high slip)**.
- But the **efficiency** of this type of motor will be **lower** as discussed previously.



(d)



Control of motor characteristics by cage rotor design

- So, now the question must be:

- How can we join the benefits of these two different rotor classes (*NEMA Class A and Class D*) ?

- **Advantages of NEMA Class A:**
  - ☐ The pullout torque is near synchronous speed
  - ☐ High efficiency under normal operation

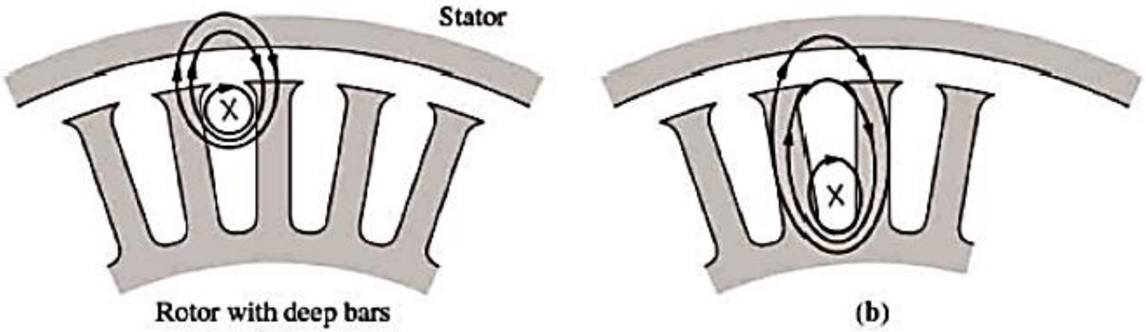
- **Advantages of NEMA Class D:**
  - ☐ The pullout torque occurs at low speeds
  - ☐ High starting torque

- The solution is to use either **deep rotor bar** or **double-cage rotor** structure.

Control of motor characteristics by cage rotor design

<u>**NEMA Design Class B: Deep rotor bars:**</u>

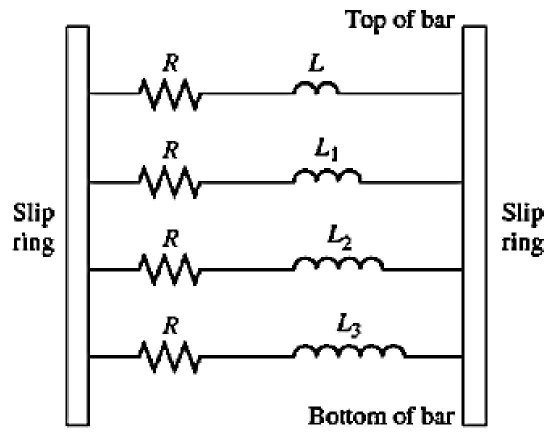
(b)



(a) (b)

rotor bar close to the stator

rotor bar far from the stator

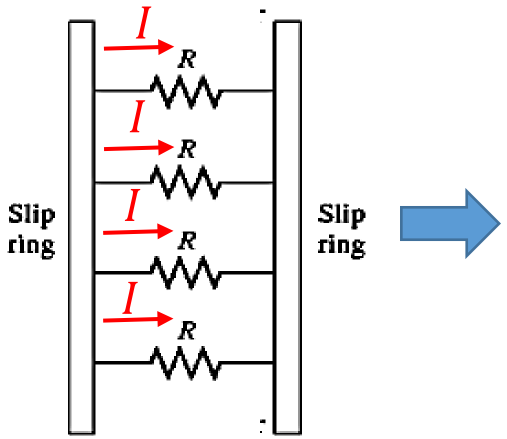


(c)

Control of motor characteristics by cage rotor design

NEMA Design Class B: Deep rotor bars:

- At **low slip (*high speed*)**, the **rotor's frequency** is **very small (fr≈0)**, and the **reactances** of all the parallel paths through the bar are **small** compared to their resistances. (**XL≈0**)
- The **impedances of all parts of the bar** are **approximately equal** (*see the figure*), so current flows through all parts of the bar equally.
- The resulting **large cross-sectional area** makes the **rotor resistance** quite small, resulting in **good efficiency** at low slips. (***Like NEMA Class A***).



$$R_{eq} = \frac{R}{4}$$

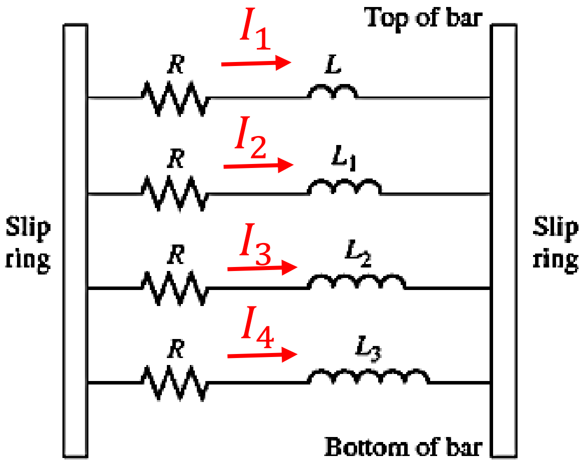
Rotor resistance becomes **very small** at **low slips**

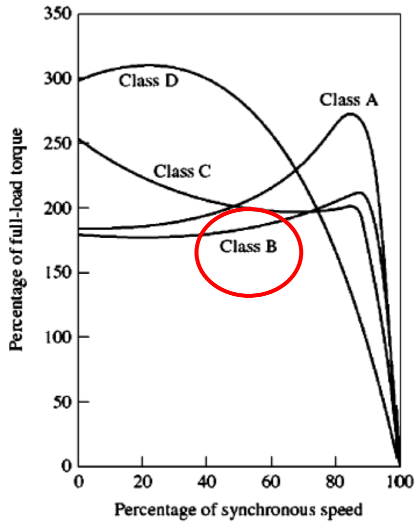
Control of motor characteristics by cage rotor design

NEMA Design Class B: Deep rotor bars:

- At **high slip** (*starting conditions*), the reactances become larger compared to the resistances in the rotor bars. Because ***fr*** is higher.
- So **major part of the rotor current** is forced to flow in the region which is **close to the top of bar**.

  $(I_1 > I_2 > I_3 > I_4)$
- Since the **effective cross section** is **lower**, the **rotor resistance** is **higher** than before. With a **high rotor resistance at starting conditions**, the **starting torque** is **relatively higher** and the **starting current** is relatively **lower**.

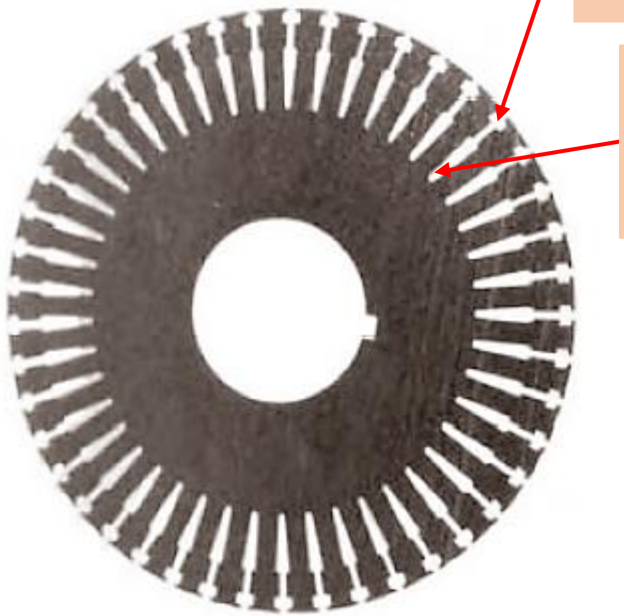




Control of motor characteristics by cage rotor design

NEMA Design Class C: Double cage rotor bars:

- It consists of a **large**, **low-resistance set of bars** buried **deeply** in the rotor and a **small**, **high-resistance set of bars** set at the **rotor surface**.
- It is similar to the deep bar rotor, except that the difference between low-slip and high-slip operation is even more exaggerated.
- At **starting conditions**, **only the small bar is effective**, and the **rotor resistance is quite high**. This high resistance results in a **large starting torque**.
- However, at normal operating speeds, **both bars are effective**, and **the resistance is almost as low as** in a deep-bar rotor. This greatly improves the efficiency under normal operation.



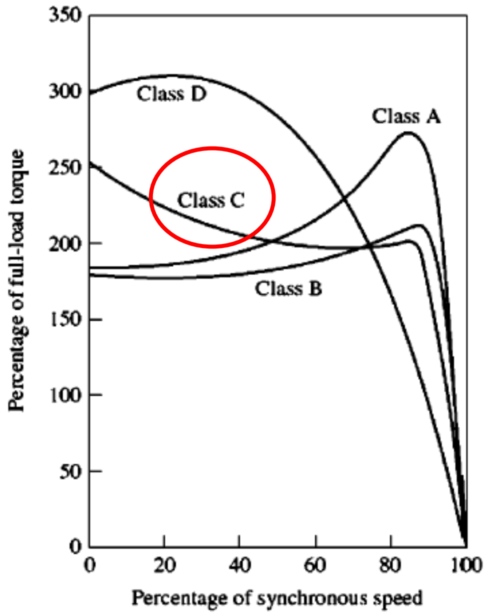
Small and **high-resistance** set of bars are located at the rotor surface

Large and **low-resistance** set of bars are located inside

Control of motor characteristics by cage rotor design

NEMA Design Class C: Double cage rotor bars:

- **Double-cage rotors** have the **disadvantage** that they are **more expensive** than the other types of cage rotors.
- But they are **cheaper** than **wound-rotor designs**.
- They allow the following features:
  - ❑ High starting torque
  - ❑ Low starting current
  - ❑ Good efficiency at normal operating conditions



Induction motor design classes

- **DESIGN CLASS A:**
  - Standard motors with a normal starting torque, normal starting current.
  - Full-load slip is less than 5 %.
  - Pullout torque is 200 to 300 % of the full-load torque and occurs at a low slip (*less than 20 %*).
  - The starting torque is at least the rated torque for larger motors and is 200 % or more of the rated torque for smaller motors.
  - Typical applications for these motors are driving fans, blowers, pumps, lathes, and other machine tools.

- **DESIGN CLASS B:**
  - They have a normal starting torque, a lower starting current, and low slip.
  - These motors produce about the same starting torque as the class A motor with about 25 % less current.
  - The pullout torque is greater than or equal to 200 % of the rated load torque, but less than that of the class A design because of the increased rotor reactance.
  - Rotor slip is still relatively low (less than 5 %) at full load.
  - Applications are similar to those for design A, but design B is preferred because of its lower starting-current requirements.

Induction motor design classes

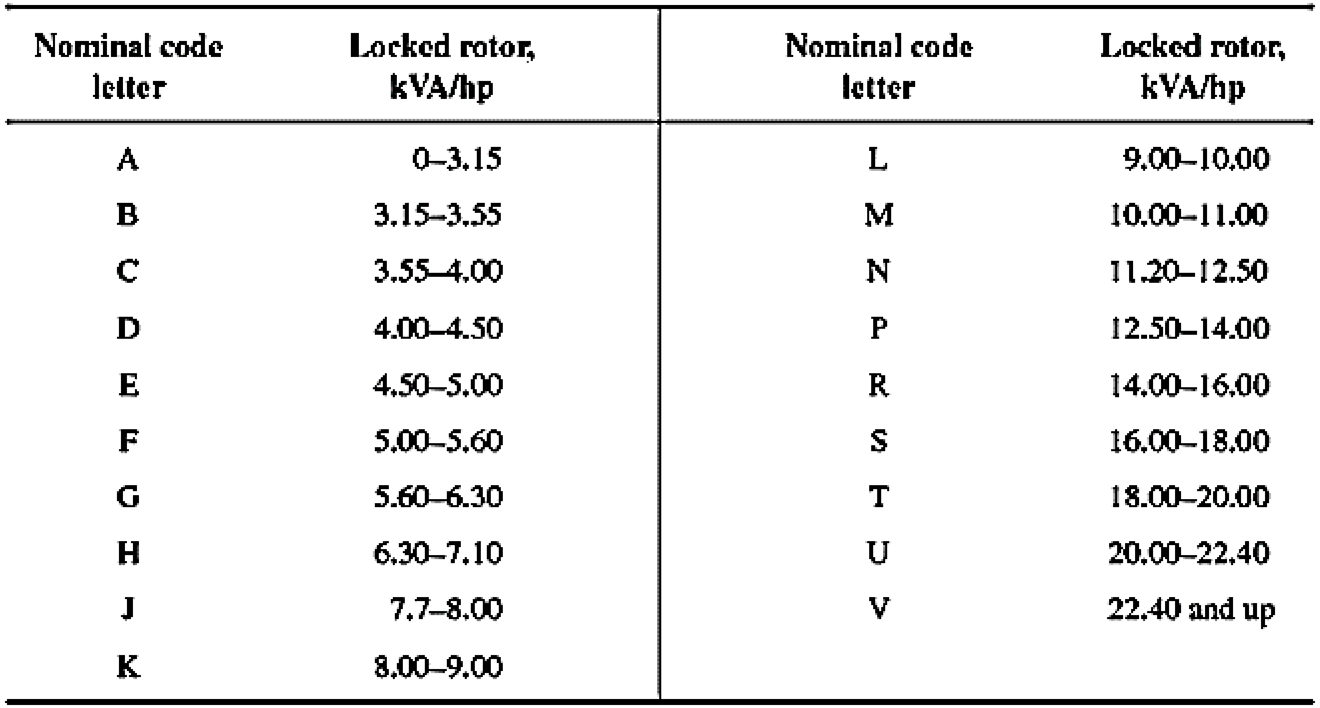
- **DESIGN CLASS C:**
  - These motors have a high starting torque with low starting currents and low slip (less than 5 %) at full load.
  - The pullout torque is slightly lower than that for class A motors, while the starting torque is up to 250 % of the full-load torque.
  - These motors are built from double-cage rotors, so they are more expensive than motors in the previous classes.
  - They are used for high-starting-torque loads, such as loaded pumps, compressors, and conveyors.

- **DESIGN CLASS D:**
  - They have a very high starting torque (275 % or more of the rated torque) and a low starting current.
  - They also have a high slip at full load.
  - They are essentially ordinary class A induction motors, but with the rotor bars made smaller and with a higher-resistance material.
  - The high rotor resistance shifts the peak torque to a very low speed. The highest torque occurs at zero speed (100 % slip).
  - Full-load slip is quite high because of the high rotor resistance. It is typically7 to 11 %.
  - These motors are used in applications requiring the acceleration of extremely high-inertia-type loads, such as large flywheels used in punch presses or shears.

Starting induction motors

- **Induction motors** can be started by **simply connecting** them to the **three-phase power source**.
- Although starting is easy, they require **high starting currents** in general.
- **This high starting current** may cause **huge voltage dip** in the **power system's voltage**.
- For **wound-rotor induction motors**, starting current can be limited by **inserting an extra resistance** in the rotor circuit during starting. This extra resistance also increases the starting torque (*mentioned previously*).
- For **squirrel-cage induction motors**, the starting current can be limited by other methods (*will be discussed soon*).
- The starting current of **squirrel-cage induction motors** depends on the **motor's rated power** and the **effective rotor resistance** at starting conditions.
- To estimate the starting current, all squirrel-cage motors have a **starting code letter on their nameplates**.

Starting induction motors



Nominal code letter	Locked rotor, kVA/hp	Nominal code letter	Locked rotor, kVA/hp
A	0–3.15	L	9.00–10.00
B	3.15–3.55	M	10.00–11.00
C	3.55–4.00	N	11.20–12.50
D	4.00–4.50	P	12.50–14.00
E	4.50–5.00	R	14.00–16.00
F	5.00–5.60	S	16.00–18.00
G	5.60–6.30	T	18.00–20.00
H	6.30–7.10	U	20.00–22.40
J	7.7–8.00	V	22.40 and up
K	8.00–9.00		

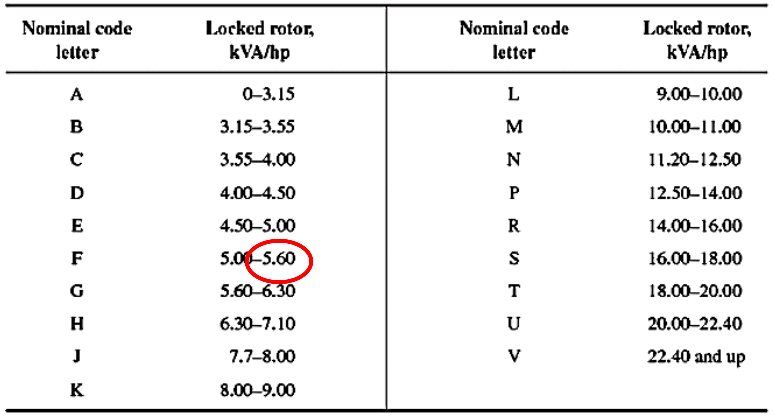
Table of NEMA code letters indicating the starting kilovolt amperes per horsepower of rating for a motor. Each code letter extends up to but does not include the lower bound of the next higher class.

Starting induction motors

- How is this table used to calculate the starting current of a given induction motor ?

Example: What is the starting current of a **15-hp**, **208-V**, **code letter-F**, three-phase induction motor?

Solution:



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C	3.55–4.00	N	11.20–12.50
D	4.00–4.50	P	12.50–14.00
E	4.50–5.00	R	14.00–16.00
F	5.00–5.60	S	16.00–18.00
G	5.60–6.30	T	18.00–20.00
H	6.30–7.10	U	20.00–22.40
J	7.7–8.00	V	22.40 and up
K	8.00–9.00		

$$S_{\text{start}} = \text{(rated horsepower)(code letter factor)}$$

$$S_{\text{start}} = (15\ \text{hp})(5.6) = 84\ \text{kVA}$$

The starting current is thus

$$I_L = \frac{S_{\text{start}}}{\sqrt{3}V_T}$$

$$= \frac{84\ \text{kVA}}{\sqrt{3}(208\ \text{V})} = 233\ \text{A}$$

Starting induction motors

- For **squirrel-cage induction motors**, the **terminal voltage of the motor** can be **reduced** during starting by using **autotransformers**.

- Then the **terminal voltage of the motor** is **increased gradually** as motor speeds up to the rated condition.

- Since the **induced torque** is **directly proportional to the square of the terminal voltage**, the **starting torque decreases as the square of the applied voltage**.

- Therefore, **only a certain amount of current reduction can be done** if the motor is required **to start under load** (*with a shaft load attached*)

$$\boxed{\tau_{\text{ind}} = \frac{3V_{\text{TH}}^2 R_2/s}{\omega_{\text{sync}}[(R_{\text{TH}} + R_2/s)^2 + (X_{\text{TH}} + X_2)^2]}}$$

Speed control of induction motors

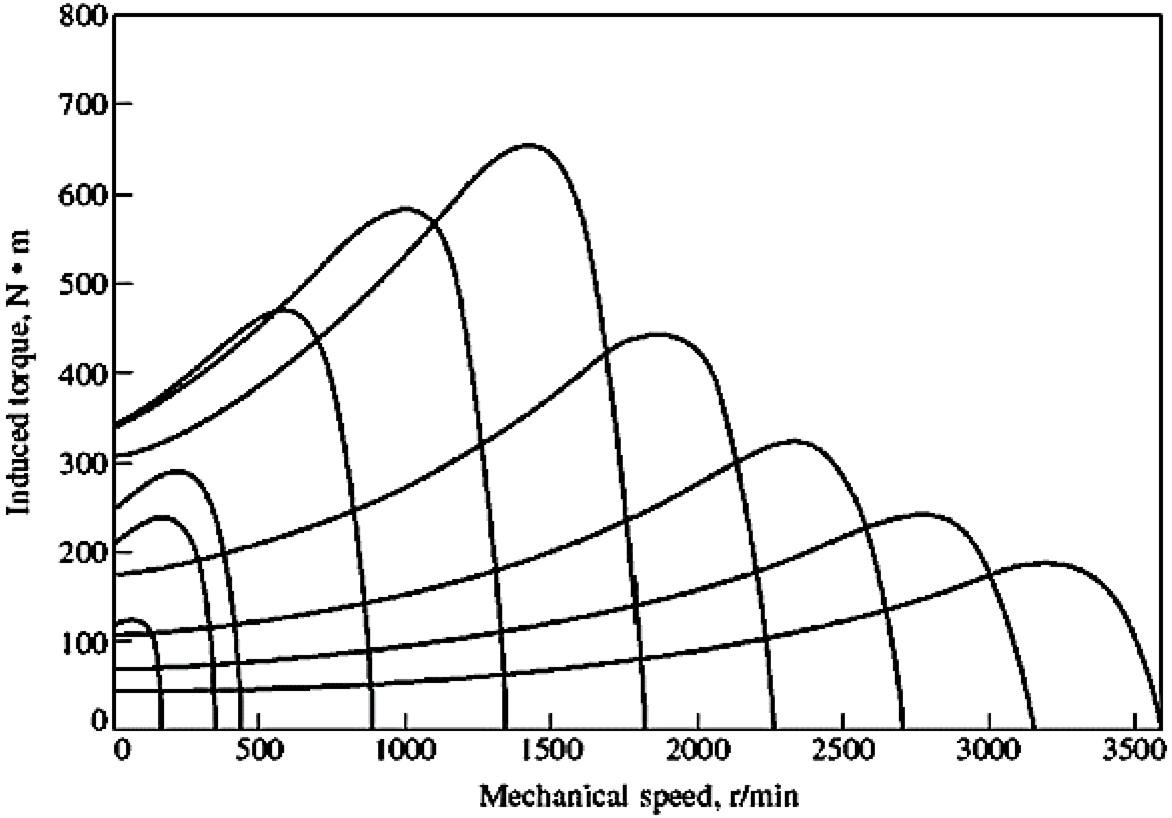
- The **synchronous speed** (*the speed of rotating magnetic field of the stator*) of an **induction motor** is given as:

$$n_{syn} = \frac{120 f_e}{P}$$

- Since **the rotor is chasing the rotating stator magnetic field**, the **rotor speed** can be controlled by **changing $\boldsymbol{n_{syn}}$**

- So, there are **two techniques** in order to **change the speed** of an **induction motor**:
  - Change f_e (Use an inverter - *variable speed drive*, *very common method used today*)
  - Change pole number P. (*Very old method, not used today anymore*)

Speed control of induction motors



The torque-speed characteristics of an induction motor for different stator frequencies.

Speed control of induction motors

- When running at speeds **below the base speed** of the motor, it is necessary **to reduce the terminal voltage applied to the stator linearly with decreasing stator frequency**.
- This process is called “***derating***”.
- If **derating** it is not done, the **steel in the core of the induction motor will saturate** and **excessive magnetization currents will flow in the machine**.
- The motor generates **low order current harmonics** which **decreases the power quality of the power system.**
- The ***proof of derating*** is given below:

$$v(t) = -N\frac{d\phi}{dt}$$ *(Faraday’s Law)*

If a voltage $v(t) = V_M \sin(wt)$ is applied to the core: the resulting flux will be:

$$\phi(t) = \frac{1}{N_P}\int v(t)\,dt$$

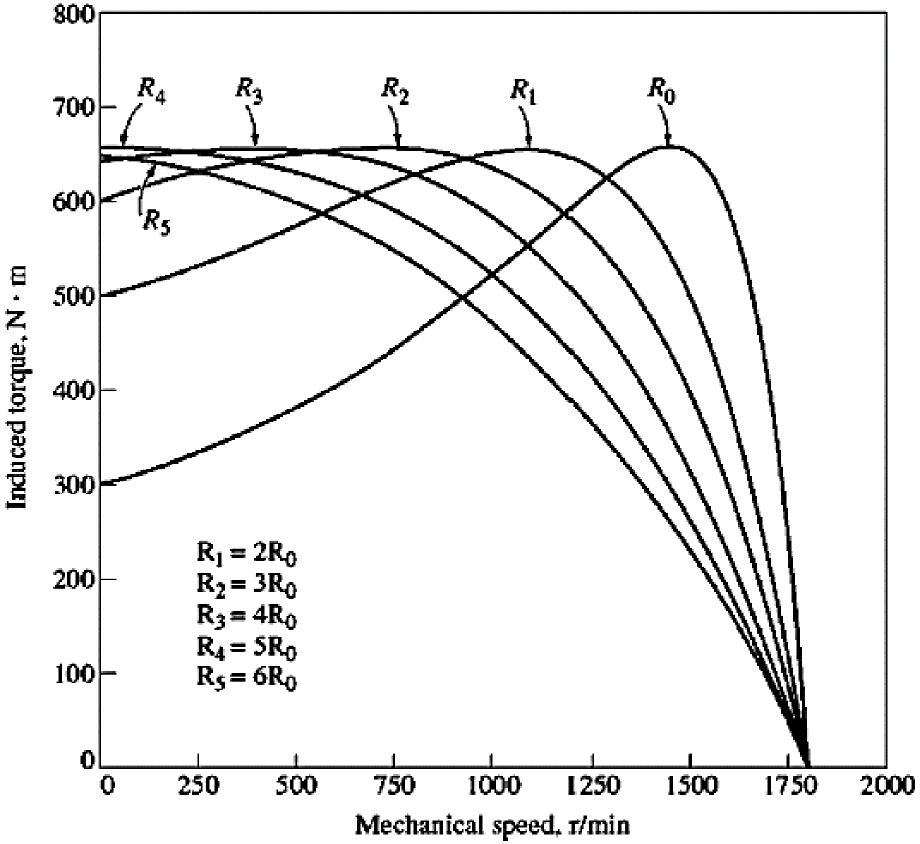
$$= \frac{1}{N_P}\int V_M \sin \omega t\,dt$$

$$\boxed{\phi(t) = -\frac{V_M}{\omega N_P}\cos \omega t}$$

For example, if ***w*** is reduced by 10%, V_M should also be reduced by 10% **to keep the flux level constant** in the core

Speed control of induction motors

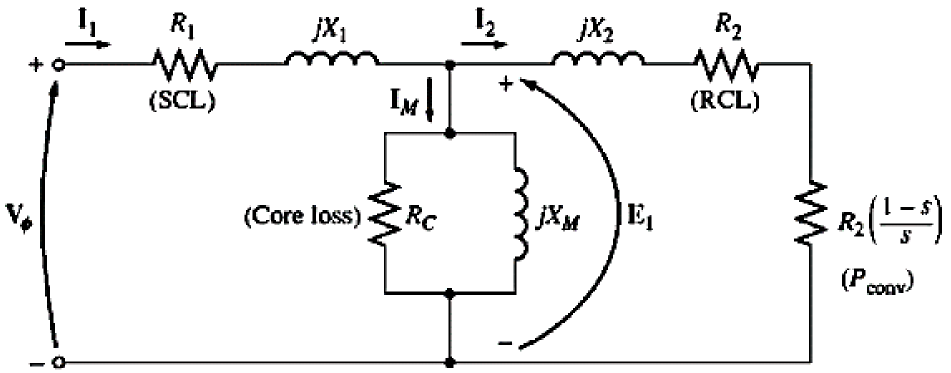
- If the motor is **wound-rotor**, speed control can also be done by **adding an extra resistance to rotor circuit**.



Speed control by varying the rotor resistance of a wound-rotor induction motor

Determining circuit model parameters

- The **equivalent circuit of an induction motor** is a **very useful tool for analyzing the motor**.



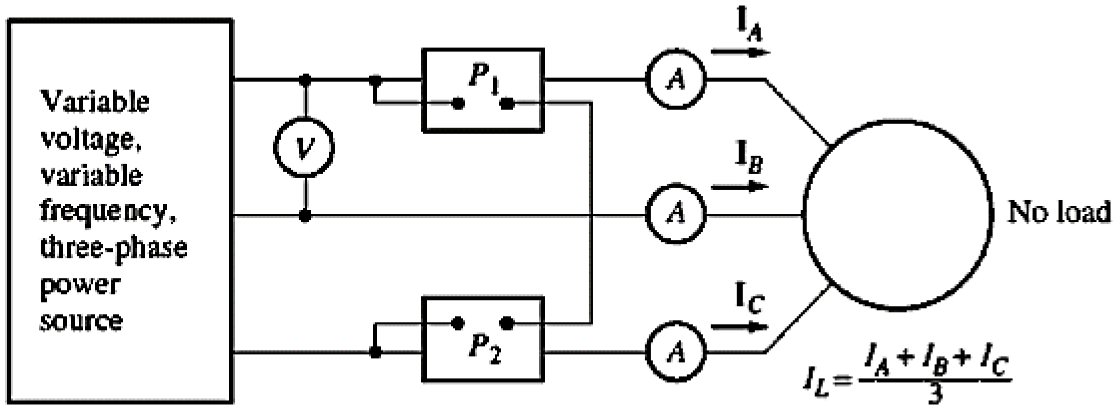
- The equivalent circuit model parameters can be found (*estimated*) by making **some tests** on the induction motor.

- These tests are as follows: (*These are similar to the tests made on a transformer*)
  - ➢ No-load test
  - ➢ DC test for stator resistance
  - ➢ Locked-rotor (Blocked-rotor) test

No-Load test

- **No-load test** of an induction motor is done **to measure the rotational losses of the motor** and **provides information about its magnetization current**.

- The test set-up is shown in the figure.



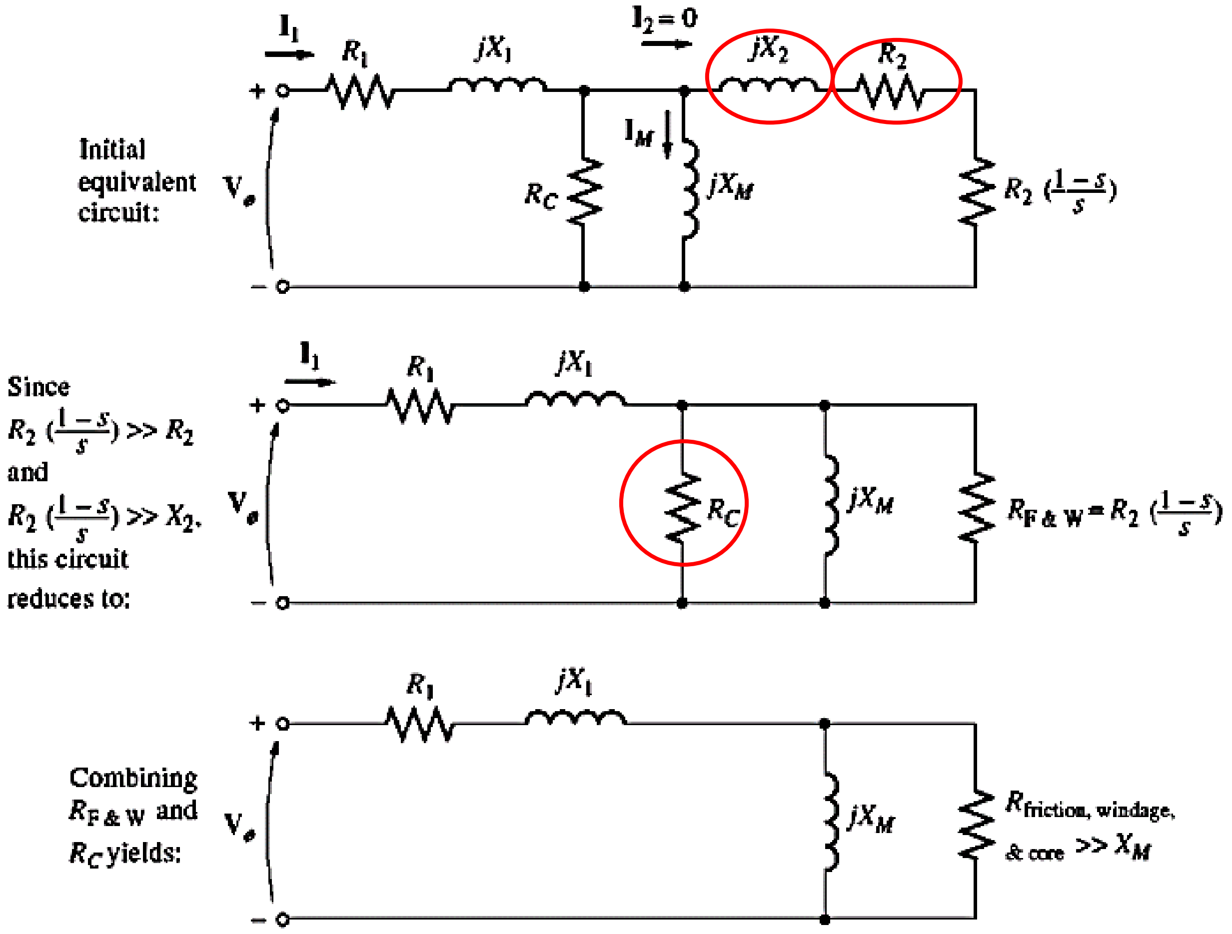
The motor is freely rotating without any load

- The **only load** on the motor is the **friction and windage losses**, so all ***Pconv*** is consumed by **mechanical losses**.

- The **slip** of the motor is **very small** (possibly as small as ***0.001 or less***).

No-Load test

- The **equivalent circuit** of the motor **under no-load test** is shown in the figure.



- X_2 is negligible (at no load, s≈0 and $f_R \cong 0$)
- $R_2\left(\frac{1-s}{s}\right) \gg R_2$ (at no load, s≈0)
- Since $R_2\left(\frac{1-s}{s}\right)$ is very high, $I_2 \cong 0$)

The equivalent circuit of the motor under no-load test

No-Load test

- Under no-load conditions, the **stator copper losses** are given by:

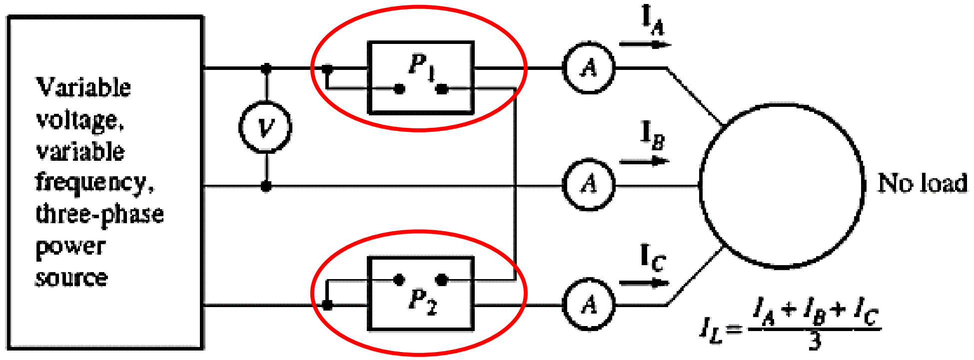
$$\boxed{P_{\mathrm{SCL}} = 3I_1^2R_1}$$

- So the **sum of wattmeter readings** will be equal to:

$$P_1 + P_2 = P_{\mathrm{in}} = P_{\mathrm{SCL}} + P_{\mathrm{core}} + P_{\mathrm{F\&W}} + P_{\mathrm{misc}}$$
$$= 3I_1^2R_1 + P_{\mathrm{rot}}$$

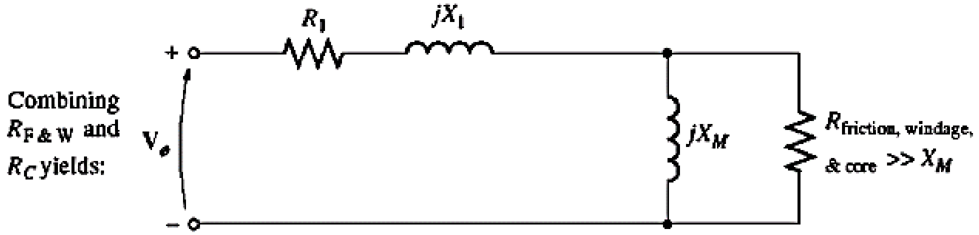
where P_{rot} is the rotational losses of the motor:

$$P_{\mathrm{rot}} = P_{\mathrm{core}} + P_{\mathrm{F\&W}} + P_{\mathrm{misc}}$$



No-Load test

- Since X_1 is much greater than the series resistance R_1
- And, X_M is much smaller than the parallel resistance,

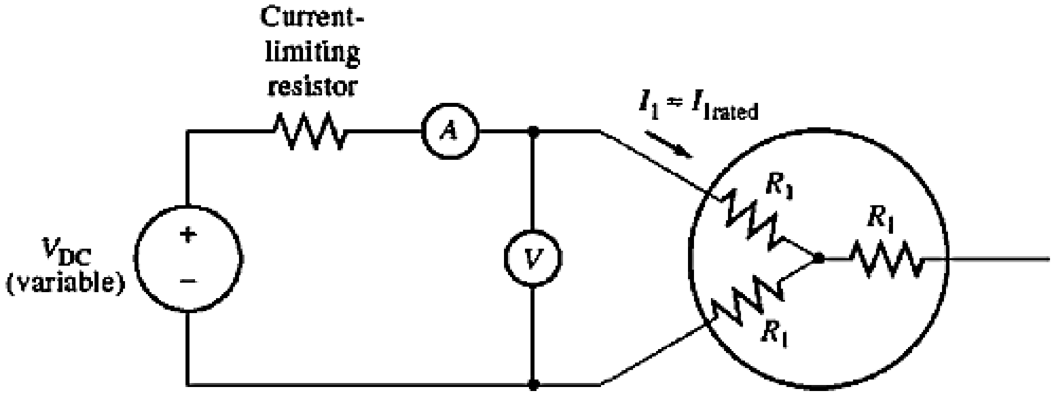


- The equivalent circuit of the induction motor <u>**under no load conditions**</u> is approximately a **purely inductive circuit**:
- So the **equivalent impedance of the motor** <u>**under no load conditions**</u> can be written as:

$$\boxed{|Z_{eq}| = \frac{V_\phi}{I_{1,nl}} \approx X_1 + X_M}$$

DC test for stator resistance

- **DC test** is applied to **stator windings to measure per-phase stator resistance**.
- To perform the test, the **current in the stator windings** is **adjusted to the rated value**.
- Since the applied voltage is DC, there will be **no induced voltage in the rotor circuit and hence no rotor current will flow** (*Faraday's Law*).
- Also, **stator reactance becomes zero** at DC (**f = 0Hz**).
- If **the stator is Y-connected**, the stator resistance can be estimated using the following equation:

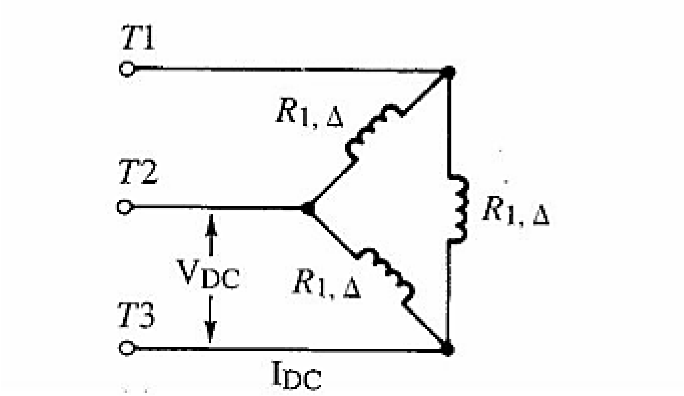


$$\boxed{R_1 = \frac{V_{DC}}{2I_{DC}}}$$

V_{DC} is the **voltmeter** reading
I_{DC} is the **ampermeter** reading

DC test for stator resistance

- If **the stator is Δ-connected**, the stator resistance can be estimated using the following equation:



$$R_{DC} = \frac{R_{1\Delta} \cdot 2R_{1\Delta}}{R_{1\Delta} + 2R_{1\Delta}} = \frac{2}{3} R_{1\Delta}$$

$$\frac{V_{DC}}{I_{DC}} = R_{DC} = \frac{2}{3} R_{1\Delta}$$

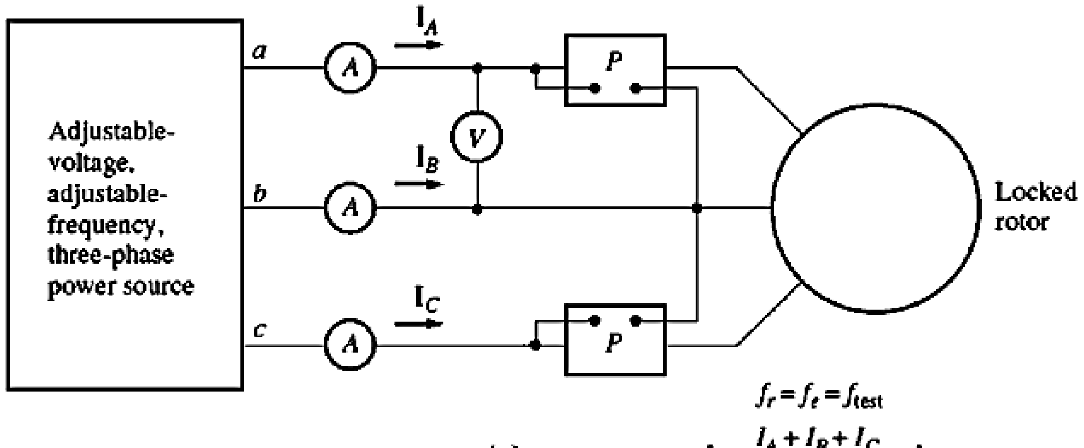
V_{DC} is the **voltmeter** reading
I_{DC} is the **ampermeter** reading

Locked-rotor test

- This test is also called «**blocked-rotor test**».

- This test corresponds to the **short-circuit** test on a transformer.

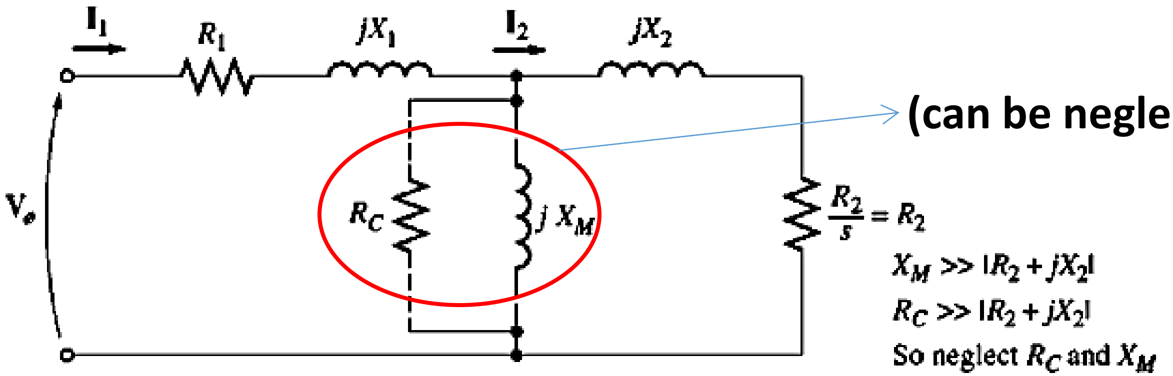
- In this test, **the rotor is locked** so that **it cannot move**, a variable AC voltage is applied to the stator, and the current flow is **adjusted to be approximately full-load value**, and the resulting voltage, current, and power are measured.



(a) $I_L = \frac{I_A + I_B + I_C}{3} = I_{L\,rated}$

Locked-rotor test

- Since **the rotor is not moving**, **the slip s = 1**, and so the rotor resistance **R2/s** becomes equal to **R2** (*a very small value*)



(can be neglected)

$X_M >> |R_2 + jX_2|$
$R_C >> |R_2 + jX_2|$
So neglect R_C and X_M

- At normal operating conditions, the **slip of most motors** is **only 2 to 4 percent**, and the **resulting rotor frequency** is in the range of **1 to 3 Hz**.

- Becasue of this situation, to obtain realistic results, typically **the frequency of the applied voltage** is **adjusted to be 25 % or less of the rated frequency**.

Locked-rotor test

- Under **locked-rotor conditions**, the **total input power of the motor** is given by:

$$P = \sqrt{3} V_T I_L \cos\theta$$

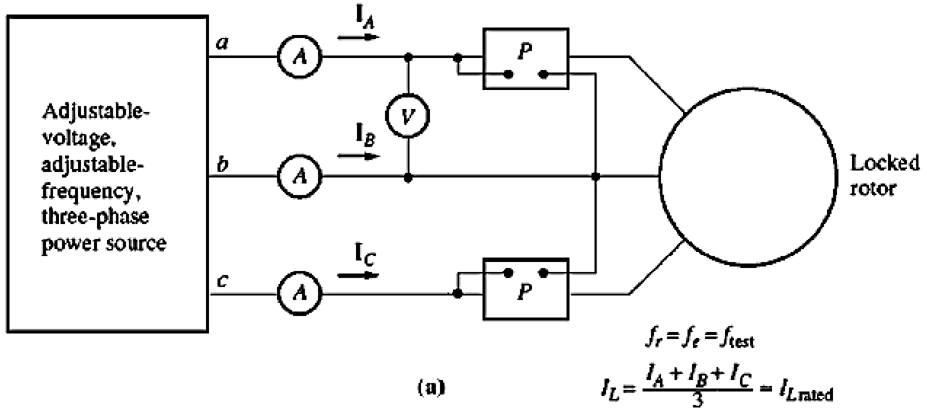
so the locked-rotor power factor can be found as

$$\boxed{\text{PF} = \cos\theta = \frac{P_{\text{in}}}{\sqrt{3} V_T I_L}}$$

and the impedance angle θ is just equal to $\cos^{-1}$ PF.

- The magnitude of the total impedance in the motor circuit:

$$\boxed{|Z_{\text{LR}}| = \frac{V_\phi}{I_1} = \frac{V_T}{\sqrt{3} I_L}}$$



Locked-rotor test

- **Locked-rotor impedance** can be written as:

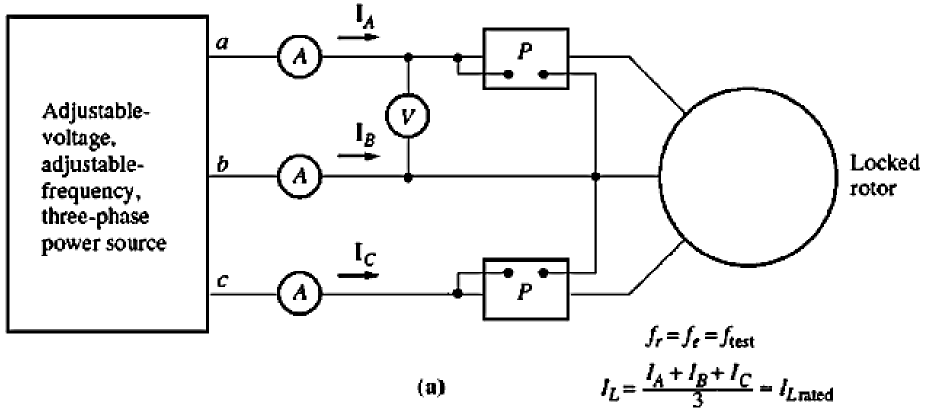
$$Z_{LR} = R_{LR} + jX'_{LR}$$
$$= |Z_{LR}|\cos\theta + j|Z_{LR}|\sin\theta$$

where,

$$\boxed{R_{LR} = R_1 + R_2}$$

$$X'_{LR} = X'_1 + X'_2$$

where **X1’** and **X2’** are the **stator** and **rotor reactances** at the **test frequency**



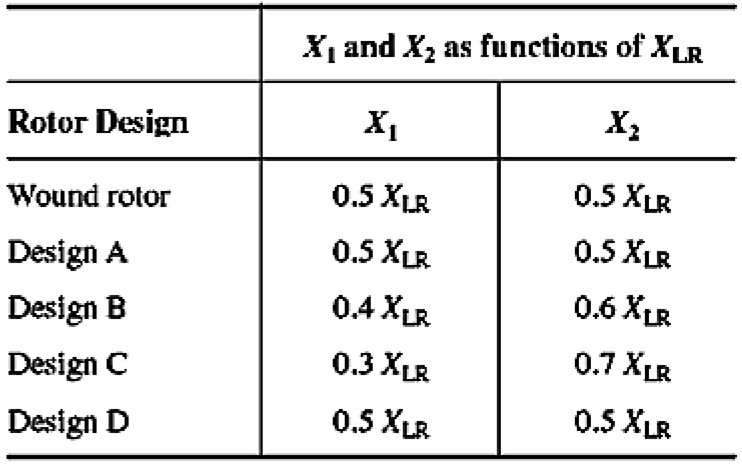
- Since the **reactance is directly proportional to the frequency**, the total equivalent reactance at the nominal operating frequency can be found as:

$$\boxed{X_{LR} = \frac{f_{rated}}{f_{test}} X'_{LR} = X_1 + X_2}$$

Locked-rotor test

- Unfortunately, there is **no simple way to separate** the contributions of the stator and rotor reactances from each other.

- Over the years, experience has shown that motors of certain design types have certain proportions between the rotor and stator reactances. The following table summarizes this experience.



	X_1 and X_2 as functions of X_{LR}	
Rotor Design	X_1	X_2
Wound rotor	$0.5\ X_{LR}$	$0.5\ X_{LR}$
Design A	$0.5\ X_{LR}$	$0.5\ X_{LR}$
Design B	$0.4\ X_{LR}$	$0.6\ X_{LR}$
Design C	$0.3\ X_{LR}$	$0.7\ X_{LR}$
Design D	$0.5\ X_{LR}$	$0.5\ X_{LR}$

Rules of thumb for dividing rotor and stator circuit reactance.

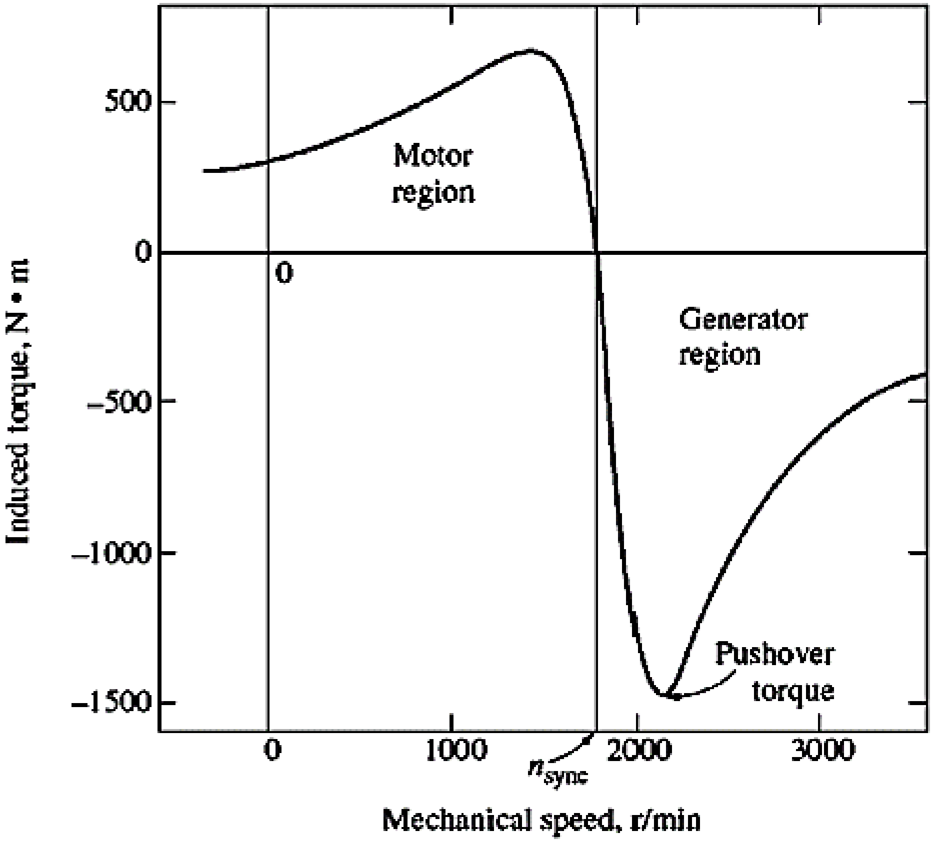
Induction generator

- If an **induction motor** is driven at a **speed greater than synchronous speed** by an **external prime mover (***such as DC motor or wind turbine, and etc…***)**, the **direction of its inducted torque will reverse** and it acts as a **generator**.

- As the torque applied to its shaft by the prime mover increases, the amount of power produced by the induction generator increases.

- There is a **maximum possible induced torque** in the **generator region**, which is known as "**pushover torque**" of the generator.

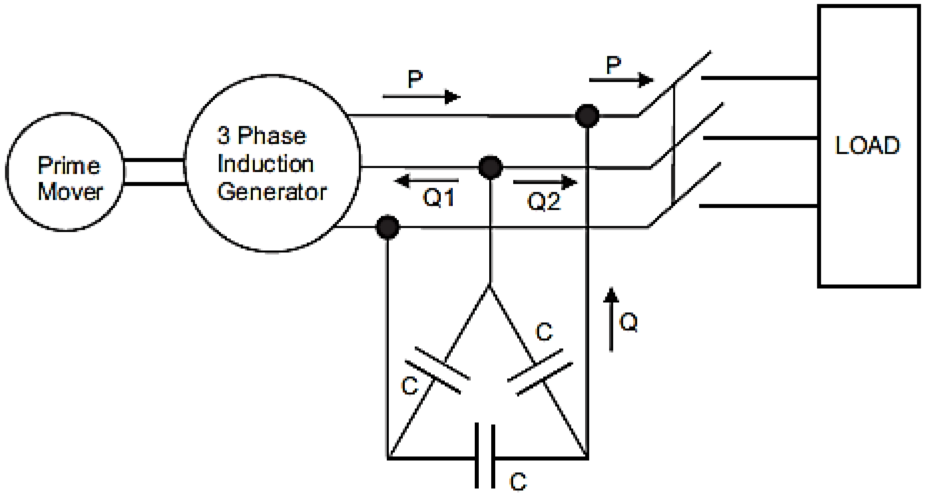
- If a **prime mover** applies **a torque greater than the pushover torque** to the shaft of an induction generator, **the generator will overspeed**.



Induction generator

Disadvantages of induction generator:

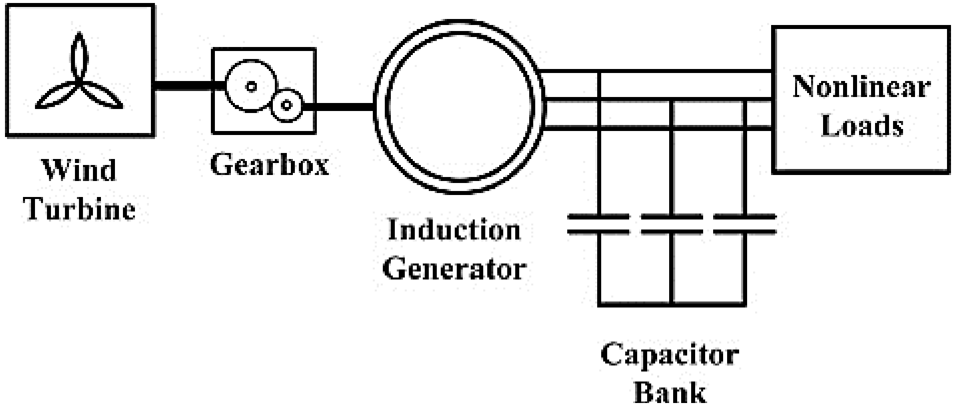
- An **induction generator** **does not have** a **separate field circuit** so **it cannot produce its own reactive power**.
- In fact, it **consumes reactive power** (*remember its equivalent circuit having many reactances*), and an **external source of reactive power must be connected to it at all times**.
- The required reactive power is usually supplied by **delta-connected capacitor banks.** (*Refer to the figure*)
- An **induction generator** **cannot control** its **output voltage** unlike a **synchronous generator.**
- The **terminal voltage** of the **induction generator** is controlled by the **external reactive power source**.
- If the **induction generator** is connected to an **infinite bus**, its **terminal voltage** is maintained by the **infinite bus**.



Induction generator

Advantages of induction generator:

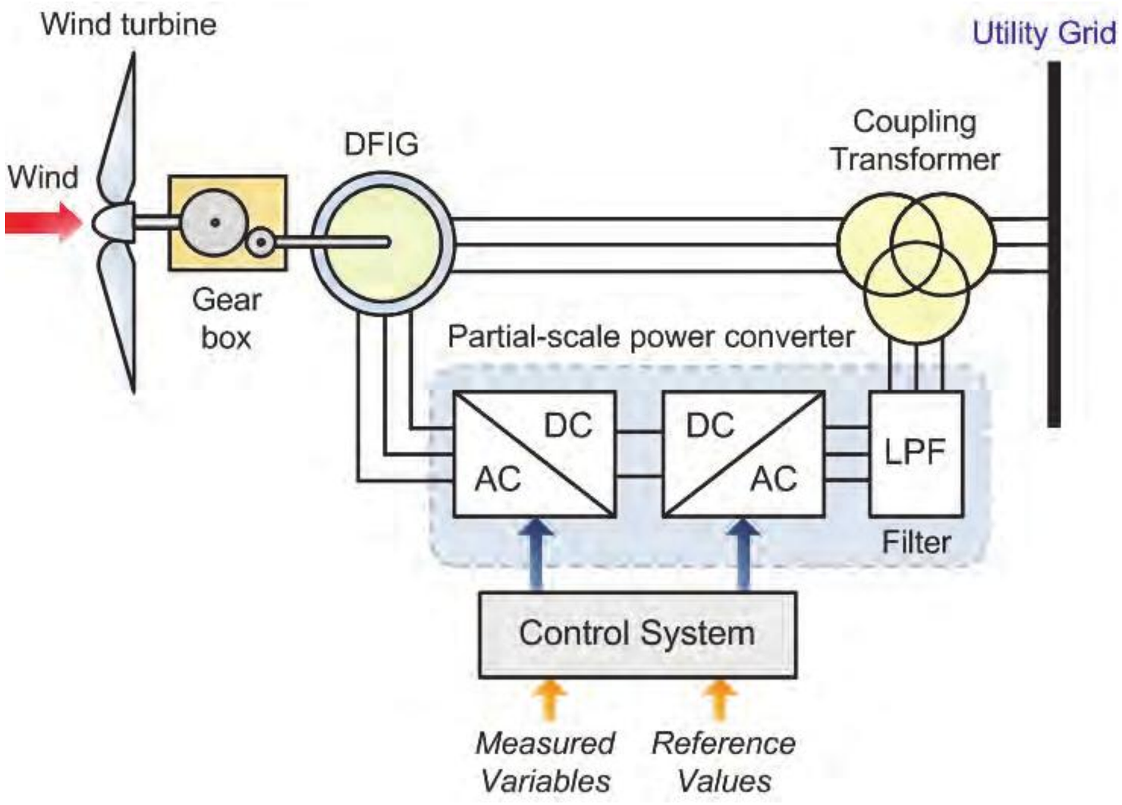
- The **induction generator** is a **very simple generator**.
- It **does not require** a **separate field circuit**.
- It **does not require** **synchronization procedures** unlike a **synchronous generator**.
- It **does not have to be** driven **continuously** at a **fixed speed** unlike a **synchronous generator**.
- As long as the **machine's speed** is some value **greater than its synchronous speed**, the machine will act as a **generator**.
- Self-excited induction generators (SEIG) are easy and cheap to apply for wind energy applications (*Refer to the figure*)



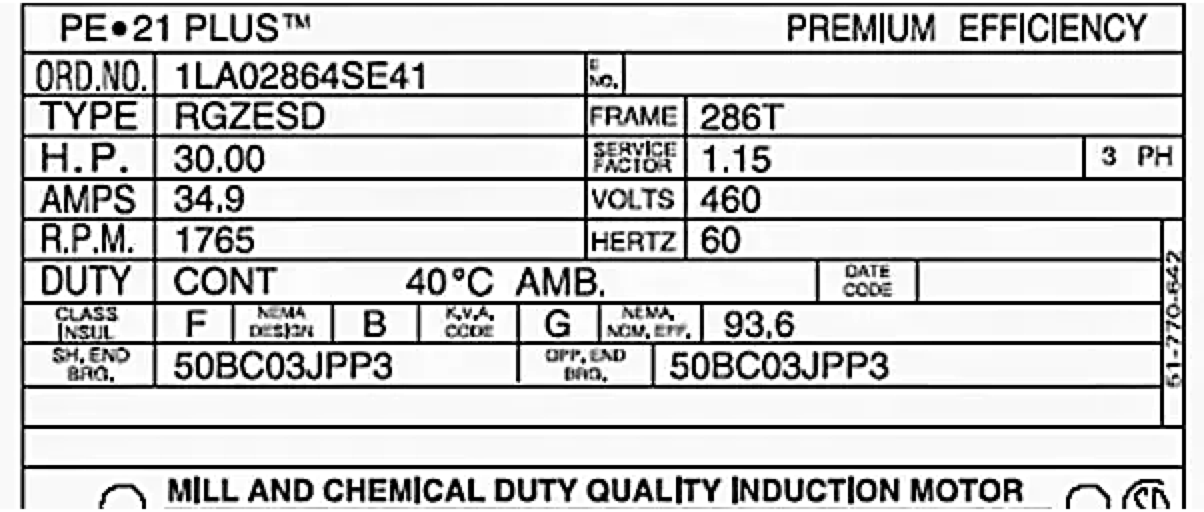
Source: Saber Mohamed Saleh Salem, "Study of wind turbine based self-excited induction generator under nonlinear resistive loads as a step to solve the Egypt electricity crisis", Computers & Electrical Engineering, Vol. 51, 2016, pp. 1-11.

Induction generator

- **Doubly fed induction generators** (**DFIG**) are very popular nowadays.
- **DFIGs** can operate at **variable wind speeds** (*sub- or super synchronously*). However, **SEIGs** operate at only **constant wind speed.**
- **DFIGs** can **capture wind power better** than **SEIGs**
- **DFIGs** can control **reactive and active power separately**. (*This function is not available in SEIGs*).
- **DFIGs** are **more expensive** than **SEIGs**, because they need power converters (*Refer to the figure*).
- However **DFIGs** are preferred more than **SEIGs** because of **many advantages**, some of which are listed above.



Induction motor ratings



PE•21 PLUS™ PREMIUM EFFICIENCY

ORD.NO.	1LA02864SE41	E NO.					
TYPE	RGZESD	FRAME	286T				
H.P.	30.00	SERVICE FACTOR	1.15	3 PH			
AMPS	34.9	VOLTS	460				
R.P.M.	1765	HERTZ	60				
DUTY	CONT 40°C AMB.	DATE CODE					
CLASS INSUL.	F	NEMA DESIGN	B	K.V.A. CODE	G	NEMA NOM. EFF.	93.6
SH. END BRG.	50BC03JPP3	OPP. END BRG.	50BC03JPP3				

61-770-642

MILL AND CHEMICAL DUTY QUALITY INDUCTION MOTOR

The nameplate of a typical induction motor

END OF CHAPTER 4

INDUCTION MOTORS