CHAPTER 13

TURBOMACHINERY

13.1 INTRODUCTION

In nature, energy may exist in various forms. **Hydraulic energy** is the type of energy possessed by a fluid and it may be in the form of kinetic, pressure, potential, strain or thermal energy. **Mechanical energy** is the type of energy associated with the moving or rotating parts of machines which transmit power. The basic purpose of fluid machinery is to convert hydraulic energy to mechanical energy or mechanical energy to hydraulic energy.

The first classification of fluid machinery is based on the direction of energy transfer. In the first category, the input is the hydraulic energy and it is converted into the mechanical energy in the form of a rotating shaft or a moving part of a machine. Therefore, work is done by the fluid and this type of fluid machinery is known as **power producing fluid machinery**. In the other category, the input is the mechanical energy and it is transformed into the hydraulic energy in the form of a moving fluid, sometimes at a higher pressure and temperature. In this case, work is done on the fluid and such machines are referred to as **power absorbing fluid machinery**. This type of classification is shown in Figure 13.1.

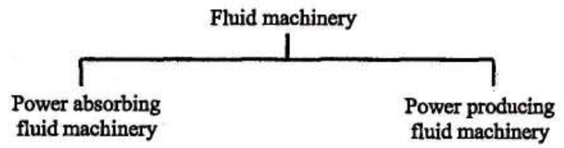


Figure 13.1 Classification of fluid machinery according to the direction of energy transfer

Another classification of fluid machinery is according to the principle of mechanical operation, as indicated in Figure 13.2. In this case, fluid machinery can be divided into two main categories: positive displacement fluid machinery, which are referred to as the static type and turbomachines, which are referred to as the dynamic type. In **positive displacement** machines, the fluid is first drawn or forced into a finite space bounded by mechanical parts and is sealed in by some mechanical means. The fluid is then forced out or allowed to flow out from the space and the cycle is repeated. As a result, the flow is intermittent and the flow rate depends on the dimensions of the space and frequency with which the space is filled and emptied. The developed pressure and work done are basically due to the static forces rather than the dynamic forces. Typical examples include the common tire pump and human heart in which the work is done by the device on the fluid. Internal combustion engine is another example to positive displacement fluid machinery in which the work is done by the fluid. In **turbomachines,** the fluid flows freely between the inlet and outlet of the machine without any intermittent sealing of the fluid. All of these turbomachines have a freely and continuously rotating part known as a **runner, impeller** or **rotor**, which allows uninterrupted flow of fluid through it. Therefore, the energy transfer between the rotor and the fluid is continuous as a result of the rate of change of angular momentum. Rotation of the rotor produces dynamic effects that either add energy to the fluid or remove energy from the fluid.

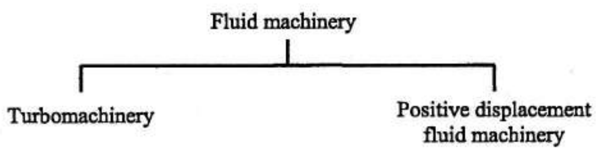


Figure 13.2 Classification of fluid machinery according to the principle of mechanical operation

It is possible to classify the turbomachines from several different viewpoints. The first type of classification is based on the type of the working fluid, as shown in Figure 13.3. The power absorbing turbomachines operating with an incompressible working fluid are referred to as **pumps** when the aim is to increase the pressure of the liquid and **propellers** when the aim is to increase the velocity of the liquid. The power absorbing turbomachines operating with a compressible fluid are referred with several different names. They are referred to as **fans,** if they impart motion to a gas with a small change in the pressure, **blowers**, if they impart substantial velocity and pressure to a gas and

compressors, if they transmit power to a gas to obtain high pressure with a small change in velocity. The power producing turbomachines operating with an incompressible fluid are referred to as **Pelton wheels,** if the available kinetic energy of the fluid is converted to the mechanical energy and **hydraulic turbines,** if the available pressure energy of the fluid is converted to the mechanical energy. In the case of power producing turbomachines operating with a compressible working fluid, they are known as **gas turbines** or **steam turbines,** if mechanical energy is obtained from the available pressure energy of the fluid and **windmills,** if mechanical energy is obtained from the available kinetic energy of the fluid.

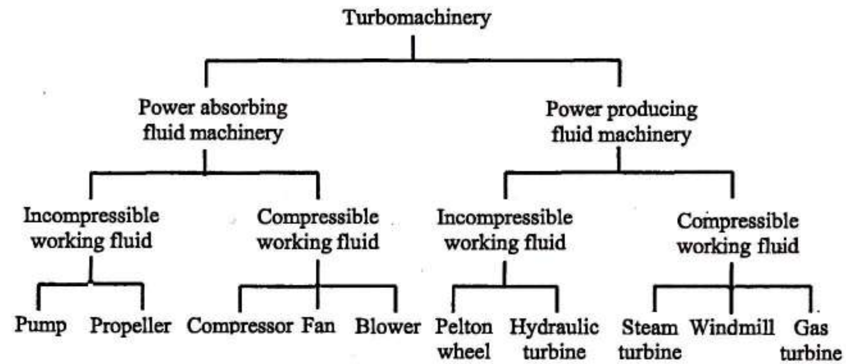


Figure 13.3 Classification of turbomachinery based on the type of the working fluid

The direction of fluid flow with respect to the plane of rotation of the impeller distinguishes the different classes of turbomachines as axial, mixed and radial flow machines. Both pumps and turbines can be axial flow, radial flow or mixed flow.

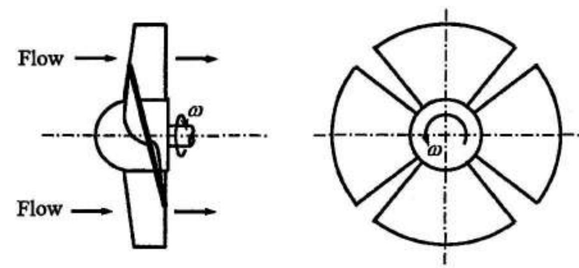


Figure 13.4 An axial flow impeller

In axial flow turbomachines, the direction of fluid flow is along the axis of rotation and perpendicular to the impeller, as shown in Figure 13.4.

In radial flow pumps, sometimes referred to as centrifugal pumps, although the fluid approaches the impeller axially, it turns at the inlet of the pump so that the flow in the impeller is in the plane of rotation of the impeller. A typical radial flow pump impeller is shown in Figure 13.5. However, in radial flow turbines, the fluid enters the impeller radially and leaves it axially.

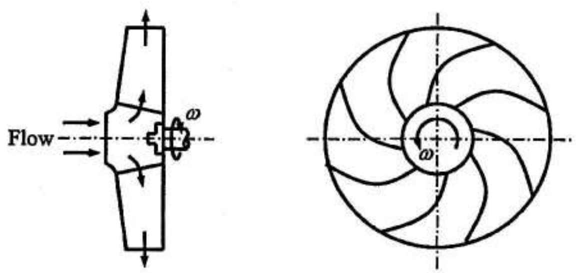


Figure 13.5 A centrifugal pump impeller

In mixed flow turbomachines, the fluid flow through the impeller is partly axial and partly radial. In a mixed flow pump, the impeller is conical and the direction of flow leaving the impeller is somewhere between axial and radial. A typical mixed flow pump impeller is shown in Figure 13.6. However, in a mixed flow turbine, the direction of flow entering the impeller is in between radial and axial directions.

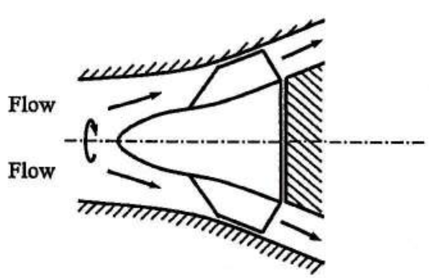


Figure 13.6 A mixed flow pump impeller

The classification of turbomachines, which is based on the direction of flow is presented in Figure 13.7. The cased power absorbing turbomachines can be radial, axial or mixed type, while the uncased power absorbing turbomachines are usually axial flow propellers. The power producing turbomachines are usually classified as the impulse and reaction turbines. The static pressure remains constant in **impulse turbines**, which are of axial turbomachines. However, there is expansion across **reaction turbines** with a corresponding decrease in the static pressure. Reaction turbines can be radial, axial or mixed type.

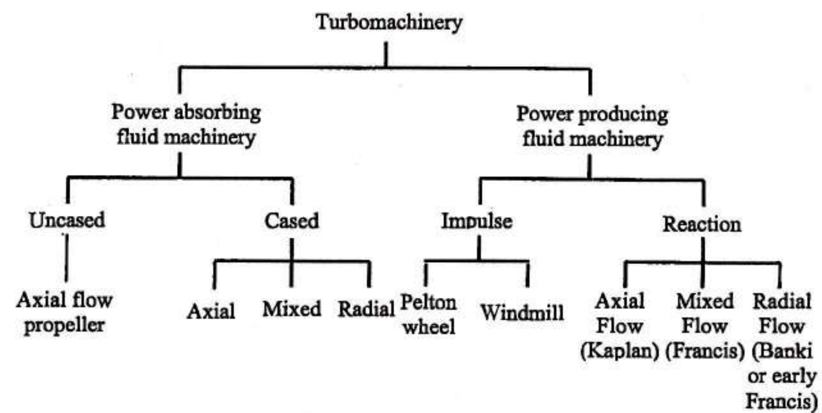


Figure 13.7 Classification of turbomachinery based on the direction of flow

A road map for this chapter is presented in Figure 13.8. The discussion starts with the application of fundamental relations to the flow through an arbitrary turbomachine by using one-dimensional approximation. Velocity triangles for pumps and turbines are then introduced and conditions for maximum energy transfer are discussed. Energy relations through pumps and turbines are analyzed and the general performance characteristics for pumps and turbines are considered. The discussion continues with the similitude in turbomachines with special emphasis to the performance characteristics at similar operating points. The same turbomachine operating at different rotational speeds and geometrically similar turbomachines operating at the same rotational speed are considered. The specific speed or type number is defined for the classification, comparison and design of pumps and turbines. Pumps are then classified as radial, axial and mixed flow according to the geometry of flow passage and analyzed in detail. This is followed by the classification of turbines as impulse and reaction turbines. The reaction turbines are further classified as radial (Francis) and axial (Kaplan) turbines. Since pumps and turbines operate in a pipe system, their performances are closely related to the losses in the system of pipes connected to them. For this reason, pumps and turbines operating in pipe systems are analyzed and concept of cavitation is introduced. This discussion is extended to pump and pipe system combinations. System characteristic is introduced and the operating point is defined. Finally, the series and parallel combination of pumps operating in a pipe system are considered and the pump selection procedure is examined.

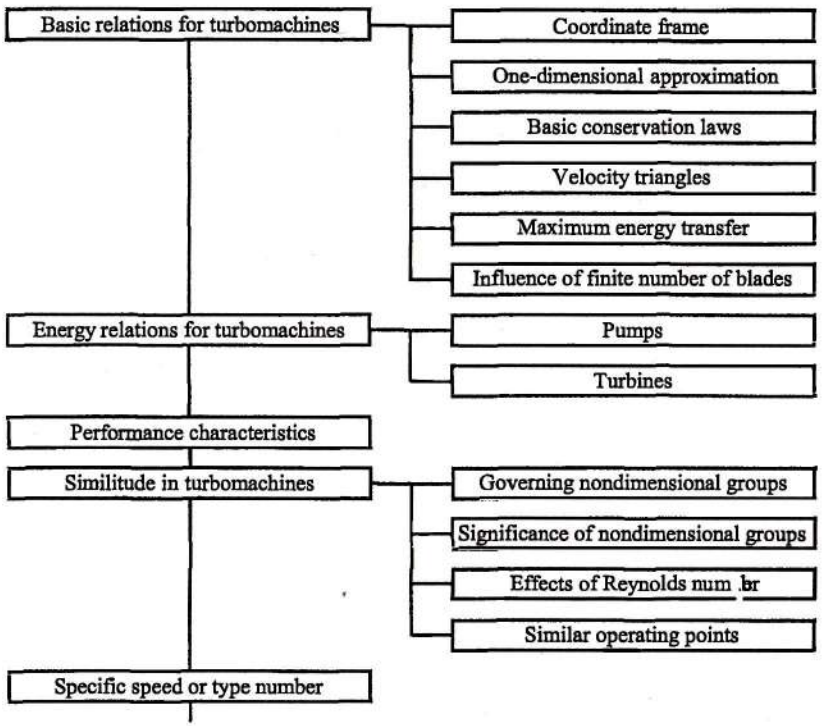


Figure 13.8 Road map for Chapter 13

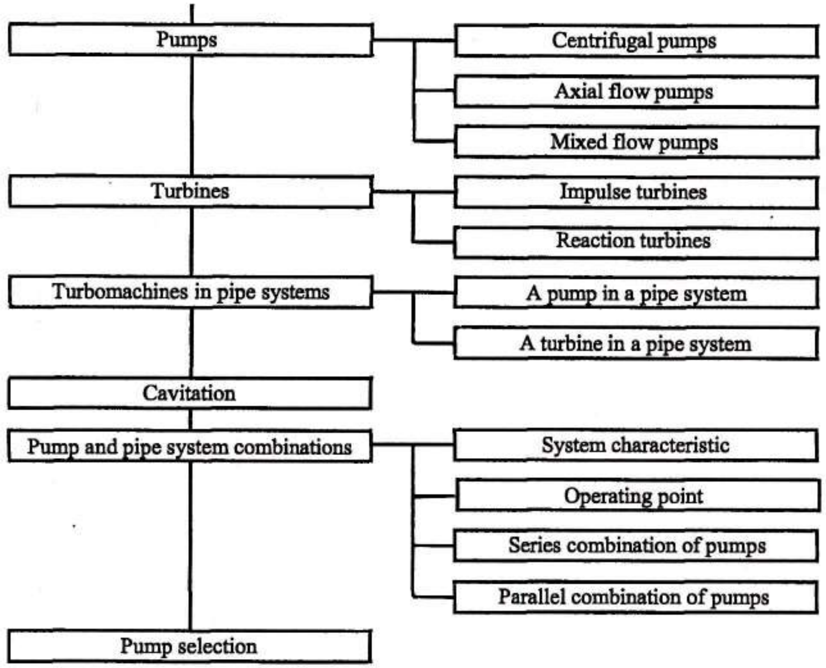


Figure 13.8 Road map for Chapter 13 (continued)

13.2 FUNDAMENTAL RELATIONS FOR THE FLOW THROUGH AN ARBITRARY TURBOMACHINE

13.2.1 Coordinate Frame for the Description of Flow

The analysis of three-dimensional flow through the impeller of an arbitrary turbomachine is very complex. The velocity of the fluid at point P is a function of the three positional coordinates r, θ and z in the cylindrical coordinate system, as shown in Figure 13.9 and given by

$$\mathbf{V} = V_r\mathbf{i}_r + V_\theta\mathbf{i}_\theta + V_z\mathbf{i}_z \tag{13.1}$$

where V_r, V_θ and V_z are the radial, tangential and axial components of the velocity, respectively. The plane of constant θ is known as the **meridional plane**, while the plane of constant r is known as the **surface of revolution.** One should note that the surface of revolution is not always cylindrical, but it may have a flared shape, as shown in Figure 13.9. The blades of turbomachines are cut out into this plane and extend in the radial direction. The velocity distribution is dependent upon the number of blades, their shapes and thicknesses and the width of the impeller. The velocity component, which is tangent to the flared cylindrical surface, is known as the meridional velocity and it can be given as

$$V_m = \sqrt{V_r^2 + V_z^2} \tag{13.2}$$

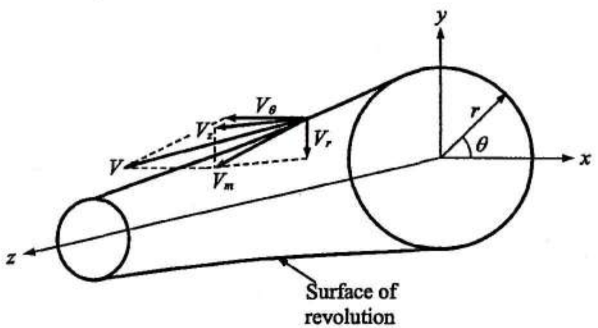


Figure 13.9 Flow through an arbitrary turbomachine

13.2.2 One-dimensional Approximation

The one-dimensional theory simplifies the problem considerably with the aid of the following assumptions:

(i) Since the pressure difference across the blades is responsible for producing the torque, the thickness of blades is not important and can assumed to be zero. In this case, the blades can be regarded as surfaces of discontinuity and there is a sudden change in pressure and velocity as one passes from one side of the blades to the other. The pressure difference across the blades can then be replaced by imaginary body forces acting on the fluid and producing the torque.

(ii) As the number of blades tends towards infinity, the strength of discontinuity across each blade decreases and tends to zero. This is equivalent of assuming axisymmetric flow through the impeller. Axisymmetric flow means that the flow properties on all planes of constant θ, that is the meridional plane, are the same.

(iii) The flow can be assumed to be uniform over each cross-section. This is valid if the thickness of the streamtube is very small when compared to the radius of the streamtube.

13.2.3 Fundamental Relations for the Flow Through an Arbitrary Turbomachine

Consider the flow through the control volume in an arbitrary turbomachine, as shown in Figure 13.10. The basic conservation laws can now be applied to this control volume.

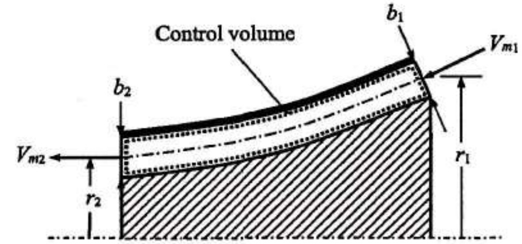


Figure 13.10 Control volume for the flow across an arbitrary turbomachine

a) Continuity Equation: For the steady flow, the integral form of the continuity equation is given by Equation (4.5) as

$$\int_A \rho(\mathbf{V}.\mathbf{n})\mathrm{d}A = 0$$

The fluid flows in through area, A_1, of the control surface and flows out through area, A_2, of the control surface. No flow can take place through other surfaces of the control volume, since they are formed by streamlines and $\mathbf{V}.\mathbf{n} = 0$ on these surfaces. Therefore, Equation (4.5) reduces to

$$\int_{A_1} \rho_1(\mathbf{V}_1.\mathbf{n}_1)\mathrm{d}A + \int_{A_2} \rho_2(\mathbf{V}_2.\mathbf{n}_2)\mathrm{d}A = 0$$

Since the flow is one-dimensional, areas A_1 and A_2 are perpendicular to velocities $\mathbf{V}_1$ and $\mathbf{V}_2$, respectively, then $\mathbf{V}_1.\mathbf{n}_1 = -V_{m1}$ and $\mathbf{V}_2.\mathbf{n}_2 = V_{m2}$. Hence,

$$-\int_{A_1} \rho_1 V_{m1}\mathrm{d}A + \int_{A_2} \rho_2 V_{m2}\mathrm{d}A = 0$$

For a one-dimensional flow, properties are uniform over each cross section and

$$\dot{m} = \rho_1 V_{m1} A_1 = \rho_2 V_{m2} A_2$$

Noting that $A_1 = 2\pi r_1 b_1$ and $A_2 = 2\pi r_2 b_2$ with b_1 and b_2 being the width of the blades at the inlet and outlet, respectively.

$$\dot{m} = 2\pi \rho_1 V_{m1} r_1 b_1 = 2\pi \rho_2 V_{m2} r_2 b_2 = \text{constant} \tag{13.3a}$$

In the case of one-dimensional flow of an incompressible fluid

$$Q = 2\pi V_{m1} r_1 b_1 = 2\pi V_{m2} r_2 b_2 = \text{constant} \tag{13.3b}$$

b) Conservation of Angular Momentum: For a steady flow, the tangential component of the angular momentum equation (4.43) is

$$T_\theta = \int_{A_1} \rho_1 r_1 V_{\theta 1} (\mathbf{V}_1 . \mathbf{n}_1) \mathrm{d}A + \int_{A_2} \rho_2 r_2 V_{\theta 2} (\mathbf{V}_2 . \mathbf{n}_2) \mathrm{d}A$$

which can be rearranged to yield

$$T_\theta = \int_{A_2} \rho_2 r_2 V_{\theta 2} V_{m2} \mathrm{d}A - \int_{A_1} \rho_1 r_1 V_{\theta 1} V_{m1} \mathrm{d}A$$

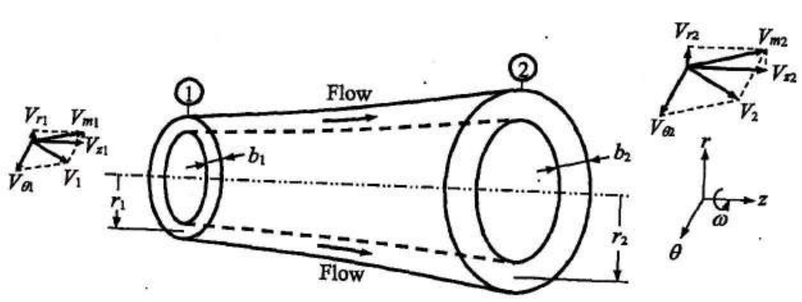
For uniform properties over each cross-section

$$T_\theta = \rho_2 r_2 V_{\theta 2} V_{m2} A_2 - \rho_1 r_1 V_{\theta 1} V_{m1} A_1$$

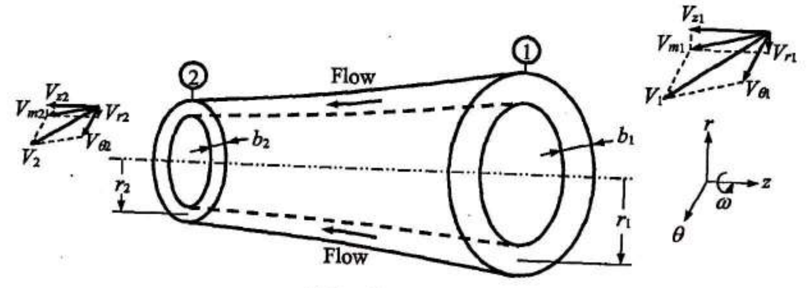
Now, using the continuity equation (13.3a)

$$T_\theta = \dot{m}(r_2 V_{\theta 2} - r_1 V_{\theta 1}) \tag{13.4}$$

which is known as the **Euler's turbine equation.** In this equation, subscript 1 indicates the inlet section and subscript 2 indicates the outlet section of the control volume. The velocity components at the inlet and outlet of a pump and a turbine are shown in Figures 13.11a and 13.11b, respectively. For a pump, the rate of angular momentum at the outlet, $\dot{m} r_2 V_{\theta 2}$, is greater than the rate of angular momentum at the inlet, $\dot{m} r_1 V_{\theta 1}$, so that the torque in the tangential direction is positive. However for a turbine, the rate of angular momentum at the outlet, $\dot{m} r_2 V_{\theta 2}$, is smaller than the rate of angular momentum at the inlet, $\dot{m} r_1 V_{\theta 1}$, so that the torque in the tangential direction is negative.



(a) Pump



(b) Turbine

Figure 13.11 Components of velocity for a pump and a turbine

c) The First Law of Thermodynamics: When the first law of thermodynamics (4.70a) for one-dimensional and steady flow of an incompressible fluid is combined with the definition of enthalpy (4.62), one can obtain

$${}_1\dot{Q}_2 + {}_1\dot{W}_2 + \dot{m}\left(u_1 + \frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1\right) = \dot{m}\left(u_2 + \frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2\right)$$

When there is no heat transfer, the internal energy of incompressible fluids is constant and the above equation becomes

$${}_1\dot{W}_2 + \dot{m}\left(\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1\right) = \dot{m}\left(\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2\right)$$

Noting that $\dot{m} = \rho Q$, $h_{t1} = p_1/(\rho g) + V_1^2/(2g) + z_1$ and $h_{t2} = p_2/(\rho g) + V_2^2/(2g) + z_2$

$${}_1\dot{W}_2 = \rho g Q(h_{t2} - h_{t1})$$

The theoretical head increase across a turbomachine can be defined as $h_{th} = h_{t1} - h_{t2}$ and the rate of work can be denoted as the power P so that

$$P = \rho g Q h_{th} \tag{13.5}$$

d) The Second Law of Thermodynamics: For incompressible flows without heat transfer, there is no need for explicit consideration of the second law of thermodynamics. However, when compressibility and heat transfer are present, one should be careful so that the second law of thermodynamics is not violated.

For incompressible flows, the angular momentum equation and the first law of thermodynamics can be combined to yield a very useful relation. To obtain this relation, first note that the power is the product of the torque and angular velocity, so that

$$P = T_\theta \omega = \dot{m}\omega(r_2 V_{\theta 2} - r_1 V_{\theta 1}) \tag{13.6}$$

with the aid of Equation (13.4). If the impeller rotates about the z-axis with an angular rotation positive in the z-direction, the rotation will be counterclockwise. Hence, the power is positive for a pump and negative for a turbine. Now, combining Equations (13.5) and (13.6), one can obtain

$$h_{th} = \frac{\omega}{g}(r_2 V_{\theta 2} - r_1 V_{\theta 1}) \tag{13.7}$$

Since the impeller is a rotating member, any point on the impeller has a peripheral velocity U, in the tangential direction, which is given as

$$U = \omega r \tag{13.8}$$

Hence

$$h_{th} = \frac{1}{g}(U_2 V_{\theta 2} - U_1 V_{\theta 1}) \tag{13.9}$$

The sign convention in the above equation is implicit and the positive signed head is a result of the head increase in the flow direction, while the negative signed head is a result of the head decrease in the flow direction.

13.2.4 Velocity Triangles for Pumps and Turbines

If the velocity of the fluid relative to the rotating impeller is denoted by **W**, the relation between the absolute and relative velocities at any location on the impeller can be expressed as

$$\mathbf{V} = \mathbf{U} + \mathbf{W} \tag{13.10}$$

The corresponding velocity triangle is shown in Figure 13.12. The angles that the absolute and relative velocities make with the tangential direction are the absolute fluid angle, α_f, and the relative fluid angle, β_f, respectively.

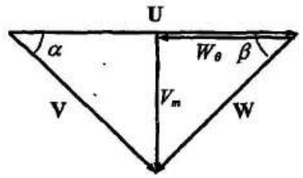


Figure 13.12 Velocity triangle on a rotating impeller

(a) Axial Flow Pump: The velocity triangles at the inlet and outlet of an axial flow pump are shown in Figure 13.13. The fluid approaches the rotating blades of the impeller at a relative velocity of W_1 and leaves them at a relative velocity of W_2. When there are infinite number of blades on the impeller, the fluid and blade angles are equal, that is $\beta_{1f} = \beta_{1v} = \beta_1$ and $\beta_{2f} = \beta_{2v} = \beta_2$. The subscripts f and v indicate the fluid and blade,

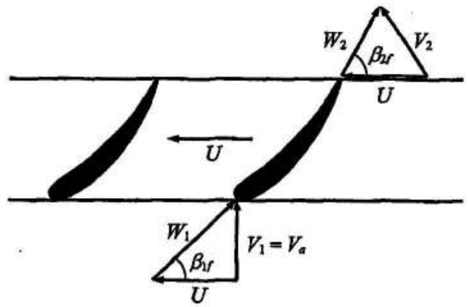


Figure 13.13 Inlet and outlet velocity triangles for an axial flow pump

respectively. The axial pump in Figure 13.13 does not have inlet guide vanes that is there are no stationary blades to direct the fluid to the impeller. As a result, $\alpha_1 = 90^0$. Also, one should note that the peripheral and axial velocities are constant for an axial flow turbomachine.

b) Axial Flow Turbine: For an axial flow turbine, the velocity triangles at the inlet and outlet of an impeller are shown in Figure 13.14.

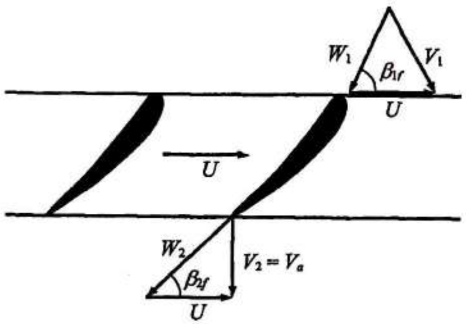


Figure 13.14 Inlet and outlet velocity triangles for an axial flow turbine

(c) Radial Flow Pump: The velocity triangles at the inlet and outlet of a radial flow pump are shown in Figure 13.15. In this case, the peripheral velocities at the inlet and outlet are not the same, that is $U_1 \neq U_2$, due to the different radii at the inlet and outlet. For the radial pump in Figure 13.15, there is no inlet whirl since there are no inlet guide vanes.

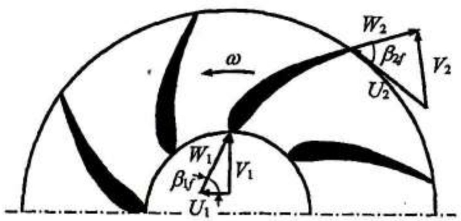


Figure 13.15 Inlet and outlet velocity triangles for a radial flow pump

(d) Radial Flow Turbine: For a radial flow turbine, the velocity triangles at the inlet and outlet of an impeller are shown in Figure 13.16.

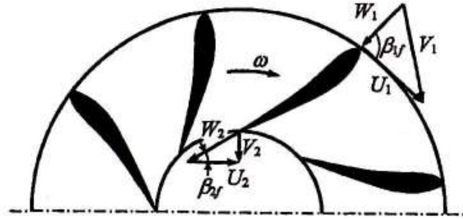


Figure 13.16 Inlet and outlet velocity triangles for a radial flow turbine

13.2.5 Maximum Energy Transfer in Turbomachines

13.2.5.1 A Pump without Inlet Guide Vanes

In a pump, the maximum energy transfer takes place, if there are no inlet guide vanes. The inlet guide vanes are the stationary blades just before the impeller, changing the direction of the free flow of the liquid before the entry to the impeller. In this case, $V_{\theta 1} = 0$ and there is no inlet whirl with $\alpha_1 = 90^0$ and $V_{m1} = V_1$, as shown in Figure 13.17. Then, Equation (13.9) becomes,

$$(h_{th})_{max} = \frac{U_2 V_{\theta 2}}{g}$$

But, $V_{\theta 2} = U_2 - W_{\theta 2}$, so that

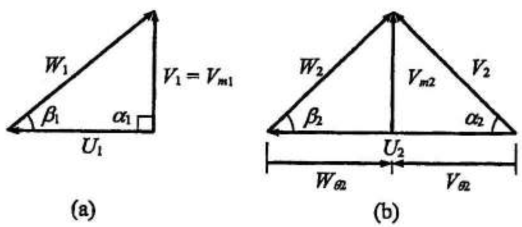


Figure 13.17 (a) Inlet and (b) discharge velocity diagrams for a pump with no inlet guide vanes

$$(h_{th})_{max} = \frac{U_2(U_2 - W_{\theta 2})}{g} \tag{13.11}$$

which is valid both for axial and radial pumps.

13.2.5.2 A Turbine without Outlet Whirl

In a turbine, the no outlet whirl condition is established by designing the blades so that $V_{\theta 2} = 0$, as shown in Figure 13.18. In this case, maximum negative head is achieved. Then, Equation (13.9) becomes

$$(h_{th})_{max} = -\frac{U_1 V_{\theta 1}}{g}$$

But, $V_{\theta 1} = U_1 - W_{\theta 1}$, so that

$$(h_{th})_{max} = -\frac{U_1(U_1 - W_{\theta 1})}{g} \tag{13.12}$$

13.2.6 Influence of Finite Number of Blades

The equations that are derived this far are valid with the axisymmetric flow assumption which also implies that there are infinite number of blades. The presence of infinite number of blades implies that the fluid is guided perfectly well through the impeller by invisible blades of finite length.

The existence of finite number of blades on an impeller results in the following two differences:

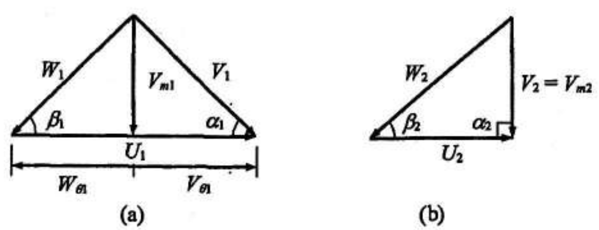


Figure 13.18 (a) Inlet and (b) discharge velocity diagrams for a turbine with no outlet whirl

(i) The existence of solid vanes in the flow area causes blockage and increases the meridional velocity for a given volumetric flow rate.

(ii) When there are finite number of vanes, the fluid is not guided perfectly well so that the fluid and blade angles are not the same. Therefore, it is impossible to assume that the fluid leaves the impeller with the same angle as the blades.

Example 13.1

A centrifugal pump runs at 1500 rpm and discharges 0.05 m^3/s of water. The inlet and exit radii of the impeller are 0.05 m and 0.15 m, respectively, while the width of the blades at the inlet and exit are 0.03 m and 0.01 m, respectively. The blade angles at the inlet and exit are 60° and 45°, respectively. Assuming that the fluid and blade angles are equal, find the

a) theoretical head of the pump,

b) angle at the inlet between the absolute velocity and tangential direction and

c) maximum possible theoretical head of the pump.

Solution

a) The rotational speed of the pump is

$$\omega = \frac{2\pi N}{60} = \frac{(2)(\pi)(1500\,\text{rev/min})}{60\,\text{s/min}} = 157.1\,\text{rad/s}$$

Then, the peripheral velocities at the inlet and exit are

$$U_1 = \omega r_1 = (157.1\,\text{rad/s})(0.05\,\text{m}) = 7.855\,\text{m/s}$$

and

$$U_2 = \omega r_2 = (157.1\,\text{rad/s})(0.15\,\text{m}) = 23.57\,\text{m/s}$$

respectively. Now, the meridional velocities at the inlet and exit of the impeller are

$$V_{m1} = \frac{Q}{2\pi r_1 b_1} = \frac{0.05\,\text{m}^3/\text{s}}{(2)(\pi)(0.05\,\text{m})(0.03\,\text{m})} = 5.305\,\text{m/s}$$

and

$$V_{m2} = \frac{Q}{2\pi r_2 b_2} = \frac{0.05\,\text{m}^3/\text{s}}{(2)(\pi)(0.15\,\text{m})(0.01\,\text{m})} = 5.305\,\text{m/s}$$

respectively. From the velocity triangles given in Figure 13.19, the tangential component of the absolute velocities at the inlet and exit are

$$V_{\theta 1} = U_1 - V_{m1}\cot\beta_1 = 7.855\,\text{m/s} - (5.305\,\text{m/s})(\cot 60^0) = 4.792\,\text{m/s}$$

and

$$V_{\theta 2} = U_2 - V_{m2}\cot\beta_2 = 23.57\,\text{m/s} - (5.305\,\text{m/s})(\cot 45^0) = 18.27\,\text{m/s}$$

respectively. Now, one can find the theoretical head by using Equation (13.9)

$$h_{th} = \frac{1}{g}(U_2V_{\theta 2} - U_1V_{\theta 1}) = \frac{(23.57\,\text{m/s})(18.27\,\text{m/s}) - (7.855\,\text{m/s})(4.792\,\text{m/s})}{(9.81\,\text{m/s}^2)}$$

$$= \underline{40.06\,\text{m}}$$

b) The inlet angle between the absolute velocity and tangential direction can be determined as

$$\alpha_1 = \tan^{-1}\frac{V_{m1}}{V_{\theta 1}} = \tan^{-1}\frac{5.305\,\text{m/s}}{4.792\,\text{m/s}} = 47.91^0$$

One should note that the fluid is directed towards the impeller by inlet guide vanes probably since α_1 is different than 90⁰. The theoretical head can be increased by adjusting the inlet guide vanes so that α_1 is greater.

c) The maximum possible theoretical head can be obtained where there are no inlet guide vanes that is $V_{\theta 1} = 0$.

$$(h_{th})_{max} = \frac{U_2V_{\theta 2}}{g} = \frac{(23.57\,\text{m/s})(18.27\,\text{m/s})}{9.81\,\text{m/s}^2} = \underline{43.90\,\text{m}}$$

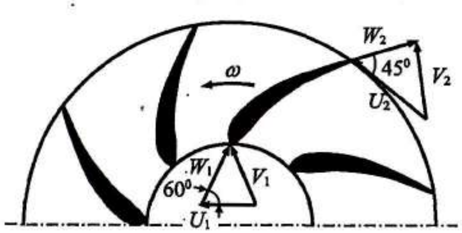


Figure 13.19 Sketch for Example 13.1

13.3 ENERGY RELATIONS THROUGH AN ARBITRARY TURBOMACHINE

All hydraulic machines convert one form energy into the other and losses always occur during these energy conversion processes. A machine becomes more and more efficient as these losses becomes smaller and smaller. The efficiency of a machine is defined as the ratio of the power output of the machine to the power input into it. However, hydraulic machines are usually very complex and there are a number of parts through which the fluid moves. For this reason, it is more convenient to consider the losses in each component as well as as the overall loss and express each of these loss components in the efficiency form. In this section, these losses in the components of the hydraulic machinery will be considered one by one for pumps and turbines separately.

13.3.1 Pumps

13.3.1.1 Head

When all losses are neglected, the head delivered to the fluid is the theoretical head, h_{th}. As the fluid passes through the blade passages, it receives energy from the moving blades. There are two major sources of energy loss in an impeller. The first one is due to the contact between the fluid moving over the solid surfaces which gives rise to boundary layer development and viscous losses. The other type of energy loss is due to separation which occurs as the fluid changes its direction. In addition to these two types of losses, there may also be energy losses due to the secondary flows within the impeller because of the pressure distribution across it.

In most hydraulic machines, the impeller is surrounded by a stationary casing and the fluid passes through the parts of the casing before it enters the impeller and after leaving it. Hence, head losses due to friction and separation also occur in the casing.

If all of these head losses are denoted by h_ℓ, then the head, h, delivered to the fluid becomes

$$h = h_{th} - h_\ell \tag{13.13}$$

13.3.1.2 Volumetric Flow Rate

The volumetric flow rate through the impeller is not the same as the volumetric flow rate through the pump. All of the fluid passing through the impeller does not go to the discharge end, but some of it flows back to the suction side through the clearances between the impeller and casing. This is due to the fact that the pressure at the discharge side is greater than the pressure at the suction side. Hence, the impeller always handles a greater volumetric flow rate than the discharge by the pump. If Q_i is the volumetric flow rate going through the impeller, the actual volumetric flow rate, Q, discharged by the pump is

$$Q = Q_i - Q_\ell \tag{13.14}$$

where Q_ℓ is the volumetric flow rate leaking back to the suction side through the clearances.

13.3.1.3 Power

In a pump, the shaft power, P_s, can not be transmitted to the impeller without any loss. Certainly, there are mechanical losses of energy in the bearings and sealing glands and they can be denoted as $(P_\ell)_{mf}$. Also, it is customary to include the losses due to the disc friction which is also referred to as the windage loss. This is equivalent to the power required to rotate the impeller at a required velocity without any work being done by the impeller on the fluid. This is only possible when the impeller does not have any blades and this power loss, $(P_\ell)_{df}$, takes care of the friction between the outer surfaces of the impeller and the surrounding fluid. Then, the mechanical power loss, $(P_\ell)_m$, is

$$(P_\ell)_m = (P_\ell)_{mf} + (P_\ell)_{df} \tag{13.15}$$

The power imparted to the impeller is referred to as the internal power, P_i, and is given by

$$P_i = P_s - (P_\ell)_m = \rho g Q_i h_{th} \tag{13.16}$$

In addition to these, there are power losses due to viscous effects, separation, secondary flows and leakage within the pump. All of these losses are referred to as hydraulic power loss, $(P_\ell)_h$. Finally, the power delivered to the fluid is

$$P_f = P_i - (P_\ell)_h = \rho g Q h \tag{13.17}$$

The relation between the shaft power and power delivered to the fluid is

$$P_f = P_s - (P_\ell)_m - (P_\ell)_h \tag{13.18}$$

The summary of energy balance for the pump is indicated in Figure 13.20.

13.3.1.4 Efficiency

The overall efficiency, η, of a pump is defined as the ratio of the power delivered to the fluid to the power input to the shaft

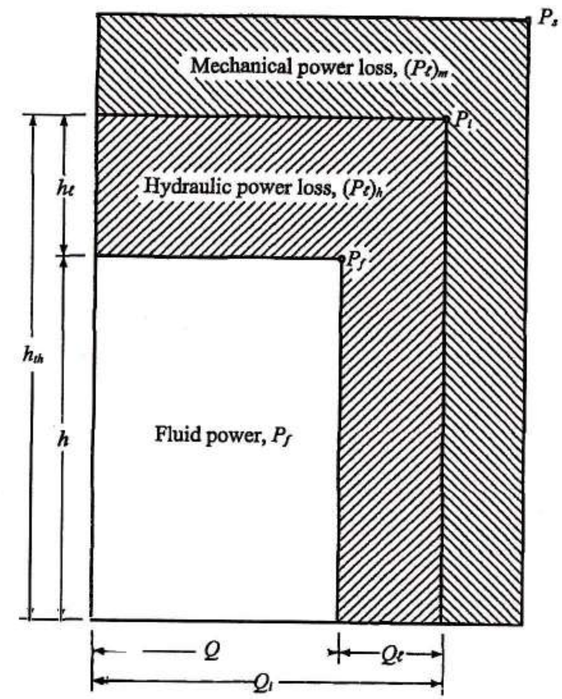


Figure 13.20 Energy balance for a pump and summary of efficiencies

$$\eta = \frac{P_f}{P_s} = \frac{\rho g Q h}{P_s} \tag{13.19}$$

The mechanical efficiency, η_m, is the ratio of the power supplied to the impeller to the shaft power

$$\eta_m = \frac{P_i}{P_s} = \frac{\rho g Q_i h_{th}}{P_s} \tag{13.20}$$

The hydraulic efficiency, η_h, is the ratio of the head delivered to the fluid to the theoretical that would have delivered to the fluid if there were no losses

$$\eta_h = \frac{h}{h_{th}} \tag{13.21}$$

The volumetric efficiency, η_v, is the ratio of the volumetric flow rate through a pump to the volumetric flow rate through the impeller

$$\eta_v = \frac{Q}{Q_i} = \frac{Q_i - Q_\ell}{Q_i} \tag{13.22}$$

The internal efficiency, η_i, is the ratio of the power delivered to the fluid to the power imparted to the impeller.

$$\eta_i = \frac{P_f}{P_i} = \frac{\rho g Q h}{\rho g Q_i h_i} = \frac{Q}{Q_i}\frac{h}{h_i} = \eta_v \eta_h \tag{13.23}$$

It is now possible to show that the overall efficiency is equal to the product of all the component efficiencies

$$\eta = \frac{P_f}{P_s} = \frac{P_f}{P_i}\frac{P_i}{P_s} = \eta_m \eta_i = \eta_m \eta_h \eta_v \tag{13.24}$$

13.3.2 Turbines

13.3.2.1 Head

In hydraulic turbines, the impeller is usually surrounded by a stationary casing. For this reason, the fluid passes through the parts of this casing before it enters the impeller and after it leaves the impeller. Therefore, losses occur due to viscous effects and separation.

When the fluid passes through the blade passages of the impeller of a turbine, loss of energy occurs because of the viscous effects between the solid surfaces and fluid, and due to the separation during the change of direction of the fluid in the impeller. In addition to these, there may be losses due to the secondary flows in the impeller.

If all of these head losses in the impeller are denoted by, h_ℓ, the theoretical head, h_{th}, delivered to the impeller is

$$h_{th} = h - h_\ell \tag{13.25}$$

13.3.2.2 Volumetric Flow Rate

One should note that the volumetric flow rate through the impeller is not the same as the volumetric flow rate through the turbine. All of the fluid passing through the turbine does not go through the impeller, but some of it leaks through the clearances between the impeller and casing. If Q indicates the volumetric flow rate through the turbine, the volumetric flow rate, Q_i, passing through the impeller is

$$Q_i = Q - Q_\ell \tag{13.26}$$

where Q_ℓ is the volumetric flow rate leaking through the clearances.

13.3.2.3 Power

In a turbine, the available fluid power, P_f, is

$$P_f = \rho g Q h \tag{13.27}$$

This power cannot be transmitted to the shaft without any loss. There are power losses due to viscous effects, separation, secondary flows and leakage within the pump. All of these losses are referred to as hydraulic power loss, $(P_\ell)_h$. so that the power imparted to the impeller is the internal power, P_i,

$$P_i = P_f - (P_\ell)_h = \rho g Q_i h_i \tag{13.28}$$

This power imparted to the impeller cannot be transmitted to the shaft due to the mechanical losses in the bearings and sealing glands. In addition to this, it is customary to include the losses due to the disc friction into the mechanical power losses. Then, the mechanical power loss, $(P_\ell)_m$, is

$$(P_\ell)_m = (P_\ell)_{mf} + (P_\ell)_{df} \tag{13.29}$$

Then, the power transmitted to the shaft is

$$P_s = P_i - (P_\ell)_m \tag{13.30}$$

The summary of energy balance for a turbine is indicated in Figure 13.21.

13.3.2.4 Efficiency

The overall efficiency, η, of a turbine is defined as the ratio of the power delivered to the shaft to the available fluid power

$$\eta = \frac{P_s}{P_f} = \frac{P_s}{\rho g Q h} \tag{13.31}$$

The hydraulic efficiency, η_h, is the ratio of the theoretical head to the head possessed by the fluid

$$\eta_h = \frac{h_{th}}{h} \tag{13.32}$$

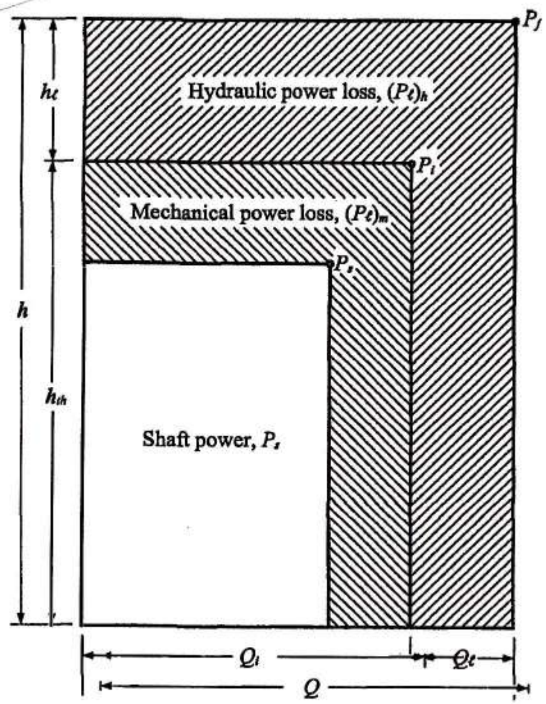


Figure 13.21 Energy balance for a turbine and summary of efficiencies

The volumetric efficiency, η_v, is the ratio of the volumetric flow rate through the impeller to the volumetric flow rate through the turbine, that is

$$\eta_v = \frac{Q_i}{Q} = \frac{Q - Q_\ell}{Q} \tag{13.33}$$

The internal efficiency, η_i, is the ratio of the power imparted to the impeller to the available fluid power.

$$\eta_i = \frac{P_i}{P_f} = \frac{\rho g Q_i h_{th}}{\rho g Q h} = \frac{Q_i}{Q}\frac{h_{th}}{h} = \eta_v \eta_h \tag{13.34}$$

The mechanical efficiency, η_m, is the ratio of the shaft power to the power imparted to the impeller

$$\eta_m = \frac{P_s}{P_i} = \frac{P_s}{\rho g Q_i h_{th}} \tag{13.35}$$

It is now possible to show that the overall efficiency is equal to the product of all the component efficiencies

$$\eta = \frac{P_s}{P_f} = \frac{P_s}{P_i}\frac{P_i}{P_f} = \frac{P_s}{P_i}\frac{\rho g Q_i h_{th}}{\rho g Q h} = \eta_m \eta_i = \eta_m \eta_h \eta_v \tag{13.36}$$

Example 13.2

A radial pump rotating at 1470 rpm produces a head of 6.5 m at a volumetric flow rate of 3800 lt/ min. The impeller, which is shown in Figure 13.22, has an inlet diameter of 80 mm and an outlet diameter of 200 mm. The blade width at the inlet and outlet are 25 mm and 10 mm, respectively. The blade angles at the inlet and outlet are 60^0 and 45^0, respectively. The blade and fluid angles may be assumed to be the same. There are no inlet guide vanes. If the mechanical efficiency is 80 percent, determine the power required to drive the pump. The working fluid is water with a density of 1000 kg/m^3.

Solution

The rotational speed of the pump can be calculated as

$$\omega = \frac{2\pi N}{60} \frac{(2)(\pi)(1470\ \text{rev/min})}{60\ \text{s/min}} = 153.9\ \text{rad/s}$$

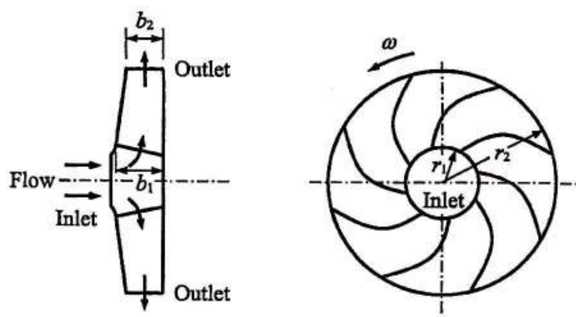


Figure 13.22 Sketch for Example 13.2

while the peripheral velocity at the inlet is

$$U_1 = \omega r_1 = (153.9\,\text{rad/s})(0.04\,\text{m}) = 6.156\,\text{m/s}$$

One can obtain the meridional velocity at the inlet of the pump as

$$V_{m1} = U_1 \tan\beta_1 = (6.156\,\text{m/s})(\tan 60^0) = 10.66\,\text{m/s}$$

Now, the volumetric flow rate passing through the pump impeller can be evaluated as

$$Q_i = V_{m1}\pi d_1 b_1 = (10.66\,\text{m/s})(\pi)(0.08\,\text{m})(0.025\,\text{m}) = 0.06698\,\text{m}^3/\text{s}$$

The actual volumetric flow rate through the pump is given and it is

$$Q = (3800\,\text{lt/min})(10^{-3}\,\text{m}^3/\text{lt})/(60\,\text{s/min}) = 0.06333\,\text{m}^3/\text{s}$$

Then, the volumetric efficiency of the pump is

$$\eta_v = \frac{Q}{Q_i} = \frac{0.06333\,\text{m}^3/\text{s}}{0.06698\,\text{m}^3/\text{s}} = 0.9455$$

Now, the meridional velocity at the outlet of the pump is

$$V_{m2} = \frac{Q_i}{\pi d_2 b_2} = \frac{0.06698\,\text{m}^3/\text{s}}{(\pi)(0.2\,\text{m})(0.01\,\text{m})} = 10.66\,\text{m/s}$$

while the peripheral velocity at the outlet is

$$U_2 = \omega r_2 = (153.9\,\text{rad/s})(0.1\,\text{m}) = 15.39\,\text{m/s}$$

One can calculate the component of the absolute velocity in the tangential direction as

$$V_{\theta 2} = U_2 - V_{m2}\cot\beta_2 = 15.39\,\text{m/s} - (10.66\,\text{m/s})(\tan 45^0) = 4.73\,\text{m/s}$$

so that the theoretical head of the pump is

$$h_{th} = \frac{U_2 V_{\theta 2}}{g} \quad \frac{(15.39\,\text{m/s})(4.73\,\text{m/s})}{9.81\,\text{m/s}^2} = 7.421\,\text{m}$$

The hydraulic efficiency can now be evaluated as

$$\eta_h = \frac{h}{h_{th}} = \frac{6.5\,\text{m}}{7.421\,\text{m}} = 0.8759$$

so that the overall efficiency is

$$\eta = \eta_v \eta_h \eta_m = (0.9455)(0.8759)(0.8) = 0.6625$$

Finally, the power required to drive the pump is

$$P = \frac{\rho g h Q}{\eta} \quad \frac{(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(6.5\,\text{m})(0.06333\,\text{m}^3/\text{s})}{0.6625} = \underline{6.096\,\text{kW}}$$

13.4 PERFORMANCE CHARACTERISTICS

The fluid quantities involved in hydraulic machines are the volumetric flow rate, Q and head, h, while the mechanical quantities associated with the machine are the size of the machine, d, rotational speed, N, power, P and efficiency, η. Although all of these quantities have equal importance, emphasis should be placed on a specific number of these quantities, which are different for pumps and turbines.

In a pump, the output at a given speed is the volumetric flow rate of the fluid and the head delivered to the fluid. Then, the fundamental characteristic of a pump is a plot of the head against the volumetric flow rate at a constant speed. However, this performance can only be achieved by a power input, which involves efficiency due to energy transfer. For this reason, it is also useful to plot the power, P, and efficiency, η, against the volumetric flow rate, Q. A typical set of performance characteristics for a pump is shown in Figure 13.23.

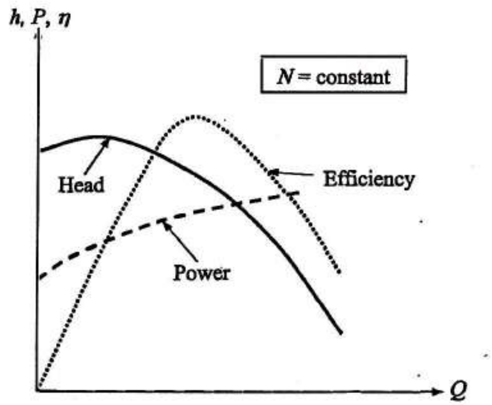


Figure 13.23 A typical set of performance characteristics for a pump

For a turbine, the output is the power developed at a constant head for a given speed. For this reason, the fundamental characteristic is a plot of the power against the speed at a constant head. In this case, the volumetric flow rate of the fluid is the input. Hence, to complete the set of characteristics, the volumetric flow rate and efficiency are plotted against the speed. A typical set of performance characteristics for a turbine is shown in Figure 13.24.

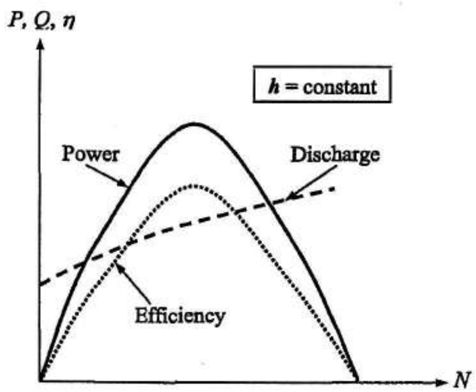


Figure 13.24 A typical set of performance characteristics for a turbine

13.5 SIMILITUDE IN TURBOMACHINES

13.5.1 Governing Nondimensional Parameters

The independent physical variables, which describe the performance of a turbomachine can be given as the

(i) volumetric flow rate, Q,
(ii) energy per unit weight or head, h,
(iii) dimension of the machine such as the diameter, d,
(iv) power of the machine, P
(v) density of the fluid, ρ,
(vi) viscosity of the fluid, μ, and
(vii) rotational speed of the impeller, ω.

Since the energy per unit weight, h, and diameter, d, of the machine have the same dimensions of length, the energy per unit mass, gh, is used instead of the energy per unit weight, h, in order to make a distinction between them. Hence, the functional relation can be expressed as

$$f(Q, gh, d, P, \rho, \mu, \omega) = 0 \tag{13.37}$$

The dimensions of the physical parameters involved can be given as $[Q] = L^3/T$, $[gh] = L^2/T^2$, $[d] = L$, $[P] = ML^2/T^3$, $[\rho] = M/L^3$, $[\mu] = M/(LT)$ and $[\omega] = 1/T$.

Since the primary dimensions which are involved in the dimensions of the governing parameters, are the length, L, time, T, and mass, M, the number of primary dimensions is $m = 3$.

Now, one can select three repeating parameters so that the number of these repeating variables is equal to the number of primary dimensions, which are used in expressing the dimensions of all parameters involved. The repeating parameters may now be chosen as the diameter, d, rotational speed, ω and density, ρ, which are the geometric, kinematic and dynamic variables. One should also note that these repeating variables contain all the primary dimensions.

According to the Buckingham-Pi theorem, the number of nondimensional parameters is $n - m = 7 - 3 = 4$.

The four nondimensional parameters, π_Q, π_h, π_P and π_μ can now be set up by multiplying the unselected parameters Q, gh, P and μ, with the repeating variables d, ρ and ω, which are raised to unknown exponents. Hence

$$\pi_Q = Q\rho^a\omega^b d^c$$

$$\pi_h = (gh)\rho^e\omega^g d^h$$

$$\pi_P = P\rho^i\omega^j d^k$$

$$\pi_\mu = \mu\rho^l\omega^p d^r$$

where $a, b, c, e, g, h, i, j, k, l, p$ and r are the unknown exponents.

The dimensional equation for the nondimensional parameter π_Q can now be given as

$$[\pi_Q] = (L^3T^{-1})(ML^{-3})^a(T^{-1})^b(L)^c = M^0L^0T^0$$

The equations for the unknown exponents a, b and c can now be set up so that the sum of the exponents of each primary dimension is zero. Therefore,

for M: $a = 0$

for T: $-1 - b = 0$

for L: $3 - 3a + c = 0$

These equations can then be solved for the unknown exponents to yield $a = 0$, $b = -1$ and $c = -3$, so that the nondimensional parameter π_Q becomes

$$\pi_Q = Q\rho^0\omega^{-1}d^{-3} = \frac{Q}{\omega d^3} \tag{13.38}$$

which is often referred to as the **flow coefficient**. Now, the dimensional equation for the nondimensional parameter π_h can be given as

$$[\pi_h] = (L^2T^{-2})(ML^{-3})^e(T^{-1})^g(L)^h = M^0L^0T^0$$

Again, the equations for the unknown exponents e, g and h can now be set up, so that the sum of the exponents of each primary dimension will be zero. Therefore,

for M: $e = 0$

for T: $-2 - g = 0$

for L: $2 - 3e + h = 0$

These equations can then be solved for the unknown exponents to yield $e = 0$, $g = -2$ and $h = -2$, so that the nondimensional parameter π_h becomes

$$\pi_h = (gh)\rho^0\omega^{-2}d^{-2} = \frac{gh}{\omega^2 d^2} \tag{13.39}$$

which is often referred to as the **head coefficient**. At this point, the dimensional equation for the nondimensional parameter π_P can be given as

$$[\pi_P] = (ML^2T^{-3})(ML^{-3})^i(T^{-1})^j(L)^k = M^0L^0T^0$$

Again, the equations for the unknown exponents i, j and k can now be set up, so that the sum of the exponents of each primary dimension will be zero. Therefore,

for M: $1 + i = 0$

for T: $-3 - j = 0$

for L: $2 - 3i + k = 0$

which can then be solved simultaneously to yield $i = -1$, $j = -3$ and $k = -5$, so that the nondimensional parameter π_P becomes

$$\pi_P = P\rho^{-1}\omega^{-3}d^{-5} = \frac{P}{\rho\omega^3 d^5} \tag{13.40}$$

which is often referred to as the **power coefficient**. Finally, the dimensional equation for the nondimensional parameter π_μ can be expressed as

$$[\pi_\mu] = (ML^{-1}T^{-1})(ML^{-3})^l(T^{-1})^p(L)^r = M^0L^0T^0$$

The equations for the unknown exponents l, p and r can now be given as

for M: $1 + l = 0$

for T: $-1 - p = 0$

for L: $-1 - 3l + r = 0$

The above equations can then be solved simultaneously to yield $l=-1$, $p=-1$ and $r=-2$, so that the nondimensional parameter π_μ becomes

$$\pi_\mu=\mu\rho^{-1}\omega^{-1}d^{-2}=\frac{\mu}{\rho\omega d^2} \tag{13.41}$$

which is the reciprocal of the Reynolds number.

Then, the functional relationship between the nondimensional parameters π_Q, π_h, π_P and π_μ can be established as

$$F=F\left(\frac{Q}{\omega d^3},\frac{gh}{\omega^2 d^2},\frac{P}{\rho\omega^3 d^5},\frac{\mu}{\rho\omega d^2}\right) \tag{13.42}$$

However, it is not possible to establish the equality of all of these four nondimensional terms in practice. If one assumes that the efficiencies are the same at two operating conditions, equality of the first three nondimensional terms, namely, π_Q, π_h and π_P can be established.

The first three nondimensional terms are also referred to as the **affinity laws**. The last nondimensional term, π_μ, is the reciprocal of the Reynolds number and it is used to correct the similarity for viscous effects.

The more common use of similitude in turbomachinery is during the scaling down a prototype to build up its model of smaller size, so that relatively inexpensive experiments can be performed in the laboratory.

13.5.2 Physical Significance of Nondimensional Groups

The physical significance of the nondimensional π terms and their combinations can be given as follows:

The first π term is proportional to the following expression:

$$\pi_Q=\frac{Q}{\omega d^3}\propto\frac{Q}{\omega r d^2}\propto\frac{Q}{Ud^2}\propto\frac{Q}{UA}\propto\frac{V_m}{U} \tag{13.43}$$

Therefore, π_Q = constant means that the ratio V_m/U is constant in two different operating conditions.

Similarly, π_h can be arranged in the following manner:

$$\pi_h=\frac{gh}{\omega^2 d^2}\propto\frac{gh}{\omega^2 r^2}\propto\frac{gh}{U^2}\propto\frac{UV_\theta}{U^2}\propto\frac{V_\theta}{U} \tag{13.44}$$

Hence, the ratio V_θ/U is constant in two different operating conditions, when π_h is constant.

The equality of the first two π terms implies that the velocity triangles at two different operating conditions are similar since the ratios V_m/U and V_θ/U are the same.

It is possible to combine the nondimensional π terms to obtain new nondimensional parameters as

$$(\pi_\eta)_p=\frac{\pi_Q\pi_h}{\pi_P}=\frac{\left(\frac{Q}{\omega d^3}\right)\left(\frac{gh}{\omega^2 d^2}\right)}{\left(\frac{P}{\rho\omega^3 d^5}\right)}=\frac{\rho gQh}{P}=\eta_p \tag{13.45}$$

and

$$(\pi_\eta)_t=\frac{\pi_P}{\pi_Q\pi_h}=\frac{\left(\frac{P}{\rho\omega^3 d^5}\right)}{\left(\frac{Q}{\omega d^3}\right)\left(\frac{gh}{\omega^2 d^2}\right)}=\frac{P}{\rho gQh}=\eta_t \tag{13.46}$$

When π_Q, π_h and π_P are the same at two different operating conditions of geometrically similar machines, the efficiencies of these two machines are the same at these particular operating conditions.

13.5.3 Effect of Reynolds Number on Similarity

When the effect of the Reynolds number is neglected, the efficiencies of the model and prototype turbomachines are exactly the same if the first three nondimensional π terms are the same. Therefore, when the viscous effects are taken into consideration, efficiencies of the model and prototype are the same at two different operating conditions whenever

(i) the Reynolds numbers are equal and

(ii) the surface roughness and clearances are geometrically similar

in addition to the equality of the first three π terms.

The difference in efficiency at different operating conditions for the model and prototype is usually given by relations that are obtained experimentally. Such a relation is the **Ackeret's relation** and can be given as

$$\frac{1-\eta_p}{1-\eta_m}=0.5\left[1+\left(\frac{Re_m}{Re_p}\right)^{0.2}\right] \quad (13.47)$$

where η_p and η_m are the efficiencies of the prototype and model, respectively, while Re_p and Re_m denotes the Reynolds numbers for the prototype and model, respectively.

13.5.4 Performance Characteristics at Similar Operating Points

In this section, the affinity laws and conclusions reached by keeping them constant are investigated. The two important cases, when the same turbomachine is operating at different rotational speeds and when the geometrically similar turbomachines are operating at the same rotational speed, are discussed.

13.5.4.1 The Same Turbomachine Operating at Different Rotational Speeds

In this case, the diameter is no longer a variable in the affinity laws that is in the first three nondimensional π terms. Then, the affinity laws reduce to

$$\pi_Q'=\frac{Q}{N} \quad (13.48)$$

$$\pi_h'=\frac{h}{N^2} \quad (13.49)$$

and

$$\pi_P'=\frac{P}{N^3} \quad (13.50)$$

where N is the rotational speed of the turbomachine in rpm.

Example 13.3

The performance characteristics of a centrifugal pump, which is running at 750 rpm, are given in Table 13.1. It is desirable to predict the performance of the same pump when it is running at a speed of 900 rpm.

Table 13.1 Performance characteristics of the centrifugal pump in Example 13.3 at a speed of 750 rpm

Point	1A	1B	1C	1D	1E	1F	1G	1H
Q (m³/s)	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
h (m)	40	41	41	40	38	34	26	15
P (kW)	0	114.9	134.1	152.9	175.4	196.2	204.1	187.3
η (%)	0	35	60	77	85	85	75	55

Solution

By using the modified affinity laws, π_Q', $\pi_h'$and π_P', which are given by Equations (13.48), (13.49) and (13.50), respectively, it is possible to determine the h versus Q, P versus Q and η versus Q characteristics at a speed of $N_2 = 900$ rpm from the corresponding known characteristics at a speed of $N_1 = 750$ rpm. The performance characteristics of this pump at 750 rpm are shown by solid circles in Figure 13.25.

If the pump is operating at point 1D with a speed of $N_1 = 750$ rpm delivers a volumetric flow rate of $Q_{1D} = 0.3$ m³/s, generates a head of $h_{1D} = 40$ m and consumes a power of $P_{1D} = 152.9$ kW at an efficiency of $\eta_{1D} = 77$ percent, the corresponding point 2D, which is a similar operating point at a speed of $N_2 = 900$ rpm will be obtained by the application of the modified affinity laws as

$$Q_{2D}=\frac{N_2}{N_1}Q_{1D}=\frac{900\text{ rpm}}{750\text{ rpm}}(0.3\text{ m}^3/\text{s})=0.36\text{ m}^3/\text{s}$$

$$h_{2D}=\left(\frac{N_2}{N_1}\right)^2 h_{1D}=\left(\frac{900\text{ rpm}}{750\text{ rpm}}\right)^2(40\text{ m})=57.60\text{ m}$$

$$P_{2D}=\left(\frac{N_2}{N_1}\right)^3 P_{1D}=\left(\frac{900\text{ rpm}}{750\text{ rpm}}\right)^3(152.9\text{ kW})=264.2\text{ kW}$$

Finally, the overall efficiency of the pump is defined as the ratio of the power delivered to the fluid to the shaft power, that is $\eta = \rho gQh/P$. If this expression is applied to points 1D and 2D, then

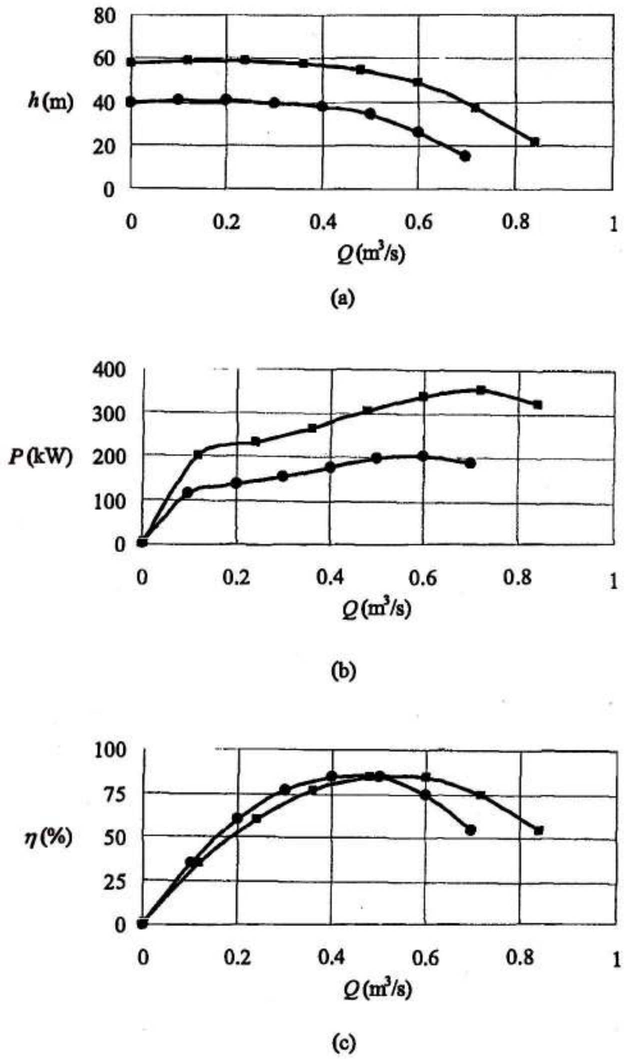


Figure 13.25 Performance characteristics of the centrifugal pump in Example 13.3 at 750 rpm (solid circles) and 900 rpm (solid squares)

Table 13.2 Performance characteristics of the centrifugal pump in Example 13.3 at a speed of 900 rpm

Point	2A	2B	2C	2D	2E	2F	2G	2H
Q (m³/s)	0.0	0.12	0.24	0.36	0.48	0.60	0.72	0.84
h (m)	57.60	59.04	59.04	57.60	54.72	48.96	37.44	21.60
P (kW)	0	198.6	231.7	264.2	303.1	339.0	352.7	323.7
η (%)	0	35	60	77	85	85	75	55

$$\eta_{2D} = \eta_{1D}\frac{P_{1D}}{P_{2D}}\frac{h_{2D}}{h_{1D}}\frac{Q_{2D}}{Q_{1D}} = 0.77\frac{152.9\,\text{kW}}{264.2\,\text{kW}}\frac{57.60\,\text{m}}{40\,\text{m}}\frac{0.36\,\text{m}^3/\text{s}}{0.3\,\text{m}^3/\text{s}} = 0.77$$

One should note that although η_{2D} is the same as η_{1D}, its position in Figure 13.25c is changed since it is now plotted against Q_{2D}.

If this procedure is now applied to the remaining points, the results presented in Table 13.2 are obtained. Also, h versus Q, P versus Q and η versus Q characteristics at a speed of $N_2 = 900$ rpm are shown by the solid squares in Figure 13.25.

Example 13.4

The characteristics of a centrifugal pump at a speed of 1500 rpm are given as

$$h = 50 - 200Q - 24000Q^2$$

and

$$\eta = 60Q - 1200Q^2$$

where h is the head in m and Q is the volumetric flow rate in m³/s. It is desired to deliver 0.03 m³/s of water against a head of 36 m. In this case, determine the

a) speed of the pump,

b) efficiency of the pump and

c) power consumption of the pump.

Solution

a) From the given h versus Q characteristic, one could easily determine that the pump can only produce a head of 22.4 m at a volumetric flow rate of 0.03 m³/s. In order to operate the pump at point 2 in Figure 13.26 having a head of $h_2 = 36$ m and a volumetric flow rate of $Q_2 = 0.03$ m³/s, its rotational speed must be changed.

The locus of similar operating points in the h-Q plane for different rotational speeds can be obtained by eliminating the speed from Equations (13.48) and (13.49) as

$$h = \frac{\pi_h'}{(\pi_Q')^2} Q^2 = CQ^2$$

where $C = \pi_h'/(\pi_Q')^2$ is a constant. This constant can be determined by using operating point 2 on the locus of similar operating points as

$$C = \frac{h_2}{Q_2^2} = \frac{36\,\text{m}}{(0.03\,\text{m}^3/\text{s})^2} = 40000\,\text{s}^2/\text{m}^5$$

Then, the equation for the locus of similar operating points takes the following form:

$$h = 40000Q^2$$

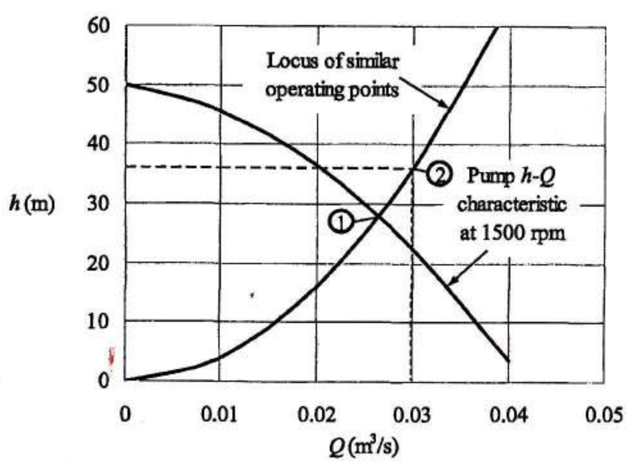


Figure 13.26 Sketch for Example 13.4

The point of intersection of the locus of similar operating points with the h versus Q characteristic of the pump at $N_1 = 1500$ rpm gives the coordinates of operating point 1. At this stage, if the equation for the locus of similar operating points is combined with the equation for the h versus Q characteristic at $N_1 = 1500$ rpm, the following quadratic equation can be obtained.

$$64000Q_1^2 + 200Q_1 - 50 = 0$$

The solution of this quadratic equation gives two solutions for the volumetric flow rate at point 1 as $Q_1 = 0.02643$ m³/s and $Q_1 = -0.02956$ m³/s. Since the negative root has no physical meaning, then

$$Q_1 = 0.02643\,\text{m}^3/\text{s}$$

The corresponding value of the head at operating point 1 is

$$h_1 = CQ_1^2 = (40000\,\text{s}^2/\text{m}^5)(0.02643\,\text{m}^3/\text{s})^2 = 27.94\,\text{m}$$

Now, the speed at operating point 2 can be calculated by using Equation (13.48) since $(\pi_Q')_1 = (\pi_Q')_2$ at the similar operating points. Then

$$N_2 = N_1 \frac{Q_2}{Q_1} = (1500\,\text{rpm})\frac{0.03\,\text{m}^3/\text{s}}{0.02643\,\text{m}^3/\text{s}} = \underline{\mathbf{1703\,rpm}}$$

At this point, one should note that the speed at operating point 2 can also be calculated by using Equation (13.49), since $(\pi_h')_1 = (\pi_h')_2$ at the similar operating points. In this case

$$N_2 = N_1 \sqrt{\frac{h_2}{h_1}} = (1500\,\text{rpm})\sqrt{\frac{36\,\text{m}}{27.94\,\text{m}}} = \underline{\mathbf{1703\,rpm}}$$

b) The efficiency at operating point 2 is equal to the efficiency at point 1, since they are similar operating points. However, the efficiency versus discharge curve is given for 1500 rpm so that

$$\eta_2 = \eta_1 = 60Q_1 - 1200Q_1^2 = (60)(0.02643\,\text{m}^3/\text{s}) - (1200)(0.02643\,\text{m}^3/\text{s})^2$$

$$= \underline{\mathbf{0.7476}}$$

c) Finally, the power consumed by the pump at operating point 2 is

$$P_2 = \frac{\rho g h_2 Q_2}{\eta} \frac{(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(36\,\text{m})(0.03\,\text{m}^3/\text{s})}{0.7476} = \underline{14.17\,\text{kW}}$$

13.5.4.2 Geometrically Similar Turbomachines Operating at the Same Rotational Speed

In this case, the rotational speed is no longer a variable in the affinity laws, so that the affinity laws become

$$\pi_Q'' = \frac{Q}{d^3} \tag{13.51}$$

$$\pi_h'' = \frac{h}{d^2} \tag{13.52}$$

and

$$\pi_P'' = \frac{P}{d^5} \tag{13.53}$$

respectively.

Example 13.5

The h versus Q characteristic of a centrifugal pump is given as

$$h = 100 - 1000Q^2$$

where h is the head in m and Q is the volumetric flow rate in m³/s. It is desired to deliver 0.1 m³/s of water against a head of 80 m. For this reason, a smaller impeller is used. Determine the percentage reduction in the diameter of the impeller.

Solution

a) From the given h versus Q characteristic, one could easily determine that the pump can produce a head of 90 m at a volumetric flow rate of 0.1 m³/s. In order to operate the pump at point 2 in Figure 13.27 having a head of h_2 = 80 m and a volumetric flow rate of Q_2 = 0.1 m³/s, the size of the impeller must be changed.

The locus of similar operating points in the h-Q plane for different rotational speeds can be obtained by eliminating the diameter from Equations (13.51) and (13.52) as

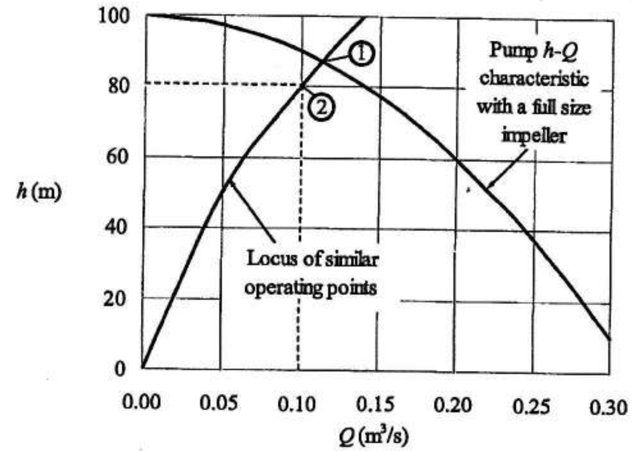


Figure 13.27 Sketch for Example 13.5

$$h = \frac{\pi_h''}{(\pi_Q'')^{2/3}} Q^{2/3} = CQ^{2/3}$$

where $C = \pi_h''/(\pi_Q'')^{2/3}$ is a constant. This constant can be determined by using point 2 on the locus of similar operating points as

$$C = \frac{h_2}{Q_2^{2/3}} = \frac{80\,\text{m}}{(0.1\,\text{m}^3/\text{s})^{2/3}} = 371.3\,\text{s}^{2/3}/\text{m}$$

Then, the equation for the locus of similar operating points takes the following form:

$$h = 371.3Q^{2/3}$$

The point of intersection of the locus of similar operating points with the h versus Q characteristic of the pump with a full size impeller gives the coordinates of operating point 1. At this stage, if the equation for the locus of similar operating points is combined with the equation for the h versus Q characteristic of the pump with a full size impeller, the following quadratic equation can be obtained.

$$1.954\text{x}10^{-5} h_1^3 + h_1 - 100 = 0$$

which can be solved to give

$$h_1 = 87.1\,\text{m}$$

The corresponding value of the volumetric flow rate at operating point 1 is

$$Q_1 = \left(\frac{h_1}{C}\right)^{3/2} = \left(\frac{87.1\,\text{m}}{371.3\,\text{s}^{2/3}/\text{m}}\right)^{3/2} = 0.1136\,\text{m}^3/\text{s}$$

Now, the ratio of the diameters can be calculated by using Equation (13.52), since $(\pi_h'')_1 = (\pi_h'')_2$ at the similar operating points. Then

$$\frac{d_2}{d_1} = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{80\,\text{m}}{87.1\,\text{m}}} = 0.9584$$

Finally, the percent reduction in the impeller diameter can be calculated as

$$\%\,\text{reduction in diameter} = \left(1 - \frac{d_2}{d_1}\right) \text{x} 100 = (1 - 0.9584)(100) = \underline{4.16\,\%}$$

13.6 THE SPECIFIC SPEED OR THE TYPE NUMBER

The performance of geometrically similar turbomachines, that is the ones belonging to the same family, is governed by the nondimensional similarity parameters and the performance of the whole family can be represented by a single plot of dimensionless parameters. In other words, when the nondimensional experimental characteristics, that is π_h versus π_Q of different sized geometrically similar turbomachines running at different speeds, are plotted on the same graph, they coincide on a single curve with some scatter, as shown in Figure 13.28. This scatter is due to the experimental errors and the Reynolds number effects. Therefore, one can compare the performance of machines belonging to different families by plotting their nondimensional characteristics on the same graph. Hence, the families of geometrically similar turbomachines can be classified by using the nondimensional type number or the specific speed.

Every turbomachine is designed for a specific duty to operate at the design point. In the case of a pump, the volumetric flow rate and head developed are specified at the design point and they represent a single point on its performance characteristic. The design point usually corresponds to the point of maximum efficiency, as shown in Figure 13.29.

Therefore, turbomachines can be compared by using the values of π_Q, π_h and π_P parameters at the design point.

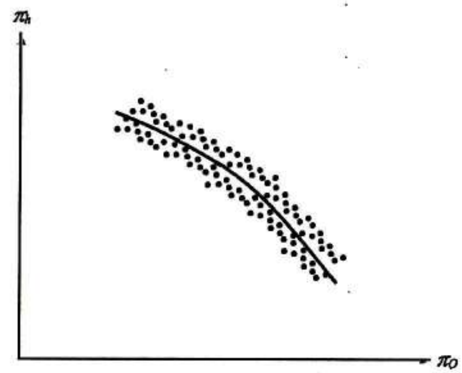


Figure 13.28 Nondimensional performance characteristic of different sized geometrically similar pumps running at different speeds

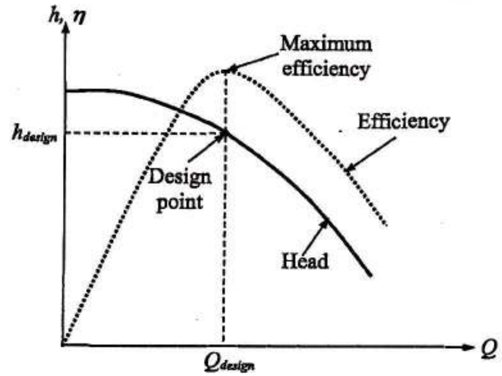


Figure 13.29 Design point of a pump

For pumps, π_Q, and π_h are the most important nondimensional parameters and their ratio indicates whether a particular pump is suitable for large or small volumetric flow rates relative to the developed head. Meanwhile, if the impeller diameter is eliminated as this ratio is obtained, the comparison will be independent of machine size. Such a ratio, known as the **type number** or the **specific speed,** N_s, is obtained by dividing π_Q raised to the power 1/2 to π_h raised to the power 3/4. Hence

$$N_s = \frac{\pi_Q^{1/2}}{\pi_h^{3/4}} = \frac{\left(\dfrac{Q}{\omega d^3}\right)^{1/2}}{\left(\dfrac{gh}{\omega^2 d^2}\right)^{3/4}} = \frac{\omega Q^{1/2}}{(gh)^{3/4}} \tag{13.54}$$

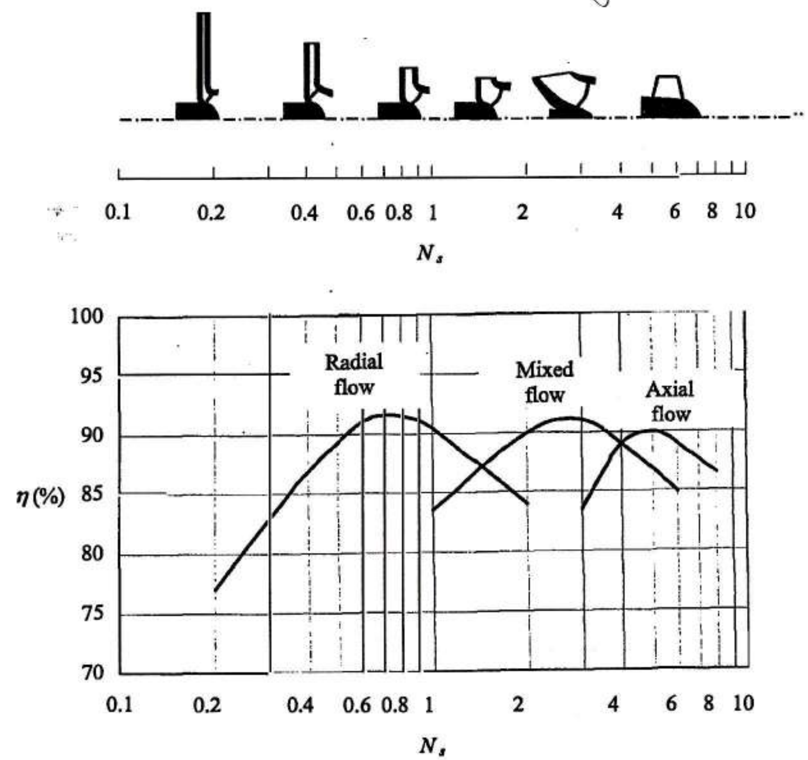


Figure 13.30 Classification of pump types with respect to the specific speed

Although, it is possible to calculate the value of the specific speed for any point on the characteristic curve, such values have no practical interest. Therefore, only the value of the specific speed at the design point referred to as the specific speed is used for the classification, comparison and design purposes. Variation of pump types with the specific speed is presented in Figure 13.30.

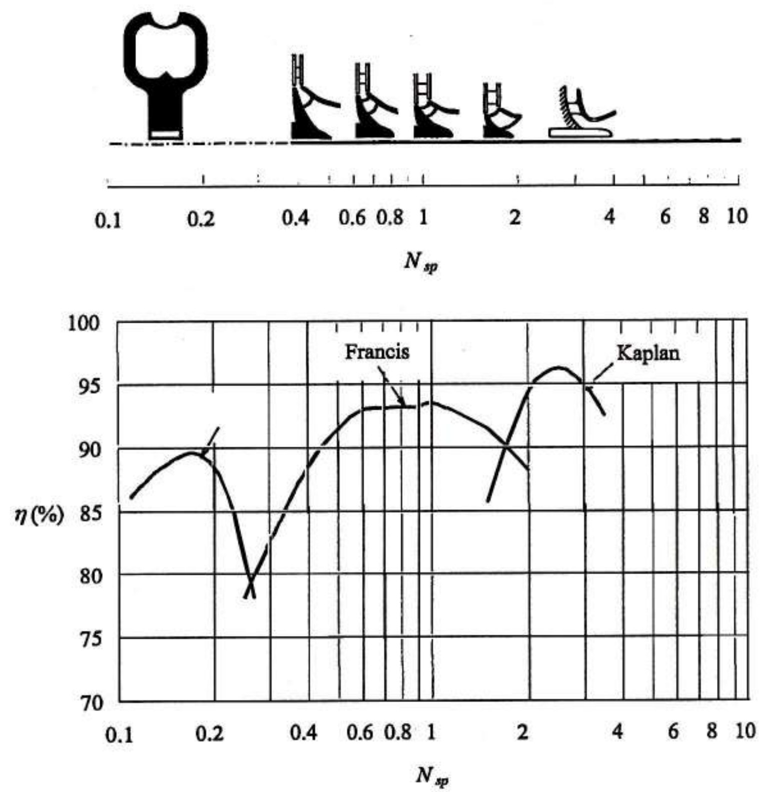


Figure 13.31 Classification of turbine types with respect to the specific speed based on power

The classification and comparison of turbines are also carried out by the specific speed. In this case, the power output is the most important variable, and the **type number** or the **specific speed based on power,** N_{sp}, can be obtained by eliminating the impeller diameter from the head and power coefficients. This ratio can be obtained by dividing π_P raised to the power of 1/2 to π_h raised to the power 5/4 as

$$N_{sp} = \frac{\pi_P^{1/2}}{\pi_h^{5/4}} = \frac{\left(\dfrac{P}{\rho\omega^3 d^5}\right)^{1/2}}{\left(\dfrac{gh}{\omega^2 d^2}\right)^{5/4}} = \frac{\omega P^{1/2}}{\rho^{1/2}(gh)^{5/4}} \tag{13.55}$$

It is possible to classify various types of turbines by using the specific speed based on power, as shown in Figure 13.31.

13.7 PUMPS

In general, it is possible to classify the pumps as radial, mixed and axial flow according to the geometry of the flow passage.

13.7.1 Centrifugal Pumps

A centrifugal pump, which is shown in Figure 13.32, is a radial flow turbomachine. They operate most efficiently in applications requiring high head rise at low volumetric flow rates. The specific speed for centrifugal pumps is in general less than 1.0. It consists of an **impeller** attached to a rotating shaft and a stationary **casing, housing** or **volute** enclosing the impeller. There are a number of curved **blades** or **vanes** attached to the impeller. As the impeller rotates, the fluid is first sucked in through the **eye** of the casing and then energy is added to the fluid by the rotating blades. The fluid with increased velocity and pressure leaves the impeller radially and discharges into the volute casing. In the volute casing, as the flow area increases, the kinetic energy of the fluid decreases resulting in an additional pressure increase. However, for large centrifugal pumps, the deceleration of the fluid is achieved by stationary diffuser guide vanes surrounding the impeller.

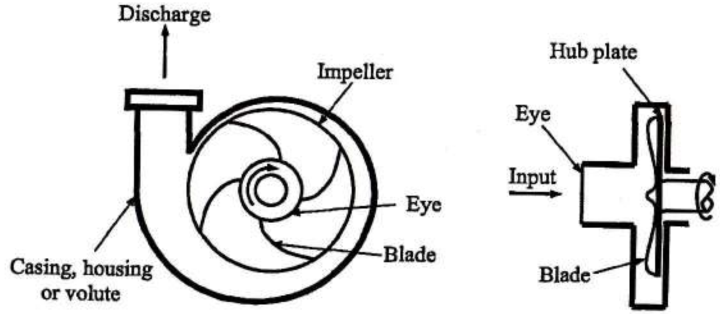


Figure 13.32 Sketch of a centrifugal pump

In general, pump impellers are of two types. In **open impellers,** the blades are arranged on a backing plate known as a **hub** and are open on the casing or **shroud** side, as shown in Figure 13.33a. However, the blades are covered both on the hub and shroud sides in an **enclosed** or **shrouded impeller,** as shown in Figure 13.33b.

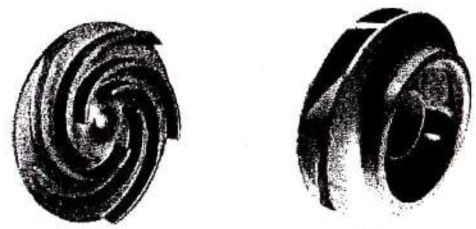


Figure 13.33 (a) An open impeller and (b) an enclosed or shrouded impeller

Pump impellers can be **single** or **double suction** type. The fluid enters through the eye on one side of a single suction impeller, while the fluid enters through the eyes on both sides of a double suction impeller. A double suction impeller looks like two single suction impellers placed back to back so that it has double width with a center plate. In this arrangement, the volumetric flow rate is doubled at the same head.

Pumps can be **single** or **multistage** type. In a single stage pump, only one impeller is mounted on the shaft, while several impellers are mounted on the same shaft in a multistage pump. The stages in a multistage pump operate in series so that the discharge from the first stage flows into the eye of the second stage, the discharge from the second stage flows into the eye of the third stage and so on. Therefore, the volumetric flow rate is the same for all stages, while the pressure of the fluid is increased over each stage. For this reason, a large pressure rise can be obtained in a multistage pump.

Centrifugal pumps may have (i) backward, (ii) radial or (iii) forward curved blades at the outlet. The inlet of the centrifugal pumps is almost designed for no whirl, that is $V_{\theta 1} = 0$. In this case, Equation (13.9) becomes

$$h = \frac{U_2 V_{\theta 2}}{g}$$

From Figure 13.15, one can observe that $V_{\theta 2} = U_2 - W_{\theta 2}$ and $\tan\beta_2 = V_{m2}/W_{\theta 2}$ so that

$$h = \frac{U_2}{g}(U_2 - V_{m2}\cot\beta_2)$$

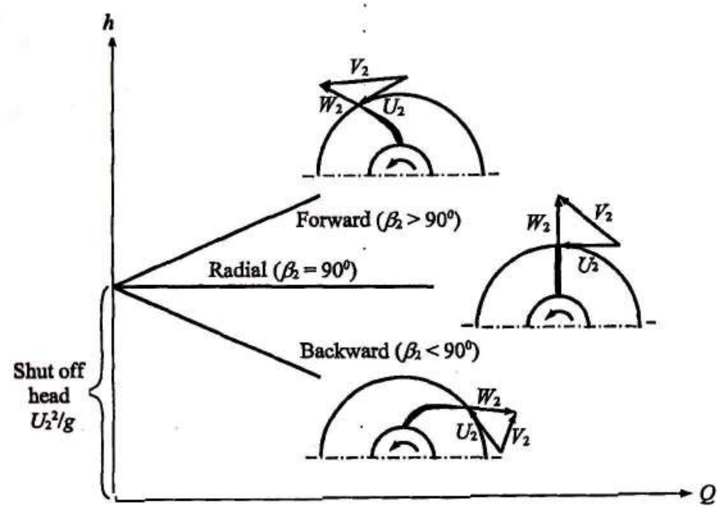


Figure 13.34 The head produced by a centrifugal pump versus the discharge for different values of the outlet blade angle

But $V_{m2} = Q/A_2$ and it is possible to express the same equation

$$h = \frac{U_2^2}{g} - \frac{U_2}{gA_2} Q\cot\beta_2$$

For a specified centrifugal pump with a known geometry and speed, U_2 and A_2 are constants so that

$$h = A - BQ\cot\beta_2$$

where $A = U_2^2/g$ and $B = U^2/(gA_2)$. The above equation is plotted in Figure 13.34 for different values of the outlet blade angle.

The value of the head corresponding to zero discharge is known as the **shut-off head** and the head produced by the centrifugal pump at this condition is U_2^2/g.

Although all three different types of blades are possible, configurations with radial and forward curved blades are not used in practice. In configurations with radial and forward curved blades, the produced absolute velocity is high and the head loss is proportional to the square of this absolute velocity. For this reason, it is recommended that $15^0 < \beta_1 < 50^0$ and $20^0 < \beta_2 < 25^0$.

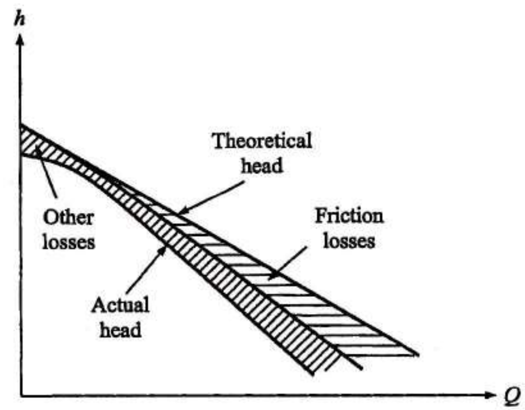


Figure 13.35 Effect of losses on the head versus discharge characteristic of a centrifugal pump

The theoretical head decreases linearly with the volumetric flow rate for centrifugal pumps with backward curved blades, as shown in Figure 13.35. However, the actual head rise is always less than the theoretical one due to several different sources of loss. Losses due to the skin friction in the blade passages are proportional to the square of the volumetric flow rate. Other losses are due to the flow through the clearance between the impeller and casing, flow separation, etc. These losses are minimum near the design flow rate.

The effect of the outlet blade angle on the performance characteristics is shown in Figure 13.36. One should note that although forward bladed impeller generates greater head at a given volumetric flow rate, the major part of this head is the velocity head. Also, the power versus discharge characteristics show different behavior for impellers with backward curved, radial and forward curved blades. In the case of an impeller with backward curved blades, the power is maximum at the design point with the maximum efficiency, but it decreases beyond this point. For this reason, the electric motor, which is used to drive the pump, can be selected at the design point. This type of power versus discharge characteristic is known as self-limiting. However, this is not the case for pumps with impellers having radial or forward curved blades for which the power requirement increases as the volumetric flow rate increases. For this reason, there are difficulties in the selection of the electric motor for these cases.

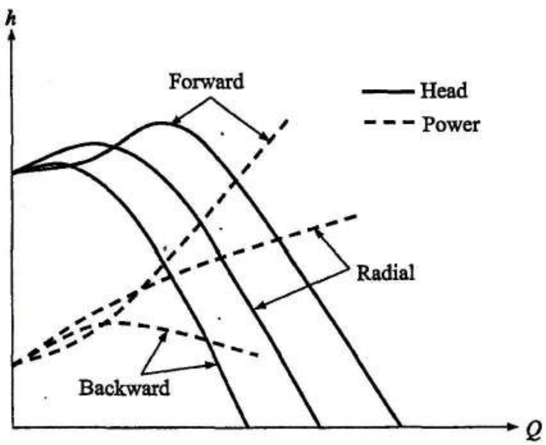


Figure 13.36 Effect of blade angle on performance characteristics

Typical performance characteristics for a centrifugal pump are presented in Figure 13.37.

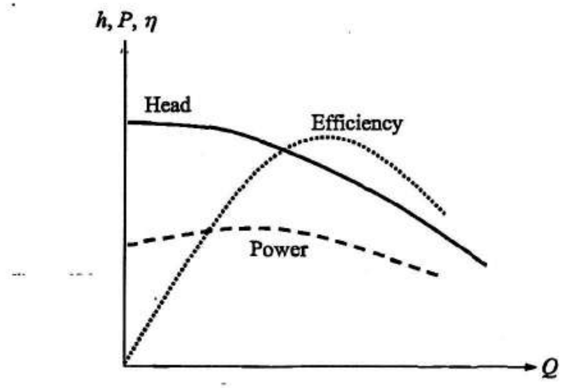


Figure 13.37 Typical performance characteristics of a centrifugal pump

13.7.2 Axial Flow Pumps

Axial flow pumps consist of an impeller, which is rotating in a concentric cylinder, as shown in Figure 13.38. In most axial flow pumps, except the cheap and inefficient ones, there are stationary guide vanes downstream of the rotating impeller so that the flow at the outlet of the pump is only in the axial direction without a swirl.

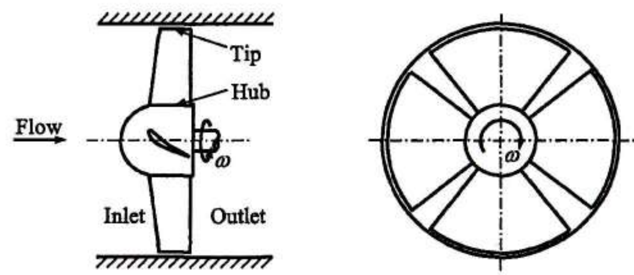


Figure 13.38 Sketch of an axial flow pump

The specific speed for axial flow pumps ranges from 2.8 to about 4.8. Axial flow pumps operate most efficiently in applications requiring low head rise at high volumetric flow rates. Typical performance characteristic for an axial flow pump is shown in Figure 13.39. If these performance characteristics are compared with the performance characteristics of a centrifugal pump in Figure 13.37, it is possible to note that there are basic differences between them. As the volumetric flow rate is decreased from the design value towards shutoff, the power requirement of the centrifugal pump decreases while the power requirement of the axial pump increases. Therefore, the power versus discharge characteristic of an axial pump is **overloading** type and the electric motor can be overloaded if the volumetric flow rate is significantly decreased from the design capacity. The head versus discharge characteristic for the axial flow pump is much steeper than that for the centrifugal pump. For this reason, there will be a large change in the head with a small change in the volumetric flow rate for axial flow pumps. However, the head versus discharge curve for the centrifugal pump is flat so that there will only be a small change in head with large changes in the volumetric flow rate. Finally, one should note that the efficiency of the axial pump is lower than that of the centrifugal pump except at the design point, since axial flow pumps have steeply descending efficiency curves. Therefore, axial flow pumps are only economical if they are operated at or very near to the design point.

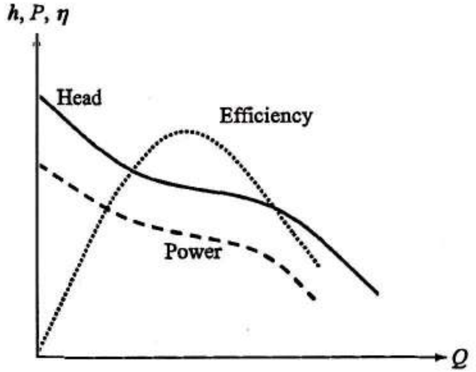


Figure 13.39 Typical performance characteristics for an axial flow pump

13.7.3 Mixed Flow Pumps

Mixed flow pumps cover the specific speeds ranging from 1.0 to 2.2 which is in between the centrifugal and axial flow pumps. The impeller of a mixed flow pump consists of a conical hub with attached blades. The blades are so arranged that the flow enters the impeller axially and leaves in a direction which is partly axial and partly radial. The fluid leaving the impeller is diffused through the stationary guide vanes and discharged axially, as shown in Figure 13.40a. Sometimes, the flow is collected by a volute casing and then discharged in a direction perpendicular to the axis of rotation of the impeller, as shown in Figure 13.40b. They usually operate at the lower end of the specific speed range for mixed flow pumps. The most important advantage of mixed flow pumps with axial discharge is that they may be arranged in multistage units to provide high pressures. The efficiency of mixed flow pumps are quite high and approach 90 percent.

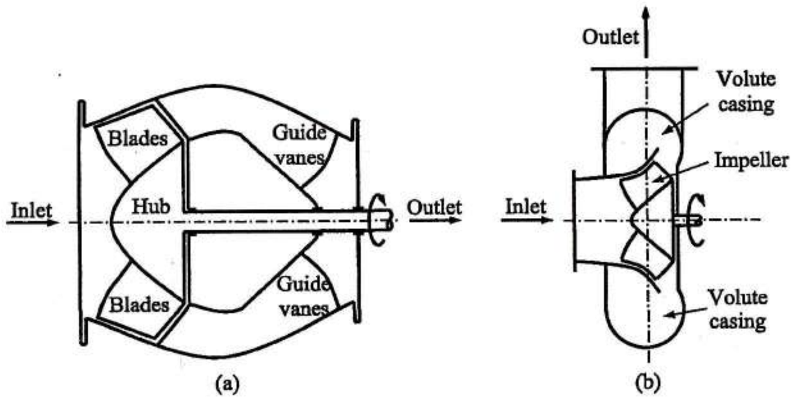


Figure 13.40 Mixed flow pump with (a) axial discharge and (b) with volute casing

13.8 TURBINES

In general, it is possible to classify the turbines as the impulse and reaction turbines.

In **impulse turbines**, the available total head of the fluid is first converted into the kinetic energy through a nozzle. The jet that is issuing from the nozzle strikes the vanes attached to the periphery of a rotating wheel. As a result of the momentum exchange between the fluid and vanes, energy is transferred to the wheel. For this reason, the kinetic energy of the fluid is reduced so that the absolute velocity of the fluid at the exit is less than the one at the inlet. Furthermore, the pressure is atmospheric throughout and the relative velocity is almost constant except the slight reduction due to the friction.

In **reaction turbines**, as the fluid passes through a ring of stationary guide vanes, part of its available total head is converted into the kinetic energy. Afterwards, the fluid having both pressure and kinetic energies enters the rotor through its periphery. In the rotor, the pressure energy is converted into the kinetic energy so that the relative velocity of the fluid increases. Hence, there exists a pressure difference across the rotor.

The energy transferred by the fluid to the turbine per unit weight of the fluid can be obtained by the application of the Bernoulli equation between the inlet and exit of the turbine as

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+h$$

In the above equation, the subscripts 1 and 2 denote the inlet and exit of the turbine. The above equation can now be rearranged as

$$h=\frac{p_1-p_2}{\rho g}+\frac{V_1^2-V_2^2}{2g} \qquad (13.56)$$

In Equation (13.56), the first term on the right hand side represents the drop in the static pressure head of the fluid across the turbine, while the second term represents the drop in the velocity head across the turbine. If a turbine is purely impulsive, then $p_1 = p_2$ and $h = (V_1^2 - V_2^2)/(2g)$. On the other hand, $V_1 = V_2$ and $h = (p_1 - p_2)/(\rho g)$ in a pure reaction turbine. For the turbines, which are not purely impulsive or purely reactive, the **degree of reaction**, R, is defined as

$$R=\frac{\text{Static pressure drop}}{\text{Total energy transfer}} \qquad (13.57)$$

The above equation can be reformulated as

$$R=\frac{\frac{p_1-p_2}{\rho g}}{h}=\frac{h-\frac{V_1^2-V_2^2}{2g}}{h}=1-\frac{V_1^2-V_2^2}{2gh} \qquad (13.58)$$

There are three important types of water turbines. These are the **Pelton wheel** which is an impulsive turbine and **Francis (radial flow)** and **Kaplan (axial flow)** turbines which are both reaction turbines.

13.8.1 Impulse Turbines

Although there are various types of impulse turbines, the Pelton wheel is the easiest one to understand. The Pelton wheel, which is shown in Figure 13.41, is named after Lester Pelton (1829-1908), an American mining engineer. The jet of water which issues from the nozzle impinges on the vanes attached to the periphery of a rotating wheel. The vanes are of double outlet section, as shown in Figure 13.42, so that the jet splits symmetrically to both sides of the vane eliminating the end thrust in the bearings. It is most efficient when operated under a large head which can be converted to a large velocity at the exit of the nozzle.

The velocity of the jet of fluid leaving the nozzle is

$$V_1=C_v\sqrt{2gh} \qquad (13.59)$$

where C_v is the velocity coefficient of the nozzle and its value is in between 0.97 and 0.99.

To calculate the torque and power developed by the Pelton turbine, one can consider the inlet and velocity diagrams in Figure 13.42. At an average radius of $r_m = 0.5(r_1 + r_2)$, the torque transmitted to the Pelton wheel can be found from Equation (13.4) as

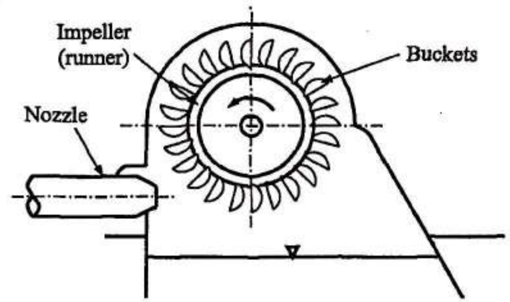


Figure 13.41 Sketch of a Pelton wheel

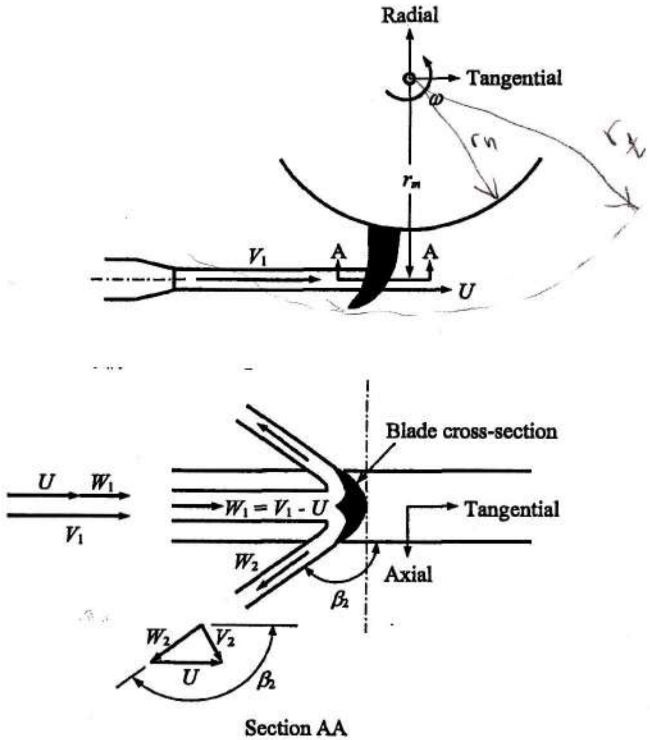


Figure 13.42 Inlet and outlet velocity diagrams for a Pelton turbine

$$T = \dot{m} r_m (V_{\theta 2} - V_{\theta 1}) \tag{13.60}$$

From the outlet velocity triangle in Figure 13.42, the tangential component of the absolute velocity at the outlet is

$$V_{\theta 2} = U - W_{\theta 2} = U - W_2 \cos(180^0 - \beta_2) = U + W_2 \cos\beta_2$$

The relative velocity at the exit can be expressed as

$$W_2 = C_b W_1$$

where C_b is the blade coefficient and it represents the reduction of the relative velocity due to friction. Noting that $W_1 = V_1 - U$ from the inlet velocity diagram of Figure 13.42

$$V_{\theta 2} = U + C_b (V_1 - U)\cos\beta_2$$

The above relation can now be substituted into Equation (13.60) with the fact that $W_{\theta 1} = V_1$ to yield

$$T = \dot{m} r_m (V_1 - U)(C_b \cos\beta_2 - 1) \tag{13.61}$$

Then the power is

$$P = T\omega = \dot{m} U (V_1 - U)(C_b \cos\beta_2 - 1) \tag{13.62}$$

since $U = \omega r_m$. Equation (13.62) indicates that there is no energy transfer when the vane velocity is zero or when it is equal to the jet velocity. Therefore, the energy transfer should be a maximum at an intermediate value of the vane velocity. For this reason, Equation (13.62) can be differentiated with respect to the peripheral velocity and then equated to zero as

$$\frac{dP}{dU} = \frac{(C_b \cos\beta_2 - 1)}{g}(V_1 - 2U)$$

or

$$U = 0.5 V_1 \tag{13.63}$$

Then, the maximum torque and power developed by the Pelton turbine can be found by substituting Equation (13.63) into Equations (13.61) and (13.62), respectively, to obtain

$$T_{max} = 0.5 \dot{m} r_m V_1 (C_b \cos\beta_2 - 1) \tag{13.64}$$

and

$$P_{max} = 0.25 \dot{m} V_1^2 (C_b \cos\beta_2 - 1) \tag{13.65}$$

One should note that the maximum torque and power in Equations (13.64) and (13.65) are negative, since $\cos\beta_2 < 0$ when $\beta_2 > 90^0$. This is basically due to the fact that the turbine extracts energy from the fluid.

The maximum theoretical efficiency of the Pelton turbine can now be defined as the ratio of the absolute value of the maximum power developed to the available kinetic energy of the jet.

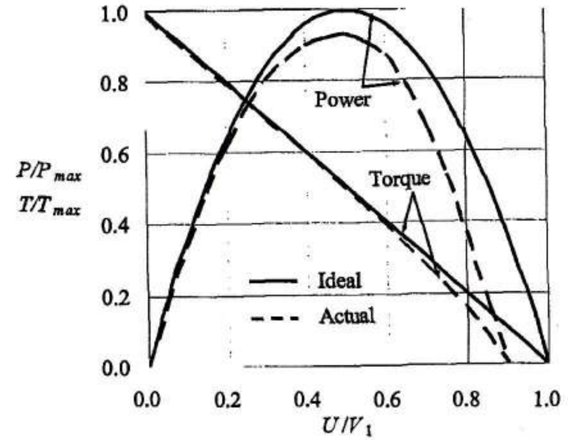


Figure 13.43 Variation of the torque and power with the ratio of the wheel speed to the jet velocity

$$\eta_{max} = \frac{|P_{max}|}{0.5\dot{m}V_1^2} = \frac{|0.25\dot{m}V_1^2(C_b \cos\beta_2 - 1)|}{0.5\dot{m}V_1^2} = \frac{1 - C_b \cos\beta_2}{2} \tag{13.66}$$

In the ideal case, when there is no friction, C_b = 1.0 and β_2 = 180°, the maximum theoretical efficiency is 100 percent. In practice, however, there is friction so that C_b is in the range of 0.80 to 0.85. Also, in order to prevent the interference of the incoming and outgoing jets, the vane angle is usually 165°. The variation of the torque and power with the ratio of the wheel speed to the jet velocity for ideal and actual cases is presented in Figure 13.43.

Example 13.6

The Pelton wheel in Figure 13.44 is driven by a jet of water, which is 50 m below the free surface of the supply reservoir. The diameter of the pipe is 0.2 m, while the diameter of the Pelton wheel is 1 m. The fluid is deflected by 165° in the buckets. Neglect the losses in the nozzle and assume that the blade coefficient is unity.

a) If the fluid flow is frictionless, determine the angular speed of the Pelton wheel and power developed by the Pelton wheel for maximum power output.

b) If the length of the supply pipe is 250 m with a friction factor of 0.02, determine the exit diameter of the nozzle, angular speed of the Pelton wheel and power developed by the Pelton wheel for maximum power output. Neglect minor head losses.

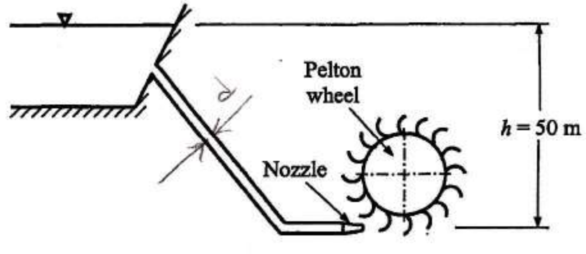


Figure 13.44 Sketch for Example 13.6

Solution

a) The velocity of the water jet can be evaluated by using Equation (13.74) as

$$V_1 = C_v\sqrt{2gh} = (1.0)\sqrt{(2)(9.81\,\text{m/s}^2)(50\,\text{m})} = 31.32\,\text{m/s}$$

For maximum power output, the relation between the wheel speed and the jet velocity is given by Equation (13.78) as

$$U = 0.5V_1 = (0.5)(31.32\,\text{m/s}) = 15.66\,\text{m/s}$$

The angular speed of the Pelton wheel is then

$$\omega = \frac{U}{R} = \frac{15.66\,\text{m/s}}{0.5\,\text{m}} = 31.32\,\text{rad/s}$$

or

$$N = \frac{60\omega}{2\pi} = \frac{(60\,\text{s/min})(31.32\,\text{rad/s})}{2\pi\,\text{rad/rev}} = \underline{\mathbf{299.1\,rpm}}$$

The mass flow rate of water can be evaluated as

$$\dot{m} = \rho V_1 \frac{\pi d^2}{4} = (1000\,\text{kg/m}^3)(31.32\,\text{m/s})\frac{(\pi)(0.2\,\text{m})^2}{4} = 984\,\text{kg/s}$$

The power developed by the turbine can now be evaluated by using Equation (13.80) as

$$P_{max} = 0.25\dot{m}V_1^2(C_b\cos\beta_2 - 1) = (0.25)(984\ \text{kg/m}^3)(31.32\ \text{m/s})^2$$

$$\times[(1.0)\cos 165^0 - 1)] = \underline{-474.4\ \text{kW}}$$

The minus sign indicates that the energy is extracted from the fluid.

b) The application of the Bernoulli equation between the free surface of the supply reservoir and the exit of the nozzle yields

$$\frac{p_{sr}}{\rho g} + \frac{V_{sr}^2}{2g} + z_{sr} = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 + f\frac{L}{d}\frac{V^2}{2g}$$

Noting that $z_{sr} = h$, $V_{sr} \cong 0$, $z_{sr} = h$, $p_{sr} = p_1 = p_{atm}$ and $z_1 = 0$ according to the datum in Figure 13.44, one can obtain

$$h = \frac{V_1^2}{2g} + f\frac{L}{d}\frac{V^2}{2g}$$

Application of the continuity equation between the pipe and nozzle exit yields

$$V\frac{\pi d^2}{4} = V_1\frac{\pi d_1^2}{4}$$

or

$$V = V_1\frac{d_1^2}{d^2}$$

As a result, the velocity of the water jet at the exit of the nozzle can be expressed as

$$V_1 = \sqrt{\frac{2gh_0}{1+\frac{fLd_1^4}{d^5}}} = \sqrt{\frac{(2)(9.81\ \text{m/s}^2)(50\ \text{m})}{1+\frac{(0.02)(250\ \text{m})d_1^4}{(0.2\ \text{m})^5}}} = \frac{31.32}{\sqrt{1+15625d_1^4}}$$

The mass flow rate of the water is then

$$\dot{m} = \rho V_1\frac{\pi d_1^2}{4} = (1000\ \text{kg/m}^3)\left[\frac{31.32}{\sqrt{1+15625d_1^4}}\right]\frac{\pi d_1^2}{4} = \frac{24599d_1^2}{\sqrt{1+15625d_1^4}}$$

so that the power developed by the turbine can be now be calculated by using Equation (13.80) as follows:

$$P_{max} = \frac{1}{4}\dot{m}V_1^2(C_b\cos\beta_2 - 1) = \frac{1}{4}\frac{24599d_1^2}{\sqrt{1+15625d_1^4}}\frac{(31.32)^2}{1+15625d_1^4}$$

$$\times[(1.0)(\cos 165^0 - 1)] = -\frac{1.186\text{x}10^7 d_1^2}{(1+15625d_1^4)^{3/2}}$$

The above equation gives the power which is maximized with respect to the vane velocity. However, it can also be maximized with respect to the nozzle exit diameter by differentiating the above expression with respect to the nozzle exit diameter and equating the result to zero as

$$\frac{dP_{max}}{dd_1} = -\left[\frac{2d_1(1+15625d_1^4)^{3/2} - 1.5(1+15625d_1^4)^{1/2}(4)(15625d_1^3)d_1^2}{(1+15625d_1^4)^{5/2}}\right]$$

$$\times(1.186\text{x}10^7) = 0$$

which can be solved for the nozzle exit diameter to yield

$$d_1 = \underline{0.07524\ \text{m}}$$

The jet velocity can now be evaluated as

$$V_1 = \frac{31.32}{\sqrt{1+(15625)(0.07524)^4}} = 25.57\ \text{m/s}$$

For maximum power output, the relation between the wheel speed and jet velocity is given by Equation (13.78) as

$$U = 0.5V_1 = (0.5)(25.57\ \text{m/s}) = 12.79\ \text{m/s}$$

The angular speed of the Pelton wheel is then

$$\omega = \frac{U}{R} = \frac{12.79\ \text{m/s}}{0.5\ \text{m}} = 25.58\ \text{rad/s}$$

or

$$N = \frac{60\omega}{(2)(\pi)} = \frac{(60\ \text{min/s})(25.58\ \text{rad/s})}{2\pi\ \text{rad/rev}} = \underline{244.3\ \text{rpm}}$$

The mass flow rate of water can be evaluated as

$$\dot{m} = \rho V_1\frac{\pi d_1^2}{4} = (1000\ \text{kg/m}^3)(25.57\ \text{m/s})\frac{\pi(0.07524\ \text{m})^2}{4} = 113.7\ \text{kg/m}^3$$

The maximum power developed by the turbine can now be evaluated by using Equation (13.79) as follows:

$$P_{max} = \frac{1}{4}\dot{m}V_1^2(C_b \cos\beta_2 - 1) = \frac{1}{4}(113.7\ \text{kg/m}^3)(25.58\ \text{m/s})^2\left[(1.0)\cos 165^0 - 1)\right]$$
$$= \underline{-36.57\ \text{kW}}$$

Again, the minus sign indicates that the energy is extracted from the fluid.

13.8.2 Reaction Turbines

In a reaction turbine, only a part of the available total head is converted into the kinetic energy before the fluid enters the runner. For this reason, both the static pressure and velocity drop during the energy transfer in the runner. There are basically two types of reaction turbines. These are the radial flow Francis turbine and axial flow Kaplan turbine.

13.8.2.1 Francis Turbine

In a Francis turbine, the fluid which is coming from the penstock completely fills the spiral casing, which is known as **volute** or **scroll**, as shown in Figure 13.45. The available total head of the fluid is partly converted into the kinetic energy in the stationary but adjustable inlet guide vanes. The fluid enters the impeller through its whole perimeter. If the inlet guide vanes surround the impeller, the fluid flows towards the center of the impeller and such a turbine is referred to as an **inward flow** type. As an alternative arrangement, the fluid may enter the inlet guide vanes at the center and flow radially outwards into the impeller surrounding the guide vanes. In this case, the turbine is referred to as an **outward flow** type. The fluid leaves the impeller through the draft tube after doing work on the impeller. Finally, the fluid joins the tailrace which is a channel for carrying away the used water.

The velocity diagrams at the inlet and exit of the impeller of a Francis turbine is shown in Figure 13.46. At the exit of the inlet guide vanes, the direction of the absolute velocity of the fluid is the same as the direction of the stationary blades. If there are no losses, this velocity is equal to the absolute velocity of the fluid at the inlet of the impeller. If the stationary guide vanes are adjusted properly, the inlet to the impeller is shockless, that is the relative velocity of the fluid entering the impeller is in the same direction as that of the blades at the inlet of the impeller. Finally, the fluid leaves the impeller in the same direction as the blades at a relative velocity of W_2, if the blade and fluid angles are exactly the same.

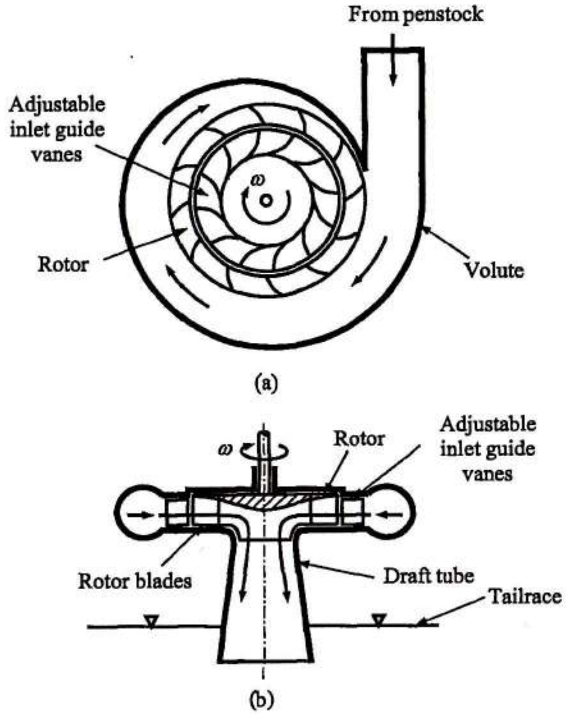


Figure 13.45 A schematic diagram of a Francis turbine

Example 13.7

An inward flow reaction turbine of the vertical shaft type having a specific speed of 0.5 operates under a net head of 55 m at an angular speed of 370 rpm, as shown in Figure 13.47. The inlet guide vanes make an angle of 20^0 with the tangent to the impeller. The impeller has a diameter of 1.2 m and a thickness 0.1 m at the inlet. The entry to the impeller is shockless with a radial velocity of 8 m/s. Water leaves the impeller without whirl at an absolute velocity of 6 m/s and enters into the draft tube. Finally, the water discharges to the tailrace at a velocity of 2 m/s. The gage pressures at the inlet of the impeller and at the inlet of the draft tube are measured to be 250 kPa and –30 kPa, respectively. The mean height of the impeller inlet and entrance to the draft tube are 3 m and 2.5 m from the water level in the tailrace, respectively. The blade thickness blocks 10 percent of the flow area at the inlet. Determine the

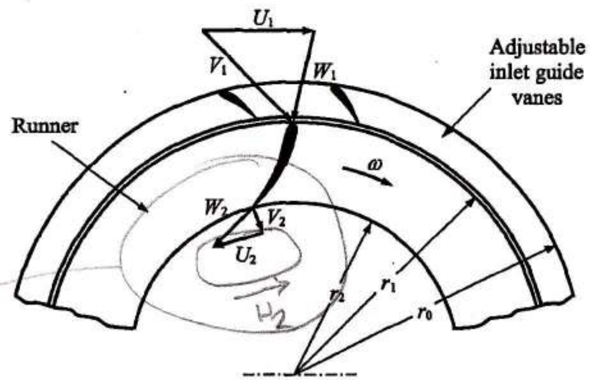


Figure 13.46 Inlet and exit velocity diagrams for a Francis turbine

a) relative fluid angle at the inlet of the impeller,
b) hydraulic efficiency,
c) head loss in the casing,
d) head loss in the impeller,
e) head loss in the draft tube,
f) overall efficiency and
g) inlet diameter of the draft tube.

Solution

a) The angular velocity of the impeller is

$$\omega = \frac{2\pi N}{60} = \frac{(2\pi \text{ rad/s})(370 \text{ rev/min})}{60 \text{ s/min}} = 38.75 \text{ rad/s}$$

The peripheral velocity at the inlet of the impeller is

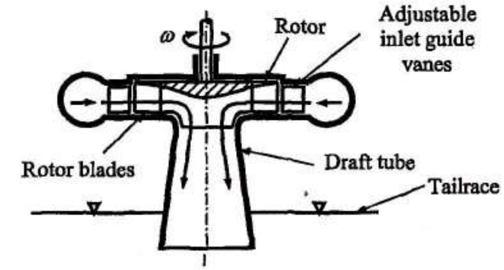


Figure 13.47 Sketch for Example 13.7

$$U_1 = \omega r_1 = (38.75 \text{ rad/s})(0.6 \text{ m}) = 23.25 \text{ m/s}$$

Using the velocity diagram at the inlet of the impeller in Figure 13.48, the absolute velocity at the inlet is

$$V_1 = \frac{V_{r1}}{\sin\alpha_1} = \frac{8 \text{ m/s}}{\sin 20^0} = 23.39 \text{ m/s}$$

while the tangential component of the absolute velocity is

$$V_{\theta 1} = V_1 \cos\alpha_1 = (23.39 \text{ m/s})(\cos 20^0) = 21.98 \text{ m/s}$$

Then, the tangential component of the relative velocity can be calculated as

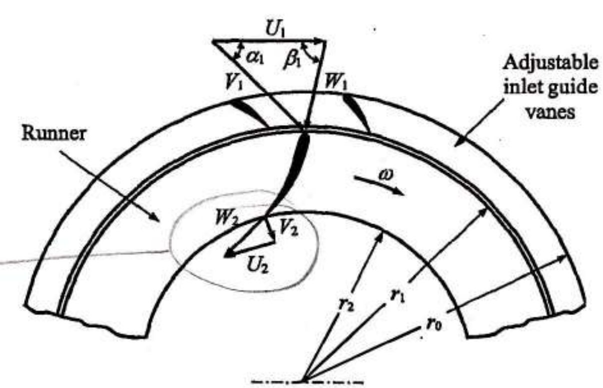


Figure 13.48 Velocity diagrams at the inlet and exit of the impeller in Example 13.7

$$W_{\theta 1} = U_1 - V_{\theta 1} = 23.25\,\text{m/s} - 21.98\,\text{m/s} = 1.27\,\text{m/s}$$

Finally, the relative velocity at the inlet of the impeller is

$$\beta_1 = \tan^{-1}\frac{V_{r1}}{W_{\theta 1}} = \tan^{-1}\frac{8\,\text{m/s}}{1.27\,\text{m/s}} = \underline{\mathbf{80.98^0}}$$

b) Since there is no outlet whirl, that is $V_{\theta 2} = 0$, the theoretical head delivered to the impeller can be evaluated by using the Euler's turbine equation (13.9) as

$$h_{th} = \frac{U_1 V_{\theta 1}}{g} = \frac{(23.25\,\text{m/s})(21.98\,\text{m/s})}{9.81\,\text{m/s}^2} = 52.09\,\text{m}$$

The hydraulic efficiency is then

$$\eta_h = \frac{h_{th}}{h} = \frac{52.09\,\text{m}}{55\,\text{m}} = \underline{\mathbf{0.9471}}$$

c) Since the net head available to the turbine represents the total head at the inlet of the casing excluding the atmospheric pressure head, the application of the Bernoulli equation between the inlet of the casing and inlet of the impeller yields

$$z_1 + \frac{p_{atm}}{\rho g} + h = \frac{p_{1g}}{\rho g} + \frac{p_{atm}}{\rho g} + \frac{V_1^2}{2g} + z_1 + h_c$$

or

$$h_c = h - \frac{p_{1g}}{\rho g} - \frac{V_1^2}{2g} = 55\,\text{m} - \frac{250000\,\text{N/m}^2}{(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)} - \frac{(23.39\,\text{m/s})^2}{(2)(9.81\,\text{m/s}^2)}$$

$$= \underline{\mathbf{1.631\,m}}$$

d) The head loss in the impeller can be evaluated by applying the Bernoulli equation between the inlet and exit of the rotor as

$$\frac{p_{1g}}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_{2g}}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_{th} + h_i$$

or

$$h_i = \frac{250000\,\text{N/m}^2}{(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)} + \frac{(23.39\,\text{m/s})^2}{(2)(9.81\,\text{m/s}^2)} + 3\,\text{m} - \frac{(6\,\text{m/s})^2}{(2)(9.81\,\text{m/s}^2)}$$

$$- \frac{-30000\,\text{N/m}^2}{(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)} - 2.5\,\text{m} - 52.09\,\text{m} = \underline{\mathbf{3.002\,m}}$$

e) The head loss in the draft tube can be found by applying the extended Bernoulli equation between the inlet of the draft tube and tailrace as

$$\frac{p_{atm}}{\rho g} + \frac{p_{2g}}{\rho g} + \frac{V_2^2}{2g} + z_2 = \frac{p_{atm}}{\rho g} + \frac{V_3^2}{2g} + z_3 + \sum h_{fdt}$$

Noting that $z_2 = 2.5$ m and $z_3 = 0$

$$\sum h_{fdt} = \frac{p_{2g}}{\rho g} + \frac{V_2^3}{2g} + z_2 - \frac{V_3^3}{2g} = \frac{-30000\,\text{N/m}^2}{(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)}$$

$$+ \frac{(6\,\text{m/s})^2}{(2)(9.81\,\text{m/s}^2)} + 2.5\,\text{m} - \frac{(2\,\text{m/s})^2}{(2)(9.81\,\text{m/s}^2)} = \underline{\mathbf{1.073\,m}}$$

f) The power delivered to the impeller can be obtained by using Equation (13.70) for the specific speed as

$$P = \frac{N_{sp}^2 (gh)^{5/2} \rho}{\omega^2} = \frac{(0.5)^2 \left[(9.81\,\text{m/s}^2)(55\,\text{m})\right]^{5/2} (1000\,\text{kg/m}^3)}{(38.75\,\text{rad/s})^2} = 1.126\,\text{MW}$$

The volumetric flow rate through the impeller can be evaluated at the impeller inlet as

$$Q = 0.9 V_{m1} \pi b_1 d_1 = (0.9)(8\,\text{m/s})(\pi)(0.1\,\text{m})(1.2\,\text{m}) = 2.714\,\text{m}^3/\text{s}$$

since there is 10 percent blockage and $V_{m1} = V_{r1}$. The overall efficiency can now be evaluated as

$$\eta = \frac{P}{\rho g Q h} = \frac{1.126\text{x}10^6\,\text{W}}{(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(2.714\,\text{m}^3/\text{s})(55\,\text{m})} = \underline{\mathbf{0.769}}$$

The inlet diameter of the draft tube is

$$d_{dt} = \sqrt{\frac{4Q}{\pi V_2}} = \sqrt{\frac{(4)(2.714\,\text{m}^3/\text{s})}{(\pi)(6\,\text{m/s})}} = \underline{\mathbf{0.7589\,m}}$$

13.8.2.2 Axial Flow Turbines (Kaplan Turbines)

The sketch of an axial flow turbine is shown in Figure 13.49. The arrangement of inlet guide vanes is similar to that of a Francis turbine. The inlet guide vane ring is located in a plane perpendicular to the shaft of the axial flow turbine so that the fluid flows in radially through the inlet guide vanes. The inlet guide vanes are used to impart whirl to the fluid which is inversely proportional to the radius. The fluid turns ninty degrees to the axial direction before entering the impeller.

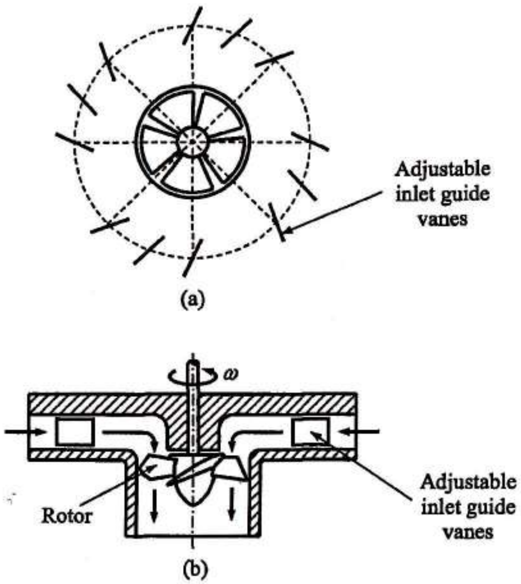


Figure 13.49 A schematic diagram of a Kaplan turbine

The velocity diagrams at the inlet and exit of the impeller of a Kaplan turbine is shown in Figure 13.50. The peripheral velocity of the blades is directly proportional to the radius and it is constant across the impeller at a given radius. Also, the axial velocity is constant, since the flow area does not change.

Example 13.8

The rotor of an axial flow turbine having stationary inlet guide vanes is rotating at 300 rpm. The hub and the tip diameters of the impeller are 0.8 m and 1.6 m, respectively. At the mean diameter, the exit blade angle of the guide vanes and inlet blade angle of the impeller are 35^0 and 25^0, respectively, relative to the tangential direction. Assume that the axial velocity is uniform at each cross-section and there is no outlet whirl. The entrance to the impeller is shockless. Determine the

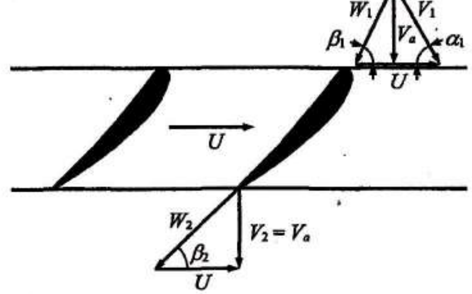


Figure 13.50 Inlet and exit velocity diagram for a Kaplan turbine

a) volumetric flow rate,

b) outlet blade angle of the impeller and

c) theoretical power output.

Solution

a) The angular velocity of the impeller is

$$\omega = \frac{2\pi N}{60} = \frac{(2\pi \text{ rad/s})(300 \text{ rev/min})}{60 \text{ s/min}} = 31.42 \text{ rad/s}$$

The mean diameter can be evaluated as

$$d_m = \frac{d_t + d_h}{2} = \frac{1.6\text{ m} + 0.8\text{ m}}{2} = 1.2\text{ m}$$

The peripheral speed at the mean diameter is constant through the impeller so that

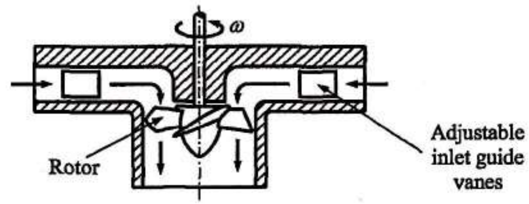


Figure 13.51 Sketch for Example 13.8

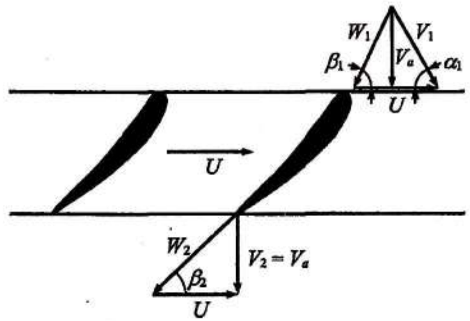


Figure 13.52 Velocity diagrams at the inlet and exit of the impeller in Example 13.8

$$U_1 = U_2 = U = \omega \frac{d_m}{2} = (31.42\ \text{rad/s})\left(\frac{1.2\ \text{m}}{2}\right) = 18.85\ \text{m/s}$$

From the velocity diagram at the inlet of the impeller in Figure 13.52, one can obtain

$$W_{\theta 1} = \frac{V_{a1}}{\tan \beta_1}$$

and

$$V_{\theta 1} = \frac{V_{a1}}{\tan \alpha_1}$$

which can be added to yield

$$U_1 = V_{\theta 1} + W_{\theta 1} = V_{a1}(\cot \alpha_1 + \cot \beta_1)$$

Then, the axial velocity at the inlet is

$$V_{a1} = \frac{U_1}{\cot \alpha_1 + \cot \beta_1} = \frac{18.85\ \text{m/s}}{\cot 35^0 + \cot 25^0} = 5.276\ \text{m/s}$$

Therefore, the volumetric flow rate is

$$Q = \frac{\pi (d_t^2 - d_h^2)}{4} V_{a1} = \frac{\pi \left[(1.6\ \text{m})^2 - (0.8\ \text{m})^2\right]}{4}(5.276\ \text{m/s}) = \underline{7.956\ \text{m}^3/\text{s}}$$

b) In an axial flow turbine, the axial velocity is constant since the flow area is constant. Hence

$$V_{a2} = V_{a1} = 5.276\ \text{m/s}$$

From the velocity diagram at the outlet of the impeller in Figure 13.52, the outlet blade angle of the impeller is

$$\beta_2 = \tan^{-1}\frac{V_{a2}}{U_2} = \tan^{-1}\frac{5.276\ \text{m/s}}{18.85\ \text{m/s}} = \underline{\mathbf{15.64^0}}$$

c) The tangential component of the absolute velocity at the inlet is

$$V_{\theta 1} = V_{a1} \cot \alpha_1 = (5.286\ \text{m/s})(\cot 35^0) = 7.549\ \text{m/s}$$

When there is no outlet whirl, the theoretical head can be evaluated by using Equation (13.9) as

$$h_{th} = \frac{U_1 V_{\theta 1}}{g} = \frac{(18.85\ \text{m/s})(7.549\ \text{m/s})}{9.81\ \text{m/s}^2} = 14.51\ \text{m}$$

Finally, the theoretical power is

$$P = \rho g Q h_{th} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(7.956\ \text{m}^3/\text{s})(14.51\ \text{m}) = \underline{\mathbf{1133\ kW}}$$

13.9 TURBOMACHINES OPERATING IN PIPE SYSTEMS

Till now, the flow through isolated turbomachines is discussed. However, pumps and turbines operate in a pipe system so that their performances are closely related to the losses in the system of pipes connected to them.

13.9.1 A Pump Operating in a Pipe System

Consider a pump, which is taking water from a suction reservoir and discharging it to a discharge reservoir at a higher elevation, as shown in Figure 13.53.

Applying the extended Bernoulli equation along a streamline between the free surface of the suction reservoir and suction side of the pump yields

$$\frac{p_{sr}}{\rho g} + \frac{V_{sr}^2}{2g} + z_{sr} = \frac{p_s}{\rho g} + \frac{V_s^2}{2g} + z_s + \sum h_{fs}$$

Noting that $z_{sr} = 0$ and $z_s = h_s$, then

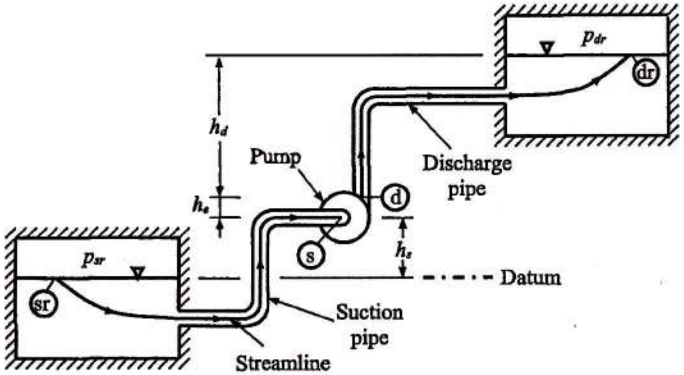


Figure 13.53 A pump operating in a pipe system

$$h_s = \frac{p_{sr} - p_s}{\rho g} + \frac{V_{sr}^2 - V_s^2}{2g} - \sum h_{fs} \tag{13.67}$$

where p_{sr} is the suction reservoir pressure, V_{sr} is the velocity at the free surface of the suction reservoir, p_s is the pump suction side pressure, V_s is the pump suction side velocity, Σh_{fs} is the total frictional head loss in the suction piping and h_s is the suction geometric head.

Similarly, application of the extended Bernoulli equation along a streamline between the discharge side of the pump and free surface of the discharge reservoir yields

$$\frac{p_d}{\rho g} + \frac{V_d^2}{2g} + z_d = \frac{p_{dr}}{\rho g} + \frac{V_{dr}^2}{2g} + z_{dr} + \sum h_{fd}$$

Noting that $z_d = h_s + h_e$ and $z_{dr} = h_s + h_e + h_d$ with respect to the datum in Figure 13.53, one can obtain the following relation:

$$h_d = \frac{p_d - p_{dr}}{\rho g} + \frac{V_d^2 - V_{dr}^2}{2g} - \sum h_{fd} \tag{13.68}$$

where p_{dr} is the discharge reservoir pressure, V_{dr} is the velocity at the free surface of the discharge reservoir, p_d is the pump discharge side pressure, V_d is the pump discharge side velocity, Σh_{fd} is the total frictional head loss in the discharge piping and h_d is the discharge geometric head.

The total geometric head can now be defined as

$$h_{gt} = h_s + h_e + h_d \tag{13.69}$$

where h_e indicates the elevation difference between the suction and discharge sides of the pump and it is negligible for small pumps. Substituting Equations (13.67) and (13.68) into Equation (13.69), one can obtain

$$h_{gt} = \frac{p_d - p_s}{\rho g} + \frac{V_d^2 - V_s^2}{2g} - \frac{p_{dr} - p_{sr}}{\rho g} - \frac{V_{dr}^2 - V_{sr}^2}{2g} - \sum h_{fd} - \sum h_{fs} + h_e \tag{13.70}$$

Now, applying the extended Bernoulli equation between the suction and discharge sides of the pump gives

$$\frac{p_s}{\rho g} + \frac{V_s^2}{2g} + z_s + h = \frac{p_d}{\rho g} + \frac{V_d^2}{2g} + z_d + h_e$$

However $z_s = h_s$ and $z_d = h_s + h_e$ with respect to the datum in Figure 13.53, one can obtain the following relation:

$$h = \frac{p_d - p_s}{\rho g} + \frac{V_d^2 - V_s^2}{2g} + h_e \tag{13.71}$$

where h is the head delivered to the fluid by the pump.

Finally, Equations (13.70) and (13.71) can be combined to yield

$$h = h_{gt} + \sum h_{fs} + \sum h_{fd} + \frac{p_{dr} - p_{sr}}{\rho g} + \frac{V_{dr}^2 - V_{sr}^2}{2g}$$

In general, the suction and discharge reservoirs are exposed to the atmosphere so that $p_{dr} = p_{sr} = p_{atm}$ and the velocity of the free surfaces in the suction and discharge reservoirs are negligible, that is $V_{dr} \cong 0$ and $V_{sr} \cong 0$. Then, the above equation becomes

$$h = h_{gt} + \sum h_{fs} + \sum h_{fd} \tag{13.72}$$

Therefore, the head deliverd by the pump is used to overcome the frictional losses and to raise the fluid by an elevation equal to the geometric head.

13.9.2 A Turbine Operating in a Pipe System

A turbine is fed with water from a lake through a penstock, as shown in Figure 13.54. After energy transfer in the turbine, the water passes through the draft tube and discharges to the tailwater.

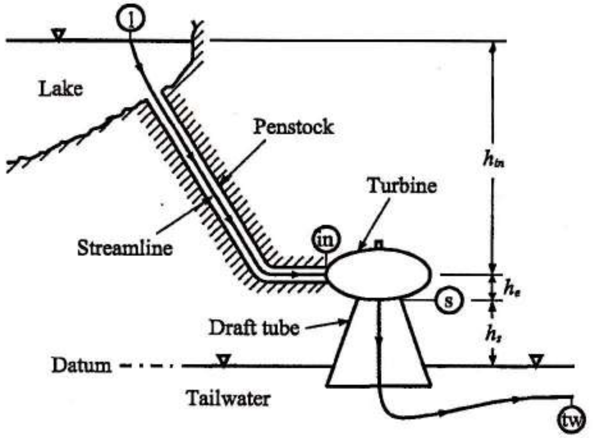


Figure 13.54 A turbine operating in a pipe system

Applying the extended Bernoulli equation along a streamline between the free surface of the lake and inlet side of the turbine yields

$$\frac{p_l}{\rho g}+\frac{V_l^2}{2g}+z_l=\frac{p_{in}}{\rho g}+\frac{V_{in}^2}{2g}+z_{in}+\sum h_{fps}$$

Noting that $z_l = h_{in} + h_e + h_s$ and $z_{in} = h_e + h_s$, then

$$h_{in}=\frac{p_{in}-p_l}{\rho g}+\frac{V_{in}^2-V_l^2}{2g}+\sum h_{fps} \tag{13.73}$$

where p_l is the pressure at the free surface of the lake, V_l is the velocity at the free surface of the lake, p_{in} is the pressure at the turbine inlet, V_{in} is the velocity at the turbine inlet, Σh_{fps} is the total frictional head loss in the penstock and h_{in} is the inlet geometric head.

Similarly, application of the extended Bernoulli equation along a streamline between the suction side of the turbine and free surface of the tailwater yields

$$\frac{p_s}{\rho g}+\frac{V_s^2}{2g}+z_s=\frac{p_{tw}}{\rho g}+\frac{V_{tw}^2}{2g}+z_{tw}+\sum h_{fdt}$$

Noting that $z_s = h_s$ and $z_{tw} = 0$, the above equation becomes

$$h_s=\frac{p_{tw}-p_s}{\rho g}+\frac{V_{tw}^2-V_s^2}{2g}+\sum h_{fdt} \tag{13.74}$$

where p_{tw} is the pressure at the free surface of the tailwater, V_{tw} is the velocity at the free surface of the tailwater, p_s is the pressure at the suction side of the turbine, V_s is the velocity at the suction side of the turbine, Σh_{fdt} is the total frictional head loss in the draft tube and h_s is the suction geometric head.

The total geometric head can now be defined as

$$h_{gt}=h_s+h_e+h_{in} \tag{13.75}$$

where h_e indicates the elevation difference between the inlet and suction sides of the turbine and it is negligible for small turbines.

Now applying the extended Bernoulli equation between the lake surface and tailwater gives

$$\frac{p_l}{\rho g}+\frac{V_l^2}{2g}+z_l=\frac{p_{tw}}{\rho g}+\frac{V_{tw}^2}{2g}+z_{tw}+\sum h_{fdt}+\sum h_{fps}+h_t$$

However, $z_l = h_{in} + h_e + h_s = h_{gt}$ and $z_{tw} = 0$, and the head delivered to the turbine by the fluid, h_t, becomes

$$h_t=h_{gt}+\frac{p_l-p_{tw}}{\rho g}+\frac{V_l^2-V_{tw}^2}{2g}-\sum h_{fdt}-\sum h_{fps}$$

At this point, one should note that $p_l = p_{tw} = p_{atm}$ and the velocity in the lake is negligible. However, the velocity of the tailwater is usually nonzero and it has to be taken into account. The term $V_{tw}^2/(2g)$ is called the **carry over loss**. In this case, the above equation becomes

$$h_t=h_{gt}-\sum h_{fdt}-\sum h_{fps}-\frac{V_{tw}^2}{2g} \tag{13.76}$$

In practice, the losses in the draft tube and carry over loss are considered as a part of the losses in the turbine and the head, that is available at the inlet of the turbine, is defined as the **net head**, h,

$$h = h_{gt} - \sum h_{fps} \tag{13.77}$$

13.10 CAVITATION

Cavitation is the local vaporization of a liquid when the absolute pressure falls to a value equal to or lower than the vapor pressure of the liquid at the local temperature. In this case, small bubbles of vapor are formed and boiling starts, while the dissolved air in the liquid is released. The combination of boiling and air release is referred to as the cavitation.

In practice, cavitation starts when the pressure is slightly lower than the vapor pressure of the liquid. Although the actual mechanism that causes the cavitation is not yet understood, it seems to be related to the presence of microscopic gas nuclei. These gas nuclei give rise to the formation of bubbles during cavitation inception.

During a flow, as the velocity increases, the pressure decreases and if it falls to a sufficiently low level, cavitation occurs with the formation of bubbles. These bubbles first grow and then flow with the fluid to the regions of higher pressure where they collapse producing pressure waves. As a result of these pressure waves, the local pressure may be as high as 4 MPa which is accompanied by an increase in the local temperature by as much as 750° C.

There are two important consequences of cavitation. These are as follows:

(i) The pressure waves, which originate during the collapse of bubbles, may damage the surrounding structure.

(ii) The vigorous mixing in the cavitation region dissipates energy and decreases the total pressure.

In turbomachines, cavitation may occur in regions where the pressure is minimum. In the case of pumps, the minimum pressure occurs at the suction side or the inlet of pumps. However, it is the outlet side or the suction side for turbines.

Net positive suction head, *NPSH*, represents the difference between the total head at the suction side of pumps or turbines and the head corresponding to the vapor pressure.

Therefore

$$NPSH = \frac{p_s}{\rho g} + \frac{V_s^2}{2g} - \frac{p_v}{\rho g} \tag{13.78}$$

where p_s is the suction side pressure, V_s is the suction side velocity and p_v is the vapor pressure at the suction temperature.

Net positive suction head for a pump can be obtained by applying the extended Bernoulli equation along the streamline in Figure 13.55 between the suction reservoir and suction side of the pump as,

$$\frac{p_{sr}}{\rho g} + \frac{V_{sr}^2}{2g} + z_{sr} = \frac{p_s}{\rho g} + \frac{V_s^2}{2g} + z_s + \sum h_{fs}$$

The suction reservoir is exposed to the atmosphere so that $p_s = p_{atm}$ and its cross-sectional area is very large when compared to the cross-sectional area of the suction pipe so that $V_{sr} \cong 0$. Finally, $z_{sr} = 0$ and $z_s = h_s$ according to the datum in Figure 13.55. Hence

$$\frac{p_s}{\rho g} + \frac{V_s^2}{2g} = \frac{p_{atm}}{\rho g} - h_s - \sum h_{fs}$$

In this case, Equation (13.93) becomes

$$NPSH = \frac{p_{atm} - p_v}{\rho g} - h_s - \sum h_{fs} \tag{13.79}$$

On the other hand, the net positive suction head for a turbine can be obtained by the application of the extended Bernoulli equation along the streamline in Figure 13.56 between the suction side of the turbine and tailwater to yield

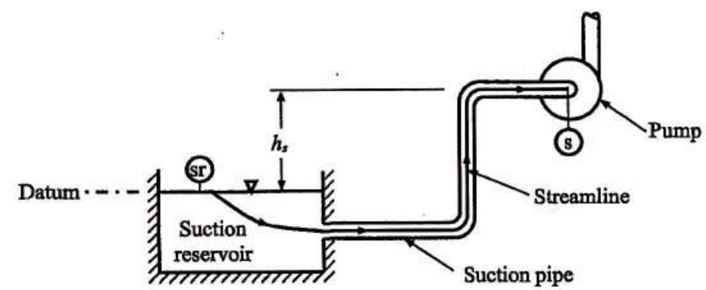


Figure 13.55 Determination of the net positive suction head for a pump

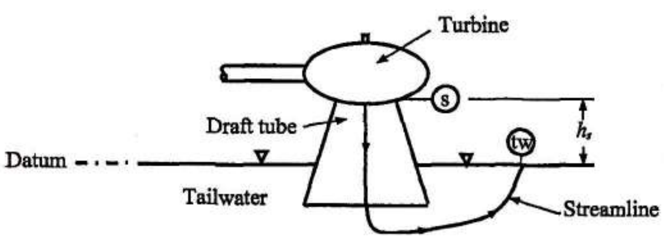


Figure 13.56 Determination of the net positive suction head for a turbine

$$\frac{p_s}{\rho g}+\frac{V_s^2}{2g}+z_s=\frac{p_{tw}}{\rho g}+\frac{V_{tw}^2}{2g}+z_{tw}+\sum h_{fdt}$$

The tailwater is exposed to the atmosphere so that $p_{tw} = p_{atm}$. If the carry over loss is neglected, $V_{tw} \cong 0$. Finally, $z_s = h_s$ and $z_{tw} = 0$, according to the datum in Figure 13.56. Hence

$$\frac{p_s}{\rho g}+\frac{V_s^2}{2g}=\frac{p_{atm}}{\rho g}-h_s+\sum h_{fdt}$$

In this case, Equation (13.93) becomes

$$NPSH=\frac{p_{atm}-p_v}{\rho g}-h_s+\sum h_{fdt} \tag{13.80}$$

The net positive suction head can be nondimensionalized as

$$\sigma=\frac{NPSH}{h} \tag{13.81}$$

where σ is the **Thoma cavitation coefficient**. This factor is constant for similar flows in similar turbomachines.

At the point where the cavitation starts, that is at the **cavitation inception point**

$$\sigma=\sigma_c \tag{13.82}$$

where σ_c is the **critical Thoma cavitation factor**. Therefore, to prevent cavitation the required condition is $\sigma > \sigma_c$.

Similar to the specific speed, suction specific speed, S, can be defined as

$$S=\frac{\omega Q^{1/2}}{(gNPSH)^{3/4}} \tag{13.83}$$

Cavitation can be prevented when the suction specific speed is less than its critical value, that is $S < S_c$.

The relation between the Thoma cavitation coefficient, suction specific speed and specific speed can be obtained by combining Equations (13.81), (13.83) and (13.54) to yield

$$N_s = S\sigma^{3/4} \tag{13.84}$$

For centrifugal pumps, the critical value of the suction specific speed is $2.7 \le S_c \le 4.0$. For practical purposes, the critical value of the suction specific speed can be taken as 3.0.

Example 13.9

A centrifugal pump delivers a head of 80 m and a volumetric flow rate of 0.5 m^3/s when it is rotating at 1470 rpm. The pump takes water with a vapor pressure of 4 kPa from a reservoir where the atmospheric pressure is 100 kPa, as shown in Figure 13.57. The frictional head loss at the suction side is 3 m. If the acceptable value of the suction specific speed is 3.0, determine the elevation of the pump inlet relative to the water level.

Solution

The rotational speed of the pump is

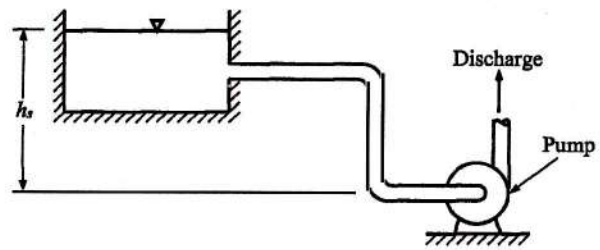


Figure 13.57 Sketch for Example 13.9

$$\omega = \frac{2\pi N}{60} = \frac{(2\pi \text{ rad/rev})(1470 \text{ rev/min})}{(60 \text{ s/min})} = 153.9 \text{ rad/s}$$

The net positive suction head for the pump can be evaluated by using the definition of suction specific speed given by Equation (13.98) as

$$NPSH = \left(\frac{\omega}{S}\right)^{4/3} \frac{Q^{2/3}}{g} = \left(\frac{153.9 \text{ rad/s}}{3}\right)^{4/3} \frac{(0.5 \text{ m}^3/\text{s})^{2/3}}{9.81 \text{ m/s}^2} = 12.24 \text{ m}$$

Now, using the definition of the net positive suction head given by Equation (13.94), the suction geometric head can be obtained as

$$h_s = \frac{p_{atm} - p_v}{\rho g} - NPSH - \sum h_{fs}$$

$$= \frac{100000 \text{ N/m}^2 - 4000 \text{ N/m}^2}{(9.81 \text{ m/s}^2)(1000 \text{ kg/m}^3)} - 12.24 \text{ m} - 3 \text{ m} = \underline{-5.454 \text{ m}}$$

The minus sign indicates that the inlet of the pump must be at least 5.454 m below the water level in the suction reservoir.

13.11 PUMP AND PIPE SYSTEM COMBINATIONS

In this section, operation of one or more pumps in a pipe system is considered.

13.11.1 System Characteristic

Consider the pump which is transporting water from a suction reservoir to a discharge reservoir at a higher elevation, as shown in Figure 13.58.

If the extended Bernoulli equation is applied between the free surface of suction and discharge reservoirs, one can obtain

$$\frac{p_{sr}}{\rho g} + \frac{V_{sr}^2}{2g} + z_{sr} + h = \frac{p_{dr}}{\rho g} + \frac{V_{dr}^2}{2g} + z_{dr} + \sum h_f$$

From Figure 13.58, one should note that $z_{sr} = 0$ and $z_{dr} = h_{gt}$ and the suction and discharge reservoirs are exposed to atmosphere so that $p_{dr} = p_{sr} = p_{atm}$. Also, the cross-sectional areas of suction and discharge reservoirs are very large so that the velocities at the free surface of these reservoirs are negligible, that is $V_{sr} \cong 0$ and $V_{dr} \cong 0$. Hence

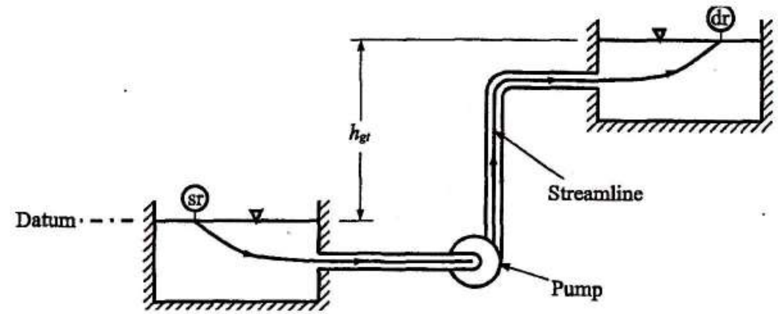


Figure 13.58 A pump transporting water between two reservoirs

$$h = h_{gt} + \sum h_f \tag{13.85}$$

In the above equation, Σh_f represents the total frictional head loss in the suction and discharge pipes due to major and minor losses and can be expressed as

$$\sum h_f = \sum_i f_i \frac{L_i}{d_i} \frac{V_i^2}{2g} + \sum_i k_i \frac{V_i^2}{2g}$$

But using the continuity equation $V_i = Q/A_i$, so that the above equation can be expressed as

$$\sum h_f = \sum_i f_i \frac{L_i}{d_i} \frac{Q^2}{2gA_i^2} + \sum_i k_i \frac{Q^2}{2gA_i^2}$$

Now, defining the resistance coefficient, K, as

$$K = \sum_i f_i \frac{L_i}{d_i} \frac{1}{2gA_i^2} + \sum_i k_i \frac{1}{2gA_i^2}$$

the total frictional head loss becomes

$$\sum h_f = KQ^2 \tag{13.86}$$

Finally, substituting Equation (13.86) into Equation (13.85), one can obtain

$$h = h_{gt} + KQ^2 \tag{13.87}$$

The above relation is known as the **system characteristic** and it is shown in Figure 13.59.

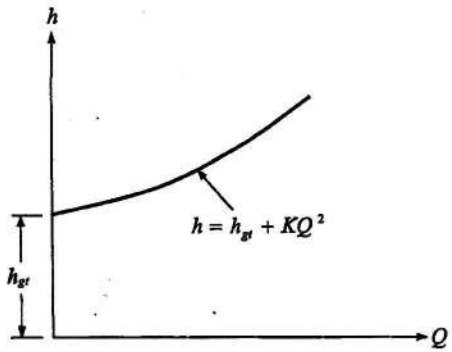


Figure 13.59 A typical system characteristic

The system characteristic can be altered in two different ways. These are as follows:

(i) When the geometric head is increased from h_{gt} to h_{gt}', the system characteristic shifts up, as shown in Figure 13.60. However, if the geometric head decreases from h_{gt} to h_{gt}'', the system characteristics shifts down.

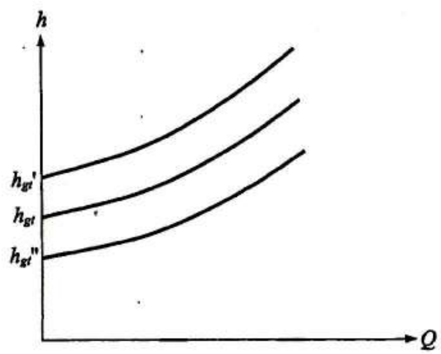


Figure 13.60 Effect of the geometric head change on the system characteristic

(ii) When the resistance coefficient is increased for example by closing a valve, the system characteristic rotates in the counterclockwise direction about point A, as indicated in Figure 13.61.

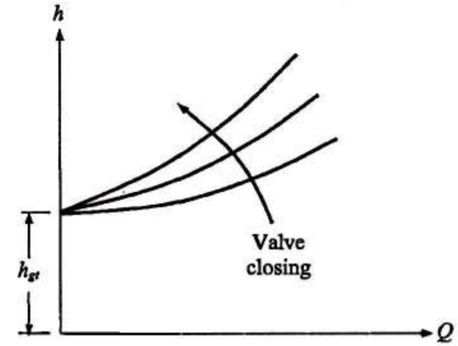


Figure 13.61 Effect of the change in resistance coefficient on the system characteristic

13.11.2 Operating Point

The point of intersection of the system characteristic and pump characteristic represents the operating point, as shown in Figure 13.62. This is the point on the system characteristic at which the system operates and at the same time, it is the point on the pump characteristic at which the pump operates.

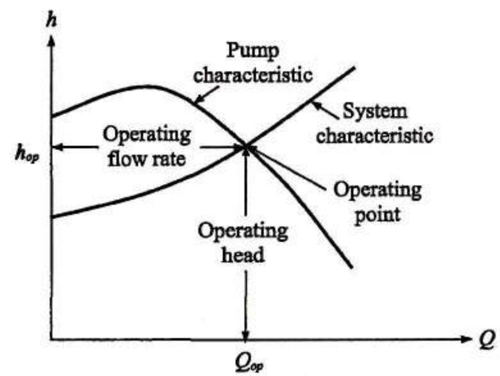


Figure 13.62 Operating point of a pump

Example 13.10

A centrifugal pump is used to transport water between two reservoirs, as shown in Figure 13.63. The performance characteristic of the centrifugal pump can be approximated by

$$h = 50 - 20000Q^2$$

and

$$\eta = 64Q - 1280Q^2$$

where h is the head in m and Q is the discharge in m^3/s. The total length of the suction and discharge pipes is 100 m and they both have a diameter of 0.1 m. Assuming a friction factor of 0.02 and neglecting minor head losses, determine the power required to drive the pump.

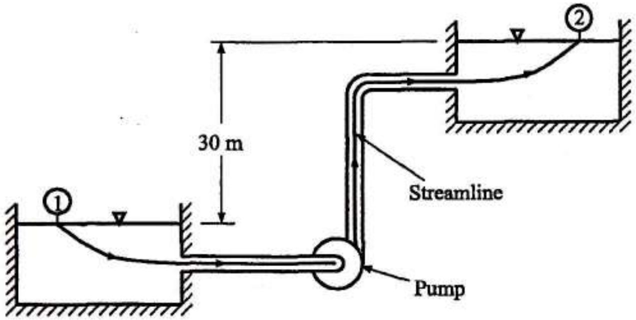


Figure 13.63 Sketch for Example 13.10

Solution

The system characteristic can be determined by applying the extended Bernoulli equation along the streamline in Figure 13.63 between points on the free surfaces of suction and discharge reservoirs as

$$\frac{p_{sr}}{\rho g} + \frac{V_{sr}^2}{2g} + z_{sr} + h = \frac{p_{dr}}{\rho g} + \frac{V_{dr}^2}{2g} + z_{dr} + \sum h_f$$

Since the cross-sectional area of suction and discharge reservoirs are very large when compared to the cross-sectional area of the pipes, $V_{sr} \cong 0$ and $V_{dr} \cong 0$. Meanwhile, both

reservoirs are exposed to the atmosphere so that $p_{sr} = p_{dr} = p_{atm}$. Also, $z_{sr} = 0$ and $z_{dr} = h_{gt}$, so that

$$h = h_{gt} + \sum h_f = h_{gt} + f\frac{L}{d}\frac{V^2}{2g}$$

However, $V = Q/A = 4Q/(\pi d^2)$, so that the system characteristic is

$$h = h_{gt} + \frac{8fL}{\pi^2 g d^5}Q^2 = 30\,\text{m} + \frac{(8)(0.02)(100\,\text{m})}{(\pi)^2(9.81\,\text{m/s}^2)(0.1\,\text{m})^5}Q^2 = 30 + 16525Q^2$$

The point of intersection of the head-discharge characteristic of the pump with the system characteristic gives the operating point as

$$h = 50 - 20000Q_{op}^2 = 30 + 16525Q_{op}^2$$

which can be solved to yield

$$Q = 0.0234\ \text{m}^3/\text{s}$$

This operating point is shown in Figure 13.64. Then, the head and efficiency corresponding to this volumetric flow rate are

$$h = 50 - 20000Q_{op}^2 = 50 - (20000)(0.0234\ \text{m}^3/\text{s})^2 = 39.05\ \text{m}$$

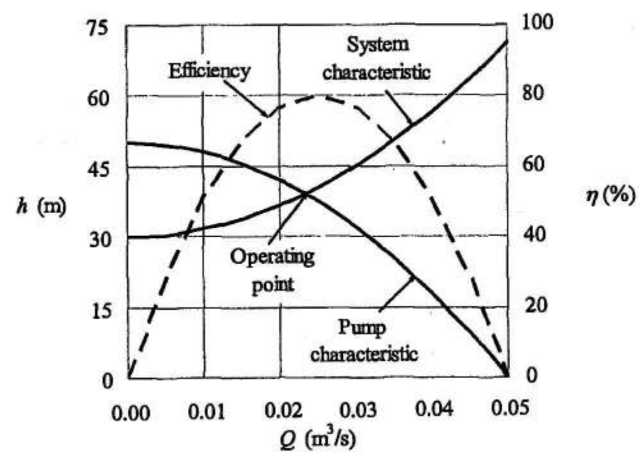


Figure 13.64 Operating point for Example 13.10

and

$$\eta = 64Q_{op} - 1280Q_{op}^2 = (64)(0.0234\,\text{m}^3/\text{s}) - (1280)(0.0234\,\text{m}^3/\text{s})^2 = 0.7967$$

Finally, the power required to drive the pump is

$$P = \frac{\rho g Q_{op} h_{op}}{\eta_{op}} = \frac{(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.0234\,\text{m}^3/\text{s})(39.05\,\text{m})}{0.7967} = \underline{\underline{11.25\,\text{kW}}}$$

13.11.3 Series Combination of Pumps

When the head supplied by a single pump can not meet the head increase required by the system, pumps are usually combined in series. In Figure 13.65, pumps A and B are combined in series for transporting water from the suction reservoir to the discharge reservoir. In this case, the total head that is supplied to the fluid is equal to the sum of the heads supplied by each pump, that is

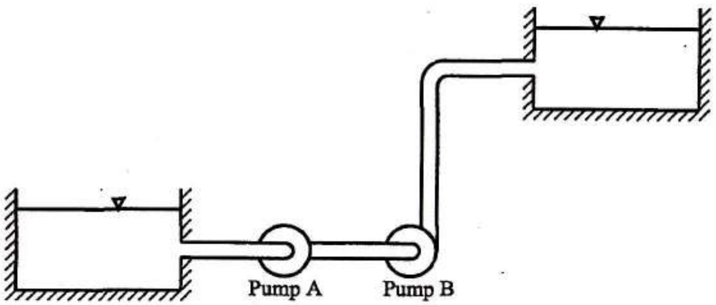


Figure 13.65 Two pumps which are connected in series

$$h = h_A + h_B \tag{13.88}$$

while the volumetric flow rate of the fluid passing through each pump is the same, so that

$$Q = Q_A = Q_B \tag{13.89}$$

The combined pump characteristic can be obtained by adding the head of each pump at a constant volumetric flow rate, as indicated by the dashed curve in Figure 13.66. The point

of intersection of this combined pump characteristic with the system characteristic gives the operating point for the configuration, which is shown in Figure 13.65. One should note that pump B can not operate below the line $h = h_c$ in Figure 13.65 and it acts as a resistance to the flow. The power required to drive these two pumps which are connected in series, is equal to the sum of the powers required to drive each of these pumps. Hence

$$P = P_A + P_B \tag{13.90}$$

or using the definition of power which is required to drive a pump

$$\frac{\rho g Q h}{\eta} = \frac{\rho g Q_A h_A}{\eta_A} + \frac{\rho g Q_B h_B}{\eta_B}$$

Introducing Equation (13.104) into the above equation, the overall efficiency of the serially combined pumps becomes

$$\eta = \frac{h}{\dfrac{h_A}{\eta_A} + \dfrac{h_B}{\eta_B}} \tag{13.91}$$

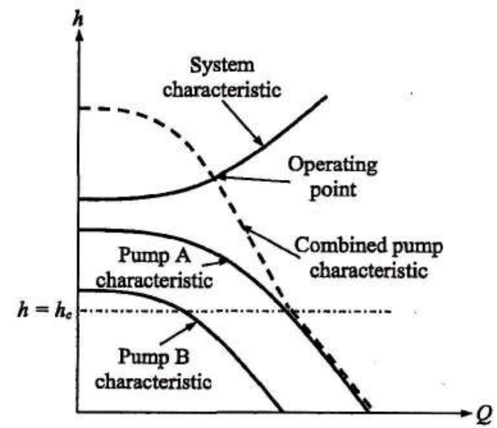


Figure 13.66 Determination of the operating point for the configuration in Figure 13.65

Example 13.11

Two identical centrifugal pumps are combined in series in order to transport water between two reservoirs, as shown in Figure 13.67. The total length of the suction and discharge pipes is 120 m and they both have a diameter of 0.12 m. The friction factor in both pipes is 0.022. The performance characteristics of each pump can be approximated by

$$h = 40 - 16000Q^2$$

and

$$\eta = 64Q - 1280Q^2$$

where h is the head in m and Q is the discharge in m³/s. Neglecting the minor losses, determine the power required to drive the two pumps.

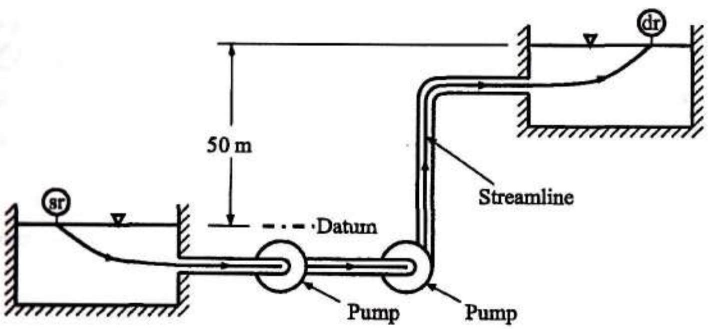


Figure 13.67 Sketch for Example 13.11

Solution

The system characteristic can be determined by applying the extended Bernoulli equation along the streamline in Figure 13.67 between the free surfaces of suction and discharge reservoirs as

$$\frac{p_{sr}}{\rho g} + \frac{V_{sr}^2}{2g} + z_{sr} + h_A + h_B = \frac{p_{dr}}{\rho g} + \frac{V_{dr}^2}{2g} + z_{dr} + \sum h_f$$

Since the cross-sectional area of suction and discharge reservoirs are very large when compared to the cross-sectional area of the pipes, $V_{sr} \cong 0$ and $V_{dr} \cong 0$. Meanwhile, both reservoirs are exposed to the atmosphere so that $p_{sr} = p_{dr} = p_{atm}$. Also, $z_{sr} = 0$ and $z_{dr} = h_{gt}$, so

that

$$h = h_A + h_B = h_{gt} + \sum h_f = h_{gt} + f\frac{L}{d}\frac{V^2}{2g}$$

In the above equation, h represents the total head supplied by pumps A and B. Noting that $V = Q/A = 4Q/(\pi d^2)$, the system characteristic is

$$h = h_{gt} + \frac{8fL}{\pi^2 g d^5}Q^2 = 50 + \frac{(8)(0.022)(120\text{ m})}{\pi^2(9.81\text{m/s}^2)(0.12\text{ m})^5}Q^2 = 50 + 8766Q^2$$

The combined pump characteristic, which is shown in Figure 13.68, can be obtained by adding the head developed by each pump at the same volumetric flow rate as

$$h = h_A + h_B = 2(40 - 16000Q^2) = 80 - 32000Q^2$$

The point of intersection of the combined pump characteristic with the system characteristic gives the operating point as

$$h_{op} = 80 - 32000Q_{op}^2 = 50 + 8766Q_{op}^2$$

or

$$Q_{op} = Q_A = Q_B = 0.02713\text{ m}^3/\text{s}$$

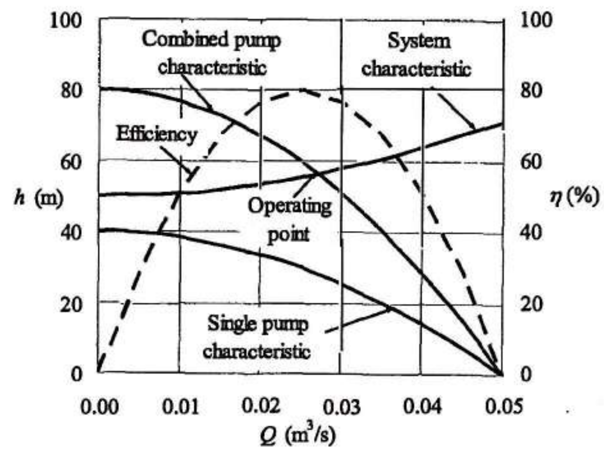


Figure 13.68 Determination of the operating point for Example 13.11

At the operating point, the head delivered by each pump is

$$h_A = h_B = 40 - 16000 Q_{op}^2 = 40 - (16000)(0.02713\ \mathrm{m^3/s})^2 = 28.22\ \mathrm{m}$$

while the efficiency of each pump is

$$\eta_A = \eta_B = 64 Q_{op} - 1280 Q_{op}^2 = (64)(0.02713\ \mathrm{m^3/s}) - (1280)(0.02713\ \mathrm{m^3/s})^2$$
$$= 0.7942$$

Hence, the power required to drive each pump is

$$P = \frac{\rho g Q_A h_A}{\eta_A} = \frac{\rho g Q_B h_B}{\eta_B} = \frac{(1000\ \mathrm{kg/m^3})(9.81\ \mathrm{m/s^2})(0.02713\ \mathrm{m^3/s})(28.22\ \mathrm{m})}{0.7942}$$
$$= 9.457\ \mathrm{kW}$$

while the power required to drive both pumps is

$$P_{total} = 2P = (2)(9.457\ \mathrm{kW}) = \underline{18.91\ \mathrm{kW}}$$

13.11.4 Parallel Combination of Pumps

When the changes in the required volumetric flow rate of a system are quite large, pumps are usually combined in parallel to meet this requirement economically. In Figure 13.69, pumps A and B are combined in parallel for transporting water from the suction reservoir to the discharge reservoir. In this case, the total head supplied to the fluid is equal to the head supplied by each of the two pumps, that is

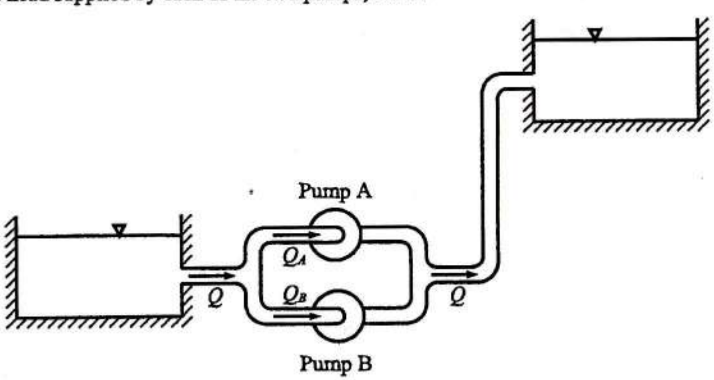


Figure 13.69 Two pumps which are connected in parallel

$$h = h_A = h_B \tag{13.92}$$

while the volumetric flow rate of the fluid passing through the main branch is equal to sum of the volumetric flow rates passing through each pump, that is

$$Q = Q_A + Q_B \tag{13.93}$$

The combined pump characteristic can be obtained by adding the volumetric flow rate delivered by each pump at a constant head, as indicated by the dashed curve in Figure 13.70. The point of intersection of this combined pump characteristic with the system characteristic gives the operating point for the configuration in Figure 13.69. One should note that pump B can not operate to the left of the line $Q = Q_c$ in Figure 13.70. The power required to drive these two pumps, which are connected in parallel, is equal to the sum of the powers required to drive each of these individual pumps. Hence

$$P = P_A + P_B \tag{13.94}$$

Using the definition for the power required to drive a pump

$$\frac{\rho g Q h}{\eta} = \frac{\rho g Q_A h_A}{\eta_A} + \frac{\rho g Q_B h_B}{\eta_B}$$

Introducing Equation (13.107) into the above equation, the overall efficiency of the pumps combined in parallel is

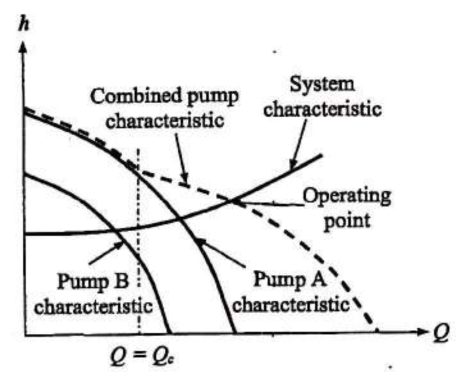


Figure 13.70 Determination of the operating point for the configuration in Figure 13.69

$$\eta = \frac{Q}{\dfrac{Q_A}{\eta_A} + \dfrac{Q_B}{\eta_B}} \tag{13.95}$$

Example 13.12

Two identical centrifugal pumps are combined in parallel in order to transport water from a suction reservoir to a discharge reservoir at a higher elevation, as shown in Figure 13.71. The performance characteristics of each pump can be approximated by

$$h = 50 - 20000Q^2$$

and

$$\eta = 64Q - 1280Q^2$$

where h is the head in m and Q is the volumetric flow rate in m³/s. The length of each branch is negligible when compared to the total length of the suction and discharge piping of 150 m. All pipes have a diameter of 0.2 m and the friction factor is 0.018 in all pipes. Neglecting the minor losses, determine the power required to drive the two pumps.

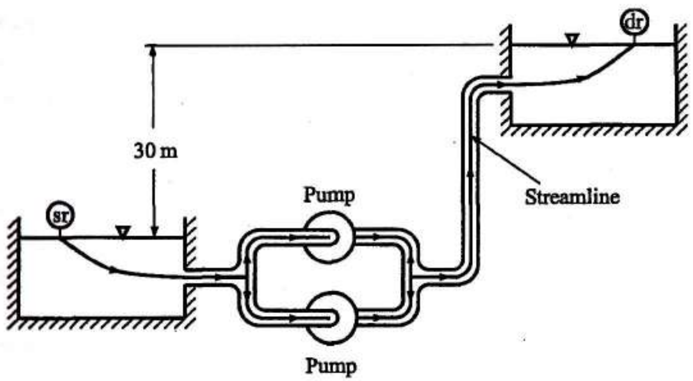


Figure 13.71 Sketch for Example 13.12

Solution

The system characteristic can be determined by applying the extended Bernoulli equation along the streamline in Figure 13.71 between the free surfaces of suction and discharge reservoirs as

$$\frac{p_{sr}}{\rho g} + \frac{V_{sr}^2}{2g} + z_{sr} + h = \frac{p_{dr}}{\rho g} + \frac{V_{dr}^2}{2g} + z_{dr} + \sum h_f$$

Since the cross-sectional area of suction and discharge reservoirs are very large when compared to the cross-sectional area of the pipes, $V_{sr} \cong 0$ and $V_{dr} \cong 0$. Meanwhile, both reservoirs are exposed to the atmosphere so that $p_{sr} = p_{dr} \cong p_{atm}$. Also, $z_{sr} = 0$ and $z_{dr} = h_{gt}$, so that

$$h = h_{gt} + \sum h_f = h_{gt} + f\frac{L}{d}\frac{V^2}{2g}$$

In the above equation, h, represents the head supplied by either one of the two pumps. Noting that $V = Q/A = 4Q/(\pi d^2)$, the system characteristic is

$$h = h_{gt} + \frac{8fL}{\pi^2 g d^5}Q^2 = 30\,\text{m} + \frac{(8)(0.018)(150\,\text{m})}{(\pi)^2(9.81\,\text{m}^2/\text{s})(0.2\,\text{m})^5}Q^2 = 30 + 697.2Q^2$$

where $Q = Q_A + Q_B$ represent the sum of the volumetric flow rates delivered by each pump. The combined pump caharacteristic, which is shown in Figure 13.72, can be obtained by adding the volumetric flow rate delivered by each pump at the same head as

$$Q = Q_A + Q_B = 2\sqrt{\frac{50-h}{20000}}$$

or

$$h = 50 - 5000Q^2$$

The point of intersection of the combined pump characteristic with the system characteristic gives the operating point as

$$h_{op} = 50 - 5000Q_{op}^2 = 30 + 697.2Q_{op}^2$$

or

$$Q_{op} = 0.05925\,\text{m}^3/\text{s}$$

Therefore, the volumetric flow rate delivered by each pump is

$$Q_A = Q_B = \frac{Q_{op}}{2} = \frac{0.05925\,\text{m}^3/\text{s}}{2} = 0.02963\,\text{m}^3/\text{s}$$

At this volumetric flow rate, the head delivered by each pump is

$$h_A = h_B = 50 - 20000\left(\frac{Q_{op}}{2}\right)^2 = 50 - (20000)(0.02963\,\text{m}^3/\text{s})^2 = 32.44\,\text{m}$$

and the efficiency of each pump is

$$\eta_A = \eta_B = (64)(0.02963\,\text{m}^3/\text{s}) - (1280)(0.02963\,\text{m}^3/\text{s})^2 = 0.7726$$

Hence, the power required to drive each pump is

$$P_A = P_B = \frac{\rho g Q_A h_A}{\eta_A} = \frac{\rho g Q_B h_B}{\eta_B}$$

$$= \frac{(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.02963\,\text{m}^3/\text{s})(32.44\text{m})}{0.7726} = 12.21\text{kW}$$

while the power required to drive both pumps is

$$P_{tot} = P_A + P_B = 12.21\,\text{kW} + 12.21\,\text{kW} = \underline{\mathbf{24.42\,kW}}$$

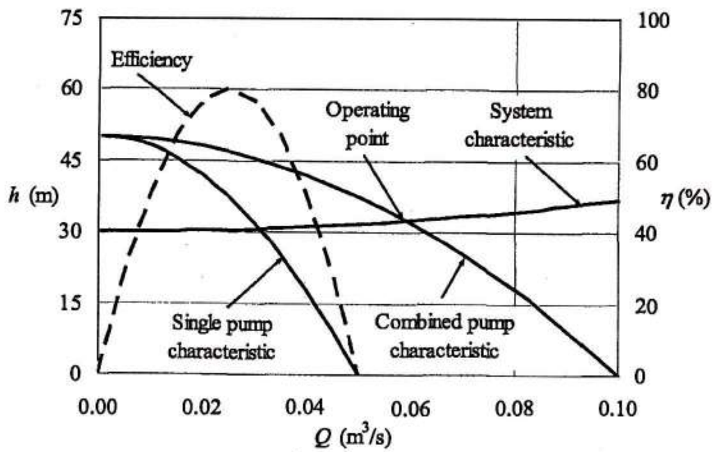


Figure 13.72 Determination of the operating point for Example 13.12

13.12 PUMP SELECTION

In general, it is desirable to select a pump which delivers a required head and volumetric flow rate. Therefore, these are the two important quantities that a pump must

deliver when it operates in a system. In addition to these, there may be some other constraints on the pump to be selected. These are the

(i) pump speed if the prime mover and its speed are known,

(ii) minimum operating efficiency,

(iii) static lift which may affect the pump location to prevent cavitation

(iv) type of fluid to be handled since the working fluid may contain solid particles, acids, etc.,

(v) type of pump such as centrifugal or axial due to the ease of installation in a system,

(vi) space available to the pump,

(vii) maximum allowable noise level and

(viii) nonoverloading power characteristics.

Certainly, all of these restrictions may influence the pump selection. However, one can theoretically select any pump type if these restrictions are disregarded. In this case, the only difference will be in the size and operating speed of the pump. If the type number is low, the size of the pump will be large and the operating speed will be low. On the other hand, if the type number is high, the size of the pump will be small and the operating speed will be high.

If the only specification given is the head and volumetric flow rate, the type number of the pump

$$N_s = \frac{\omega\sqrt{Q}}{(gh)^{3/4}}$$

is directly proportional to the rotational speed. Therefore, the type number of the pump can be fixed by the selection of the rotational speed.

Usually, pumps are driven by alternating current (a.c) electric motors whose speed depends on the alternating current line frequency and number of poles. The synchronous speed of an a.c. electric motor can be given as

$$N_{syn} = \frac{2f}{n}60 \tag{13.96}$$

where N_{syn} is the synchronous speed in rpm, f is the line frequency and n is the number of poles. However, the nominal speed is less than the synchronous speed due to the slip. The synchronous and nominal speeds of a.c. electric motors are presented in Table 13.1 for a line frequency of 50 Hz.

Table 13.1 Synchronous and nominal speeds for a line frequency of 50 Hz

No of poles	N_{syn}	N_{nom}
2	3000	2900
4	1500	1450
6	1000	960
8	750	720
10	600	575
12	500	480
14	450	410
16	375	360

If an a.c. electric motor is used to drive the pump, there are different values of the nominal speeds that can be selected giving a number of choices for the pump types. However, any of the restrictions that are considered above may result in the elimination of some of these choices.

Final selection is usually based on a proper balance between the initial investment and operation costs. For example, if a pump with a low type number is selected, the size of the pump will be large requiring a high initial investment. However, it can be more efficient so that the operation cost will be low. Hence, although such a pump is expensive initially, its selection can be justified since it pays off in the long run.

When the nominal speed is fixed, the type number can be calculated which identifies the pump type. The final step is the determination of the size of the pump that is the diameter of the impeller. In general, the performance characteristic of a pump is specified in terms of the nondimensional parameters, namely the head coefficient, π_h and the flow coefficient, π_Q. In this case, the specific speed given by Equation (13.54) can be expressed in terms of the head and flow coefficients with the aid of Equations (13.38) and (13.39) as

$$N_s = \frac{\sqrt{\pi_Q}}{(\pi_h)^{3/4}} \tag{13.97}$$

However, one should note that it is very difficult to find a pump having a type number which is identical to the calculated one. The perfect solution to this problem would be to design a new pump which exactly meets the given requirements. But, this is a very costly process and it can only be justified if the pump in consideration is very large and/or a large number of them is required. If this is not the case, the pump having a type number greater than the calculated one. In this case, the pump will not operate at its design point but slightly to the right of it to prevent the instabilities.

Example 13.13

It is required to pump water from a storage basin at a volumetric flow rate of 0.4 m^3/s into a reservoir, whose water level is 35 m above the water level in the storage basin, as shown in Figure 13.73. The frictional head loss in the suction and discharge piping are 7 m and 23 m, respectively. The atmospheric pressure is 100 kPa, while the vapor pressure of water is 2 kPa. There are three pumps which are driven by an a.c electric motor, whose characteristics are shown in Figure 13.74. Select a pump to meet the given requirements and specify its size, speed, efficiency, power consumption and its location relative to the water level in the storage basin.

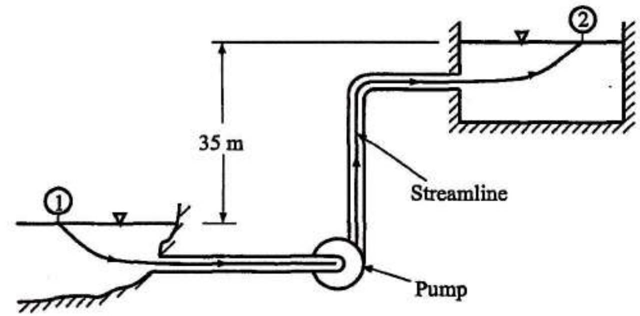
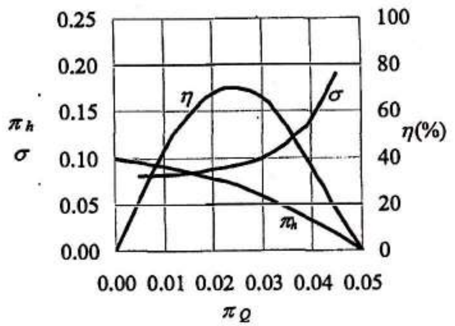
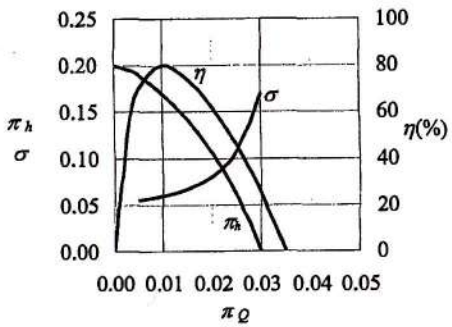


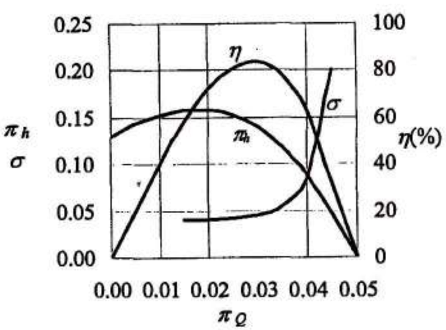
Figure 13.73 Sketch for Example 13.13



(a) Pump A



(b) Pump B



(c) Pump C

Figure 13.74 Pump performance characteristics for Example 13.13

Solution

The type numbers of the pumps whose characteristics are given in Figure 13.74 can be calculated by considering the value of the head and discharge coefficients at the point of maximum efficiency.

For pump A, one can obtain $(\pi_h)_A = 0.07$ and $(\pi_Q)_A = 0.025$ at the point of maximum efficiency from Figure 13.74a. Then, the type number of pump A can be calculated by using Equation (13.112) as

$$(N_s)_A = \frac{(\pi_Q)_A^{1/2}}{(\pi_h)_a^{3/4}} = \frac{\sqrt{0.025}}{(0.07)^{3/4}} = 1.162$$

The Thoma cavitation parameter at the point of maximum efficiency can also be obtained from Figure 13.74a as $\sigma_A = 0.09$. The same procedure can now be repeated for pumps B and C. These results are presented in Table 13.2.

Table 13.2 Type numbers for pumps in Example 13.13

	π_Q	π_h	N_s	σ
Pump A	0.025	0.07	1.162	0.09
Pump B	0.010	0.17	0.3777	0.06
Pump C	0.030	0.14	0.7568	0.04

In order to determine the required head, one can apply the extended Bernoulli equation between points 1 and 2 along the streamline in Figure 13.73 as

$$h + \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \sum h_{fs} + \sum h_{fd}$$

Since points 1 and 2 are both exposed to the atmosphere, $p_1 \cong p_2 \cong p_{atm}$. Also, $V_1 \cong 0$ and $V_2 \cong 0$, since the cross-sectional areas of storage basin and discharge reservoir are quite large when compared to the cross-sectional area of the suction and discharge pipes. According to the chosen datum $z_1 = 0$ and $z_2 = h_{gt2}$, so that

$$h = h_{gt} + \sum h_{fs} + \sum h_{fd} = 35\text{ m} + 7\text{ m} + 23\text{ m} = 65\text{ m}$$

Then, the required type number can be calculated as

$$(N_s)_{req} = \frac{\omega_{nom}\sqrt{Q}}{(gh)^{3/4}} = \frac{\omega_{nom}\sqrt{0.4\,\text{m}^3/\text{s}}}{\left[(9.81\,\text{m/s}^2)(65\,\text{m})\right]^{3/4}} = 0.004984\omega_{nom}$$

For different nominal speeds of a.c electric motors, the required type numbers are presented in Table 13.3.

Table 13.3 Required type numbers for different nominal speed of a.c electric motors

N_{nom}(rpm)	ω_{nom}(rad/s)	$(N_s)_{req}$
2900	303.7	1.514
1450	151.8	0.7566
960	100.5	0.5009
720	75.40	0.3758
575	60.21	0.3001
480	50.27	0.2506
410	42.94	0.2140
360	37.70	0.1879

At this point, if the required type numbers for different motor speeds are compared with the type numbers of the three available pumps, one can determine that there are two matches. These are (i) pump B running at a speed of 720 rpm and (ii) pump C running at a speed of 1450 rpm.

One can first consider pump B. The impeller diameter of this pump can be calculated from the flow coefficient as

$$d = \left(\frac{Q}{\omega\pi_Q}\right)^{1/3} = \left(\frac{0.4\,\text{m}^3/\text{s}}{(75.40\,\text{rad/s})(0.01)}\right)^{1/3} = 0.8095\,\text{m}$$

Pump B is operating at its maximum efficiency point, since there is an exact match between its type number and required type number. Hence, the efficiency is 80 percent from Figure 13.74b. Then, the consumed power is

$$P = \frac{\rho gQh}{\eta} = \frac{(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.4\,\text{m}^3/\text{s})(65\,\text{m})}{0.8} = 318.8\,\text{kW}$$

At the point of maximum efficiency, Thoma cavitation parameter is $\sigma = 0.06$ from Figure 13.74b. Then, the net positive suction head is

$$NPSH = \sigma h = (0.06)(65\,\text{m}) = 3.9\,\text{m}$$

The suction geometric head can now be evaluated with the aid of Equation (13.92) as

$$h_s = \frac{p_{atm} - p_v}{\rho g} - NPSH - \sum h_s = \frac{100000\,\text{N/m}^2 - 2000\,\text{N/m}^2}{(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)} - 3.9\,\text{m} - 7\,\text{m}$$
$$= -0.9102\,\text{m}$$

Therefore, pump B must be located below the water level in the storage basin in order to prevent cavitation. The same calculations can now be repeated for pump C. The results are presented in Table 13.4 together with pump B.

Table 13.4 Results for pump B and C in Example 13.13

	Pump B	Pump C
π_Q	0.01	0.03
π_h	0.17	0.14
N_s	0.3777	0.7566
N (rpm)	720	1450
ω (rad/s)	75.40	151.8
d (m)	0.8095	0.4145
η	0.80	0.85
P (kW)	318.8	300
σ	0.06	0.04
$NPSH$ (m)	3.9	2.6
h_s (m)	-0.9192	0.3898

At this point, if the results for pumps B and C are compared, one could observe that the impeller diameter of pump C is smaller than the impeller diameter of pump B. In addition to this, pump C is more efficient and consumes less power. In order to prevent cavitation pump B must be placed below the water level in the storage basin, while pump C can be located above ground. Therefore, it is better to **select pump C.**

13.13 SUMMARY

This chapter is devoted to the discussion of the turbomachinery. The discussion starts with the classification of fluid machinery according to direction of energy transfer direction as power producing and power absorbing fluid machinery. This is followed by the classification of fluid machinery according to principle of mechanical operation as positive displacement fluid machinery and turbomachinery. The fundamental relations are applied to the flow through an arbitrary turbomachine by using one-dimensional approximation. Velocity triangles for pumps and turbines are then introduced and conditions for maximum energy transfer. During this analysis, the axisymmetric flow assumption is used which implies that there are infinte number of blades in the flow passage and the fluid is guided perfectly well through the runner. The losses in the components of hydraulic machinery are considered and energy relations through pumps and turbines are analysed. The general performance characteristics for pumps and turbines are discussed. The discussion continues with the similitude in turbomachines with special emphasis to the performance characteristics at similar operating points. The same turbomachine operating at different rotational speeds and geometrically similar turbomachines operating at the same rotational speed are considered. The specific speed or type number is introduced for the classification, comparison and design of pumps and turbines. Pumps are then classified as radial, axial and mixed flow according to the geometry of flow passage. This is followed by the classification of turbines as impulse and reaction turbines. In impulse turbines, the available head is first converted to kinetic energy and then transferred to the rotor while the pressure across the rotor is almost constant. In reaction turbines, only a part of the total head is converted to kinetic energy in the stationary guide vanes, which is then transferred to rotor with a decrease in static pressure. The reaction turbines are further classified as radial (Francis) and axial (Kaplan) turbines according to the geometry of flow passage. Since pumps and turbines operate in pipe system, their performances are closely related to the losses in the system of pipes connected to them. For this reason, pumps and turbines operating in pipe systems are analyzed. The concept of cavitation, which is an important design parameter, is also considered. The pump and pipe system combinations are then discussed, system characteristic introduced and the operating point is defined. The series and parallel combination of pumps operating in a pipe system are analyzed. Finally, the restrictions, which influence the pump selection, are examined.

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In V3.3 the followings not included here in V4.0
13.51

V3.2 Pr#

PROBLEMS

13.1 **13.1** A centrifugal pump having an impeller diameter of 20 cm is rotating at 2950 rpm. The blade angle at the outlet is 70^0, while the meridional velocity at the outlet is 4 m/s. Determine the theoretical head of the pump, if there is no inlet whirl.

13.2 **13.2** A centrifugal pump having an impeller outside diameter of 0.2 m delivers 0.02 m^3/s of water when it is rotating at 1450 rpm. Blades are extending up to the inlet eye where the hub and tip radii are 3 cm and 6 cm, respectively. There are no inlet guide vanes and the blade angle at the exit is 20^0. The meridional velocity is constant throughout the impeller. Assuming that the fluid and blade angles are equal and neglecting friction, determine the head of the pump.

13.3 **13.3** A centrifugal pump produces a head of 25 m at a volumetric flow rate of 0.15 m^3/s, when it is rotating at 1470 rpm. The inlet and exit diameters of the impeller are 0.15 m and 0.4 m, respectively. The blade width at the inlet is 0.025 m, while the blade angle at the inlet is 60^0. The meridional velocity across the impeller remains the same. The blade and fluid angles are the same. Assuming axisymmetric flow and neglecting the frictional effects, determine the

a) tangential component of the absolute velocity at the inlet,

b) blade angle at the outlet,

c) blade width at the outlet and

d) shut-off head.

13.4 A pump delivers a head of 40 m at a volumetric flow rate of 0.1 m^3/s. The leakage in the pump is 0.05 m^3/s. If the hydraulic and mechanical efficiencies are 0.85 and 0.9, determine the shaft power.

13.5 The net head of a radial inflow turbine is 80 m. The overall efficiency of the turbine is 85 percent, while the mechanical efficiency is 98 percent. The leakage through the gap between the rotor and casing is 3 percent of the incoming flow. The head loss in the draft tube is 1 m and the velocity of the fluid at the exit of the draft tube is 1.5 m/s. Determine the internal head loss in the turbine.

13.6 The net head of a radial flow turbine having a hydraulic efficiency of 80 percent is 40 m. The inlet guide vane angle is 20^0, while the relative blade angle at the inlet is 90^0. The inner and outer diameters of the rotor are 1.0 m and 0.5 m, respectively. There is no outlet whirl and the blade angles are equal to the fluid angles. Assuming that the meridional velocity is constant across the rotor. Determine the

a) angular speed of the rotor and

b) relative fluid angle at the exit.

13.7 A centrifugal fan with an impeller diameter of 0.8 m delivers 2.5 m^3/s of air when it is rotating at 800 rpm. At this operating condition, it creates a pressure difference of 700 Pa. The volumetric efficiency is 95 percent and 40 percent of the theoretical head is dissipated in the fan. The width of the impeller at the outlet is 10 cm and there are no inlet guide vanes. The density of air is 1.2 kg/m^3. Determine the relative fluid angle at the outlet of the impeller.

13.8 A centrifugal pump produces a head of 40 m at a volumetric flow rate of 0.025 m^3/s, when it is running at 2950 rpm. There are no inlet guide vanes. Blades are extending up to the inlet eye where the hub and tip radii are 2 cm and 4 cm, respectively. The diameter and width of the impeller at the outlet are 20 cm and

1.2 cm, respectively. If the volumetric and hydraulic efficiencies are 95 percent and 80 percent, determine the

a) inlet relative fluid angle at the mean radius and

b) outlet relative fluid angle.

13.9 A centrifugal pump produces a head of 17 m at a volumetric flow rate of 0.04 m^3/s when it is rotating at 1470 rpm. The loss due to the leakage is 0.001 m^3/s and the mechanical loss due to friction is 0.5 kW. The inlet and outlet diameters of the impeller are 10 cm and 25 cm, respectively, while the width of the impeller at the inlet and exit are 2 cm and 1 cm, respectively. The relative fluid angle at the outlet is 30^0. There are no inlet guide vanes. Determine the

a) relative fluid angle at the inlet,

b) absolute fluid angle at the outlet,

c) absolute velocity at the outlet,

d) hydraulic efficiency and

e) overall efficiency.

13.10 A centrifugal pump produces a head of 20 m, when it runs at 750 rpm. The meridional velocity is 2 m/s and it is constant across the impeller. The fluid angle at the outlet of the impeller is 40^0. There is no inlet whirl and the entry is shockless. The hydraulic efficiency of the pump is 85 percent. The ratio of the outlet to inlet diameter of the impeller is 2.5. The width of the impeller at the outlet is 1.6 cm. Neglecting leakage and mechanical losses, determine the

a) diameter of the impeller,

b) inlet blade angle,

c) volumetric flow rate,

d) width of the impeller at the inlet and

e) power consumption of the pump.

13.11 An axial flow fan having a diameter of 1.5 m is operating at a speed of 1450 rpm. In this case, the axial velocity of the air passing through the impeller is 10 m/s. A model of this fan having a scale ratio of 5 is to be tested in the laboratory at a speed of 2900 rpm.

a) Determine the axial velocity of the air in the model, if the effect of the Reynolds number is neglected.

b) If the model is to be tested in a large pressure vessel, find the required pressure. The dependence of the absolute viscosity of air on the pressure can be neglected. The prototype is designed to operate in the atmosphere with a pressure of 100 kPa. The temperature of the air is the same for the prototype and model.

13.12 A centrifugal pump is designed to deliver a head of 60 m at a volumetric flow rate of 0.8 m^3/s when it is running at 1470 rpm. The performance of this pump is to be determined in the laboratory by using a model. The power consumption of the model is 10 kW when it is delivering 0.1 m^3/s of water. The efficiencies of the prototype and model are both 80 percent. Determine the

a) rotational speed of the model and

b) length scale.

13.13 A centrifugal pump with an impeller diameter of 0.2 m is designed to deliver a head of 10 m at a volumetric flow rate of 0.1 m^3/s when it is rotating at 1500 rpm. It is desirable to operate this pump at 1750 rpm to deliver the same head. For this reason, the diameter of the impeller is changed without disturbing the geometric similitude.

a) Determine the diameter of the modified impeller.

b) Determine the volumetric flow rate of the pump with the modified impeller.

13.14 A propeller type wind turbine having a tip diameter of 2.4 m is designed to run at 150 rpm when the average wind speed is 5 m/s. A model of this turbine having a tip diameter of 0.6 m is tested in a wind tunnel where the air speed is 12 m/s. The density of the air is 1.2 kg/m^3. Determine the

a) rotational speed of the model,

b) power output of the full scale turbine, if the power output of the model is measured to be 2 kW and

c) power output of the full scale turbine when it is operating at 200 rpm.

13.15 A centrifugal pump operating at 1000 rpm is delivering 18.2 m^3/h of water at a total pressure difference of 1030 kPa between inlet and outlet, and requires 6.0 kW of shaft power. Determine the volumetric flow rate, head, shaft power and efficiency at the similar operating point of the pump when it is operating at 1500 rpm. (METU 2013)

13.16 A turbine is designed to operate under a net of 80 m when the volumetric flow rate of water is 2 m^3/s. The performance of this design is to be predicted in the laboratory by using a 1/4 scale model. The model turbine is operating under a head of 10 m and its efficiency is 80 percent. Determine the

a) volumetric flow rate for the model and

b) efficiency of the prototype.

13.17 A 2.4 m diameter hydraulic turbine is designed to run at 430 rpm under the head of 289 m with a flow rate of 42 m^3/s. To estimate the power produced by this turbine, a model is constructed in a laboratory where the head is 10 m. To simulate the design operation, an experiment is performed on the model while running at 640 rpm.

a) What should be the flow rate in the experiment?

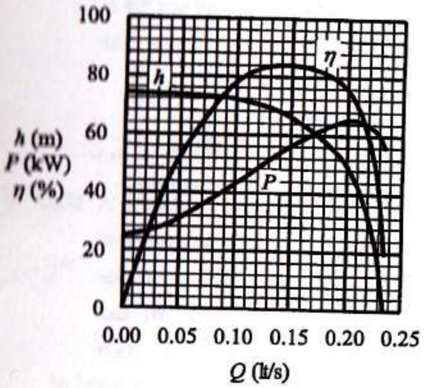
b) If the shaft power is measured as 10.2 kW in this experiment, estimate the power produced by the designed turbine. (METU 1999)

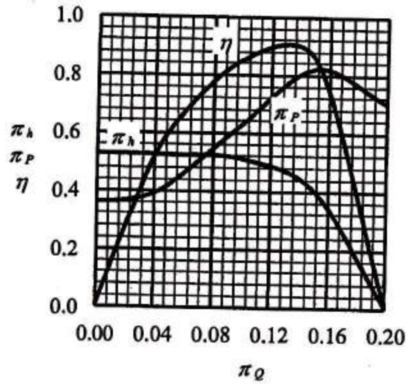
13.18 A 3.5 m-diameter hydraulic turbine is designed to operate at 212 rpm under the head of 30 m with the flow rate of 79.7 m^3/s. A 1:10 scale model of this turbine is constructed and a simulation test is planned to estimate the power of the designed (prototype) turbine. The head of the model test is to be 2.5 m.

a) Determine the speed of the model turbine and the water flow rate for the simulation test. Can you realize full-simulation? Comment.

b) If the shaft power of the model is measured as 5.02 kW as the result of the above test, estimate the shaft power of the prototype by using Ackeret's correlation. (METU 2003)

13.19 Show that "When π_Q, π_h and π_P are the same at two different operating conditions of geometrically similar turbomachines, the efficiencies of these two machines are the same at these particular operating conditions". A 200 mm diameter centrifugal pump (Pump A) operating at 1200 rpm is geometrically similar to a 300 mm diameter pump (Pump B). Dimensional and non-dimensional characteristics of pump B while operating at 1000 rpm are shown below. Determine the discharge (Q), provided head rise (h) and the shaft power for pump A working at maximum efficiency. Working fluid is water. (METU 2006)





Problem 13.19

13.20 A multistage pump is formed by mounting five identical impellers on the same shaft. It delivers a head of 100 m at a volumetric flow rate of 0.02 m^3/s when it is operating at 1450 rpm. When one of the impellers is broken, it is removed. In order to obtain the same head from the four stage pump, determine the required rotational speed and the volumetric flow rate. Neglect the Reynolds number effects.

13.21 The performance characteristic of a pump operating at 1450 rpm can be approximated by

$$h = 50 - 4000Q^2 \quad \text{and} \quad \eta = 32Q - 320Q^2$$

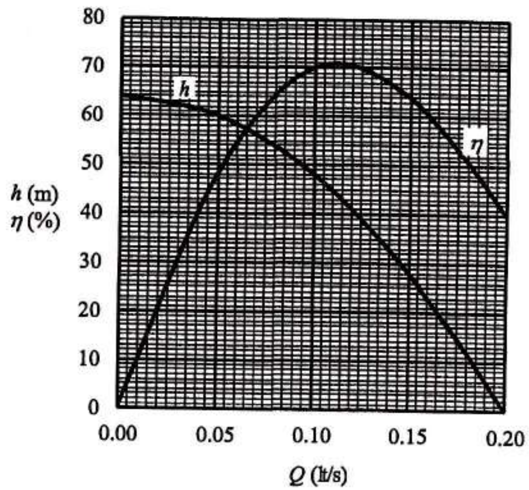
where h is in m and Q is in m^3/s. It is desirable to pump water against a head of 45 m at a volumetric flow rate of 0.05 m^3/s. Determine the

a) rotational speed of the pump,
b) efficiency of the pump and
c) power consumption of the pump.

13.22 A pump with the given characteristics below, is operating at its design point at 1200 rpm.

a) The system requirements change such that the pump must now deliver water with a head of 69.12 m at a flow rate of 0.12 m^3/s. This is achieved by changing the speed of the pump. Neglecting the Reynolds number effects, determine the new pump speed.

b) Calculate the percent change in pump efficiency, when the pump speed is changed as explained in part (a). (METU 2007)



Problem 13.22

13.23 The performance characteristics of a pump operating at 1450 rpm can be approximated by

$$h = 50 - 4000Q^2 \quad \text{and} \quad \eta = 32Q - 320Q^2$$

where h is in m and Q in m^3/s. It is desirable to pump water against a head of 35 m at a volumetric flow rate of 0.05 m^3/s. Determine the

a) percentage reduction in the diameter of the pump impeller,
b) efficiency of the pump and
c) power consumption of the pump.

13.24 To determine the characteristics of a pump, a series of experiments are conducted while the pump is working at 800 rpm and the following data is collected.

Q (m³/s)	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07
h (m)	41	40	39	37	34	28	17	0
η (%)	0	31	55	71	76	70	50	0

For a certain application we need to select a pump that will deliver 0.04 m³/s of water against a head of 50 m. Possibilities are to use

a) the above pump but operated at a rotational speed other than 800 rpm. Find the efficiency of the pump. Find the power required to run it.

b) a pump operated at 800 rpm which is geometrically similar to the pump mentioned above. Find the efficiency of this pump and the power required to run it. (METU 2005)

13.25 A fan is designed to produce a pressure difference of 120 mm water at a volumetric flow rate of 2 m³/s when it is operating at 2900 rpm. If the efficiency of the fan is 70 percent, determine its type. The density of the air is 1.2 kg/m³.

13.26 A pump is to be designed to deliver a head of 100 m at a volumetric flow rate of 0.5 m³/s when it is operating at 720 rpm. Determine the type of the pump.

13.27 Practicing engineers do not bother to change rpm to radians per second or Q to cubic meter per second or to include gravity with head. A British engineer prefers a practical form:

$$(N_s)_B = \frac{N(\text{rpm})\sqrt{Q(\text{gpm})}}{\left[h(\text{ft})\right]^{3/4}}$$

while a German one prefers:

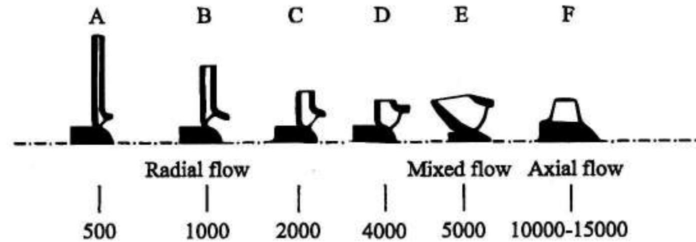
$$(N_s)_G = \frac{N(\text{rpm})\sqrt{Q(\text{rpm})}}{\left[h(\text{m})\right]^{3/4}}$$

If the dimensionless specific speed is N_s, one can show that $(N_s)_B = 2735N_s$

a) Rewrite the specific speed scale in$(N_s)_G$. Note that British scale is used in the figure.

b) Which type of pump should be selected, if the task is to obtain 90 m head with a 9000 lpm flow rate by using an AC motor running at 1500 rpm.

c) If the same task is going to be employed on the moon, would the selection be changed? Explain (METU 1999)



Problem 13.27

13.28 A centrifugal pump having an impeller diameter of 0.2 m is operating at a speed of 1450 rpm. The blade width and meridional velocity are 0.02 m and 5 m/s, respectively. There is no inlet whirl and the blades are radial at the outlet. If the hydraulic efficiency is 100 percent, determine the specific speed of the pump.

13.29 A turbine is designed to produce 150 kW at a net head of 10 m when it is running at 350 rpm. The efficiency of the turbine is 75 percent.

a) Determine the type of the turbine.

b) If the turbine is operated under a net head of 8 m, determine the rotational speed of the turbine, volumetric flow rate of water and power produced by the turbine.

13.30 A centrifugal pump delivers 0.3 m³/s of water when it is operating at 1450 rpm. The outlet diameter and width of the impeller are 0.25 m and 0.04 m, respectively. The blade angle at the outlet is 30°. Assuming that blade and fluid angles are equal, determine the

a) absolute velocity at the outlet and

b) absolute fluid angle at the outlet.

13.31 A three stage centrifugal pump delivers 0.05 m^3/s of water when it is running at 1000 rpm. The mechanical, volumetric and hydraulic efficiencies of the pump are 90 percent, 95 percent and 80 percent, respectively. At the outlet, the impeller diameter and width are 0.45 and 0.03 m, respectively. The blade angle at the outlet is 40^0 and there are no inlet guide vanes. The blades block the 6 percent of the flow area. Assuming that the blade and fluid angles are the same, determine the

a) head developed,
b) power required to drive the pump and
c) specific speed.

13.32 A centrifugal pump is rotating at 1450 rpm. The inlet and outlet diameters of the impeller are 0.5 m and 0.2 m, respectively. The blade width at the inlet and outlet are 0.04 m and 0.02 m, respectively. The blade angles at the inlet and outlet are 20^0 and 10^0, respectively. There are no inlet guide vanes. Neglecting losses and thickness of vanes, determine the

a) volumetric flow rate,
b) theoretical head,
c) power developed and
d) pressure rise across the impeller.

13.33 A centrifugal pump delivers 17 m of head at a volumetric flow rate of 0.1 m^3/s when it is rotating at 720 rpm. The inner and outer diameters of the impeller are 0.2 m and 0.4 m, respectively. The blade angle at the outlet is 45^0. The flow area of 0.05 m^2 is constant throughout the impeller. The water enters the impeller without shock.

a) Determine the hydraulic efficiency.

b) Find the absolute fluid angle at the inlet when the volumetric flow rate is reduced to 0.6 m^3/s, while the rotational speed remains the same.

13.34 An axial flow pump handling water consumes 75 kW at a volumetric flow rate of 0.8 m^3/s when it is rotating at 250 rpm. The hub and tip diameters of the impeller are 0.4 m and 0.8 m, respectively. There are guide vanes at the downstream of the impeller. The hydraulic efficiency is 85 percent, while the overall efficiency is 75 percent. Assuming that the blade and fluid angles are equal, determine the

a) inlet blade angle of the impeller,
b) outlet blade angle of the impeller and
c) inlet blade angle of the guide vanes at the mean diameter.

13.35 A Pelton turbine having a mean bucket diameter of 2 m is operating at a rotational speed of 300 rpm under a net head of 110 m. The velocity coefficient of the nozzle is 0.97, while the diameter of the nozzle is 0.15 m. The buckets deflect the jet of fluid by an angle of 165^0. Neglecting friction, determine the

a) power developed by the wheel and
b) hydraulic efficiency.

13.36 A Pelton wheel develops a power of 6 MW under a net head of 100 m when it is rotating at a speed of 180 rpm. The ratio of the wheel diameter to the jet diameter is 8. The velocity coefficient of the nozzle is 0.97 and the hydraulic efficiency is 85 percent. If the ratio of the wheel speed to the jet velocity is 0.45, determine the

a) volumetric flow rate,
b) wheel diameter,
c) jet diameter,
d) required number of nozzles and
e) specific speed.

13.37 A Pelton turbine operates under a net head of 600 m. The wheel diameter is 1.5 m, while the jet diameter is 0.1 m. The jet is deflected through an angle of 165^0 in the buckets and the relative velocity is reduced by 15 percent due to the friction. The fluid leaves the nozzle without whirl. If the velocity coefficient of the nozzle is 0.97, determine the

a) rotational speed of the Pelton wheel,
b) force exerted by the jet on the buckets,
c) power developed and
d) efficiency of the buckets.

13.38 A Pelton wheel having a diameter of 0.4 m is driven by a water jet with a velocity of 50 m/s. The water jet is deflected by 165^0 in the buckets and the relative velocity is reduced by 15 percent. If the Pelton wheel is rotating at 350 rpm, determine the

a) hydraulic efficiency and

b) maximum possible hydraulic efficiency.

13.39 A vertical shaft inward flow reaction turbine operates under a net head of 100 m. The vertical distance between the turbine entrance and tailrace is 5 m. The peripheral velocity at the entry is 25 m/s and the radial velocity of 8 m/s is constant throughout the impeller. There is no inlet whirl. The hydraulic losses are 4 m in the inlet guide vanes, 7 m in the runner and 1 m in the draft tube. The velocity of water in the tailrace is 2 m/s. Determine the

a) inlet guide vane angle,

b) blade angle at the inlet of the impeller,

c) pressure head at the inlet of the impeller and

d) pressure head at the outlet of the impeller.

13.40 An inward flow reaction turbine operating under a net head of 20 m produces 150 kW when it is rotating at 150 rpm. At the inlet, the peripheral velocity is 15 m/s and the radial velocity is 5 m/s. There is no inlet whirl and the overall efficiency is 80 percent. If the hydraulic efficiency is 85 percent, determine the

a) outer diameter of the impeller,

b) inlet guide vane angle,

c) blade angle at the inlet of the impeller and

d) volumetric flow rate.

13.41 A Francis turbine having an overall efficiency of 80 percent operates under a net head of 50 m. The inner and outer diameters of the runner are 0.4 m and 0.6 m, respectively, while the thickness at the outer rim is 0.04 m. The blade angles at the inlet and outlet are 80^0 and 15^0, respectively. 6 percent of the flow area is blocked by the blades. There is no outlet whirl. The hydraulic efficiency is 90 percent, while the overall efficiency is 80 percent. Determine the

a) rotational speed and

b) power delivered.

13.42 A Francis turbine produces 70 MW at a volumetric flow rate of 120 m^3/s, when it is running at 150 rpm. Water is supplied to the turbine by a penstock with a diameter of 5 m. The impeller diameter of the turbine is 3.5 m and the blade height at the entry is 0.8 m. The inlet diameter of the draft tube is 4 m and the velocity of the water at the tailrace is 3 m/s. The hydraulic efficiency is 90 percent. The static pressure head in the penstock just before the entry to the runner is 60 m where the elevation from the tailrace is 4 m. The loss in the draft tube is 25 percent of the velocity head at the inlet of the draft tube. The exit of the runner is 2.5 m above the water level in the tailrace and water leaves the runner without whirl. Determine the

a) overall efficiency,

b) absolute fluid angle at the inlet of the runner and

c) pressure at the inlet of the draft tube.

13.43 The hub and tip diameters of the impeller of an axial flow turbine are 0.8 m and 1.6 m, respectively. The axial turbine is running at 240 rpm. The blade angle at the exit of the inlet guide vanes is 30^0, while the blade angle at the inlet of the impeller is 45^0. There is no inlet whirl. Determine the

a) volumetric flow rate,

b) blade angle of the impeller at the outlet and

c) theoretical power output if the whirl is independent of radius.

13.44 An axial flow turbine develops 20 MW under a head of 20 m when it is running at 150 rpm. The hub and tip diameters of the impeller are 4 m and 2 m, respectively. There is no outlet whirl. If the hydraulic efficiency is 90 percent and the overall efficiency is 85 percent, determine the inlet and outlet blade angles of the impeller at the mean radius.

13.45 A pump transports water at a volumetric flow rate of 0.12 m^3/s from a suction reservoir to a discharge reservoir. The hydraulic and volumetric efficiencies of the pump are 80 percent and 95 percent, respectively. The shaft power required to drive the pump is 40 kW and mechanical friction loss is 2 kW. The head losses in the suction and discharge pipes are 2 m and 6 m, respectively. The diameters of suction and discharge pipes are both 0.15 m. The atmospheric pressure is 100 kPa.

a) Determine the elevation difference between the two reservoirs.

b) If the pressure at the suction side of the pump must be greater than 10 kPa, determine maximum elevation of the pump inlet from the water level in the suction reservoir.

13.46 A pump transports water at a volumetric flow rate of 0.12 m^3/s from a suction reservoir to a discharge reservoir. The elevation difference between the two reservoirs is 20 m. The suction side of the pump is located 2 m above the water level in the suction reservoir. The head losses in the suction and discharge pipes are 0.5 m and 2.5 m, respectively. The suction and discharge pipe diameters are 0.2 m. Neglecting the losses in the pump, determine

a) the gage pressure at the suction side and

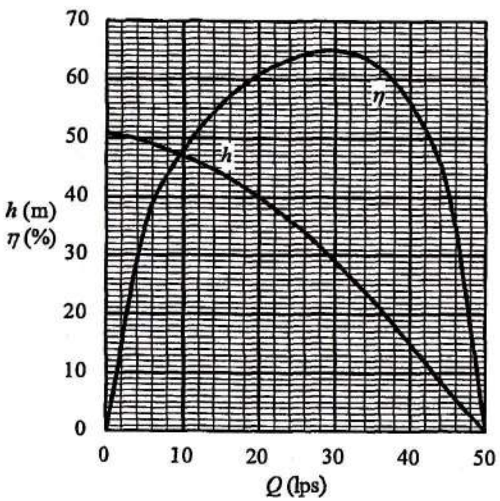
b) the shaft power.

13.47 The characteristics of a pump having a design speed of 1500 rpm are given in the figure. This pump is observed to deliver 25 l/s, when it is operated in an installation which elevates the water 15 m from an open reservoir to a large closed tank where the gage pressure is 49 kPa.

a) Find the specific speed of the pump.

b) Find the frictional head loss in the installation for the flow rate of 25 l/s.

c) Find the power consumption of the pump for the flow rate of 25 l/s. (METU 2005)



Problem 13.47

13.48 A centrifugal pumps is designed to operate at 1500 rpm in order to transport water from a suction reservoir to a discharge reservoir at a higher elevation. The performance characteristic of the pump can be approximated by

$$h = 50 - 20000Q^2 \quad \text{and} \quad \eta = 64Q - 1280Q^2$$

where h is the head in m and Q is the volumetric flow rate in m^3/s. The total length of the suction and discharge piping of 150 m. All pipes have a diameter of 0.1 m and the friction factor is 0.018 in all pipes. Neglect minor losses. Determine

a) specific speed of the pump and

b) the geometric head at the design point. (METU 1998)

13.49 The combined pump - turbine experimental set-up is shown in the figure. At the inlet section of the turbine, the cross sectional area is equal to 0.018 m^2. The pump starts to operate and the flow rate is regulated by the valve. When the turbine is operating, the static pressure (gage) head at section 1, ($p_1/\rho g$), volumetric flow rate, rotational speed of the turbine and torque on the shaft of the turbine are measured as 4.75 m, 0.0124 m^3/s, 1200 rpm and 3.97 N.m, respectively.

a) Determine the head of the turbine (neglect the losses between turbine and the water tank)

b) Determine the overall efficiency of the turbine.

c) If the total head loss in the system is 1.5, what is the head delivered by the pump to the fluid?

d) If the pump efficiency is 65 %, determine the power required to drive the pump. (METU 1998)

13.50 A centrifugal pump delivers water at a volumetric flow rate of 0.02 m^3/s when it is running at 1450 rpm. The elevation difference between the discharge and suction flanges of the pump is 50 cm, while the pressures at the suction and discharge flanges are 60 kPa and 220 kPa, respectively. The diameters of the suction and discharge pipes are 0.055 m and 0.05, respectively. The width and diameter of the impeller at the outlet are 0.015 m and 0.2 m, respectively. There are no inlet guide vanes and the relative fluid angle at the outlet is 40°.

a) Determine the overall efficiency of the pump, if the power supplied to the pump is 6 kW.

b) Determine the head loss in the pump, if the volumetric efficiency is 98 percent.

13.51 A hydraulic power station with a geometric head of 120 m discharges water to a lake at a volumetric flow rate of 1.5 m^3/s. The cross-sectional area at the exit of the turbine is 0.15 m^2, while the cross-sectional area at the exit of the draft tube is 0.165 m^2. The head losses in the penstock and draft tube are 0.5 m and 1 m, respectively. The elevation of the draft tube is 1.8 m and the pressure at its exit is atmospheric. Neglecting the losses in the turbine, determine the

a) gage pressure at the exit of the turbine and

b) shaft power.

13.52 A radial inflow turbine produces a shaft power of 3000 kW at a volumetric flow rate of 4 m^3/s at a rotational speed of 600 rpm. The elevation difference between the water level in the lake and tailwater is 100 m. The head loss in the penstock is 5 m and the leakage through the gap between the runner and casing is 0.1 m^3/s. The diameter and width of the impeller are 1 m and 0.15 m, respectively. Inlet guide vanes direct the flow by an angle of 20^0 to the tangential direction. Assuming that there is no outlet whirl, determine the

a) hydraulic efficiency and

b) mechanical efficiency.

13.53 A centrifugal pump delivers 0.05 m^3/s of water when it is running at 1450 rpm and it is placed 5 m above the water level in the suction reservoir. The suction specific speed and specific speed are 3.0 and 0.7, respectively. The losses in the suction pipe are estimated as 0.8 m. The atmospheric pressure is 100 kPa, while the vapor pressure of water at the operating temperature is 3.9 kPa. Determine whether the cavitation will occur or not.

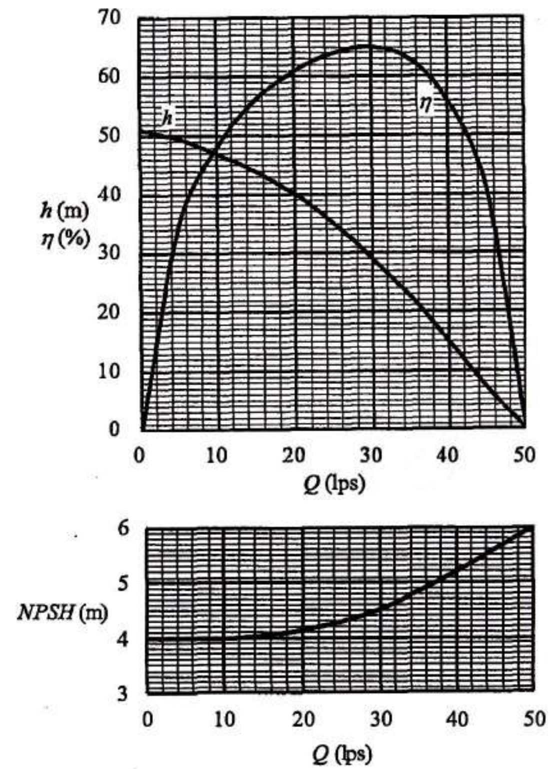
13.54 A centrifugal pump takes water from a closed tank where the pressure is 120 kPa at a volumetric flow rate of 0.1 m^3/s when it is rotating at 1450 rpm. The loss in the suction pipe is 1.5 m. The atmospheric pressure is 100 kPa and the vapor pressure of water at the operating temperature is 4 kPa. The suction specific speed of the pump is 3.0. Determine the maximum allowable elevation of the pump from the water level in the tank in order to prevent the cavitation.

13.55 The characteristics of a pump at its design speed of 2900 rpm are given below. The pump is used to pump water from a large closed tank and the tank pressure is measured by a Bourdon gage which reads 15 kPa. The elevation of the pump inlet from the tank is 7 m and the loss along the suction pipe is 2 m at the flow rate of 33 l/s. The vapor pressure of water is 3.9 kPa. The atmospheric pressure is 100 kPa.

a) Find the required net positive suction head of the pump while operating at 33 l/s.

b) Find the net positive suction head available for the flow rate of 33 l/s.

c) State whether the pump will cavitate or not at 33 l/s. (METU 2009)



Problem 13.55

13.56 A Kaplan turbine is to be operated under a net head of 12 m and a design flow rate of 15 m^3/s. The turbine is required to operate safely in a range of 0.85 to 1.15 times the design flow rate. The speed of the turbine is 300 rpm. The atmospheric pressure is 100 kPa and vapor pressure is 1.8 kPa. The acceptable value of turbine suction specific speed is 5. What will be the maximum elevation of the turbine outlet from the tail water? Neglect all losses. (METU 2006)

13.57 A Kaplan turbine operates under a net head of 10 m at a volumetric flow rate of 12 m^3/s when it is rotating at 400 rpm. The head loss in the draft tube is 4.5 m and the velocity of the tail water is 2 m/s. The atmospheric pressure is 100 kPa, while the vapor pressure of water at the operating temperature is 4 kPa. Determine the maximum elevation of the turbine outlet from the tailwater to prevent cavitation. The suction specific speed of the turbine is 4.

13.58 A centrifugal pump takes water from a lake and discharges to a reservoir at a volumetric flow rate of 0.1 m^3/s. The pump is located at the ground which is 5 m above the water level in the lake. The water level in the discharge reservoir is 25 m above the ground. The losses in the suction and discharge piping are 2 m and 5 m, respectively. The atmospheric pressure is 100 kPa and the vapor pressure of water at the operating temperature is 4 kPa. The suction specific speed for the pump is 3.0 and the efficiency of the pump is 70 percent. Determine the

a) maximum possible rotational speed to avoid cavitation and

b) power required to drive the pump.

13.59 A centrifugal pump is to be designed to discharge water at a rate of 0.08 m^3/s at the speed of 1500 rpm with an overall efficiency of 70 percent. The pump will be placed at the ground level. The water level is 4 m below the ground level and shall be pumped 20 m above the ground. Estimating head losses of 1 m and 7 m at the suction and discharge piping, respectively, at the design operation. The suction specific speed of the pump is 2.7. The atmospheric pressure is 100 kPa, while the vapor pressure of water is 3.9 kPa.

a) Is it possible to operate this pump cavitation free?

b) If there is no cavitation, calculate the shaft power. Otherwise, suggest a geometric modification to the system. (METU 1997)

13.60 Water is pumped from a large closed suction reservoir to large discharge reservoir which is open to atmosphere with a pressure of 100 kPa. The pressure in the suction reservoir is 200 kPa gage. The pump delivers water at a flow rate of 0.01 m^3/s in a 0.05 m diameter piping system. The pump is located at an elevation of 10 m from the free surface of the suction reservoir and the geometric head is 30 m. At this flow rate, the suction side head loss is 3.2 m, discharge side head loss is 7.2 m. The vapor pressure of water at the working conditions is 3500 Pa. The pump manufacturer catalog gives $NPSH_R$ for the pump at 0.01 m^3/s as 4.5 m.

a) Determine the energy given to the water by the pump per unit mass of water.

b) Determine whether the pump cavitates or not at the given working condition. (METU 2013)

13.61 A pump operating at 1450 rpm delivers water from a large suction reservoir to a large discharge reservoir and has the following characteristics:

$$h = 50 - 20000Q^2 \quad \eta = 60Q - 1200Q^2 \quad \text{and} \quad NPSH = 4 + 800Q^2$$

The suction reservoir is exposed to a pressure of 120 kPa, while the discharge reservoir is open to atmosphere with a pressure of 100 kPa. The diameter of the pipe in the piping system is constant and equal to 10 cm. The total geometric head of the system is 20 m. The suction pressure and discharge pressure of the pump are measured by Bourdon gages and found to be −40 kPa and 300 kPa, respectively, while working at 1450 rpm. The vapor pressure of water is 4 kPa.

a) Determine the specific speed of the pump.

b) Determine the flow rate delivered by the pump.

c) Determine the total head loss in the system.

d) Determine the shaft power of the pump.

e) Determine whether the pump cavitates under given working condition. (METU 2015)

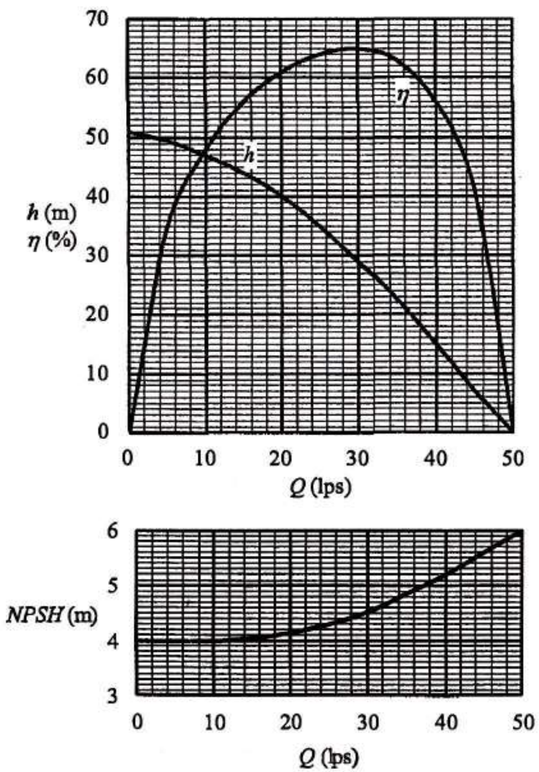
13.62 A pump with given characteristics (at 1500 rpm) delivers water from a large suction reservoir to a large discharge reservoir. Both reservoirs are open to atmosphere where the pressure is 100 kPa. The diameter of the pipe in the piping system is constant and equal to 10 cm. The total geometric head of the system is 20 m. The suction and discharge sides of the pump are at the same level. The pump suction and discharge pressures are measured by Bourdon gages and found to be –40.4 kPa and 352 kPa, respectively, while the pump is working at 1500 rpm. At the working condition, the vapor pressure of water is 4 kPa.

a) Determine the flow rate delivered by the pump.

a) Determine the total head loss in the system.

b) Determine the shaft power of the pump.

a) Determine whether the pump cavitates under given working condition. (METU 2014)



Problem 13.62

13.63 A centrifugal pump consumes 440 kW to deliver 0.8 m³/s of water when it is operating at 720 rpm. The specific speed and suction specific speed of the pump are 0.75 and 3.0, respectively. The diameter and width of the impeller at the outlet are 0.8 m and 0.06 m, respectively. The mechanical and volumetric efficiencies are 0.95 and 0.97, respectively. The pump is located 4 m above the water level in the suction reservoir which is exposed to the atmosphere. The head loss in the suction pipe is 1.5 m. The atmospheric pressure is 100 kPa, while the vapor pressure of water at the operating temperature is 4 kPa. Determine

a) the relative fluid angle at the impeller outlet and

b) whether the pump will cavitate or not.

13.64 A centrifugal pump is transporting water from a suction reservoir to a discharge reservoir. Both reservoirs are open to the atmosphere. The length of the pumping line is 300 m with an internal diameter of 0.2 m. The elevation difference between the water levels in the suction and discharge reservoirs is 5 m. The friction coefficient in the pipe is 0.018. The performance characteristics of the pump can be approximated by

$$h = 20 - 320Q^2 \quad \text{and} \quad \eta = 14Q - 70Q^2$$

where h is in m and Q is in m³/s. Determine the

a) volumetric flow rate and

b) power consumption of the pump.

13.65 The characteristics of a centrifugal pump at a speed of 1500 rpm are given as,

$$h = 50 - 200Q - 2400Q^2 \quad \text{and} \quad \eta = 60Q - 1200Q^2$$

This pump works in a system which has the characteristic of

$$h = 20 + 16300Q^2$$

where h is in meters, Q is in m³/s. The density of water is 1000 kg/m³.

a) Determine the power required to drive the pump.

b) Determine the specific speed of the pump. (METU 2000)

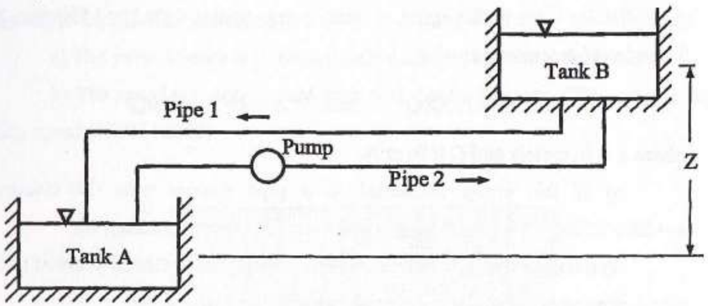
13.66 Pipe 1 which connects the two tanks has a length L_1 = 100 m, diameter D_1 = 100 mm, and friction factor f_1 = 0.025, as shown in the figure. A pump is used to deliver the water from tank A to tank B. The characteristics of the pump at 1450 rpm is given as

$$h = 20 - 12500Q^2$$

where h is in m and Q is in m³/s. The pipeline 2 at which the pump is connected has a length $L_2 = 90$ m, diameter $D_2 = 100$ mm and friction factor $f_2 = 0.03$. Water levels in tanks A and B does not change during operation. Neglecting the minor losses, find the

a) discharge flow rate of the pump and

b) elevation difference between the water levels of tanks A and B. (METU 2014)



Problem 13.66

13.50

13.67 A centrifugal pump transports water from a large suction reservoir to a large discharge reservoir through a piping system. Both reservoirs are exposed to the atmosphere. The system characteristic, when the delivery valve is fully open, is given by

$$h = 15 + 30000Q^2$$

where h is in m and Q is in m³/s. The performance characteristics of the centrifugal pump at 1450 rpm can be approximated by

$$h = 35 - 20000Q^2 \quad \text{and} \quad \eta = 70Q - 1750Q^2$$

a) Determine the geometric head, head of the pump and total loss in the system when the delivery valve is fully open.

b) Determine the power consumption of the pump when the delivery valve is fully open.

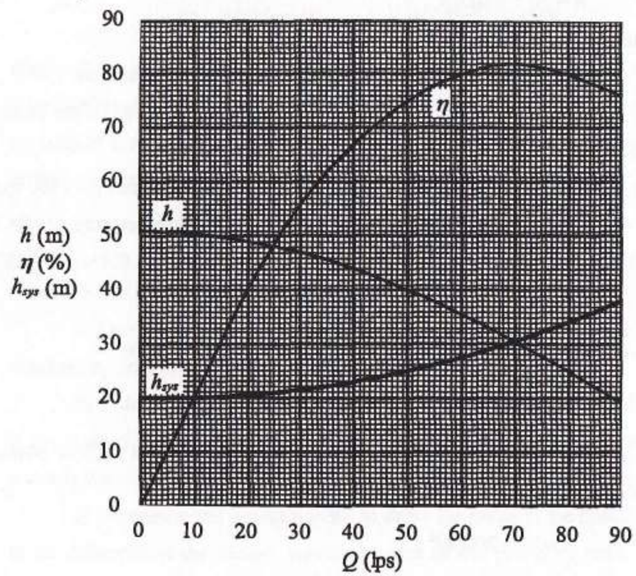
c) It is desirable to obtain a volumetric flow rate of 0.01 m³/s by changing the speed of the pump while the delivery valve is kept fully open. Determine the required speed and power consumption.

13.68 Head versus discharge and efficiency versus discharge characteristics of a water pump at 800 rpm are given in the figure. The characteristic of the system in which the pump is installed is also plotted at fully open valve conditions. It is required to obtain a flow rate of 50 lt/s. This flow rate can be obtained in various ways

a) If 50 lt/s is obtained by changing the valve closure only, find the power consumption of the pump. (Pump operates at 800 rpm)

b) If 50 lt/s is obtained by changing the speed only, find the necessary speed of the pump and the power consumption. (The valve is kept fully open) (METU 1997)

c)



Problem 13.68

13.69 The characteristics of a centrifugal pump at a speed of 1500 rpm are given as,

$$h = 50 - 200Q - 2400Q^2 \quad \text{and} \quad \eta = 60Q - 1200Q^2$$

where h is in meters, Q is in m³/s. This pump works in a system which has the characteristic of

$$h = 20 + 16000Q^2$$

If you want to run this pump at its maximum efficiency point in this given system, what would be the speed of the pump? Also determine the head, flow rate and shaft power of the pump at this operating condition. (METU 2000)

13.70 A water pump, operating at 1500 rpm, has the following characteristics:

$$h = 60 - 12000Q^2 \quad \text{and} \quad \eta = 25Q - 4700Q^3$$

where h is in meters and Q is in m³/s. This pump is connected to a system with a pressurized suction reservoir tank. When the delivery valve is completely open, its characteristic is

$$h = 30 + 3000Q^2$$

for a certain tank pressure value.

a) Find the operation point and calculate the power consumption.

b) Find the flow rate through the same system if the speed of the pump is increased by 10%.

c) To obtain 0.055 m³/s of flow rate at this increased speed (at 1650 rpm), find the necessary change, increase or decrease, in the tank pressure (while the delivery valve is kept open.) Also, calculate the shaft power in this case. (METU 2004)

13.71 The performance characteristics of a pump can be approximated by

$$h = 50 - 4000Q^2 \quad \text{and} \quad \eta = 24Q - 170Q^2$$

when it is running at 1450 rpm. The pump is operating in a system having a characteristic of

$$h = 20 + 2000Q^2$$

where h is in m and Q is in m³/s when the discharge valve is fully open. If the required volumetric flow rate reduces to 0.05 m³/s, this can be obtained in two different ways:

a) The discharge valve can be partially closed, while the pump is operating at the same speed. Determine the power consumption of the pump.

b) The pump speed can be changed, while the discharge valve is kept fully open. Determine the speed and power consumption of the pump.

13.72 "The resistance of a given pipe increases with age as deposits form, increasing the surface roughness and reducing the pipe inside diameter." As an example: After 20 years of use, the frictional loss at any given flowrate in the pipe system is multiplied by 4 due to this aging. A water pump, operating at $N = 1450$ rpm, has the following characteristics:

$$h = 50 - 12500Q^2 \quad \text{and} \quad \eta = 25Q - 4700Q^3$$

where h is in meters and Q is in m³/s.

a) If this pump connected to a pipe system with the characteristic $h = 26 + 2500Q^2$, find the flow rate and the shaft power consumption.

b) Estimate the flow rate through the pipe system after 20 years of use, if the pump remains new but the pipe is aged as described above.

c) What should be the pump speed in order to have the original flow rate through the 20-years-old piping? Also, find the power consumption at this operation. Sketch your solution on the h-Q plane. (METU 2003)

13.73 A centrifugal pump which is used to elevate water by 20 m at 1450 rpm, has the following characteristics

$$h = 50 - 10000Q^2 \quad \text{and} \quad \eta = 30Q - 300Q^2$$

When this pump is first installed in the piping system, it operates at its maximum efficiency. Due to aging deposits form on the inner surface of the pipes increasing the surface roughness and reducing the pipe inside diameter. After 30 years of usage, the frictional loss at any given flowrate in the piping system is increased by 4 times.

a) Determine the volumetric flow rate of the pump after it is first installed in the piping system.

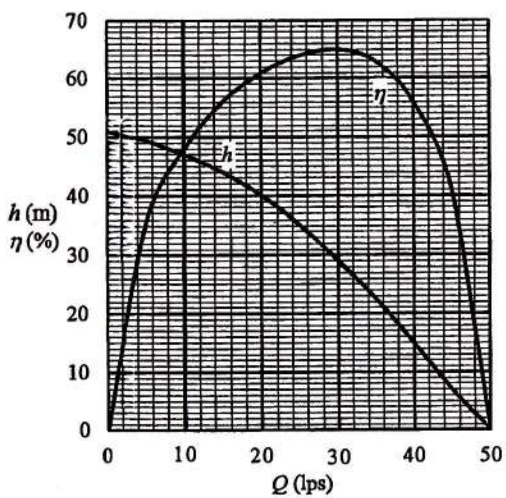
b) Determine the volumetric flow rate of the pump after 30 years of usage.

c) What should be the pump speed in order to have the original flow rate through the 30-years-old piping? Also, find the power consumption at this operation. (METU 2015)

13.74 The 600 rpm characteristics of a pump are given in the figure. This pump is installed on a system which elevates the water 32 m from a lake to an open tank. The flow rate is measured as 22 lps while the delivery valve is fully open and the pump runs at 600 rpm. Determine the power consumptions for the two different runs those are described separately:

a) The valve closure is increased only such that the loss is doubled.

b) The speed is changed only from 600 rpm to 720 rpm. (The valve is kept fully open) (METU 2009)



Problem 13.74

13.75 A pump transfers water from a suction reservoir to a discharge reservoir, as shown in the figure. The performance characteristic of a pump operating at 1450 rpm is given as

$$h = 50 - 5000Q^2 \quad \text{and} \quad \eta = 30Q - 300Q^2$$

It is desired to transfer water at a volumetric flow rate of 0.04 m^3/s through the system, which requires a head of 30 m from the pump. The elevation difference between the suction and discharge reservoirs is 20 m. This task can be achieved by changing the speed of the pump. Determine the

a) system characteristic.

b) rotational speed of the pump.

c) power consumption of the pump for changed speed in part (b) and

d) shut-off head value of the pump for the changed speed in (b). (METU 2016)

13.76 The performance characteristics of a pump at the speed of 1100 rpm are given as

$$h = 50 - 4000Q^2 \quad \text{and} \quad \eta = 32Q - 320Q^2$$

where Q is in m^3/s and h is in m. This pump is to deliver water from a lower reservoir to a higher reservoir in a system where the flow rate can be adjusted with the help of a valve in the system. The system characteristics at a particular position of the valve are

$$h = 11.8 + 863Q^2$$

where Q is in m^3/s and h is in m. It is desired to pump water at a flow rate of 0.04 m^3/s with this pump in this system.

a) Determine the power needed to drive the pump if the desired flow rate is obtained by only changing the speed of the pump.

b) Determine the power needed to drive the pump at the speed of 1100 rpm if the desired flow rate is obtained by adjusting (further opening or closing) the valve in the system. State whether the valve will be opened or closed.

c) Determine the power needed to drive the pump if the desired flow rate is to be delivered at the design operating point of the pump by both changing the pump speed and adjusting the system valve. Reynolds number effects can be neglected. (METU 2013)

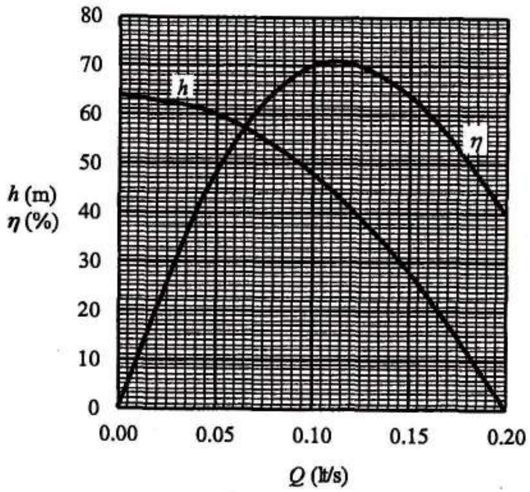
13.77 The characteristic of a pump with an impeller diameter of 0.30 m is given below, the pump is operating at its design speed of 1200 rpm.

a) Calculate the specific speed of this pump.

b) Find the power requirement for this pump to deliver water while operating in a system where the characteristic is given in the figure.

c) The flow rate requirement in the same system changes such that the pump must now deliver water at a flow rate of 0.17 m^3/s. This is achieved by selecting a similar pump of different size, while the rotational speed is kept constant. Neglecting the Reynolds number and scale effects, determine the diameter of the new pump.

d) Calculate the change in pump efficiency, when the pump diameter is changed as explained in part (c). (METU 2007)



Problem 13.77

13.78 The performance characteristics of a centrifugal pump at 1450 rpm can be approximated by

$$h = 40 - 3500Q^2 \quad \text{and} \quad \eta = 25Q - 200Q^2$$

where h is in m and Q is in m^3/s. The system characteristic is given as

$$h = 20 + 500Q^2$$

when the delivery valve is fully open.

a) It is desirable to operate this pump at the maximum efficiency, while the delivery valve is fully open. For this situation, determine the speed of the pump and its power consumption.

b) In order to operate the pump without cavitation for the conditions found in part (a), determine the maximum elevation of the pump from the water level in the suction reservoir which is open to the atmosphere. The head loss in the suction pipe is 1.5 m. The atmospheric pressure is 100 kPa, while the vapor pressure of water at the operating temperature is 4 kPa. The suction specific speed of the pump at the design point is 3.

13.79 Performance curves of a centrifugal pump at 1500 rpm and the pipe system characteristics curve are given below. As shown in the figure, the pump takes water from a closed suction reservoir, which has a pressure of 150 kPa and pumps it into a discharge reservoir, which is open to the atmosphere of pressure of 100 kPa.

a) Determine the power required to drive the pump.

b) If the losses in the suction pipe are 5.0 m, determine the maximum allowable value of suction geometric head (location of the pump from the free surface of the suction reservoir) for cavitation free operation. Vapor pressure for water is 380 Pa.

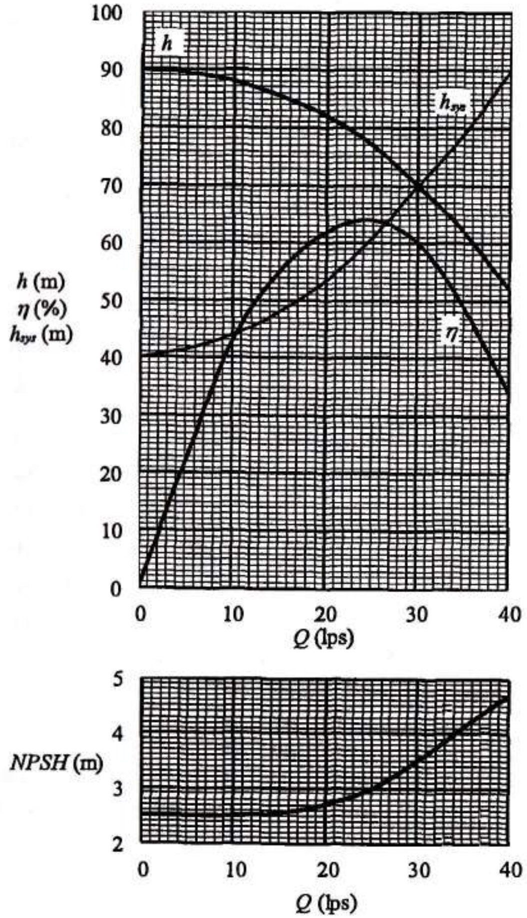
c) Determine the elevation difference between the free surfaces of suction and discharge reservoirs. (METU 1998)

13.80 The characteristics of a pump at the speed of 1500 rpm are given approximately:

$$h = 50 - 0.25Q - 0.02Q^2, \quad \eta = 0.08Q - 0002Q^2 \quad \text{and}$$

$$NPSH = 3 + 0.005Q^2$$

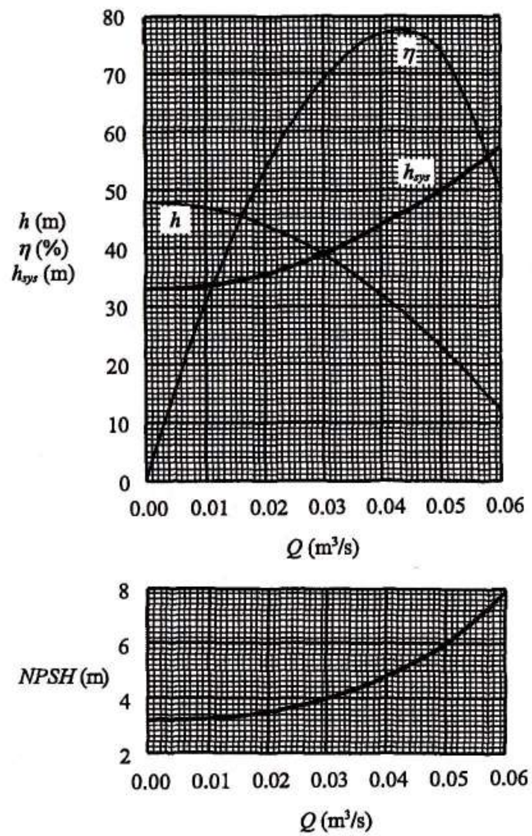
where h and $NPSH$ are in m and Q in liters per second. This pump is connected to a system in order to transport water from a suction reservoir to a discharge reservoir. The gage pressure at the large suction tank is 78.5 kPa gage. The large discharge reservoir is open to atmosphere of 101 kPa. The loss in suction piping in meters is $0.001Q^2$, while the loss in delivery piping in meters is $0.004Q^2$. The suction geometric head is 5.4 m, while the discharge geometric head is 16 m. Determine whether the operation of this pump is cavitation free in the specified installation. Take the vapor pressure of the water in the system as 3800 Pa. (METU 2000)



Problem 13.79

13.81 A pump is used to carry water between two large reservoirs. The lower reservoir is closed and kept at a pressure of -39.24 kPa gage whereas the upper reservoir is open to atmosphere. The elevation difference between the lower reservoir water level and the inlet of the pump is 2 m. The pump and the system characteristics are given in the figure. The pump characteristics correspond to the operating speed of the pump, 1400 rpm.

a) Determine the elevation difference between the water levels in the two reservoirs.



Problem 13.81

b) Determine the leakage flow rate in the pump in m^3/s, if the volumetric efficiency is 95 %.

c) If the total head at the suction side of the pump is 5 m, determine whether the pump cavitates or not. The vapor pressure of water is 3.9 kPa

d) The pump speed is changed in order to pump water at a flow rate of 0.05 m^3/s. Neglecting Reynolds number effects, determine the new speed of the pump in rpm.

e) Determine the power consumption of the pump at the new speed, in kW. (METU 2008)

13.82 The characteristics for a pump rotating at 950 rpm are given in the figure. The pump delivers water from a large suction reservoir to a large discharge reservoir through a piping system. The diameter of the pipe in the piping system is constant and equal to 10 cm. The reservoirs are open to atmosphere where the pressure is 100 kPa. The total geometric head of the system is 30 m. The suction pressure and discharge pressure of the pump are measured by Bourdon gages and found to be −50.0 kPa and 293.35 kPa, respectively, while working at 950 rpm. At the working condition, the vapor pressure of water is 4 kPa.

a) Determine the specific speed of the pump and state the type of the pump.

b) Determine the flow rate delivered by the pump.

c) Sketch the system characteristic on the above given graph.

d) Determine the total head loss in the system at the operating point.

e) Determine the shaft power of the pump at the operating point.

f) Determine whether the pump cavitates under given working condition.

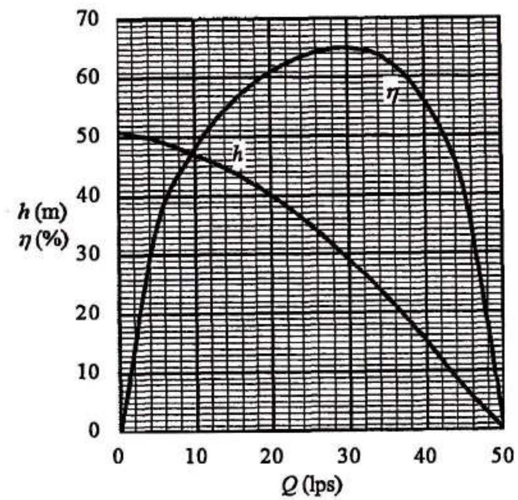
13.83 Two identical centrifugal pumps, whose performance characteristics are approximated by

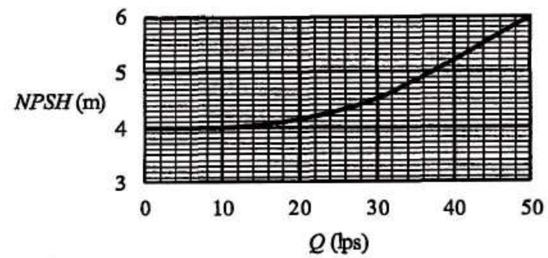
$$h = 60 - 24000Q^2 \quad \text{and} \quad \eta = 56Q - 1120Q^2$$

are connected in series. If the characteristic of the pipe system is given by

$$h = 75 + 3700Q^2$$

where h is in m and Q is in m^3/s, determine the power required to transport water.





Problem 13.82

13.84 Two identical centrifugal pumps are connected in series for transporting water from a lake to a discharge reservoir, which is open to the atmosphere. The performance characteristics of the pump can be approximated by

$$h = 64 - 10000Q^2 \quad \text{and} \quad \eta = 40Q - 500Q^2$$

where h is in m and Q is in m^3/s. The elevation difference between the water levels in the lake and discharge reservoir is 80 m. The total length of the piping is 180 m, while the internal diameter of all pipes is 0.2 m. The friction factor is 0.022. Neglecting minor losses, determine the

a) volumetric flow rate and

b) power required to operate the two pumps.

13.85 Water is being transported between two reservoirs by the use of two identical centrifugal pumps connected in series. Suction reservoir is pressurized at 50 kPa gage and discharge reservoir is open to atmosphere. Performance characteristics of each pump is given by

$$h = 70 - 9000Q^2, \quad \eta = 32Q - 360Q^2$$

where h is in meters and Q is in m³/s. Elevation difference between the water levels in the two reservoirs is 70 m. Total length of the piping is 300 m, internal diameter of all pipes is 0.2 m and the friction factor is 0.03. Neglecting minor losses, determine the power required to deliver water to the discharge reservoir. (METU 2012)

13.86 Characteristics of a five stage pump (five impellers with their diffusers are connected in series on the same shaft with a single motor) are given at its design speed of 1500 rpm. Assume that the stages are identical and they supply equal heads with the same efficiency. The characteristic of the system in which this pump is connected is also shown in the figure. If one of the stages is broken and removed, find

a) the flow rate of the four stage pump,

b) the head of the four stage pump and

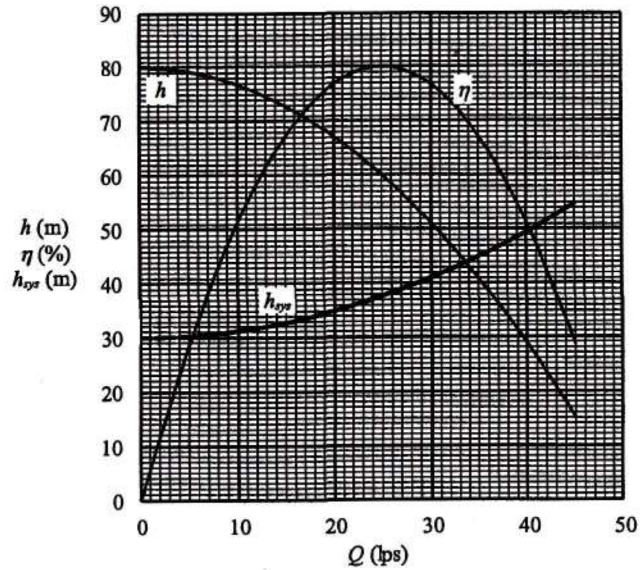
c) the power consumed by the four stage pump.

d) Also, find the specific speed on which the impeller design is based. (METU 1997)

13.87 Two centrifugal pumps A and B are connected in series to transport water from a suction reservoir to a discharge reservoir at a higher elevation, as shown in the figure. The performance characteristics of pumps A and B can be approximated by

$$h_A = 80 - 8000Q^2 \quad \text{and} \quad h_B = 40 - 16000Q^2$$

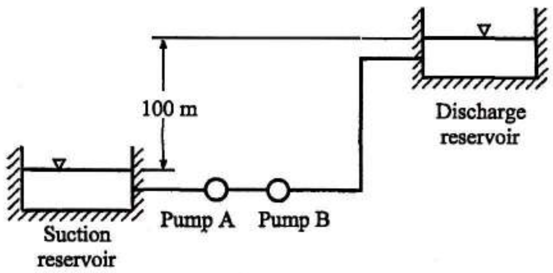
respectively, where h is the head in m and Q is the volumetric flow rate in m³/s. The free surface level of the suction reservoir does not change but the free surface level of the discharge reservoir changes.



Problem 13.86

a) The pumps are operated until the elevation difference between the discharge and suction reservoirs is 100 m. At this instant, the volumetric flow rate of water delivered to the discharge reservoir is 0.025 m³/s. Then, both pumps are stopped and water in the discharge reservoir is consumed. The pumps are restarted when the water level in the discharge reservoir decreases by 10 m. Determine the volumetric flow rate of water at this instant.

b) Determine the limiting value for h_{gt} that both pumps are in effective use. (delivered head) (METU 2000)



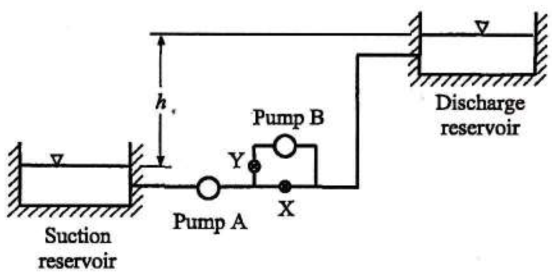
Problem 13.87

13.88 It is desired to transport water from a suction reservoir to a discharge reservoir at a higher elevation, as shown in the figure. When the elevation difference between the two reservoirs is 79.6 m, the necessary head is supplied by serially connected pumps A and B (valve X is closed and valve Y is open). When the elevation difference decreases to 40 m, it is found that pump B has no contribution to the developed head so that it should be shut down (valve Y is closed and valve X is open). Determine the power required when the elevation difference is 79.6 m. The performance characteristics of pumps A and B can be approximated by

$$h_A = 100 - 10000Q^2 \quad \text{and} \quad \eta_A = 30Q - 300Q^2$$

$$h_B = 50 - 20000Q^2 \quad \text{and} \quad \eta_B = 60Q - 1200Q^2$$

respectively, where h is the head in m and Q is the volumetric flow rate in m^3/s. (METU 2004)



Problem 13.88

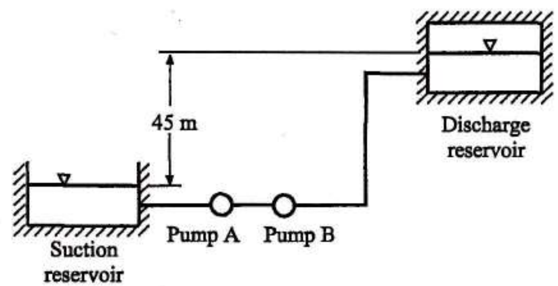
13.89 Two pumps (A and B) are connected in series and deliver water from a large suction reservoir to a large discharge reservoir. The elevation difference between the two reservoirs is 45 m. Suction reservoir is open atmosphere, while the discharge reservoir is closed and kept at a gage pressure of 42 kPa. The performance characteristics of the pumps are given as

$$h_A = 40 - 5000Q^2 \quad \text{and} \quad h_B = 40 - 3200Q^2$$

The *NPSH* characteristics are the same for both pumps and is given by

$$NPSH = 4 + 942Q^2$$

where h and *NPSH* are in m and Q is in m^3/s.The overall head loss coefficient, k, for the whole pipe system is 18, while the cross-sectional area for all pipes is 0.0314 m^2. Both pumps are placed 1 m above the water level in the suction reservoir. The head loss coefficient in the pipe connecting suction reservoir to pump A is 0.45. The frictional head loss in the pipe between the two pumps can be neglected. The vapor pressure of water at the working temperature is 3.9 kPa. Neglecting minor losses, determine whether any of the pumps will cavitate or not. (METU 2008)



Problem 13.89

13.90 Two identical centrifugal pumps, whose performance characteristics are approximated by

$$h = 50 - 20000Q^2 \quad \text{and} \quad \eta = 60Q - 1200Q^2$$

are connected in parallel. If the characteristic of the pipe system is given by

$$h = 35 + 6000Q^2$$

where h is in m and Q is in m^3/s, determine the power required to transport water.

13.91 Two identical centrifugal pumps are connected in parallel in order to deliver water from a suction reservoir to a discharge reservoir. Both reservoirs are open to the atmosphere. The performance characteristics of the pump can be approximated by

$$h = 50 - 5000Q^2 \quad \text{and} \quad \eta = 30Q - 300Q^2$$

where h is in m and Q is in m^3/s. The elevation difference between the water levels in two reservoirs is 20 m. The total length of the piping is 200 m, while the internal diameter of all pipes is 0.2 m. The friction factor is 0.018. Neglecting minor losses, determine the

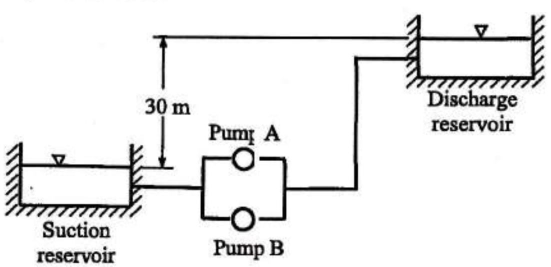
a) volumetric flow rate and

b) power required to drive the pumps.

13.92 Two pumps (A and B) are connected in parallel in order to deliver water from a suction reservoir to a discharge reservoir. Both reservoirs are open to atmosphere. The performance characteristics of the pumps at an operating speed of 1200 rpm can be approximated as below:

$$h_A = 50 - 5000Q^2 \quad \text{and} \quad \eta_A = 30Q - 300Q^2$$
$$h_B = 50 - 3200Q^2 \quad \text{and} \quad \eta_B = 20Q - 125Q^2$$

where h is in m and Q is in m^3/s. The elevation difference between the water levels in the two reservoirs is 30 m. The total length of the piping is 200 m, while the internal diameter of all pipes is 0.2 m. The friction factor is 0.018. Neglecting minor losses, determine the



Problem 13.92

a) volumetric flow rate of each pump,

b) head developed by each pump and

c) type of pump A. (METU 2006)

13.93 Two identical pumps combined in parallel are used for delivering water from a large suction reservoir to a large discharge reservoir. The discharge reservoir is closed and pressurized whereas the suction reservoir is open to atmosphere at a pressure of 100 kPa. The elevation difference between the water levels in the two reservoirs is 20 m. The speed of the pumps cannot be changed; both pumps operate at the same constant speed. The head and efficiency characteristics of a single pump at the operating speed are

$$h = 50 - 5000Q^2 \quad \text{and} \quad \eta = 30Q - 300Q^2$$

where the head is in meters and the volumetric flow rate in m^3/s. The system has a discharge valve with which the flow rate in the system can be adjusted. When the discharge valve is fully open, the system characteristics can be formulated as

$$h = 14 + 1250Q^2$$

where the head is in meters and the volumetric flow rate, in m^3/si

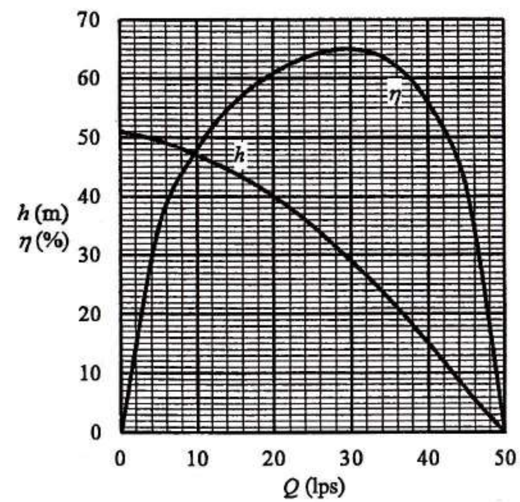
a) Determine the pressure in the discharge reservoir.

b) Determine the power consumption of each pump when pumping the maximum flow rate in the system.

c) If the pumps are to be operated at their design point, determine the corresponding flow rate in the system that can be obtained by adjusting the discharge valve.

d) If, instead of parallel combination, the two pumps were combined in series, determine the operating efficiency of each pump for maximum flow rate in the system. (METU 2014)

13.94 The characteristics of a water pump at the design speed of 1500 rpm, and the system characteristic are given in the figure. If another identical pump is connected in parallel and speeds of these pumps increased to 1650 rpm, calculate the total power consumption of the pumps. (METU 1999)



Problem 13.94

13.95 The characteristics of a pump at the speed of 1900 rpm and the system characteristic are given in the figure. A water flow rate of 45 liter/s is required through the system. Therefore another identical pump is connected in parallel and its speed is adjusted to a certain value while the other runs at 1900 rpm. Find the necessary speed of the second pump and the power consumptions of the both pumps at that desired parallel operation. (METU 2004)

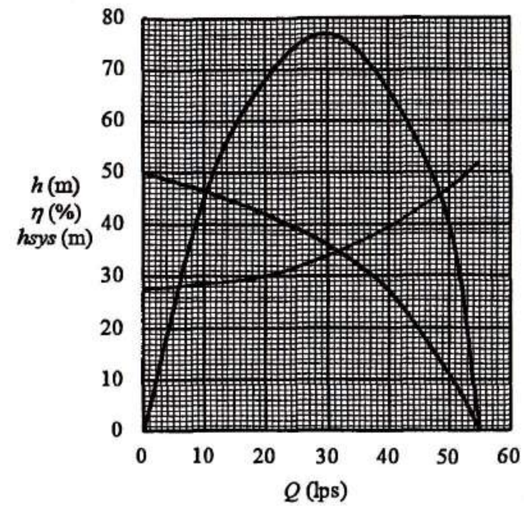
13.96 Two pumps, with identical characteristics at a given constant speed, are connected in parallel. Pump A has constant-speed motor and the other, Pump B, has variable-speed motor. The single pump performance characteristics at $N = 1450$ rpm are

$$h = 50 - 12500Q^2 \quad \text{and} \quad \eta = 48Q - 750Q^2$$

and the system characteristics is $h = 25 + 2528Q^2$ by neglecting any losses in very short individual branches. In the above expressions, h is in meters and Q is in m³/s. The required water discharge (flow rate) into the tank is 0.059 m³/s.

a) What should be the speed of the Pump B, while Pump A is running at 1450 rpm?

b) Estimate the total shaft power consumption for the operation. (METU 2003)

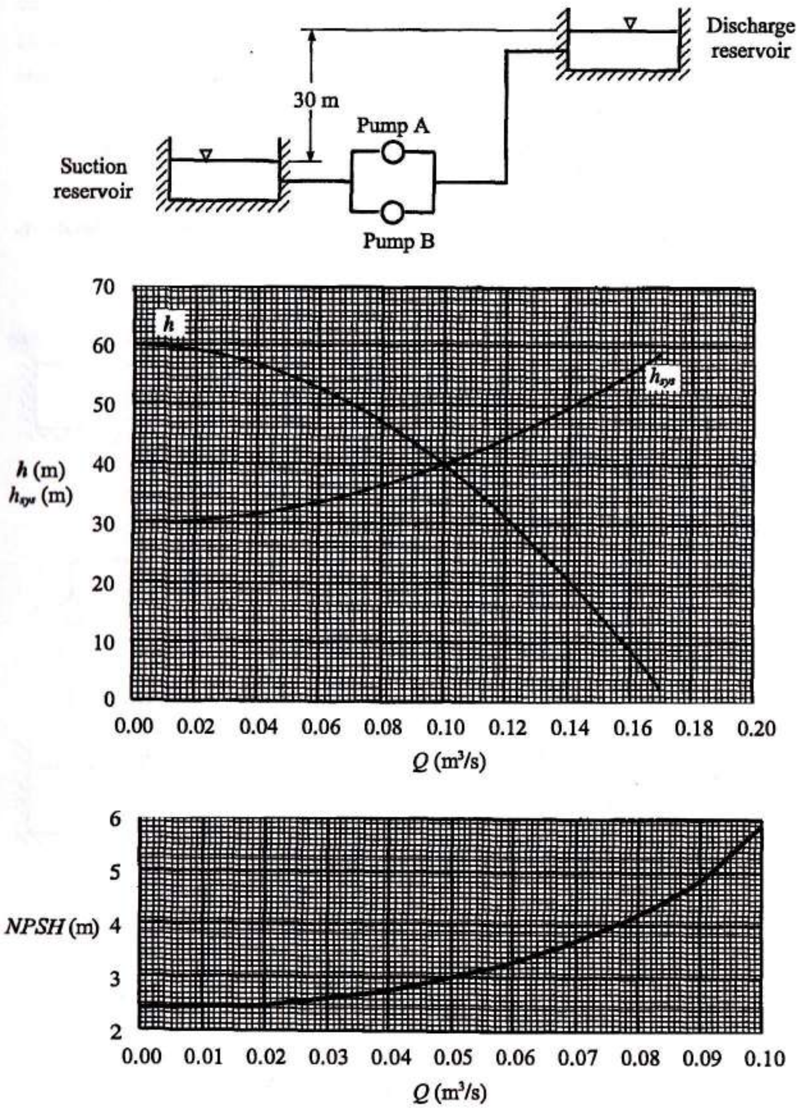


Problem 13.95

13.97 Two identical centrifugal pumps are operating in parallel to transport water from a large closed suction reservoir at a gage pressure of 20 kPa to a large open discharge reservoir at a higher elevation. System and combined pump characteristics are given in the figure. . The elevation difference between the free surfaces of suction and discharge reservoirs is 30 m.The head loss in the suction piping is 2m. The vapor pressure of water at the operating temperature is 4 kPa.

a) Determine the elevation difference between the two reservoirs.

b) Determine the maximum elevation of the pumps from the suction reservoir for no cavitation. (METU 2002)



Problem 13.97

13.98 0.12 m^3/s of water will be pumped in a pipe system. The pipe system requires a head of at least 45 m. Two identical pumps, with the characteristics given below, are available for the job.

a) Determine if a single pump can be used or not?

b) Determine if a series combination can be used or not?

h (m)	61	58	50	38	21	2
Q (m^3/s)	0	0.04	0.08	0.12	0.16	0.20
η (%)	0	48	71	70	44	0

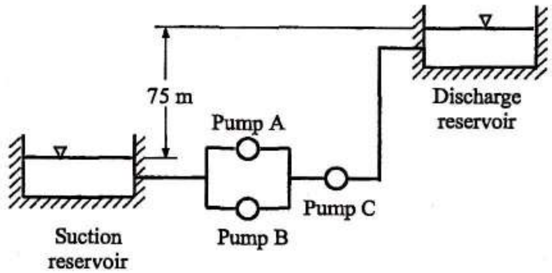
c) Determine if a parallel combination can be used or not?

d) Which of the above alternatives would provide a better solution? (METU 2006)

13.99 Three identical centrifugal pumps A, B and C are combined to transport water from a suction reservoir to a discharge reservoir at a higher elevation, as shown in the figure. The elevation difference between the free surfaces of suction and discharge reservoirs is 75 m. The performance characteristics of each pump can be approximated by

$$h = 50 - 20000Q^2 \quad \text{and} \quad \eta = 64Q - 1280Q^2$$

where h is the head in m and Q is the volumetric flow rate in m^3/s. The losses in the piping system can be approximated as $15000Q^2$. Determine the power required to drive the three pumps. (METU 1999)



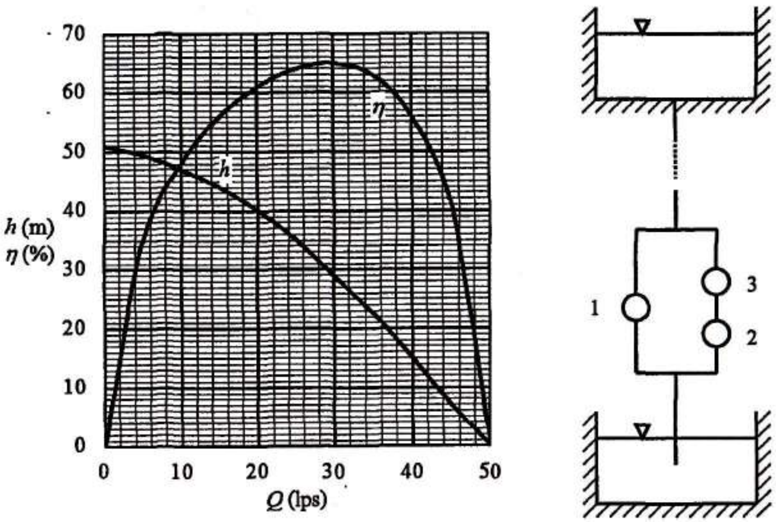
Problem 13.99

13.100 Three identical pumps are connected to a system, as shown in the figure. The characteristics of the system and one of these pumps are given in the graph. Find the total power consumed by the following pump combinations to pump water from the bottom reservoir to the top one:

a) If pumps 2 and 3 share the work equally. on the right branch,

b) If on the right branch, pump 3 does not work.

Neglect the losses in short branches and loss through pump 3 when it is shutdown. (METU 2007)



Problem 13.101