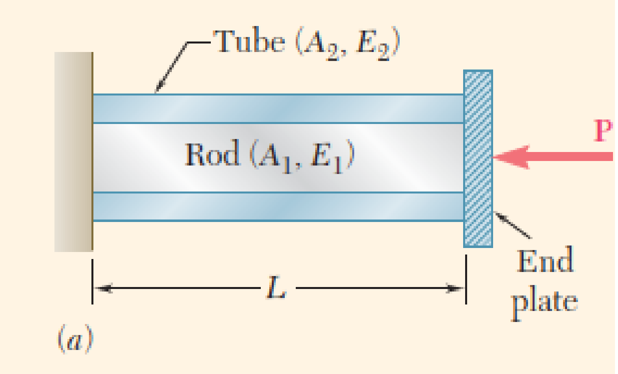
Stress and Strain
Axial Loading

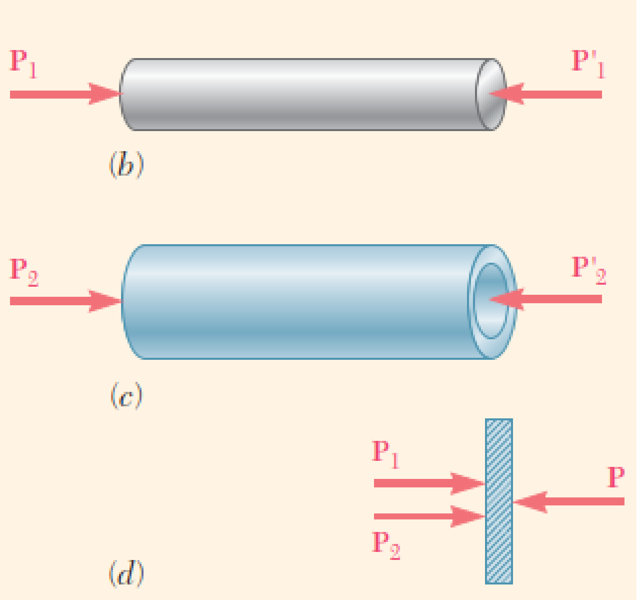
Part II

Statically Indeterminate Problems

- In the problems considered in the preceding section, we could always use free-body diagrams and equilibrium equations to determine the internal forces produced in the various portions of a member under given loading conditions. There are many problems, however, where the internal forces cannot be determined from statics alone. In most of these problems, the reactions themselves—the external forces—cannot be determined by simply drawing a free-body diagram of the member and writing the corresponding equilibrium equations. The equilibrium equations must be complemented by relationships involving deformations obtained by considering the geometry of the problem. Because statics is not sufficient to determine either the reactions or the internal forces, problems of this type are called statically indeterminate.

Concept Application 2.2





- A rod of length L, cross-sectional area A_1, and modulus of elasticity E_1, has been placed inside a tube of the same length L, but of cross-sectional area A_2 and modulus of elasticity E_2 (Fig. 2.21a).
- What is the deformation of the rod and tube when a force P is exerted on a rigid end plate as shown?

- The axial forces in the rod and in the tube are P_1 and P_2, respectively. From, Figure 2.21d.

$$P_1 + P_2 = P$$

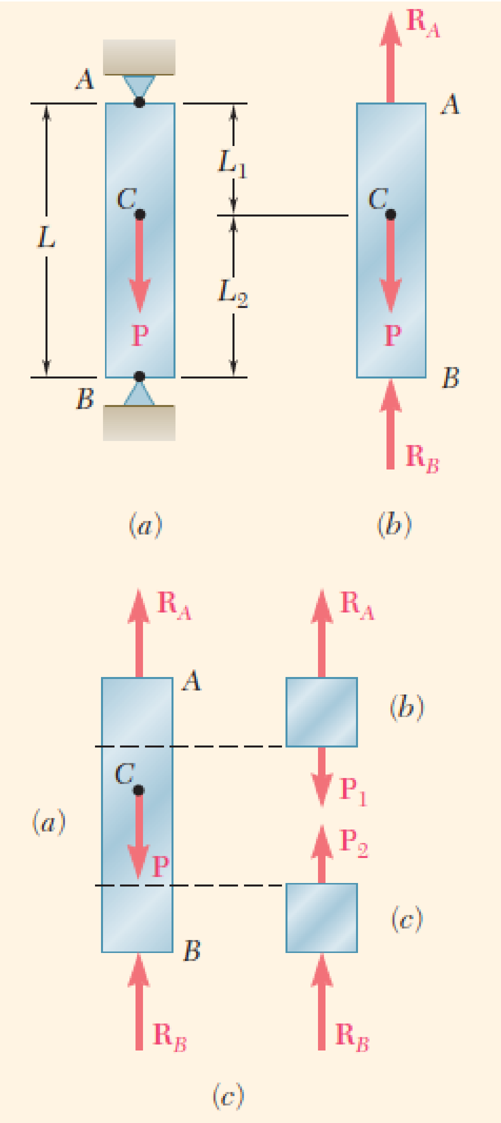
- Clearly, one equation is not sufficient to determine the two unknown internal forces P_1 and P_2. The problem is ***statically indeterminate***.
- However, the geometry of the problem shows that the deformations δ_1 and δ_2 of the rod and tube must be equal.

$$\delta_1 = \frac{P_1 L}{A_1 E_1} \qquad \delta_2 = \frac{P_2 L}{A_2 E_2} \quad \rightarrow \quad \frac{P_1}{A_1 E_1} = \frac{P_2}{A_2 E_2}$$

$$P_1 = \frac{A_1 E_1 P}{A_1 E_1 + A_2 E_2} \qquad P_2 = \frac{A_2 E_2 P}{A_1 E_1 + A_2 E_2}$$

Either of deformation equations can be used to determine the common deformation of the rod and tube.

Concept Application 2.3



- A bar AB of length L and uniform cross section is attached to rigid supports at A and B before being loaded. What are the stresses in portions AC and BC due to the application of a load P at point C (Fig. 2.22a)?
- Drawing the free-body diagram of the bar (Fig. 2.22b), the equilibrium equation is

$$R_A + R_B = P$$

- Since this equation is not sufficient to determine the two unknown reactions R_A and R_B, the problem is statically indeterminate.
- However, the reactions can be determined if observed from the geometry that the total elongation d of the bar must be zero. The elongations of the portions AC and BC are respectively δ_1 and δ_2, so

$$\delta = \delta_1 + \delta_2 = 0 \longrightarrow \delta = \frac{P_1L_1}{AE} + \frac{P_2L_2}{AE} = 0$$

- Note from the free-body diagrams shown in parts b and c of Figure that $P_1 = R_A$ and $P_2 = - R_B$. Carrying these values into deformation equation above,

$$R_AL_1 - R_BL_2 = 0$$

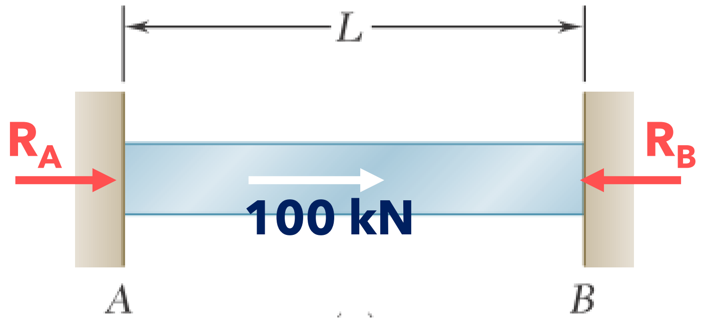
$$R_A = PL_2/L \text{ and } R_B = PL_1/L$$

$$\sigma_1 = \frac{PL_2}{AL} \qquad \sigma_2 = -\frac{PL_1}{AL}$$

Superposition Method

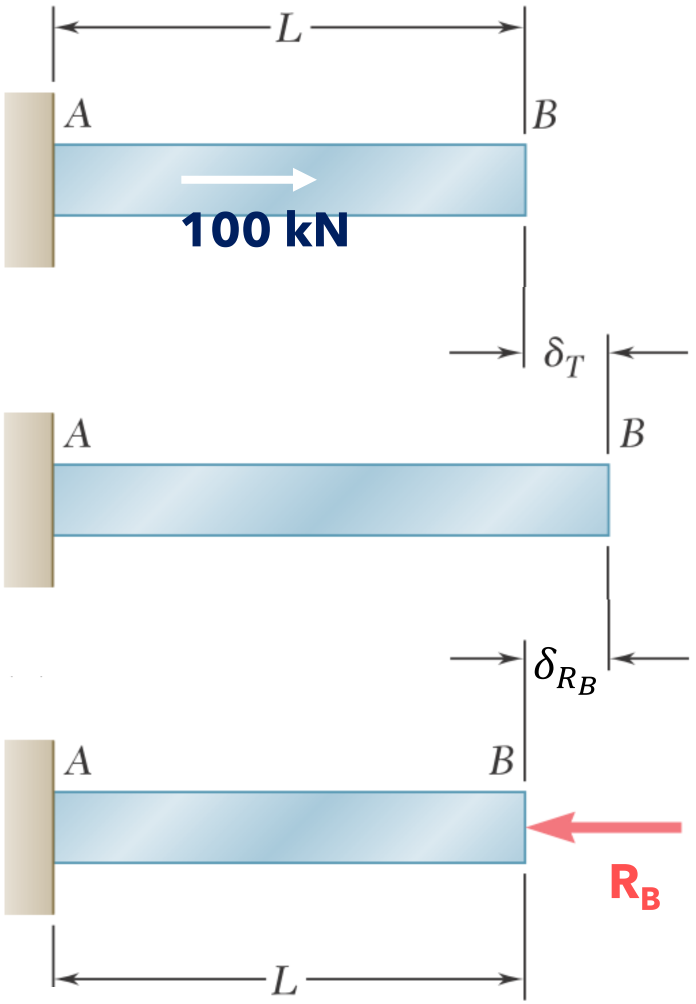
- A structure is statically indeterminate whenever it is held by more supports than are required to maintain its equilibrium.
- This results in more unknown reactions than available equilibrium equations. It is often convenient to designate one of the reactions as redundant and to eliminate the corresponding support. Since the stated conditions of the problem cannot be changed, the redundant reaction must be maintained in the solution. It will be treated as an unknown load that, together with the other loads, must produce deformations compatible with the original constraints. The actual solution of the problem considers separately the deformations caused by the given loads and the redundant reaction, and by adding—or superposing—the results obtained.

$\delta_T = 0$



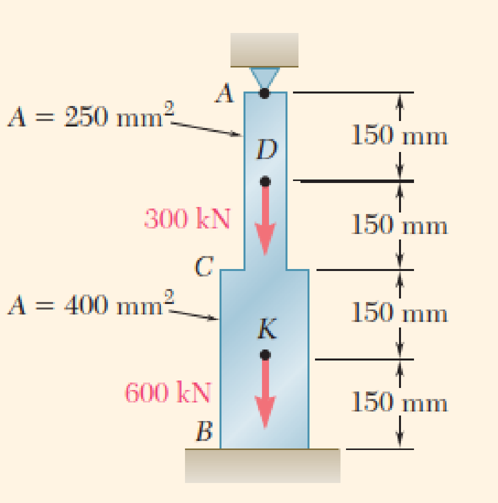
- $\sum F = R_A + 100 - R_B = 0$
- $R_B - R_A = 100$ kN

$\delta_T = 0$



- The following two conditions must be satisfied if the principle of superposition is to be applied.
  - The loading P must be linearly related to the stress σ or displacement δ that is to be determined.
  - The loading must not significantly change the original geometry or configuration of the member.

Concept Application 2.4



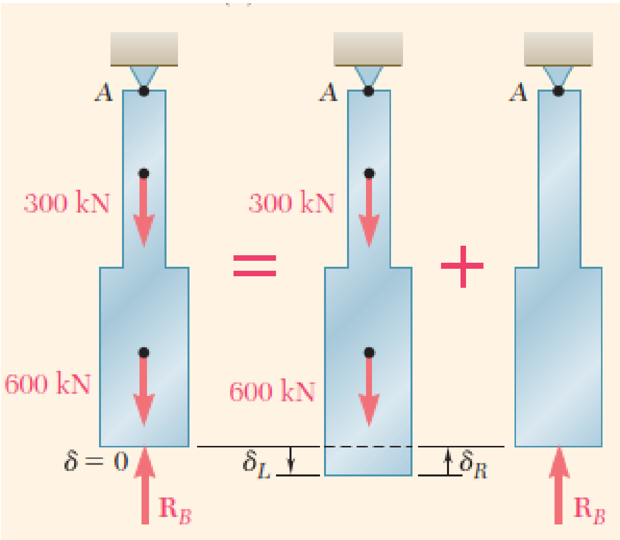
(a)

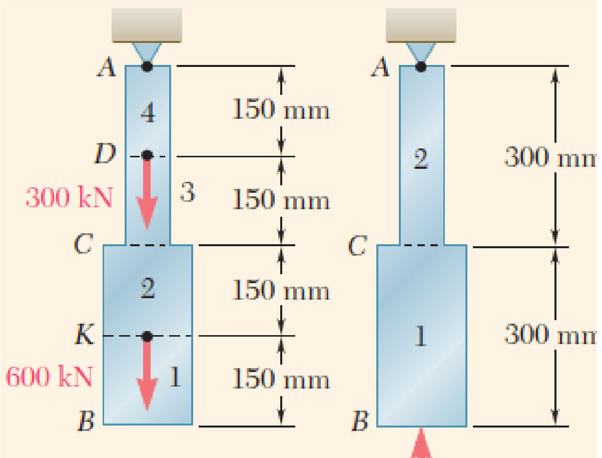
- Determine the reactions at A and B for the steel bar and loading shown in Fig. 2.23a, assuming a close fit at both supports before the loads are applied.
- We consider the reaction at B as redundant and release the bar from that support. The reaction R_B is considered to be an unknown load and is determined from the condition that the deformation δ of the bar equals zero.
  - The solution is carried out by considering the deformation δ_L caused by the given loads and the deformation δ_R due to the redundant reaction R_B (Fig. 2.23b). Dividing the bar into four portions:

$$P_1 = 0 \qquad P_2 = P_3 = 600 \times 10^3\,\text{N} \qquad P_4 = 900 \times 10^3\,\text{N}$$

$$A_1 = A_2 = 400 \times 10^{-6}\,\text{m}^2 \qquad A_3 = A_4 = 250 \times 10^{-6}\,\text{m}^2$$

$$L_1 = L_2 = L_3 = L_4 = 0.150\,\text{m}$$





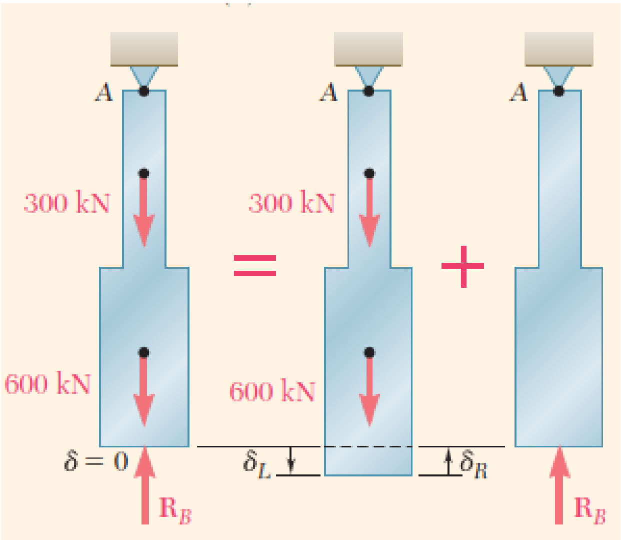
(c)

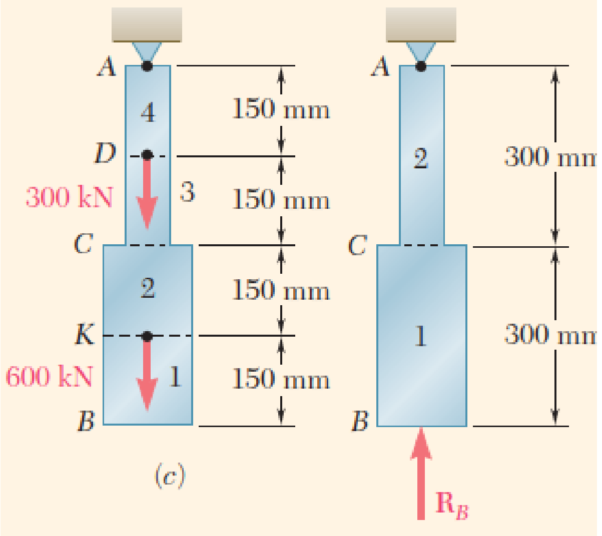
- Substituting these values into deformation equation

$$\delta_L = \sum_{i=1}^{4} \frac{P_i L_i}{A_i E} = \left(0 + \frac{600 \times 10^3\,\text{N}}{400 \times 10^{-6}\,\text{m}^2} + \frac{600 \times 10^3\,\text{N}}{250 \times 10^{-6}\,\text{m}^2} + \frac{900 \times 10^3\,\text{N}}{250 \times 10^{-6}\,\text{m}^2} \right) \frac{0.150\,\text{m}}{E}$$

$$\delta_L = \frac{1.125 \times 10^9}{E}$$

Concept Application 2.4





- Considering now the deformation δ_R due to the redundant reaction R_B, the bar is divided into two portions,

$$P_1 = P_2 = -R_B$$
$$A_1 = 400 \times 10^{-6}\ \mathrm{m}^2 \qquad A_2 = 250 \times 10^{-6}\ \mathrm{m}^2$$
$$L_1 = L_2 = 0.300\ \mathrm{m}$$

- Substituting these values into deformation equation

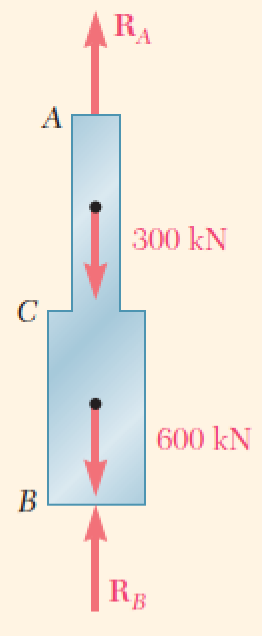
$$\delta_R = \frac{P_1 L_1}{A_1 E} + \frac{P_2 L_2}{A_2 E} = -\frac{(1.95 \times 10^3) R_B}{E}$$

- Express the total deformation δ of the bar is zero

$$\delta = \delta_L + \delta_R = 0$$

$$\delta = \frac{1.125 \times 10^9}{E} - \frac{(1.95 \times 10^3) R_B}{E} = 0$$

Concept Application 2.4



(e)

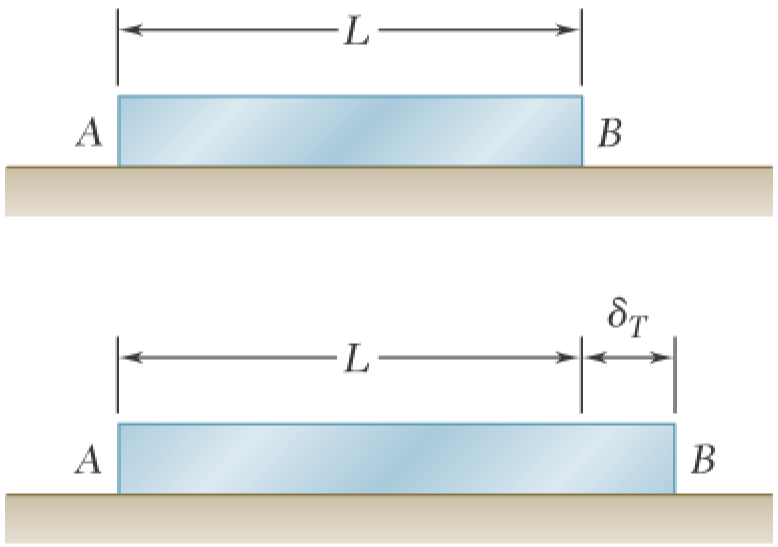
- Solving for R_B, $R_B = 577 \times 10^3 \text{ N} = 577 \text{ kN}$

- The reaction R_A at the upper support is obtained from the free-body diagram of the bar

$$+\uparrow \Sigma F_y = 0: \quad R_A - 300 \text{ kN} - 600 \text{ kN} + R_B = 0$$

$$R_A = 900 \text{ kN} - R_B = 900 \text{ kN} - 577 \text{ kN} = 323 \text{ kN}$$

Problems Involving Temperature Changes



- Consider a homogeneous rod AB of uniform cross section that rests freely on a smooth horizontal surface. If the temperature of the rod is raised by ΔT, the rod elongates by an amount δ_T that is proportional to both the temperature change ΔT and the length L of the rod. Here

$$\delta_T = \alpha(\Delta T)L$$

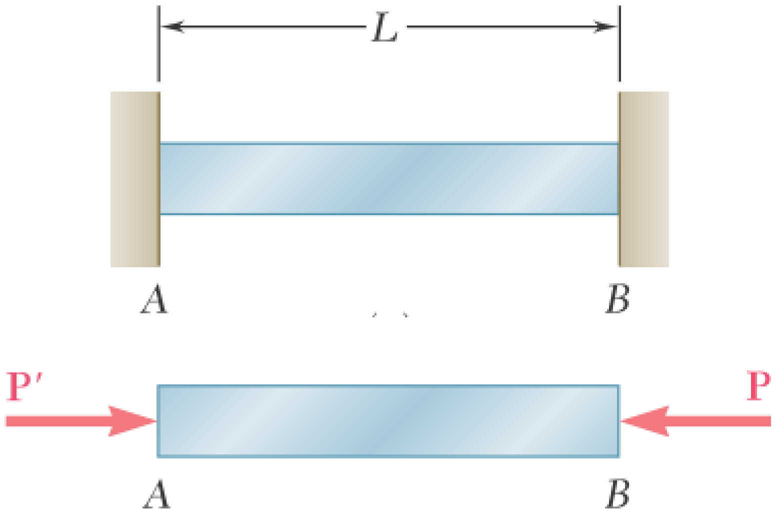
where α is a constant characteristic of the material called the ***coefficient of thermal expansion***.

Since δ_T and L are both expressed in units of length, a represents a quantity *per degree C* or *per degree F*, depending whether the temperature change is expressed in degrees Celsius or Fahrenheit.

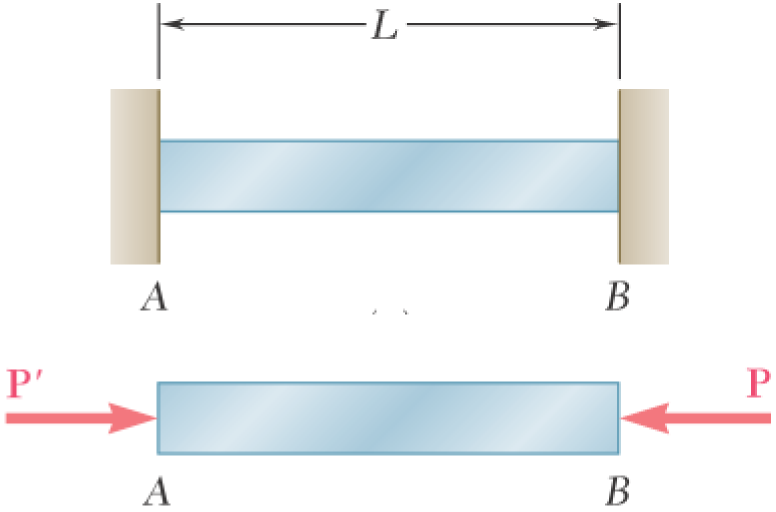
- Associated with deformation δ_T there must be a strain $\varepsilon_T = \delta_T / L$.

$$\varepsilon_T = \alpha \Delta T$$

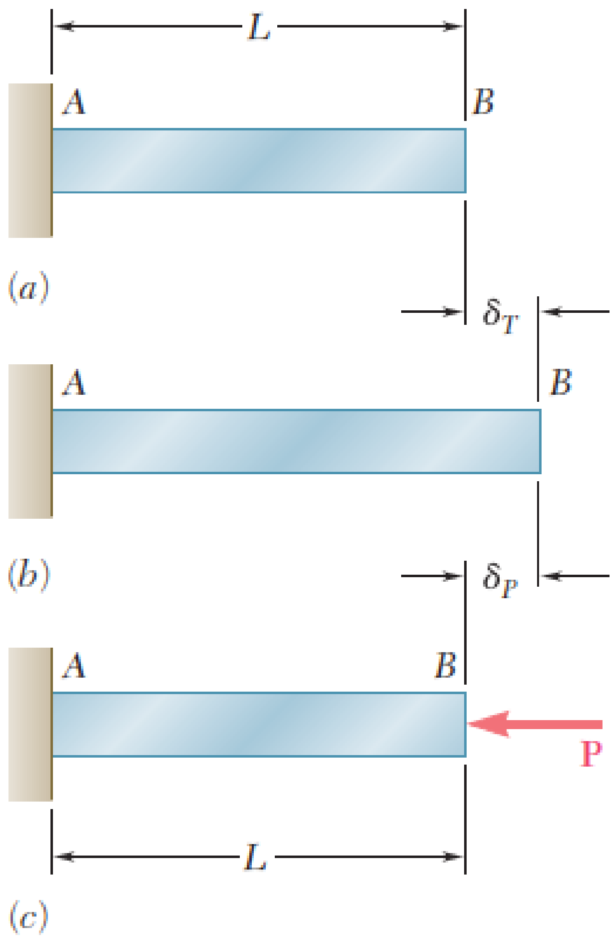
- The strain ε_T is called a ***thermal strain***, as it is caused by the change in temperature of the rod. **However, there is no stress associated with the strain $\boldsymbol{\varepsilon_T}$.**



- Assume the same rod AB of length L is placed between two fixed supports at a distance L from each other. Again, there is neither stress nor strain in this initial condition.
- If we raise the temperature by ΔT, the rod cannot elongate because of the restraints imposed on its ends; the elongation δ_T of the rod is zero. Since the rod is homogeneous and of uniform cross section, the strain ε_T at any point is $\varepsilon_T = \delta_T / L$ and thus is also zero.



- However, the supports will exert equal and opposite forces P and P' on the rod after the temperature has been raised, to keep it from elongating. It follows that a state of stress (with no corresponding strain) is created in the rod.
- The problem created by the temperature change ΔT is statically indeterminate. Therefore, the magnitude P of the reactions at the supports is determined from the condition that the elongation of the rod is zero. no corresponding strain) is created in the rod.



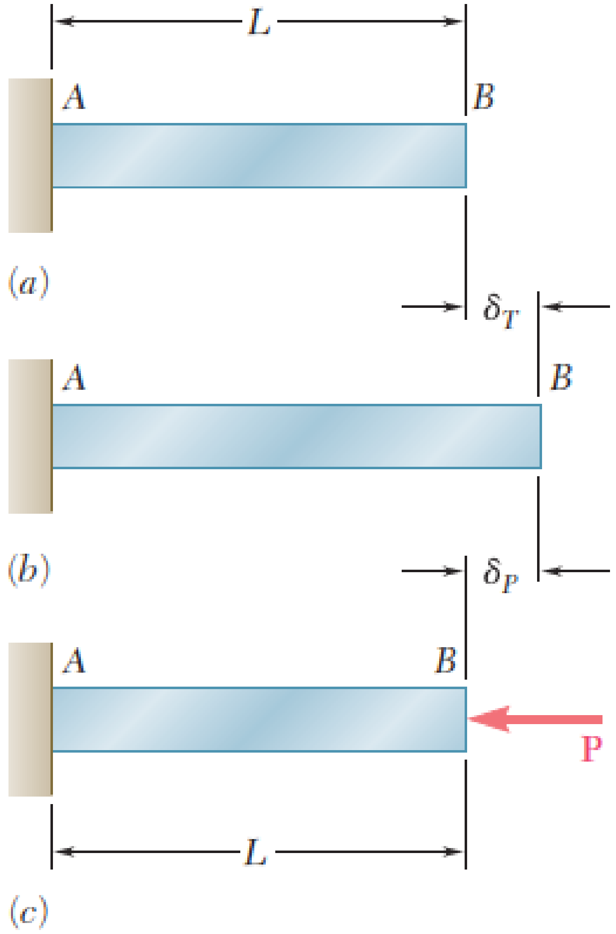
- The problem created by the temperature change ΔT is statically indeterminate. Therefore, the magnitude P of the reactions at the supports is determined from the condition that the elongation of the rod is zero.

- Using the superposition method, the rod is detached from its support B and elongates freely as it undergoes the temperature change ΔT. The corresponding elongation is

$$\delta_T = \alpha(\Delta T)L$$

- Applying now to end B the force P representing the redundant reaction, a second deformation is

$$\delta_P = \frac{PL}{AE}$$



- Expressing that the total deformation d must be zero,

$$\delta = \delta_T + \delta_P = \alpha(\Delta T)L + \frac{PL}{AE}$$

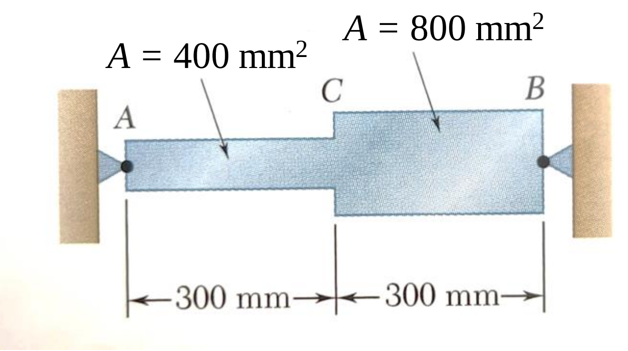
- from which

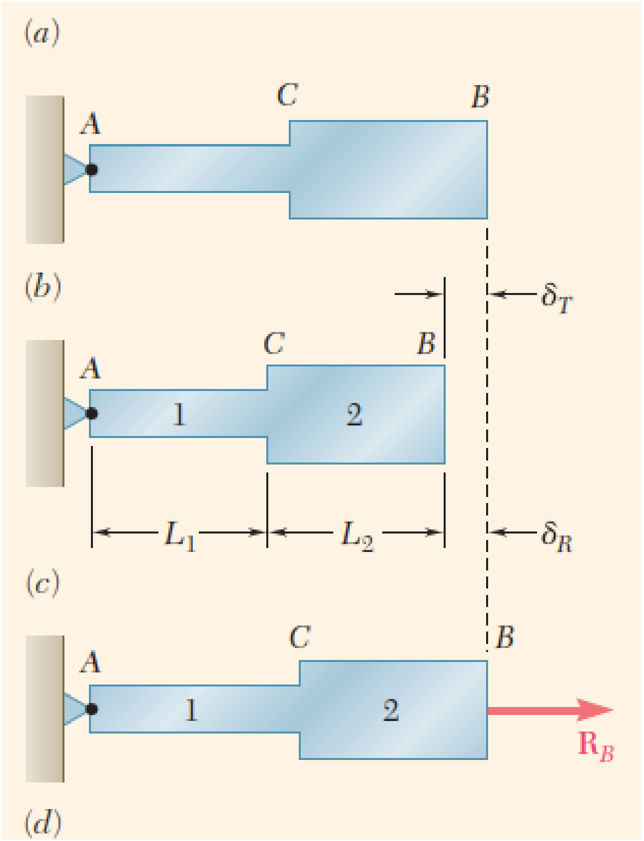
$$P = -AE\alpha(\Delta T)$$

- The stress in the rod due to the temperature change ΔT is

$$\sigma = \frac{P}{A} = -E\alpha(\Delta T)$$

Concept Application 2.5





- Determine the values of the stress in portions AC and CB of the steel bar shown (Fig. 2.28a) when the temperature of the bar is -50°C, knowing that a close fit exists at both of the rigid supports when the temperature is +25°C . Use the values E = 200 GPa and α=12*10^{-6} /°C for steel.

Since the problem is statically indeterminate, detach the bar from its support at B and let it undergo the temperature change

$$\Delta T = T_F - T_O = -50 - 25 = -75\ °C$$

The corresponding deformation $$\delta_T = \alpha(\Delta T)L = 12\text{x}10^{-6}\text{x}(-75\ °C) * 0.6\ m$$

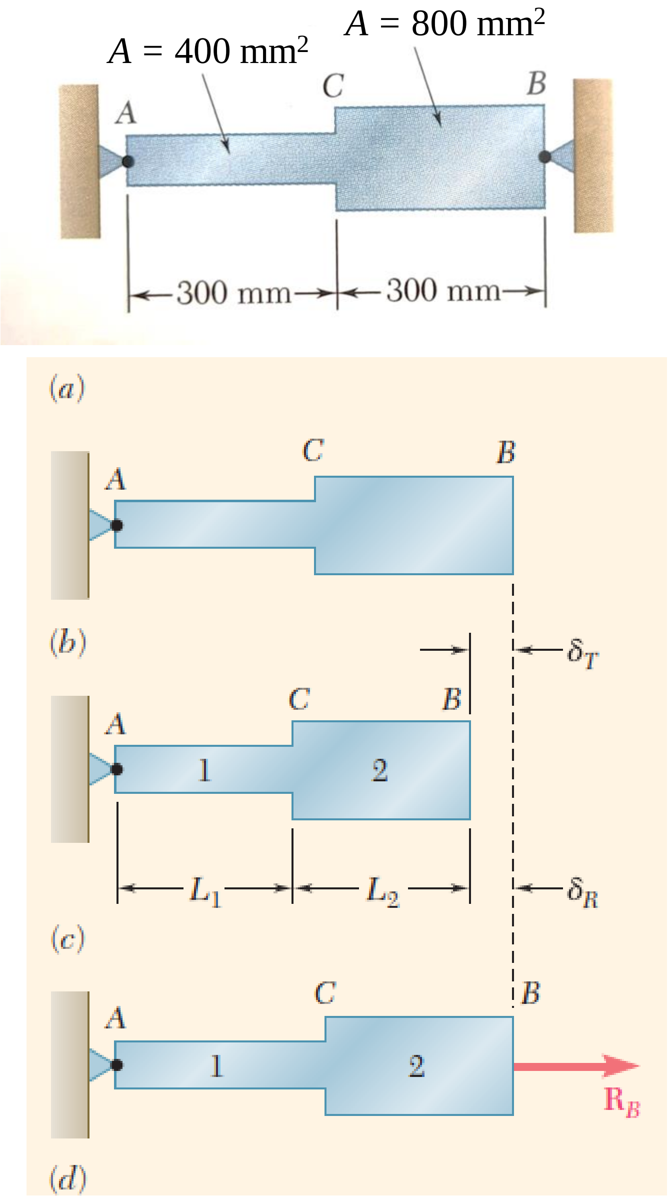
$$= -540\text{x}10^{-6}\ m$$

Applying the unknown force R_B at end B, δ_R

$$\delta_R = \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2}$$

$$= \frac{R_B}{200x10^9\ Pa}\left(\frac{0.3\ m}{400\text{x}10^{-6} m^2} + \frac{0.3\ m}{800\text{x}10^{-6} m^2}\right) = (5.625\text{x}10^{-9} m/N)\boldsymbol{R_B}$$

Concept Application 2.5



Expressing that the total deformation of the bar must be zero as a result of the imposed constraints, write

$$\delta = \delta_T + \delta_R = 0$$

$$\delta = -540x10^{-6} + (5.625x10^{-9}m/N)\boldsymbol{R_B}=0$$

$$\boldsymbol{R_B} = 96x10^3 N = 93\ kN$$

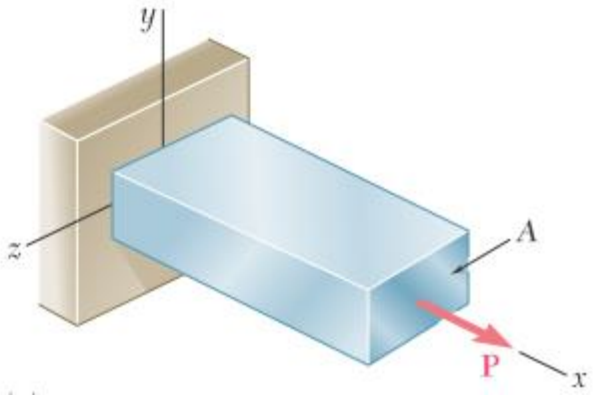
The reaction at A is equal and opposite.

Noting that the forces in the two portions of the bar are $P_1 = P_2 = 93\ kN$. Normal stress in portions AC and CB are

$$\sigma_1 = \frac{P_1}{A_1} = \frac{93x10^3\ N}{400x10^{-6}\ m^2} =240\ \text{MPa}$$

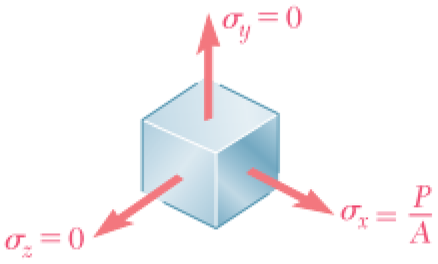
$$\sigma_2 = \frac{P_2}{A_2} = \frac{93x10^3\ N}{800x10^{-6}\ m^2} =120\ \text{MPa}$$

Poisson's Ratio

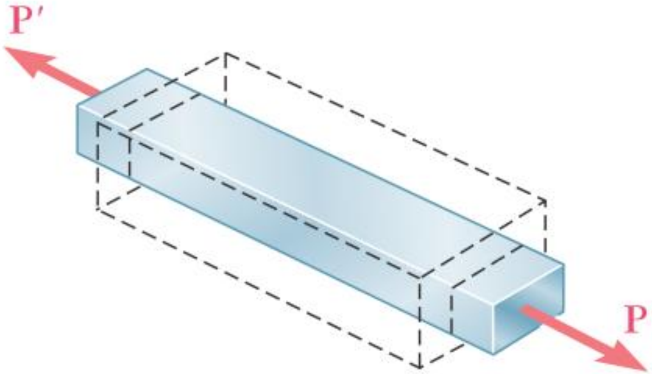


$$\sigma_x = \frac{P}{A} \text{ and } \varepsilon_x = \frac{\sigma_x}{E}$$

- The normal stresses on faces perpendicular to the y and z axes are zero: $\sigma_y = \sigma_z = 0$.



- It would be tempting to conclude that the corresponding strains ε_y and ε_z are also zero. This is not the case.



In all engineering materials, the elongation produced by an axial tensile force P in the direction of the force is accompanied by a contraction in any transverse direction.

- In this section and the following sections, all materials are assumed to be both homogeneous and isotropic (i.e., their mechanical properties are independent of both position and direction). It follows that the strain must have the same value for any transverse direction. Therefore,

$$\varepsilon_y = \varepsilon_z$$

- This common value is the lateral strain. An important constant for a given material is its **Poisson's ratio**, named after the French mathematician Siméon Denis Poisson (1781–1840) and denoted by the Greek letter ν (nu).

$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}} \qquad \nu = -\frac{\epsilon_y}{\epsilon_x} = -\frac{\epsilon_z}{\epsilon_x}$$

$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}}$$

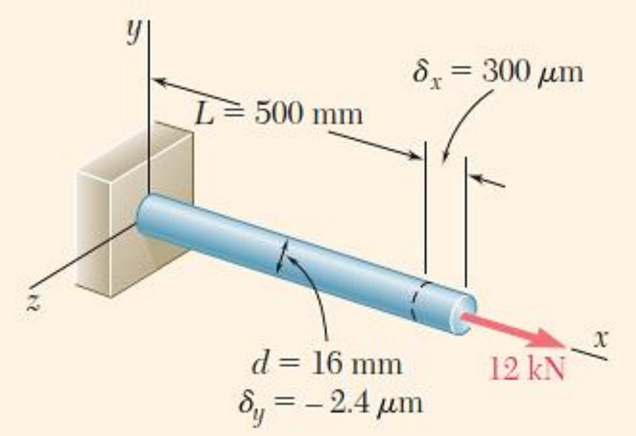
$$\nu = -\frac{\epsilon_y}{\epsilon_x} = -\frac{\epsilon_z}{\epsilon_x}$$

- Note the use of a minus sign in these equations to obtain a positive value for ν, as the axial and lateral strains have opposite signs for all engineering materials.

$$\epsilon_x = \frac{\sigma_x}{E} \qquad \epsilon_y = \epsilon_z = -\frac{\nu\sigma_x}{E}$$

* It also would be tempting, but equally wrong, to assume that the volume of the rod remains unchanged as a result of the combined effect of the axial elongation and transverse contraction (see Sec. 2.6).
** However, some experimental materials, such as polymer foams, expand laterally when stretched. Since the axial and lateral strains have then the same sign, Poisson's ratio of these materials is negative. (See Roderic Lakes, "Foam Structures with a Negative Poisson's Ratio," Science, 27 February 1987, Volume 235, pp. 1038–1040.)

Concept Application 2.7



- A 500-*mm*-long, 16-*mm*-diameter rod made of a homogenous, isotropic material is observed to increase in length by 300 μm, and to decrease in diameter by 2.4 μm when subjected to an axial 12-kN load. Determine the modulus of elasticity and Poisson's ratio of the material.

The cross-sectional area of the rod is $$A = \pi r^2 = \pi(8 \times 10^{-3}\,\text{m})^2 = 201 \times 10^{-6}\,\text{m}^2$$

Choosing the x axis along the axis of the rod (Fig. 2.31), write

$$\sigma_x = \frac{P}{A} = \frac{12 \times 10^3\,\text{N}}{201 \times 10^{-6}\,\text{m}^2} = 59.7\,\text{MPa}$$

$$\epsilon_x = \frac{\delta_x}{L} = \frac{300\,\mu\text{m}}{500\,\text{mm}} = 600 \times 10^{-6}$$

$$\epsilon_y = \frac{\delta_y}{d} = \frac{-2.4\,\mu\text{m}}{16\,\text{mm}} = -150 \times 10^{-6}$$

From Poisson's ratio equation:

$$\nu = -\frac{\epsilon_y}{\epsilon_x} = -\frac{-150 \times 10^{-6}}{600 \times 10^{-6}} = 0.25$$

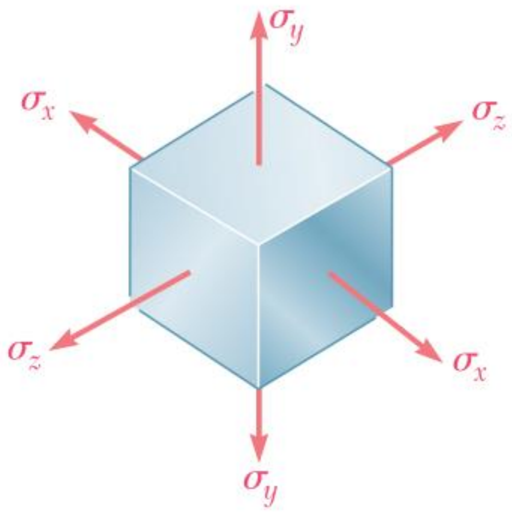
From Hooke's law, modulus of elasticty (*E*) is:

$$\sigma_x = E\epsilon_x$$

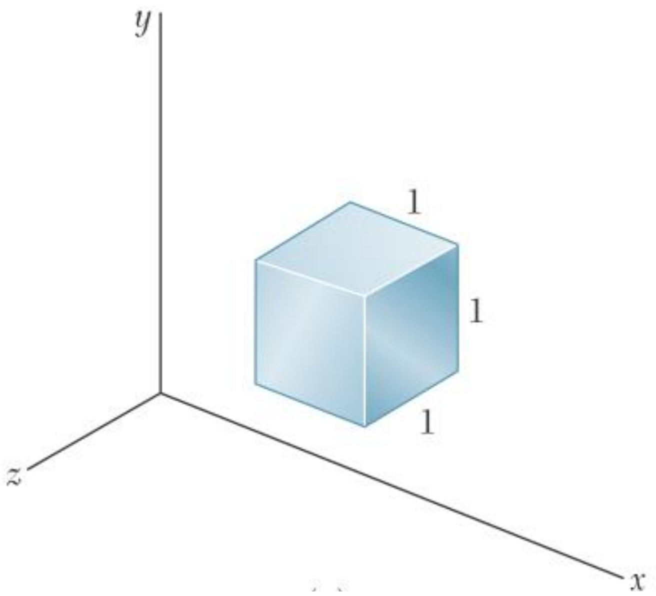
$$E = \frac{\sigma_x}{\epsilon_x} = \frac{59.7\,\text{MPa}}{600 \times 10^{-6}} = 99.5\,\text{GPa}$$

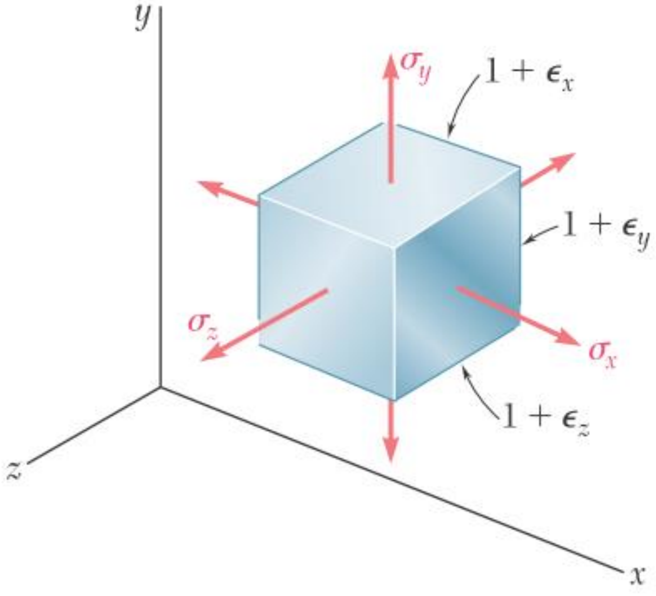
Multiaxial Loading: Generalized Hooke's Law

- All the examples considered so far in this chapter have dealt with slender members subjected to axial loads, i.e., to forces directed along a single axis.
- Consider now structural elements subjected to loads acting in the directions of the three coordinate axes and producing normal stresses σ_x, σ_y, and σ_z that are all different from zero. This condition is a ***multiaxial loading***.



Note that this is not the general stress condition, since no shearing stresses are included among the stresses.





- Consider an element of an isotropic material in the shape of a cube. Assume the side of the cube to be equal to unity, since it is always possible to select the side of the cube as a unit of length.

- Under the given multiaxial loading, the element will deform into a rectangular parallelepiped of sides equal to $1 + \varepsilon_x$, $1 + \varepsilon_y$, and $1 + \varepsilon_z$, where ε_x, ε_y, and ε_z denote the values of the normal strain in the directions of the three coordinate axes.

- In order to express the strain components ε_x, ε_y, ε_z in terms of the stress components σ_x, σ_y, σ_z, consider the effect of each stress component and combine the results. This approach will be used repeatedly in this text, and is based on the **principle of superposition**.
- This principle states that ***the effect of a given combined loading on a structure can be obtained by determining the effects of the various loads separately and combining the results***, provided that the following conditions are satisfied:
  - Each effect is linearly related to the load that produces it.
  - The deformation resulting from any given load is small and does not affect the conditions of application of the other loads.
- For multiaxial loading, the first condition is satisfied if the stresses do not exceed the proportional limit of the material, and the second condition is also satisfied if the stress on any given face does not cause deformations of the other faces that are large enough to affect the computation of the stresses on those faces.

- Considering the effect of the stress component σ_x,

$$\varepsilon_x = \frac{\sigma_x}{E} \text{ and } \varepsilon_y = -\nu\frac{\sigma_x}{E} \text{ and } \varepsilon_z = -\nu\frac{\sigma_x}{E}$$

- Similarly, the stress component σ_y,

$$\varepsilon_y = \frac{\sigma_y}{E} \text{ and } \varepsilon_x = -\nu\frac{\sigma_y}{E} \text{ and } \varepsilon_z = -\nu\frac{\sigma_y}{E}$$

- and the stress component σ_z,

$$\varepsilon_z = \frac{\sigma_z}{E} \text{ and } \varepsilon_x = -\nu\frac{\sigma_z}{E} \text{ and } \varepsilon_y = -\nu\frac{\sigma_z}{E}$$

Combining the results, the components of strain corresponding to the given multiaxial loading are

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_z}{E}$$

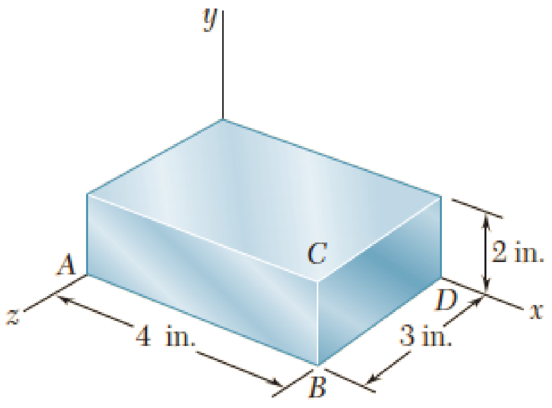
$$\varepsilon_z = \frac{\sigma_z}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$

Generalized Hooke's law for the multiaxial loading of a homogeneous isotropic material

- a positive value for a stress component signifies tension and a negative value compression

- a positive value for a strain component indicates expansion in the corresponding direction and a negative value contraction

Concept Application 2.8



The steel block shown (Fig. 2.34) is subjected to a uniform pressure on all its faces. Knowing that the change in length of edge AB is $-1.2 * 10^{-3}$ in., determine (a) the change in length of the other two edges and (b) the pressure *p* applied to the faces of the block. Assume E = $29*10^6$ psi and v=0.29.

a. Change in Length of Other Edges. Substituting $\sigma_x = \sigma_y = \sigma_z = -p$ into strain equantions in multi-axial loading case, the three strain components have the common value

$$\epsilon_x = \epsilon_y = \epsilon_z = -\frac{p}{E}(1 - 2\nu) \quad (1)$$

$$\epsilon_x = \delta_x/AB = (-1.2 \times 10^{-3}\text{ in.})/(4\text{ in.}) = -300 \times 10^{-6}\text{ in./in.}$$

$$\epsilon_y = \epsilon_z = \epsilon_x = -300 \times 10^{-6}\text{ in./in.}$$

$$\delta_y = \epsilon_y(BC) = (-300 \times 10^{-6})(2\text{ in.}) = -600 \times 10^{-6}\text{ in.}$$

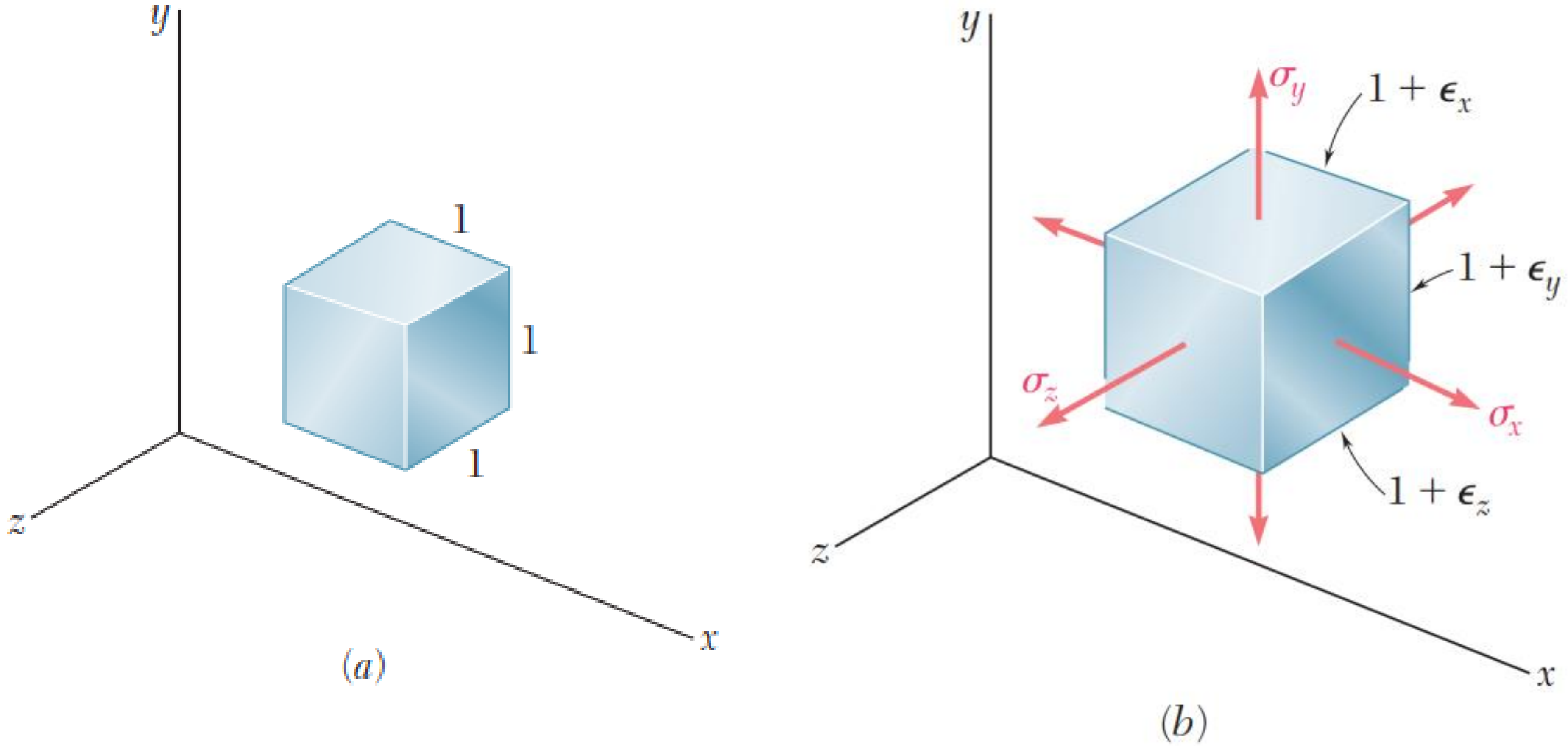
$$\delta_z = \epsilon_z(BD) = (-300 \times 10^{-6})(3\text{ in.}) = -900 \times 10^{-6}\text{ in.}$$

b. Pressure. Solving Eq. (1) for p,

$$p = -\frac{E\epsilon_x}{1 - 2\nu} = -\frac{(29 \times 10^6\text{ psi})(-300 \times 10^{-6})}{1 - 0.58}$$

$$p = 20.7\text{ ksi}$$

DILATATION AND BULK MODULUS



undeformed

deformed

Volume of deformed unity cube

$$v = (1 + \epsilon_x)(1 + \epsilon_y)(1 + \epsilon_z)$$

Since the strains are much smaller than unity, their products can be omitted

$$v = 1 + \epsilon_x + \epsilon_y + \epsilon_z$$

The change in volume *e* of the element is

$$e = v - 1 = 1 + \epsilon_x + \epsilon_y + \epsilon_z - 1$$

or

$$e = \epsilon_x + \epsilon_y + \epsilon_z$$

e represents *the change in volume per unit volume* and is called the ***dilatation*** of the material.

Substitute strains to the *e* equation

$$e = \frac{\sigma_x + \sigma_y + \sigma_z}{E} - \frac{2\nu(\sigma_x + \sigma_y + \sigma_z)}{E}$$

$$\boxed{e = \frac{1 - 2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)}$$

When a body is subjected to a uniform hydrostatic pressure *p*, each of the stress components is equal to *-p*

$$e = -\frac{3(1 - 2\nu)}{E}p \quad \text{or} \quad \boxed{e = -\frac{p}{k}}$$

The constant *k* is known as the *bulk modulus* or *modulus of compression* of the material

$$\boxed{k = \frac{E}{3(1 - 2\nu)}}$$

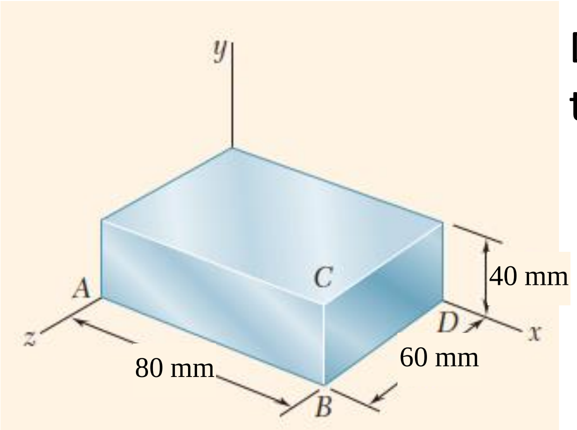
Because a stable material subjected to a hydrostatic pressure can only *decrease* in volume, the dilatation *e* is negative, and the bulk modulus *k* is a positive quantity. Referring from bulk modulus that

$$1 - 2\nu > 0 \qquad \text{or} \qquad \nu < \frac{1}{2}$$

Poisson's ratio (v) is positive for all engineering materials. Thus, for any engineering material,

$$0 < \nu < \tfrac{1}{2}$$

Concept Application 2.9



Determine the change in volume DV of the steel block shown in Figure, when it is subjected to the hydrostatic pressure p=180 MPa. Use E = 200 GPa and v=0.29.

From $k = \dfrac{E}{3(1 - 2\nu)}$ the bulk modulus of steel is

$$k = \frac{E}{3(1 - 2\nu)} = \frac{200 \text{ GPa}}{3(1 - 0.58)} = 158.7 \text{ GPa}$$

And the dilatation is $$e = -\frac{p}{k} = -\frac{180 \text{ MPa}}{158.7 \text{ GPa}} = -1.134 \times 10^{-3}$$

Since the volume V of the block in its unstressed state is $$V = (80 \text{ mm})(40 \text{ mm})(60 \text{ mm}) = 192 \times 10^3 \text{ mm}^3$$

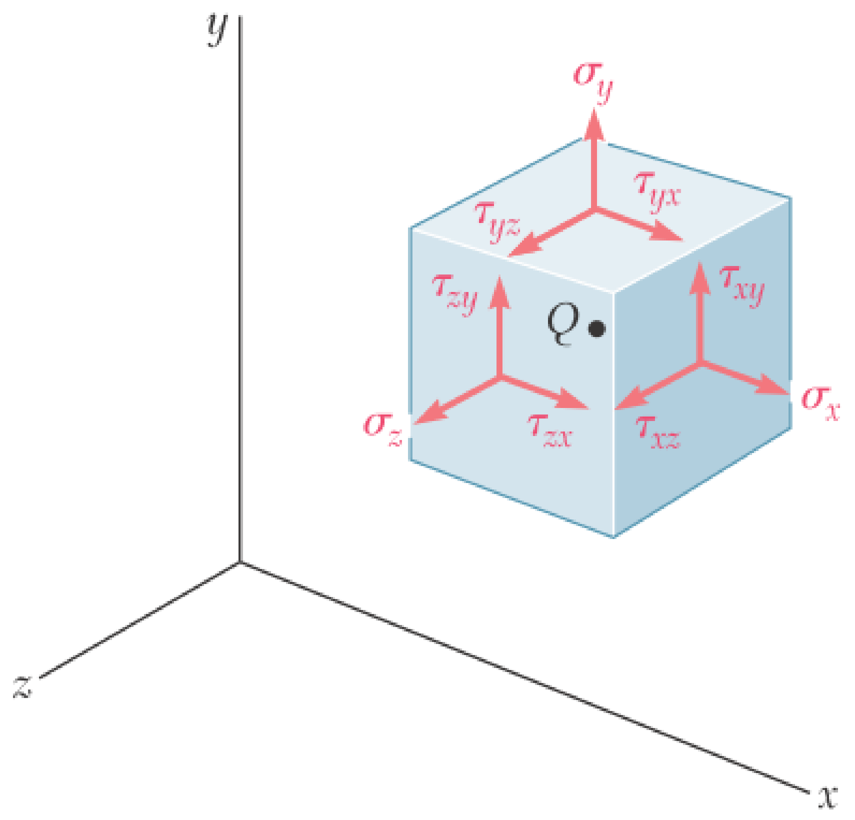
and e represents the change in volume per unit volume,

$$\Delta V = eV = (-1.134 \times 10^{-3})(192 \times 10^3 \text{ mm}^3) \qquad \Delta V = -218 \text{ mm}^3$$

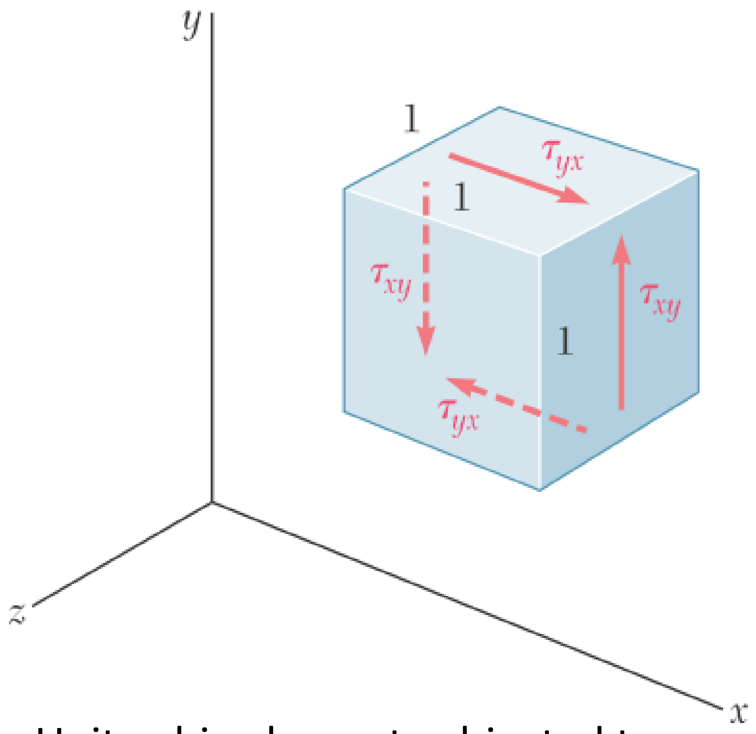
SHEARING STRAIN

- In a general stress situation, shown in figure, shearing stresses τ_{xy}, τ_{yz}, and τ_{zx} are present (as well as the corresponding shearing stresses τ_{yx}, τ_{zy}, and τ_{xz}).

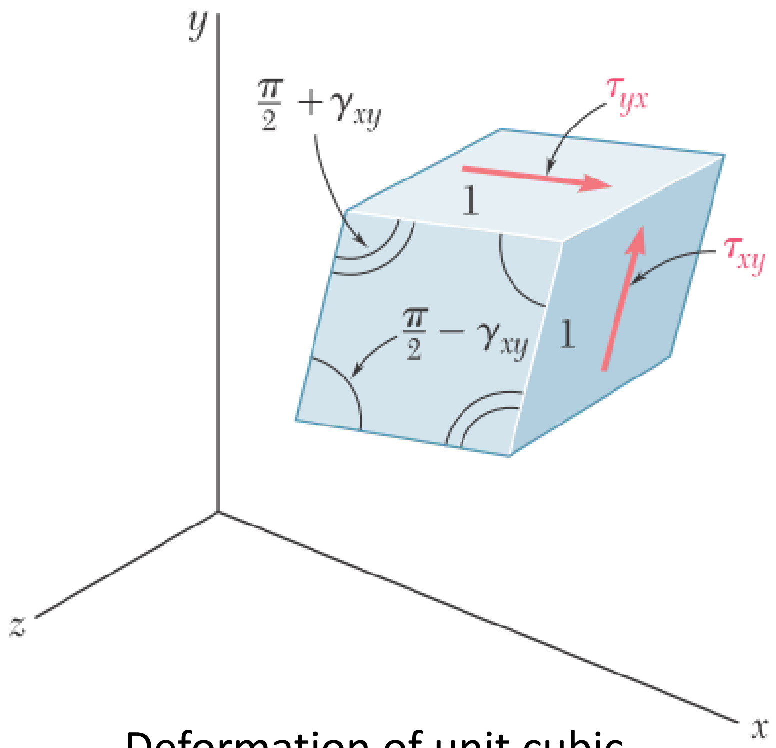
- These stresses have no direct effect on the normal strains and, as long as all the deformations involved remain small, they will not affect the derivation nor the validity of Generalized Hooke's law for the multiaxial loading.



- The shearing stresses, however, tend to deform a cubic element of material into an oblique parallelepiped.
- Consider a cubic element subjected to only the shearing stresses τ_{xy} and τ_{yx} applied to faces of the element respectively perpendicular to the x and y axes. (Recall that $\tau_{xy} = \tau_{yx}$) The cube is observed to deform into a rhomboid of sides equal to one



Unit cubic element subjected to shearing stress.

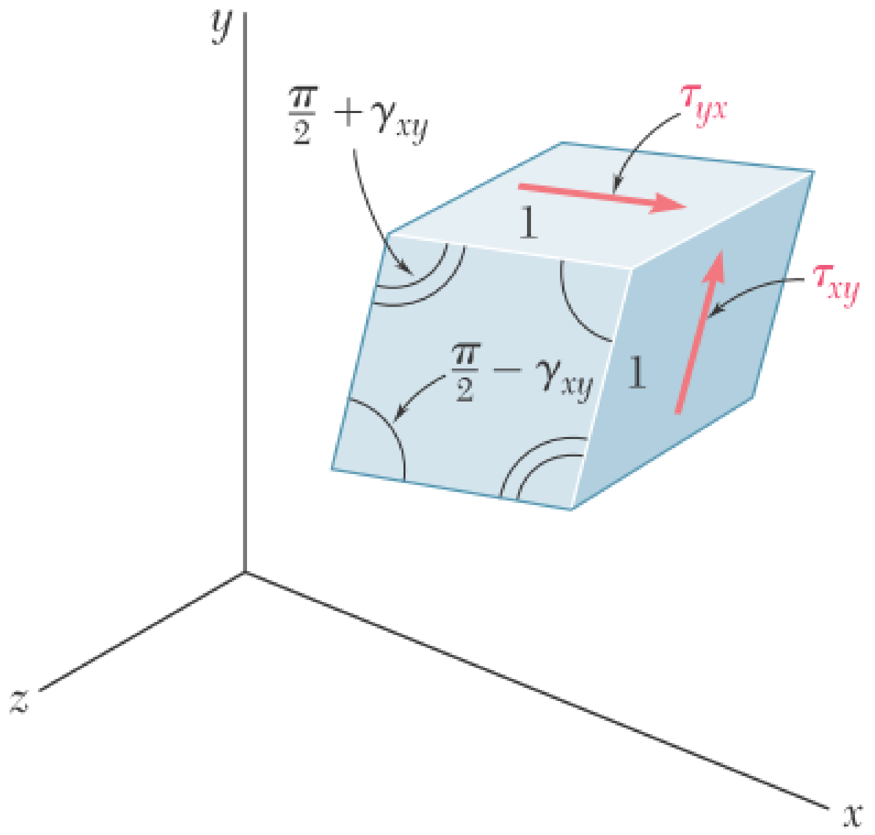


Deformation of unit cubic element due to shearing stress.

Two of the angles formed by the four faces under stress are reduced from $(\pi/2)$ to $(\pi/2 - \gamma_{xy})$, while the other two are increased from $(\pi/2)$ to $(\pi/2 + \gamma_{xy})$.

The small angle γ_{xy} (expressed in radians) defines the **shearing strain** corresponding to the x and y directions.

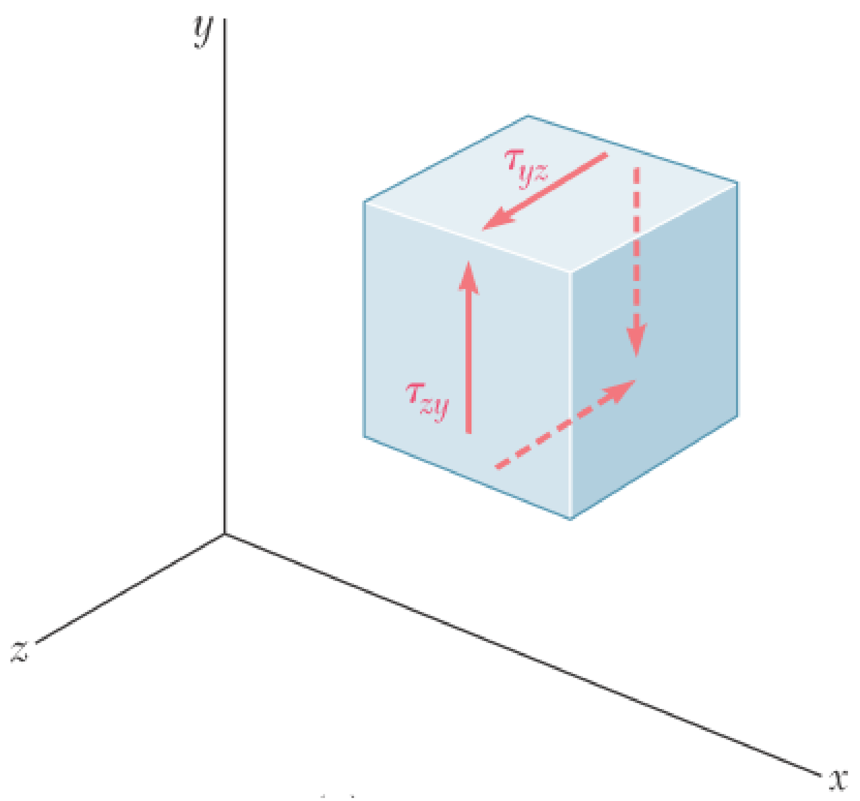
When the deformation involves a reduction of the angle formed by the two faces oriented toward the positive x and y axes, the shearing strain γ_{xy} is positive; otherwise, it is negative.

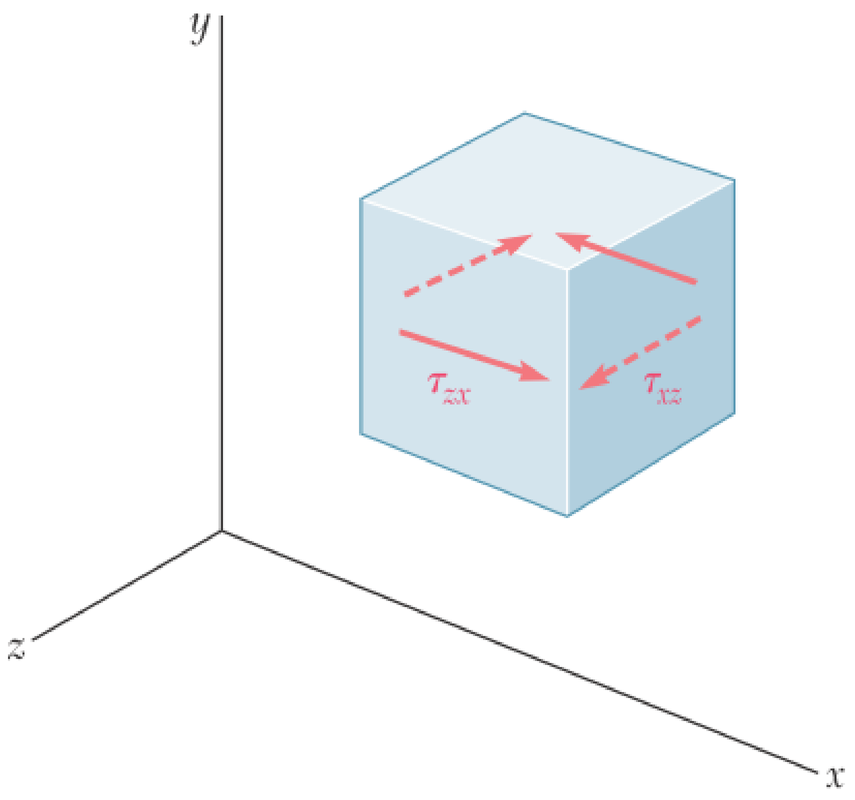


- Plotting successive values of τ_{xy} against the corresponding values of γ_{xy}, the shearing stress-strain diagram is obtained for the material. This diagram is similar to the normal stress-strain diagram from the tensile test described earlier; however, the values for the yield strength, ultimate strength, etc., are about half as large in shear as they are in tension. As for normal stresses and strains, the initial portion of the shearing stress-strain diagram is a straight line. For values of the shearing stress that do not exceed the proportional limit in shear, it can be written for any homogeneous isotropic material that

$$\tau_{xy} = G\gamma_{xy}$$

- This relationship is Hooke's law for shearing stress and strain, and the constant G is called the modulus of rigidity or shear modulus of the material.





For values of the stress that do not exceed the proportional limit, you can write two additional relationships:

$$\tau_{yz} = G\gamma_{yz} \quad \text{and} \quad \tau_{xz} = G\gamma_{xz}$$

- For the general stress condition, as long as none of the stresses involved exceeds the corresponding proportional limit, you can apply the principle of superposition and combine the results.
- The generalized Hooke's law for a homogeneous isotropic material under the most general stress condition is

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$

$$\tau_{xy} = G\gamma_{xy} \qquad \tau_{xy} = G\gamma_{xy} \qquad \tau_{xy} = G\gamma_{xy}$$

Deformations Under Axial Loading — Relation Between E, ν, And G

- For an isotropic homogenous material, two of the material constant must be determined by conducting mechanic tests, and the third one can be calculated by using the relationship :

$$G = \frac{E}{2(1+\nu)}$$