Pure Bending

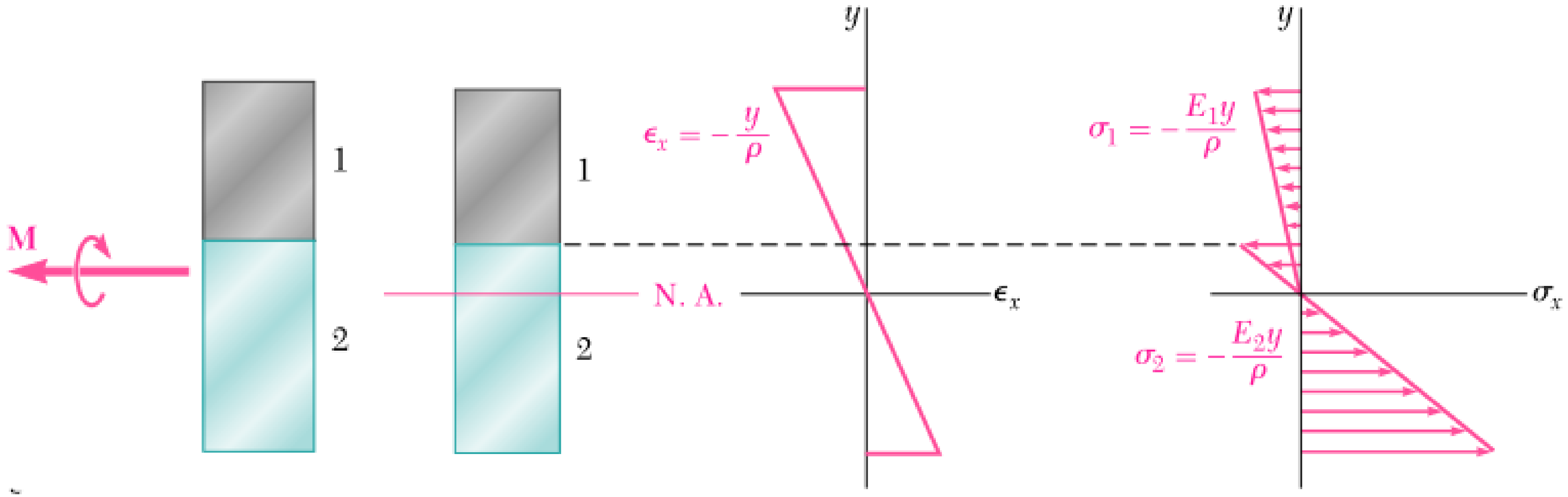
Part II

Objectives

- Understand the bending behavior
- Define the deformations, strains, and normal stresses in beams subject to pure bending
- Describe the behavior of composite beams made of more than one material
- Analyze members subject to eccentric axial loading, involving both axial stresses and bending stresses
- Review beams subject to unsymmetric bending, i.e., where bending does not occur in a plane of symmetry

Members Made Of Composite Materials

- Consider a bar consisting of two portions of different materials bonded together as shown in figure.



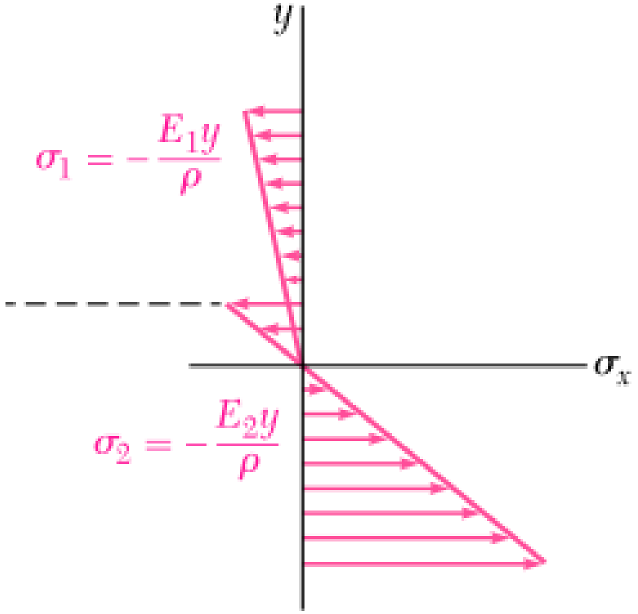
- It cannot be assumed that the neutral axis passes through the centroid of the composite section, and one of the goals of this analysis is to determine the location of this axis.

- Since the moduli of elasticity E_1 and E_2 of the two materials are different, the equations for the normal stress in each material are

$$\sigma_1 = E_1 \varepsilon_x = E_1 \frac{-y}{\rho}$$

$$\sigma_2 = E_2 \varepsilon_x = E_2 \frac{-y}{\rho}$$

- A stress-distribution curve is obtained that consists of two segments with straight lines as shown in figure.



- Following the stress equations above, the force dF_1 exerted on an element of area dA of the upper portion of the cross section is

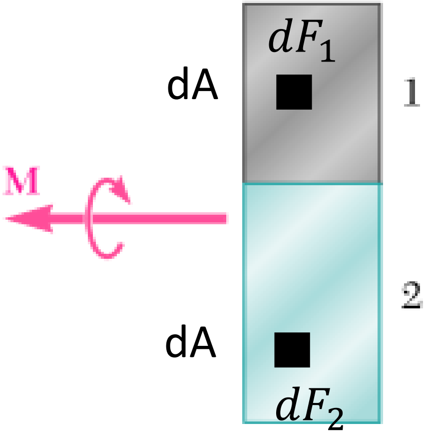
$$dF_1 = \sigma_1 dA = -\frac{E_1 y}{\rho} dA$$

- while the force dF_2 exerted on an element of the same area dA of the lower portion is

$$dF_2 = \sigma_2 dA = -\frac{E_2 y}{\rho} dA$$

- Denoting the ratio E_2/E_1 of the two moduli of elasticity by n,

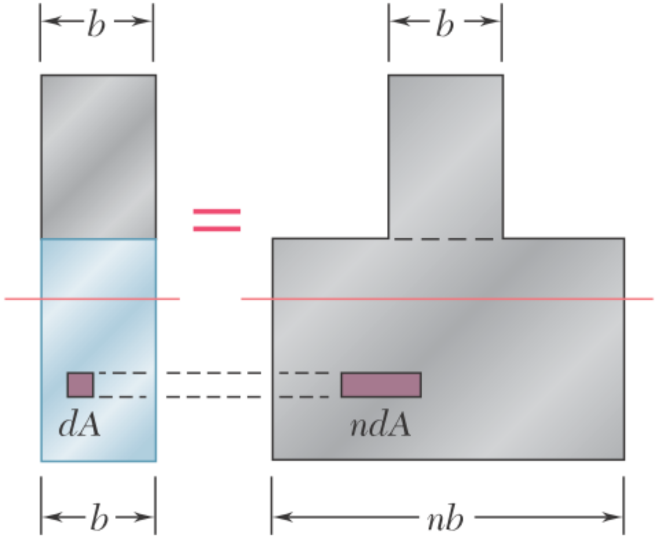
$$dF_2 = -\frac{(nE_1)y}{\rho} dA = -\frac{E_1 y}{\rho}(ndA)$$



- Comparing the equations

$$dF_1 = \sigma_1 dA = -\frac{E_1 y}{\rho} dA$$

$$dF_2 = -\frac{(nE_1)y}{\rho} dA = -\frac{E_1 y}{\rho}(ndA)$$



- we note that the same force dF_2 would be exerted on an element of area ndA of the first material. Thus, the resistance to bending of the bar would remain the same if both portions were made of the first material, provided that the width of each element of the lower portion were multiplied by the factor n.

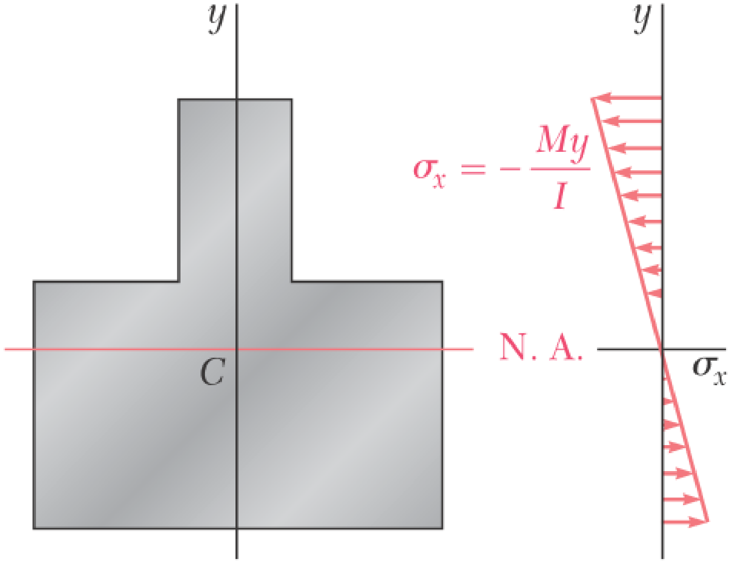
- Note that this widening (if $n > 1$) or narrowing (if $n < 1$) must be in a direction parallel to the neutral axis of the section, since it is essential that the distance y of each element from the neutral axis remain the same. This new cross section is called the transformed section of the member.

- Since the transformed section represents the cross section of a member made of a homogeneous material with a modulus of elasticity E_1, the method described in previous course can be used to determine the neutral axis of the section and the normal stress at various points. The neutral axis is drawn through the centroid of the transformed section, and the stress $\sigma_x$at any point of the corresponding homogeneous member obtained from

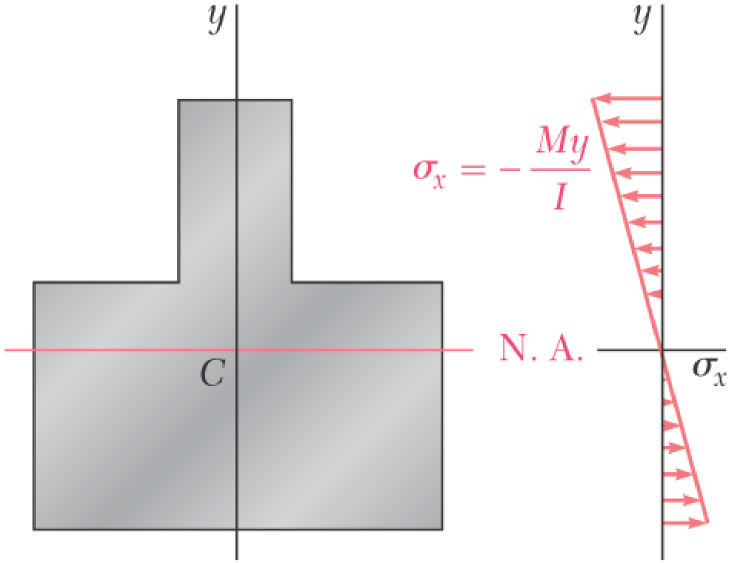
$$\sigma_x = -\frac{My}{I}$$

- where y is the distance from the neutral surface and I is the moment of

inertia of the transformed section with respect to its centroidal axis.



- To obtain the stress σ_1 at a point located in the upper portion of the cross section of the original composite bar, compute the stress σ_x at the corresponding point of the transformed section. However, to obtain the stress σ_2 at a point in the lower portion of the cross section, we must multiply by n the stress σ_x computed at the corresponding point of the transformed section. Indeed, the same elementary force dF_2 is applied to an element of area ndA of the transformed section and to an element of area dA of the original section. Thus, the stress σ_2 at a point of the original section must be n times larger than the stress at the corresponding point of the transformed section.

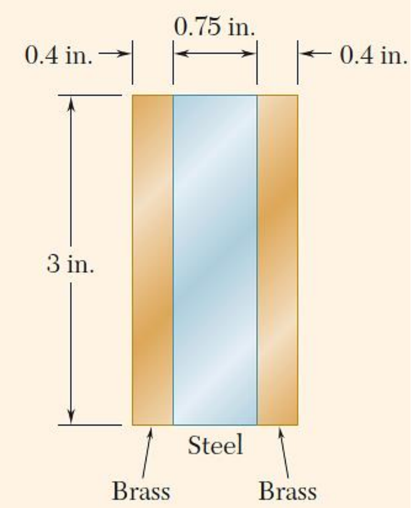


- The deformations of a composite member can also be determined by using the transformed section. We recall that the transformed section represents the cross section of a member, made of a homogeneous material of modulus E_1, which deforms in the same manner as the composite member. Therefore, using radius of curvature equation, we write that the curvature of the composite member is

$$\frac{1}{\rho} = \frac{M}{E_1 I}$$

- where I is the moment of inertia of the transformed section with respect to its neutral axis.

Concept Application 4.3



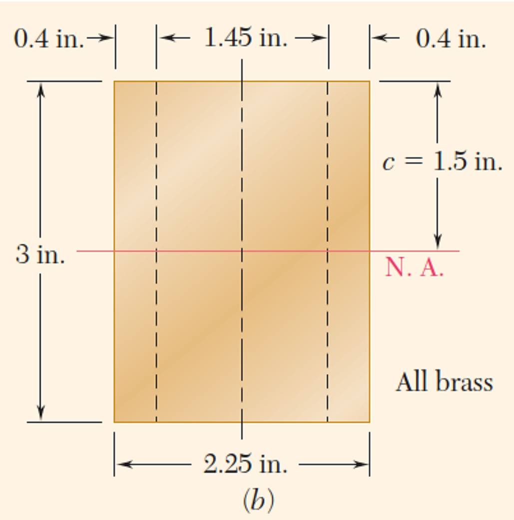
A bar obtained by bonding together pieces of steel ($E_s = 29x10^6\ psi$) and brass ($E_s = 15x10^6\ psi$) has the cross section shown. Determine the maximum stress in the steel and in the brass when the bar is in pure bending with a bending moment M = 40 kip.in.

- The transformed section corresponding to an equivalent bar made entirely of brass is shown in Fig. Since;

$$n = \frac{E_s}{E_b} = \frac{29x10^6}{15x10^6} = 1.933$$

the width of the central portion of brass, which replaces the original steel portion, is obtained by multiplying the original width by 1.933:

$$(0.75)(1.933) = 1.45\ in.$$

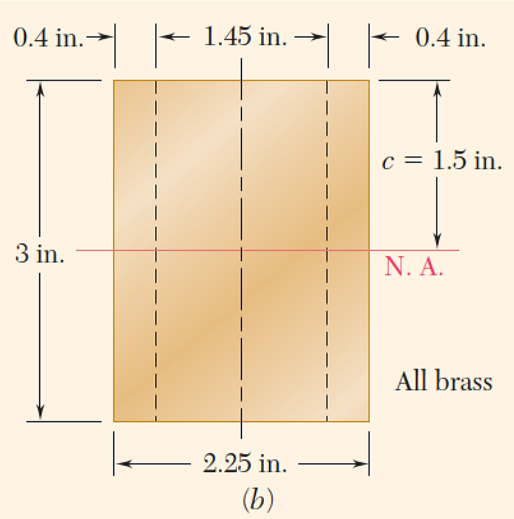


Transformed section

the moment of inertia of the transformed section about neutral axis:

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(2.25)(3)^3 = 5.063\ in^4$$

Concept Application 4.3



Transformed section

- Maximum distance from the neutral axis is c=1.5 in. Maximum normal stress in the transformed section is

$$\sigma_m = \frac{Mc}{I} = \frac{(40\ kip.in)(1.5\ in.)}{5.063\ in^4} = 11.85\ ksi$$

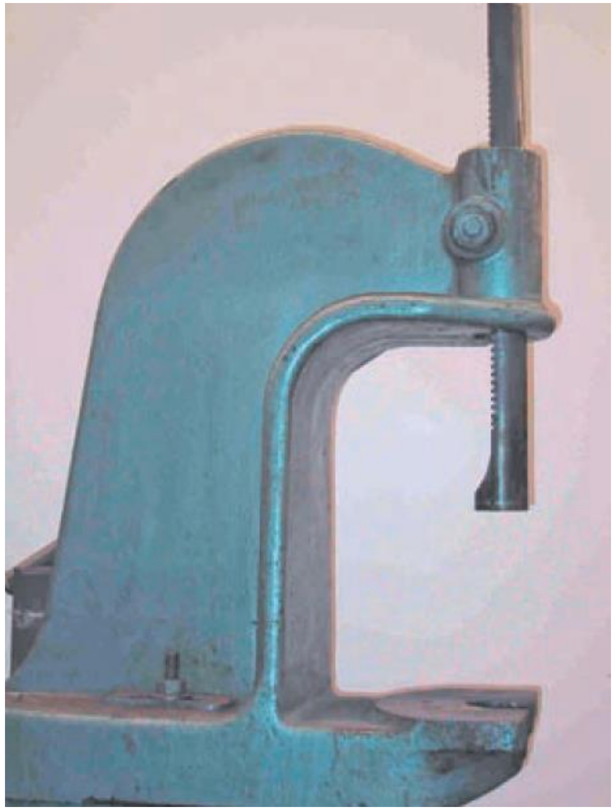
This value also represents the maximum stress in the brass portion of the original composite bar. The maximum stress in the steel portion, however, will be larger than for the transformed section, since the area of the central portion must be reduced by the factor n = 1.933. Thus,

$$(\sigma_{brass})_{max} = 11.85\ ksi$$

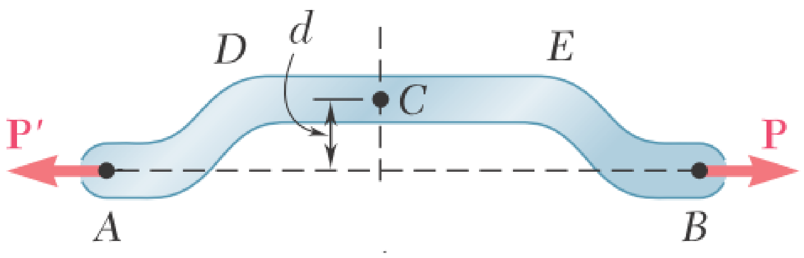
$$(\sigma_{steel})_{max} = (1.933)(11.85\ ksi) = 22.9\ ksi$$

Eccentric Axial Loading in a Plane Of Symmetry

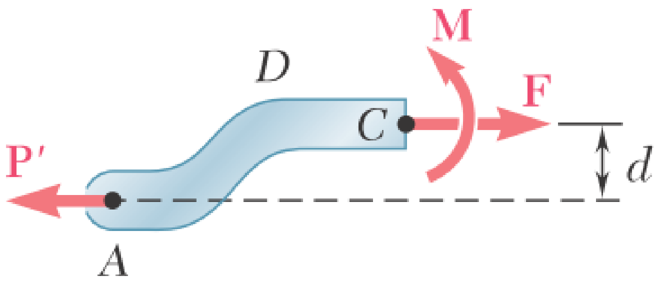
- We saw in Sec. 1.2A that the distribution of stresses in the cross section of a member under axial loading can be assumed uniform only if the line of action of the loads ***P*** and ***P’*** passes through the centroid of the cross section. Such a loading is said to be *centric*.
- Let us now analyze the distribution of stresses when the line of action of the loads does not pass through the centroid of the cross section, i.e., when the loading is *eccentric*.

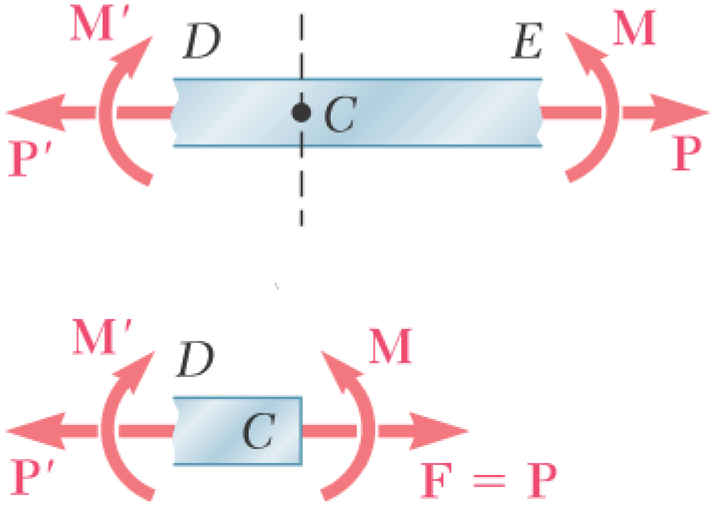


- In this section, our analysis will be limited to members that possess a plane of symmetry, and it will be assumed that the loads are applied in the plane of symmetry of the member.



- The internal forces acting on a given cross section may then be represented by a force ***F*** applied at the centroid C of the section and a couple ***M*** acting in the plane of symmetry of the member.



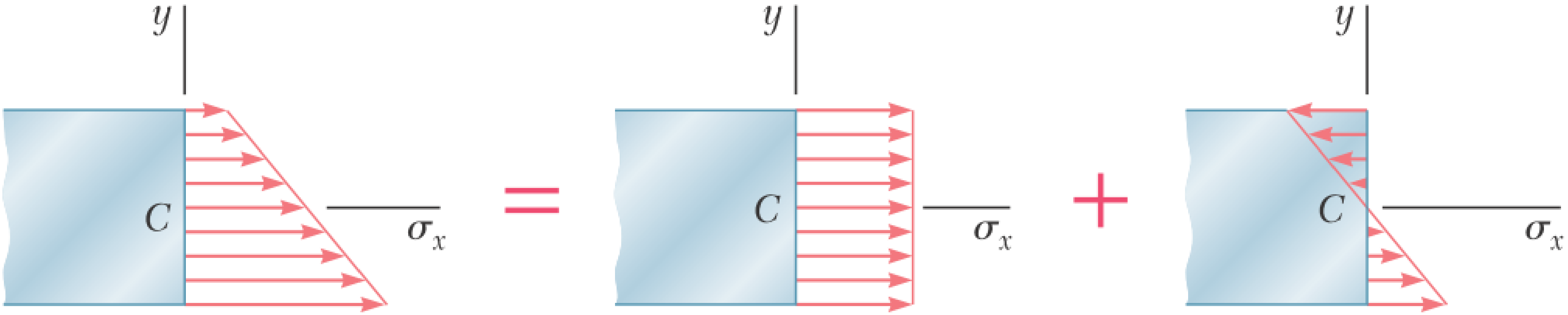


- We now observe that the internal forces in the section would have been represented by the same force and couple if the straight portion *DE* of member *AB* had been detached from *AB* and subjected simultaneously to the centric loads $\boldsymbol{P}$ and $\boldsymbol{P}'$ and to the bending couples $\boldsymbol{M}$ and $\boldsymbol{M}'$.

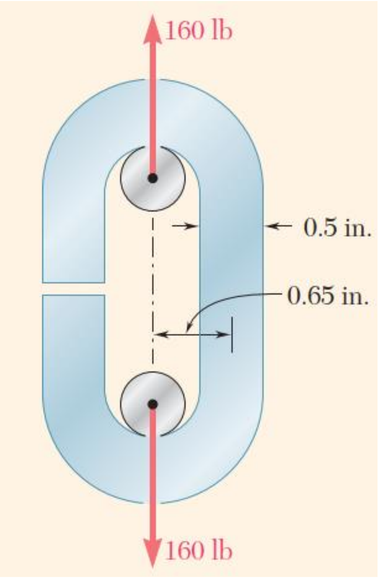
- Thus, the stress distribution due to the original eccentric loading can be obtained by superposing the uniform stress distribution corresponding to the centric loads P and P' and the linear distribution corresponding to the bending couples M and M'. Write

$$\sigma_x = (\sigma_x)_{centric} + (\sigma_x)_{bending}$$

$$\sigma_x = \frac{P}{A} - \frac{My}{I}$$

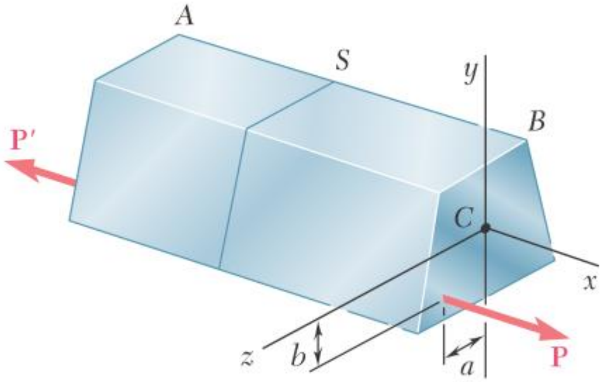


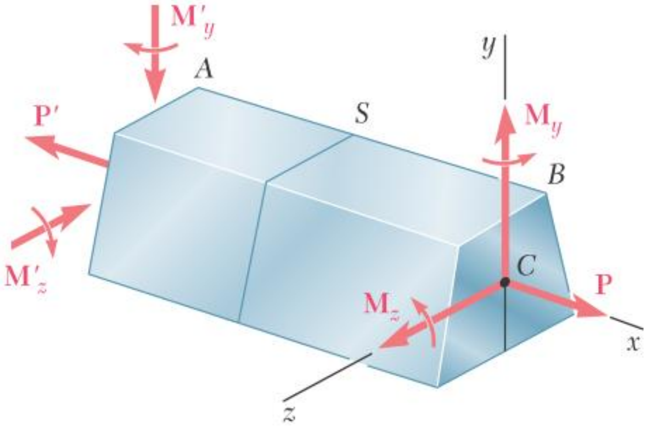
Concept Application 4.7



An open-link chain is obtained by bending low-carbon steel rods of 0.5-in. diameter into the shape shown (Fig. 4.43a). Knowing that the chain carries a load of 160 lb, determine (a) the largest tensile and compressive stresses in the straight portion of a link, (b) the distance between the centroidal and the neutral axis of a cross section.

General Case Of Eccentric Axial Loading Analysis





$$\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} - \frac{M_y z}{I_y}$$