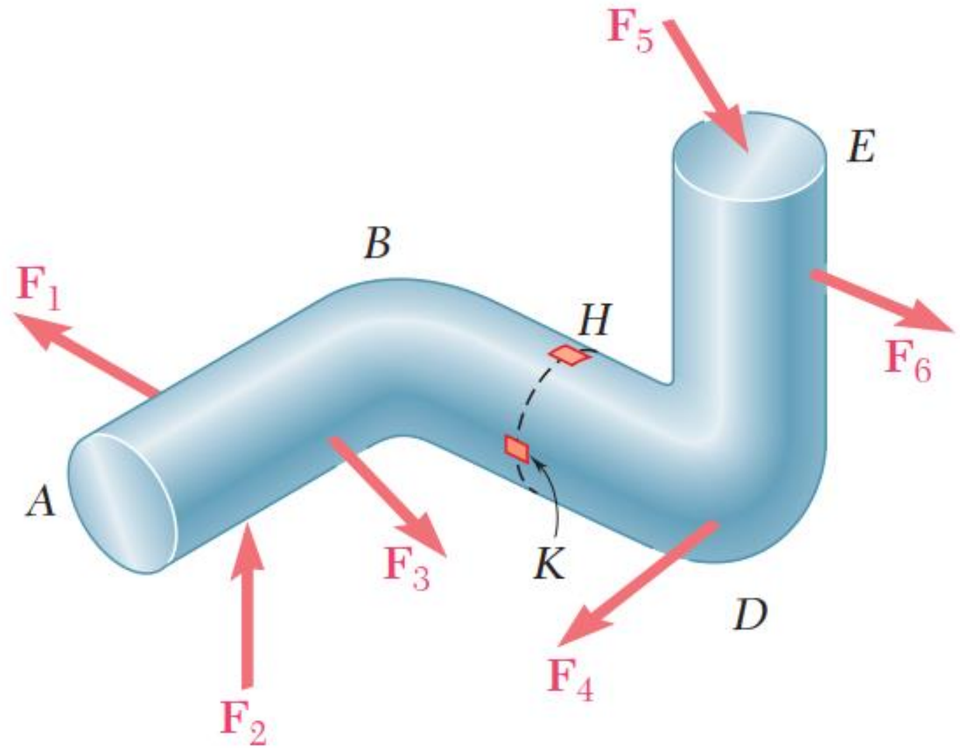
Principal Stresses under a Given Loading

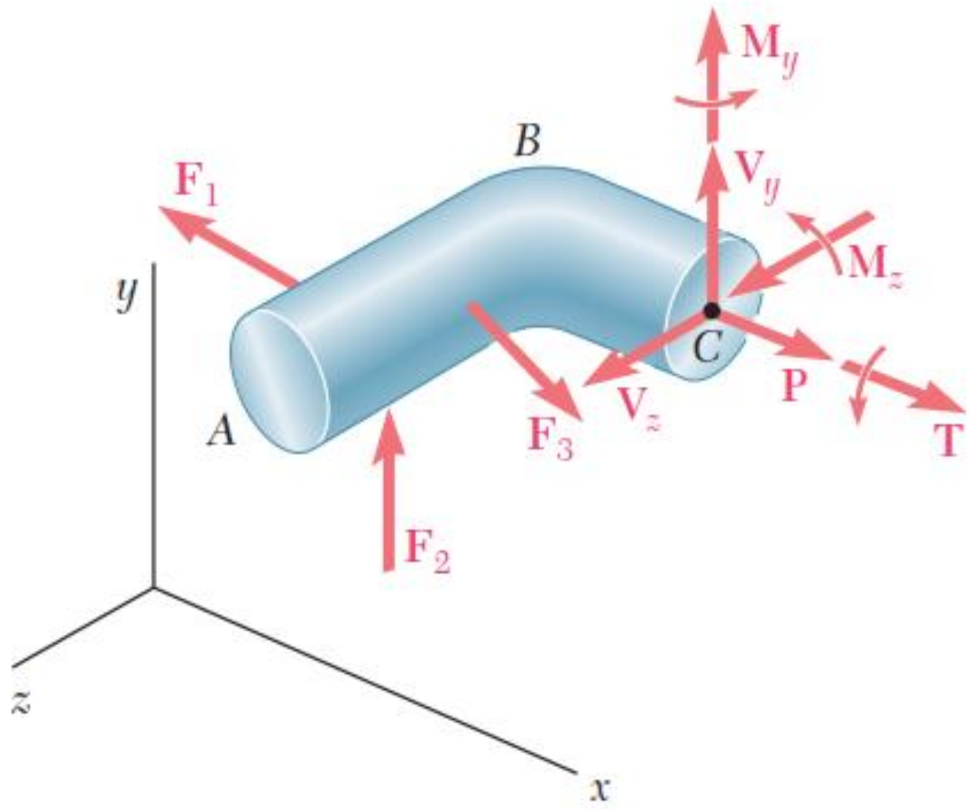
8

Objectives

- ✓ Determine stresses developed in a member's cross section when axial load, torsion, bending and shear occur simultaneously.

Stresses Under Combined Loads





Stresses Under Combined Loads

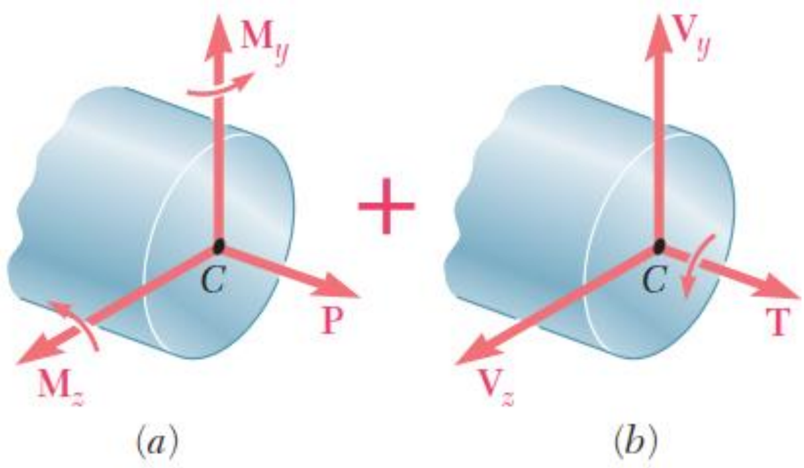


Fig. 8.17 Internal forces and couple vectors separated into (a) those causing normal stresses and (b) those causing shearing stresses.

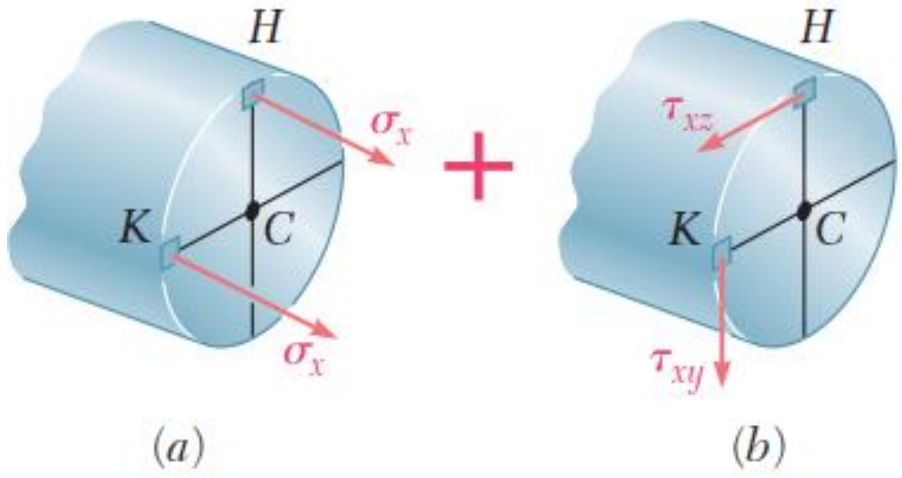


Fig. 8.18 Normal and shearing stresses at points H and K.

Stresses Under Combined Loads

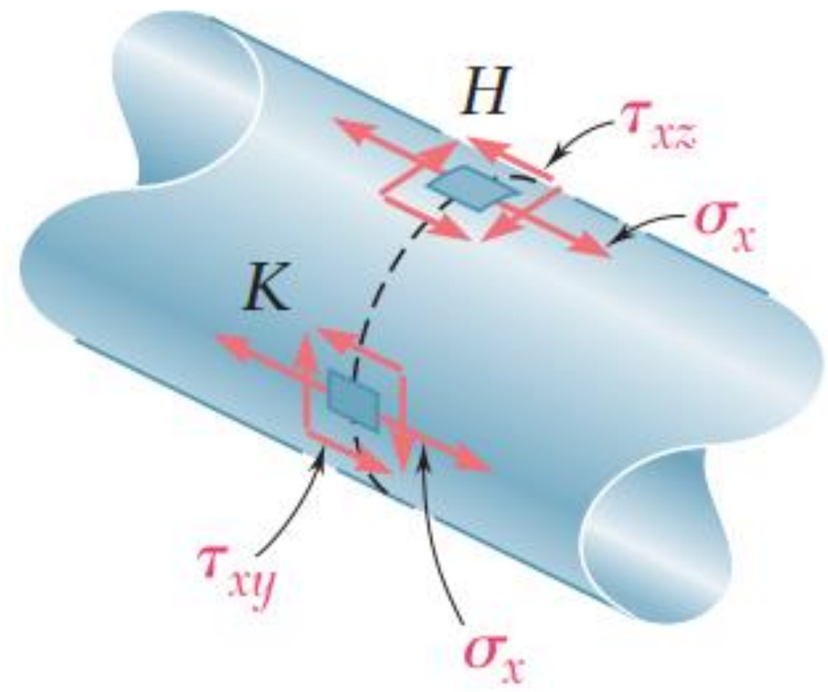
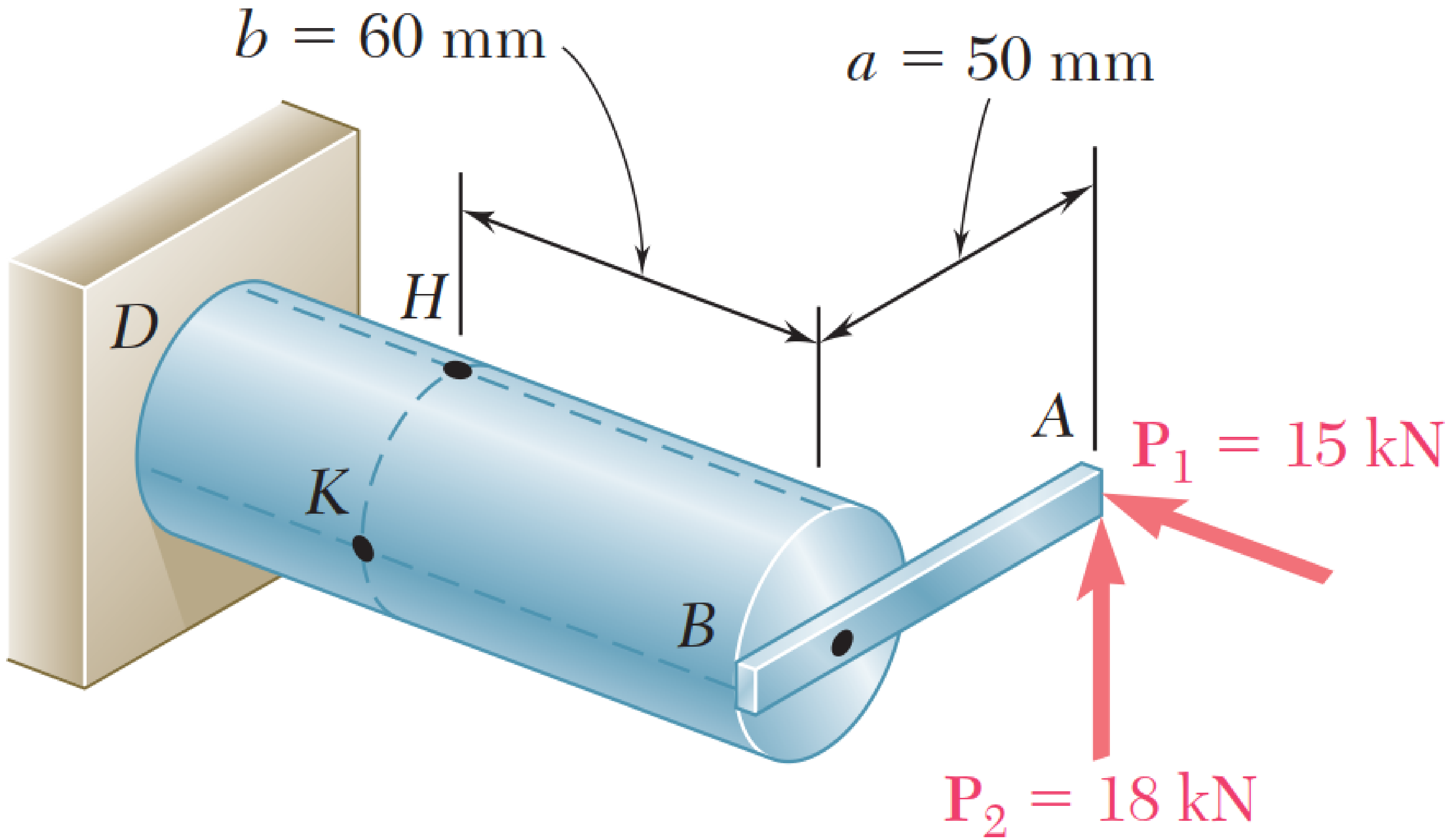


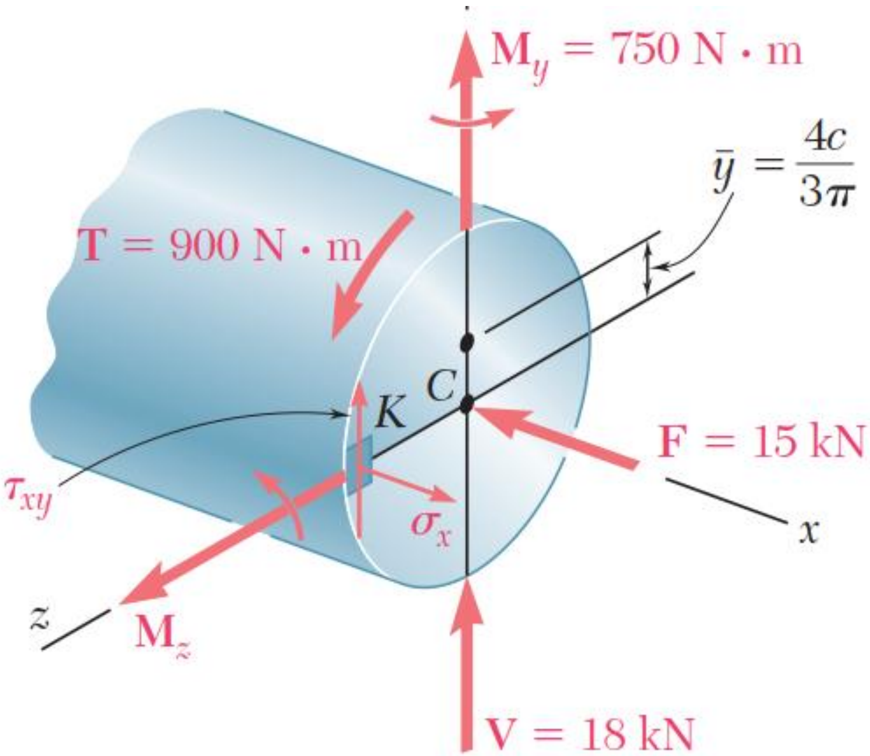
Fig. 8.19 Elements at points H and K showing combined stresses.

Example



Determine (*a*) the normal and shearing stresses at point *K* of the transverse section of member *BD* located at a distance *b* = 60 mm from end *B*, (*b*) the principal axes and principal stresses at *K*, and (*c*) the maximum shearing stress at *K*.

Solution



$$F = P_1 = 15 \text{ kN}$$

$$V = P_2 = 18 \text{ kN}$$

$$T = P_2 a = (18 \text{ kN})(50 \text{ mm}) = 900 \text{ N·m}$$

$$M_y = P_1 a = (15 \text{ kN})(50 \text{ mm}) = 750 \text{ N·m}$$

$$M_z = P_2 b = (18 \text{ kN})(60 \text{ mm}) = 1080 \text{ N·m}$$

Solution

Geometric Properties of the Section

$$A = \pi c^2 = \pi(0.020\text{ m})^2 = 1.257 \times 10^{-3}\text{ m}^2$$

$$I_y = I_z = \tfrac{1}{4}\pi c^4 = \tfrac{1}{4}\pi(0.020\text{ m})^4 = 125.7 \times 10^{-9}\text{ m}^4$$

$$J_C = \tfrac{1}{2}\pi c^4 = \tfrac{1}{2}\pi(0.020\text{ m})^4 = 251.3 \times 10^{-9}\text{ m}^4$$

$$Q = A'\bar{y} = \left(\frac{1}{2}\pi c^2\right)\left(\frac{4c}{3\pi}\right) = \frac{2}{3}c^3 = \frac{2}{3}(0.020\text{ m})^3$$

$$= 5.33 \times 10^{-6}\text{ m}^3$$

$$t = 2c = 2(0.020\text{ m}) = 0.040\text{ m}$$

Solution

Normal Stresses.

$$\sigma_x = -\frac{F}{A} + \frac{M_y c}{I_y} = -11.9\ \text{MPa} + \frac{(750\ \text{N·m})(0.020\ \text{m})}{125.7 \times 10^{-9}\ \text{m}^4}$$

$$= -11.9\ \text{MPa} + 119.3\ \text{MPa}$$

$$\sigma_x = +107.4\ \text{MPa}$$

Shearing Stresses.

$$(\tau_{xy})_V = +\frac{VQ}{I_z t} = +\frac{(18 \times 10^3\ \text{N})(5.33 \times 10^{-6}\ \text{m}^3)}{(125.7 \times 10^{-9}\ \text{m}^4)(0.040\ \text{m})}$$

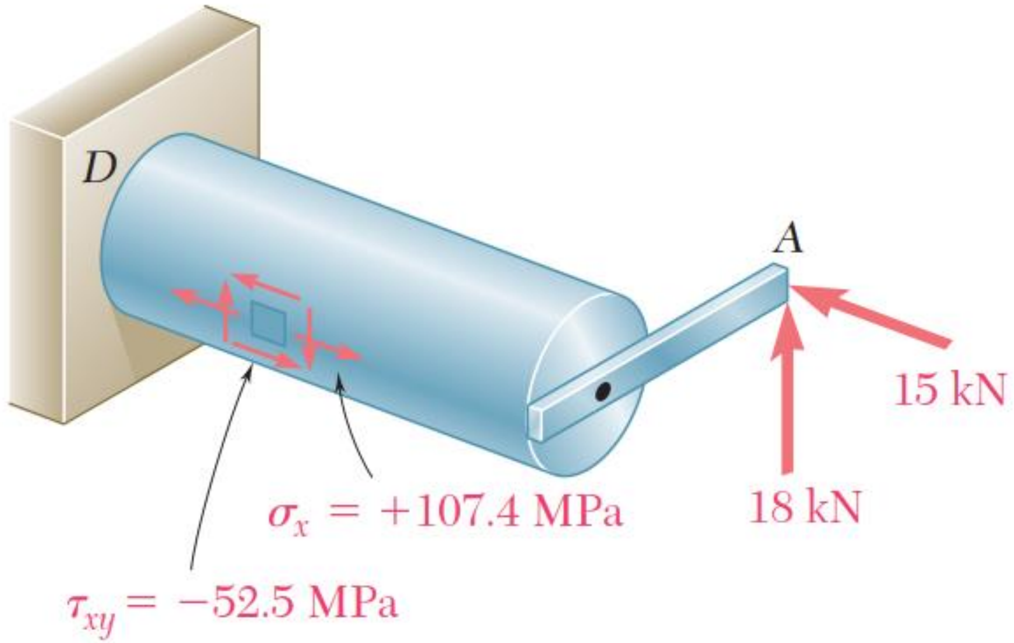
$$= +19.1\ \text{MPa}$$

$$(\tau_{xy})_{\text{twist}} = -\frac{Tc}{J_C} = -\frac{(900\ \text{N·m})(0.020\ \text{m})}{251.3 \times 10^{-9}\ \text{m}^4} = -71.6\ \text{MPa}$$

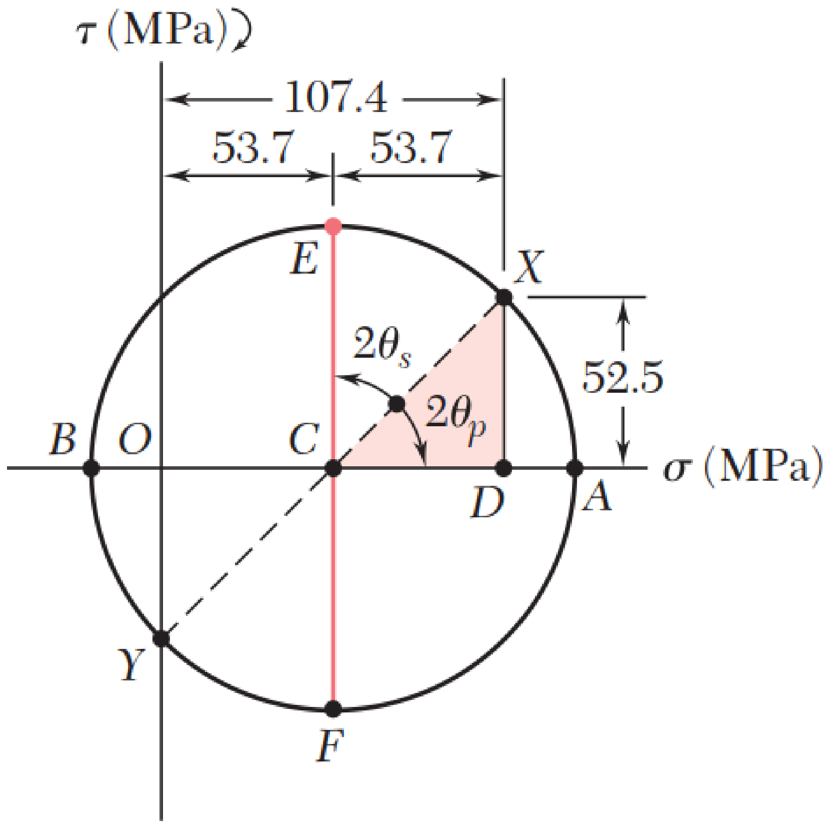
$$\tau_{xy} = (\tau_{xy})_V + (\tau_{xy})_{\text{twist}} = +19.1\ \text{MPa} - 71.6\ \text{MPa}$$

$$\tau_{xy} = -52.5\ \text{MPa}$$

Solution

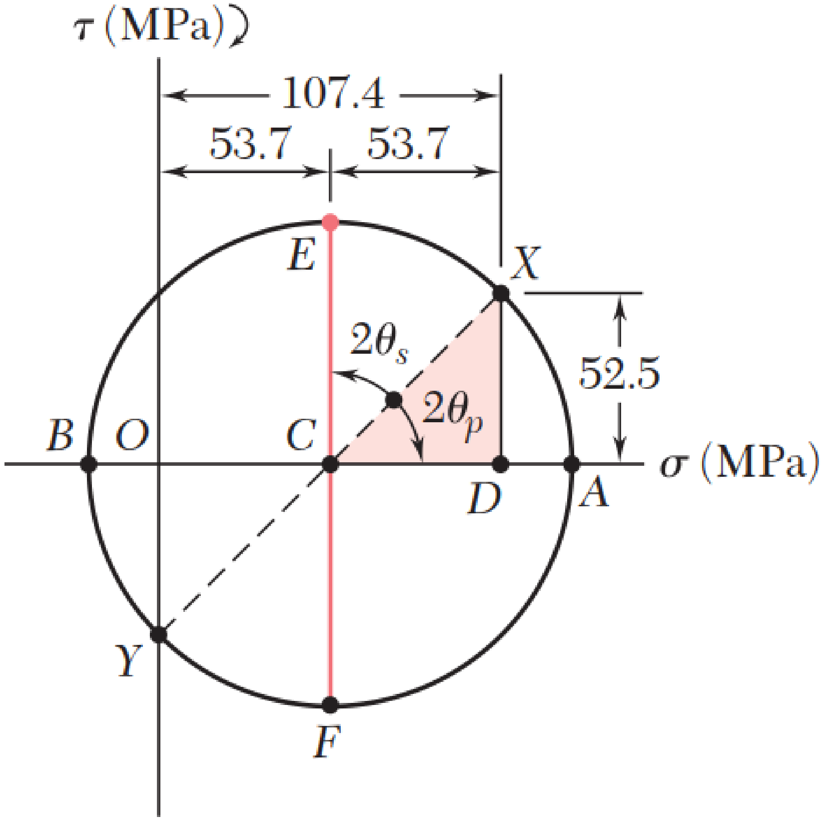


D
A
15 kN
18 kN
$\sigma_x = +107.4$ MPa
$\tau_{xy} = -52.5$ MPa



τ (MPa)
107.4
53.7
53.7
E
X
2θs
2θp
52.5
B
O
C
D
A
σ (MPa)
Y
F

Solution



Principal Planes and Principal Stresses at Point *K*.

$$\tan 2\theta_p = \frac{DX}{CD} = \frac{52.5}{53.7} = 0.97765 \qquad 2\theta_p = 44.4^\circ \circlearrowright$$

$$\theta_p = 22.2^\circ \circlearrowright$$

$$R = \sqrt{(53.7)^2 + (52.5)^2} = 75.1 \text{ MPa}$$

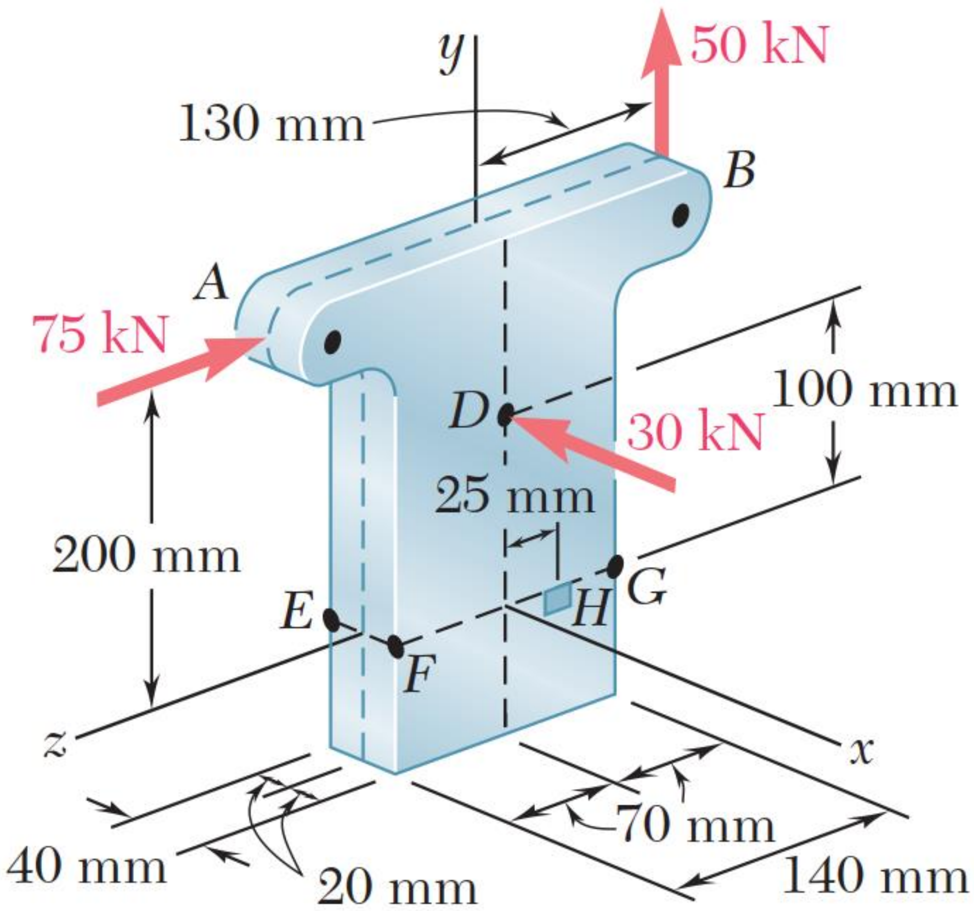
$$\sigma_{\max} = OC + R = 53.7 + 75.1 = 128.8 \text{ MPa}$$

$$\sigma_{\min} = OC - R = 53.7 - 75.1 = -21.4 \text{ MPa}$$

Maximum Shearing Stress at Point *K*.

$$\tau_{\max} = CE = R = 75.1 \text{ MPa} \qquad \theta_s = 22.8^\circ \circlearrowleft$$

Example 2



For the loading shown determine the principal stresses, principal planes, and maximum shearing stress at point *H*.

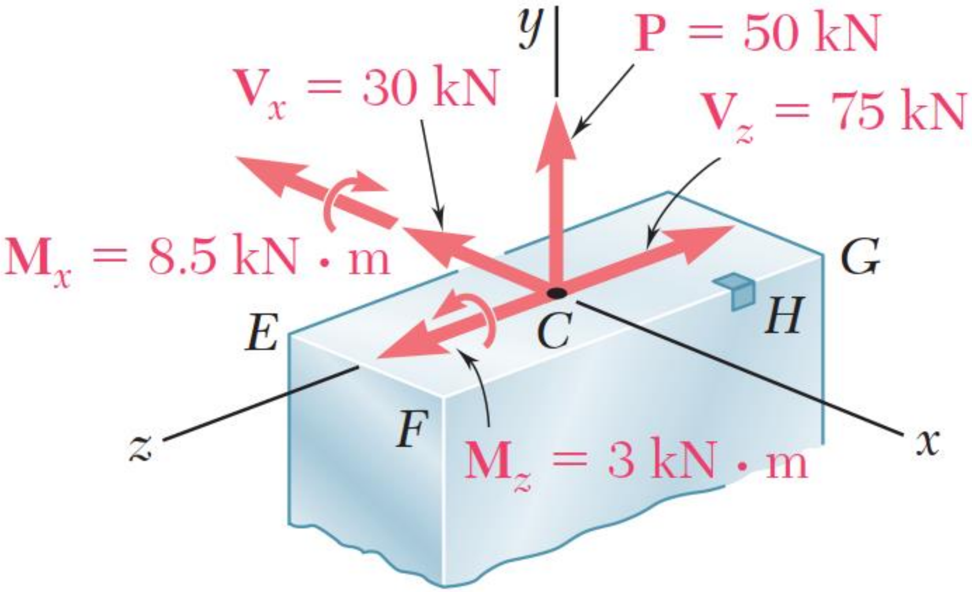
Solution

Internal Forces in Section *EFG*.

$$V_x = -30 \text{ kN} \qquad P = 50 \text{ kN} \qquad V_z = -75 \text{ kN}$$

$$M_x = (50 \text{ kN})(0.130 \text{ m}) - (75 \text{ kN})(0.200 \text{ m}) = -8.5 \text{ kN·m}$$

$$M_y = 0 \qquad M_z = (30 \text{ kN})(0.100 \text{ m}) = 3 \text{ kN·m}$$

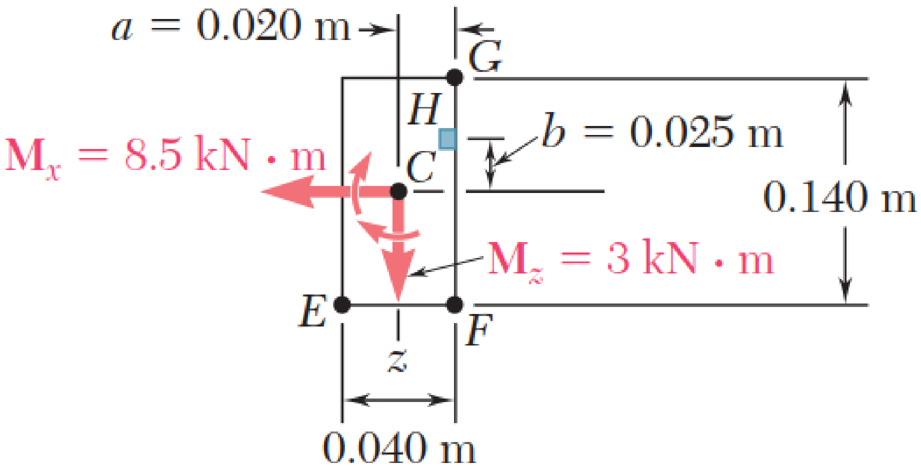


$$A = (0.040 \text{ m})(0.140 \text{ m}) = 5.6 \times 10^{-3} \text{ m}^2$$

$$I_x = \tfrac{1}{12}(0.040 \text{ m})(0.140 \text{ m})^3 = 9.15 \times 10^{-6} \text{ m}^4$$

$$I_z = \tfrac{1}{12}(0.140 \text{ m})(0.040 \text{ m})^3 = 0.747 \times 10^{-6} \text{ m}^4$$

Solution



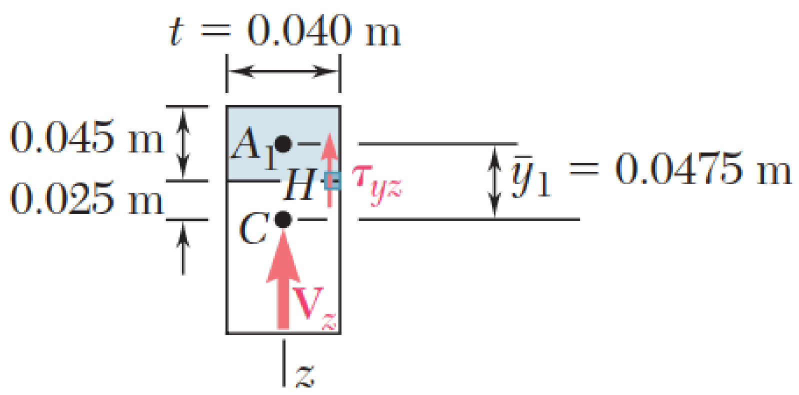
Normal Stress at *H*.

$$\sigma_y = +\frac{P}{A} + \frac{|M_z|a}{I_z} - \frac{|M_x|b}{I_x}$$

$$= \frac{50\text{ kN}}{5.6 \times 10^{-3}\text{ m}^2} + \frac{(3\text{ kN}\cdot\text{m})(0.020\text{ m})}{0.747 \times 10^{-6}\text{ m}^4} - \frac{(8.5\text{ kN}\cdot\text{m})(0.025\text{ m})}{9.15 \times 10^{-6}\text{ m}^4}$$

$$\sigma_y = 8.93\text{ MPa} + 80.3\text{ MPa} - 23.2\text{ MPa} \qquad \sigma_y = 66.0\text{ MPa}$$

Solution



Considering the shearing force $\mathbf{V}_x$, we note that Q = 0 with respect to the z axis, since H is on the edge of the cross section. Thus, $\mathbf{V}_x$ produces no shearing stress at H.

The shearing force $\mathbf{V}_z$ does produce a shearing stress at H

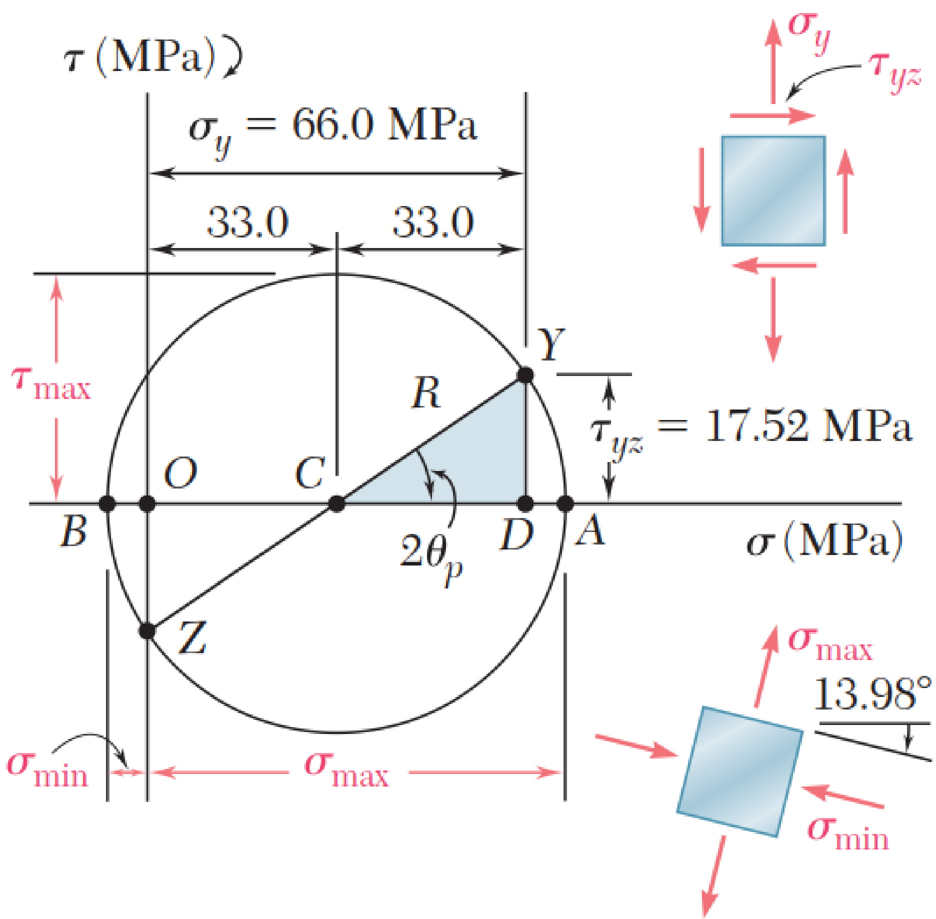
Shearing Stress at *H*.

$$Q = A_1\bar{y}_1 = [(0.040\text{ m})(0.045\text{ m})](0.0475\text{ m}) = 85.5 \times 10^{-6}\text{ m}^3$$

$$\tau_{yz} = \frac{V_z Q}{I_x t} = \frac{(75\text{ kN})(85.5 \times 10^{-6}\text{ m}^3)}{(9.15 \times 10^{-6}\text{ m}^4)(0.040\text{ m})} \qquad \tau_{yz} = 17.52\text{ MPa}$$

Solution

Principal Stresses, Principal Planes, and Maximum Shearing Stress at *H*.



$$\tan 2\theta_p = \frac{17.52}{33.0} \qquad 2\theta_p = 27.96^\circ$$

$$R = \sqrt{(33.0)^2 + (17.52)^2} = 37.4 \text{ MPa}$$

$$\sigma_{max} = OA = OC + R = 33.0 + 37.4$$

$$\sigma_{min} = OB = OC - R = 33.0 - 37.4$$

$$\theta_p = 13.98^\circ$$

$$\tau_{max} = 37.4 \text{ MPa}$$

$$\sigma_{max} = 70.4 \text{ MPa}$$

$$\sigma_{min} = -7.4 \text{ MPa}$$