Shearing Stresses in Beams and Thin-Walled Members

Part II

Shearing Stresses In Thin-walled Members

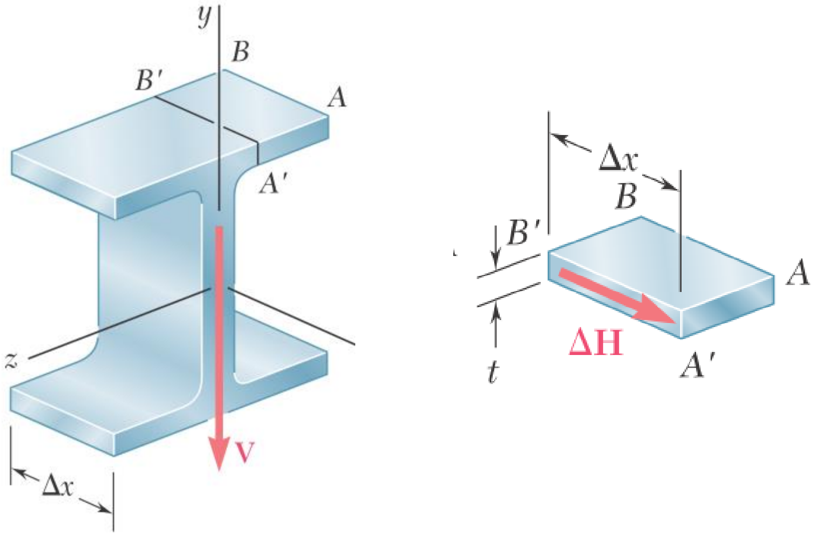
 

- The longitudinal shear and horizontal shear per unit length (shear flow) are used in this section to calculate both the shear flow and the average shearing stress in thin-walled members such as the flanges of wide-flange beams, box beams, or the walls of structural tubes

$$\Delta H = \frac{VQ}{I}\Delta x \qquad\qquad q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I}$$

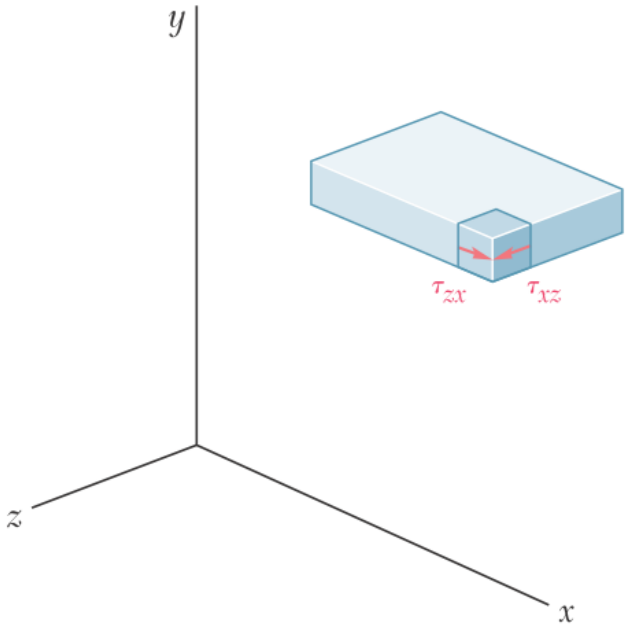
- Consider a segment of length Δx of a wide-flange beam where V is the vertical shear in the transverse section shown. Detach an element ABB'A' of the upper flange. The longitudinal shear ΔH exerted on that element can be obtained as

$$\Delta H = \frac{VQ}{I}\Delta x$$

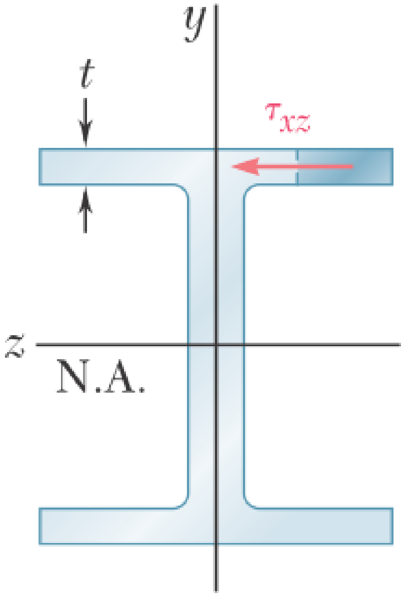


- Dividing ΔH by the area $\Delta A = t\Delta x$ of the cut, the average shearing stress exerted on the element is the same expression obtained in the previous course for a horizontal cut:

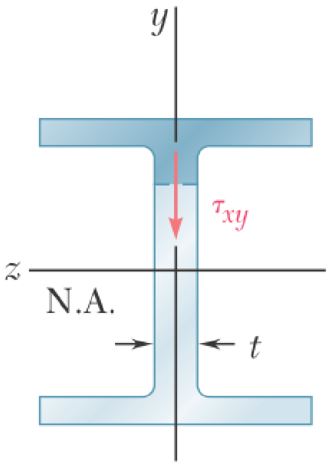
$$\tau_{ave} = \frac{VQ}{It}$$

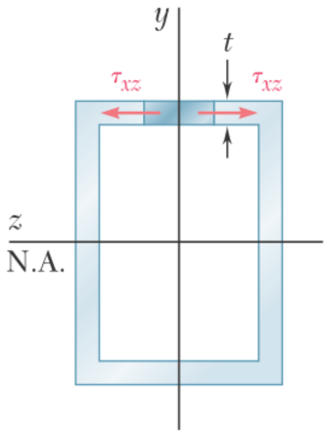


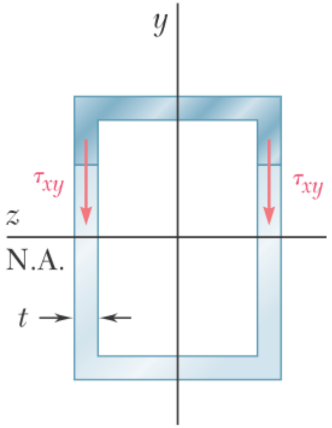
- Note that τ_{ave} now represents the average value of the shearing stress τ_{zx} over a vertical cut, but since the thickness t of the flange is small, there is very little variation of τ_{zx} across the cut.

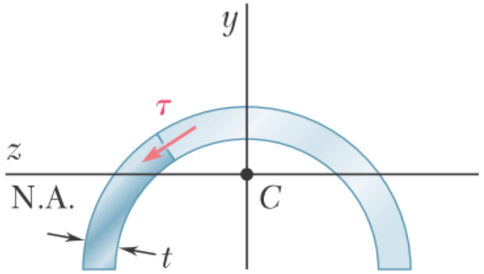


- Recalling that $\tau_{zx} = \tau_{xz}$, the horizontal component τ_{xz} of the shearing stress at any point of a transverse section of the flange can be obtained from $\tau_{ave} = \frac{VQ}{It}$, where Q is the first moment of the shaded area about the neutral axis.



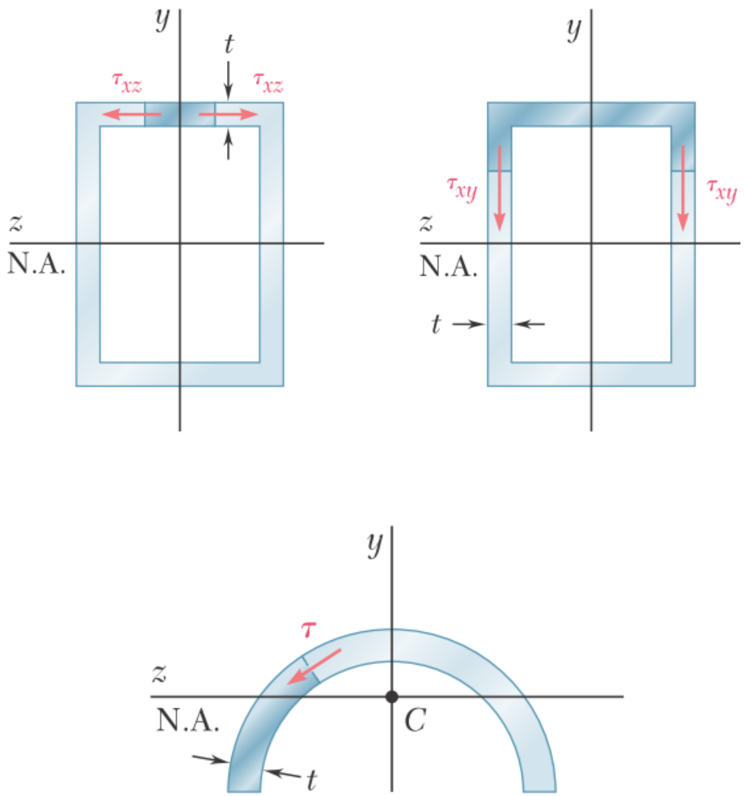






- A similar result was obtained for the vertical component τ_{xy} of the shearing stress in the web.

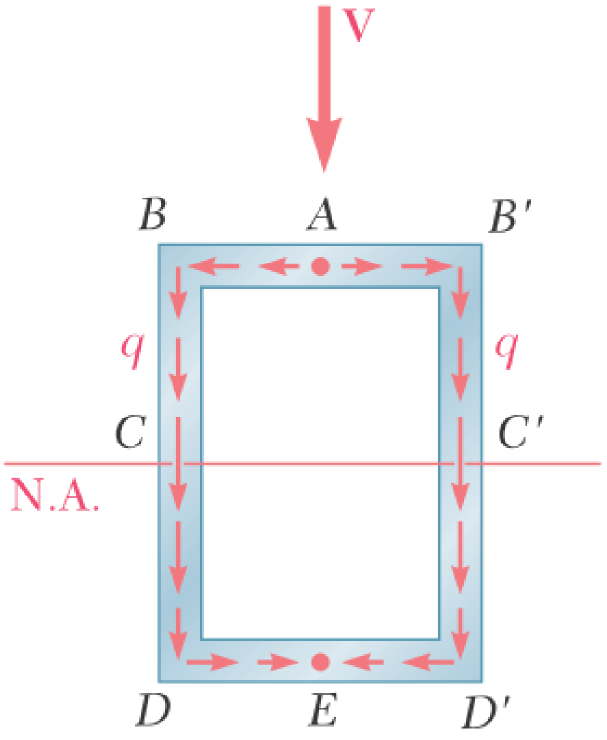
- $\tau_{ave} = \frac{VQ}{It}$ can be used to determine shearing stresses in box beams, half pipes, and other thin-walled members, as long as the loads are applied in a plane of symmetry.



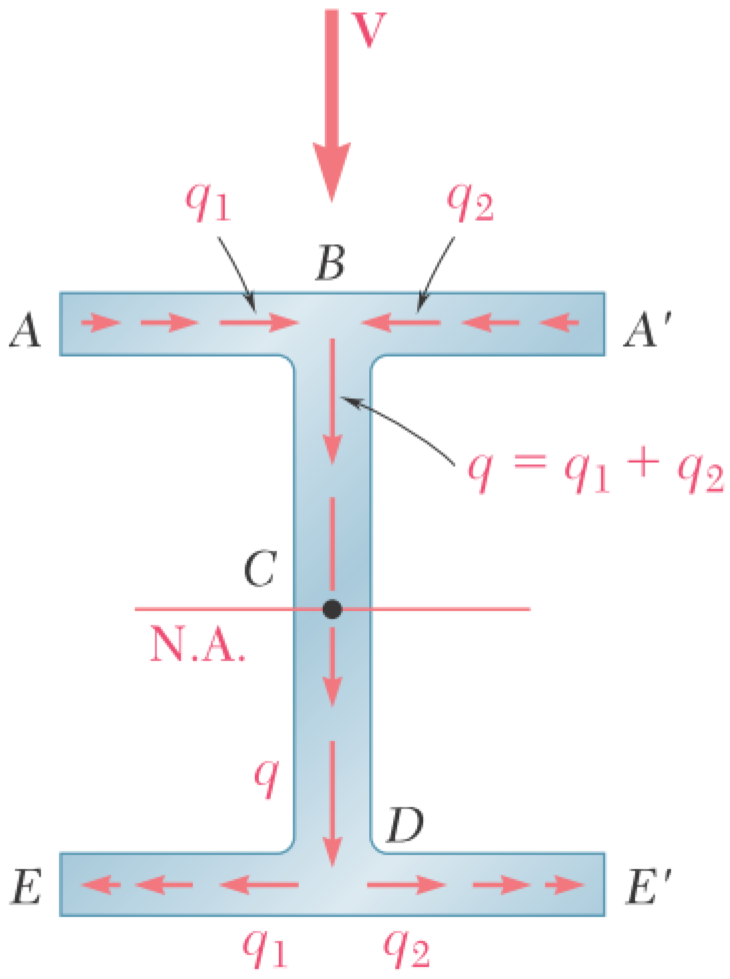
- In each case, the cut must be perpendicular to the surface of the member, and $\tau_{ave} = \frac{VQ}{It}$ will yield the component of the shearing stress in the direction tangent to that surface.

- The other component is assumed to be equal to zero, because of the proximity of the two free surfaces.

- Comparing $q = \frac{VQ}{I}$ and $\tau_{ave} = \frac{VQ}{It}$, the product of the shearing stress τ at a given point of the section and the thickness t at that point is equal to q.
- Since V and I are constant, q depends only upon the first moment Q and easily can be sketched on the section.

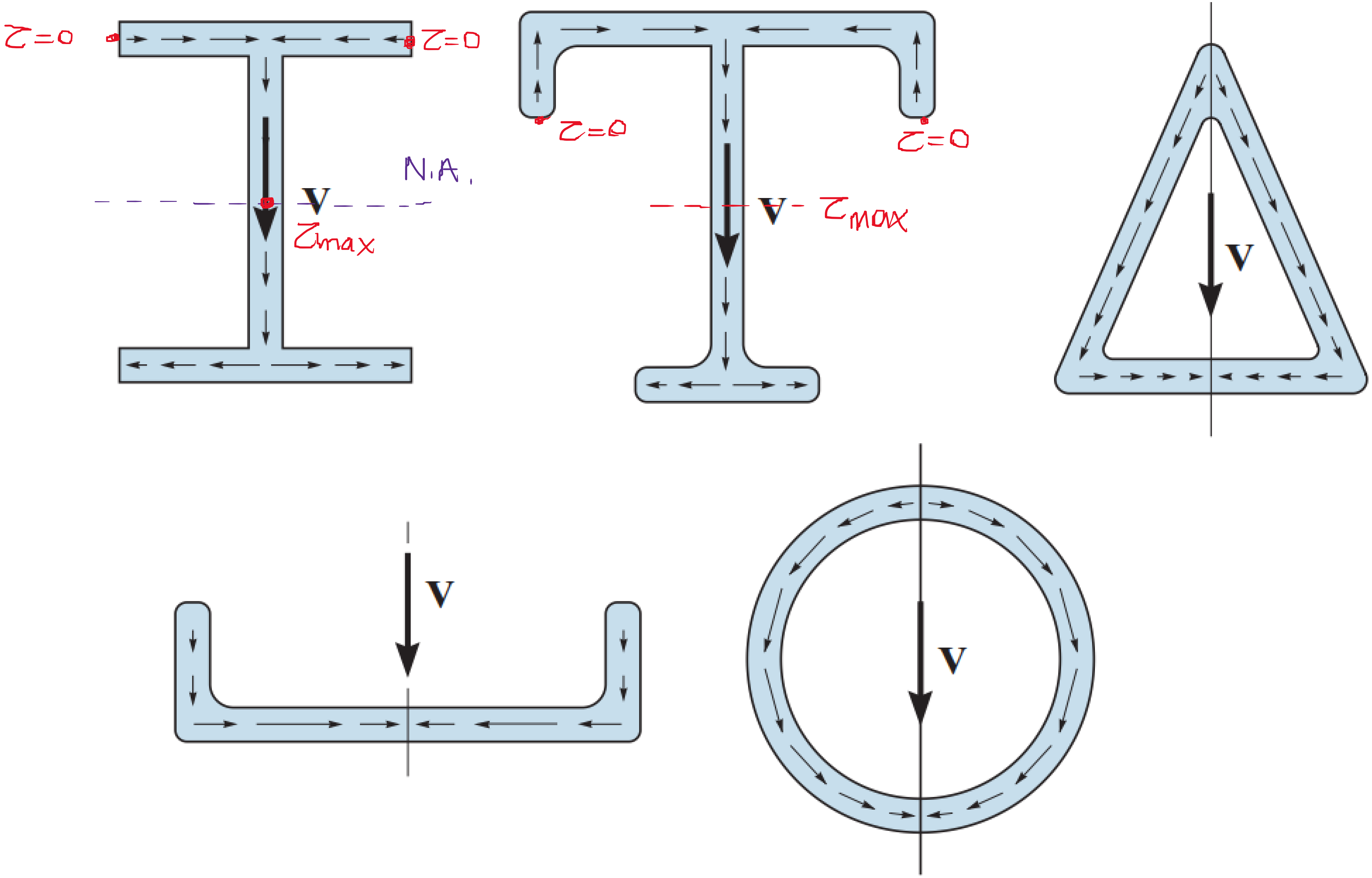


- For a box beam, q grows smoothly from zero at A to a maximum value at C and C' on the neutral axis and decreases back to zero as E is reached.
- There is no sudden variation in the magnitude of q as it passes a corner at B, D, B', or D', and the sense of q in the horizontal portions of the section is easily obtained from its sense in the vertical portions (the sense of the shear V).



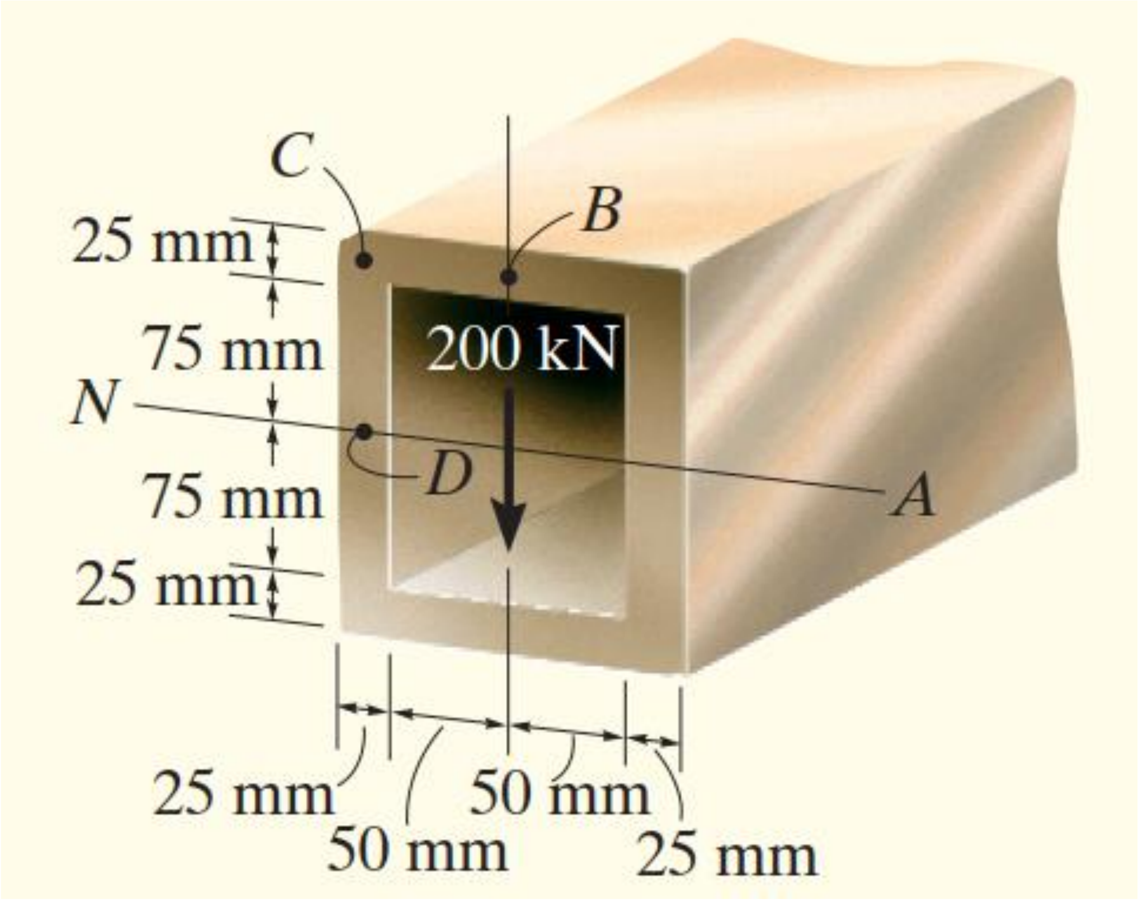
- In a wide-flange section, the values of q in portions AB and $A'B$ of the upper flange are distributed symmetrically.
- At B in the web, q corresponds to the two halves of the flange, which must be combined to obtain the value of q at the top of the web.
- After reaching a maximum value at C on the neutral axis, q decreases and splits into two equal parts at D, which corresponds at D to the two halves of the lower flange.
- The shear per unit length q is commonly called the shear flow and reflects the similarity between the properties of q just described and some of the characteristics of a fluid flow through an open channel or pipe.

Shear Flow in Common Thin-Walled Members



Example

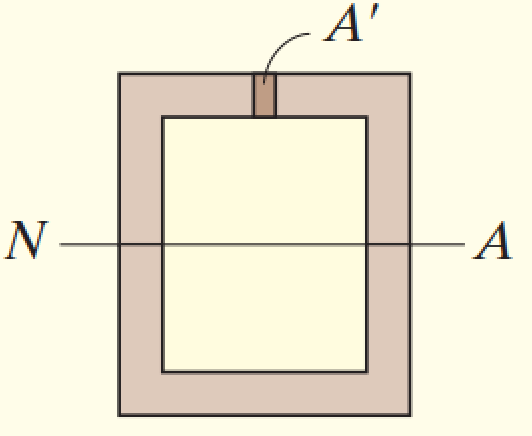
The thin-walled box beam in figure is subjected to a shear of 200 kN. Determine the variation of the shear flow throughout the cross section.



Solution

$$I = \frac{1}{12}(0.05 \text{ m})(0.175 \text{ m})^3 + 2[(0.125 \text{ m})(0.025 \text{ m})(0.0875 \text{ m})^2] = 70.18(10^{-6}) \text{ m}^4$$

At point B.

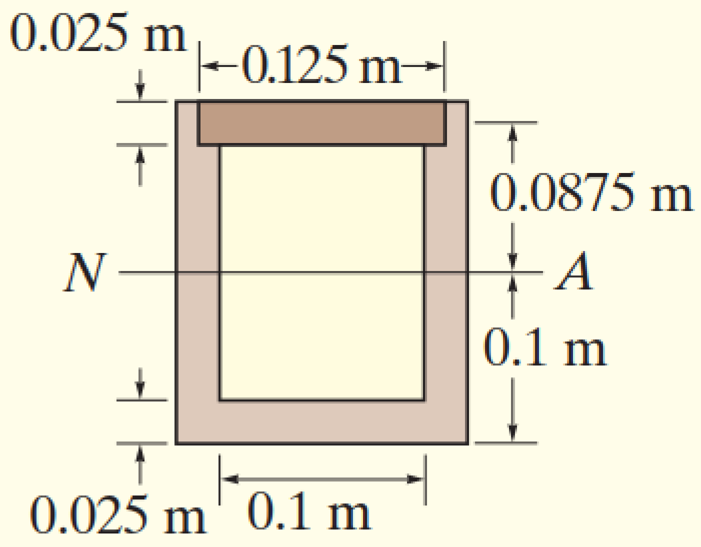


$$Q_B = \bar{y}'A' = 0$$

since $y' = 0$

$$q_B = 0$$

At point C.



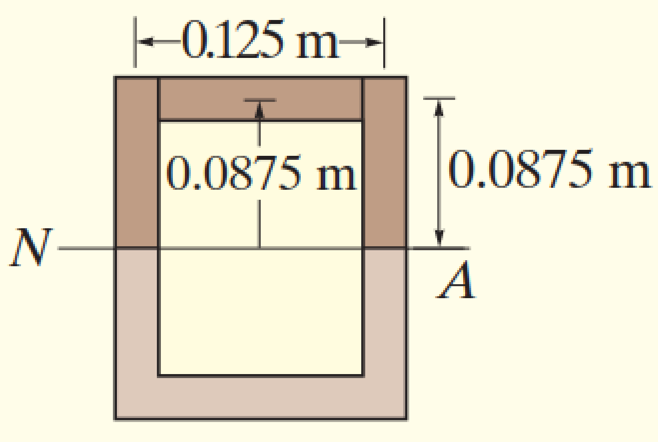
$$Q_C = \bar{y}'A' = (0.0875 \text{ m})(0.125 \text{ m})(0.025 \text{ m})$$

$$= 0.27344(10^{-3}) \text{ m}^3$$

$$q_C = \frac{1}{2}\left(\frac{VQ_C}{I}\right) = 390 \text{ kN/m}$$

Solution

At point D.

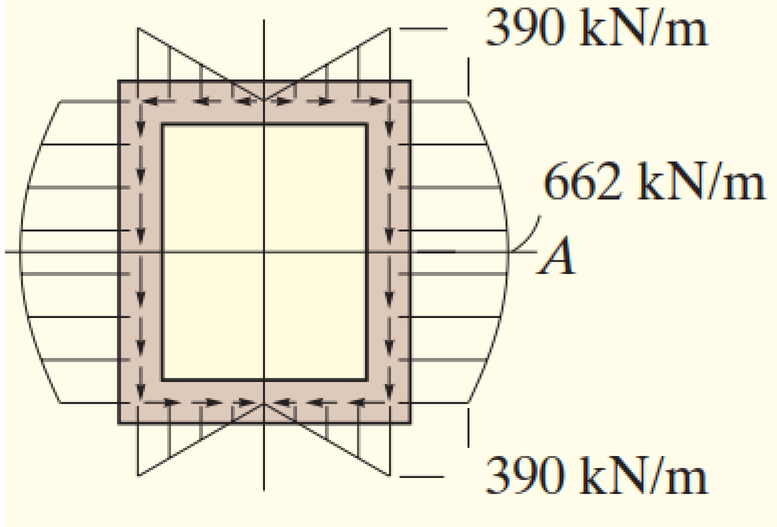


$$Q_D = \Sigma \bar{y}'A' = 2\left[\left(\frac{0.0875\text{ m}}{2}\right)(0.025\text{ m})(0.0875\text{ m})\right]$$

$$+ [0.0875\text{ m}](0.125\text{ m})(0.025\text{ m})$$

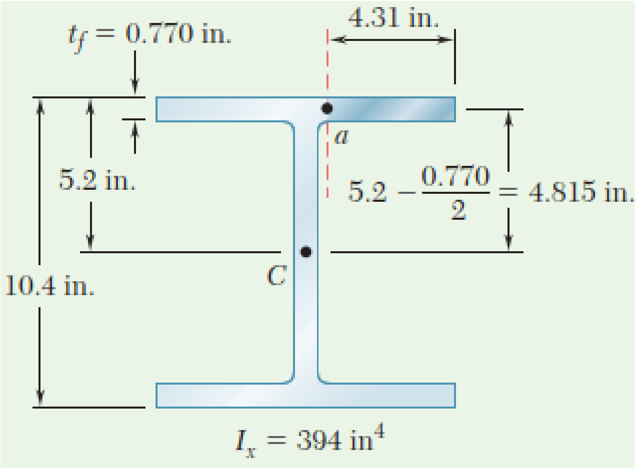
$$= 0.4648(10^{-3})\text{ m}^3$$

$$q_D = \frac{1}{2}\left(\frac{VQ_D}{I}\right) = 662\text{ kN/m}$$



Using these results, and the symmetry of the cross section, the shear-flow distribution is

Sample Problem 6.3

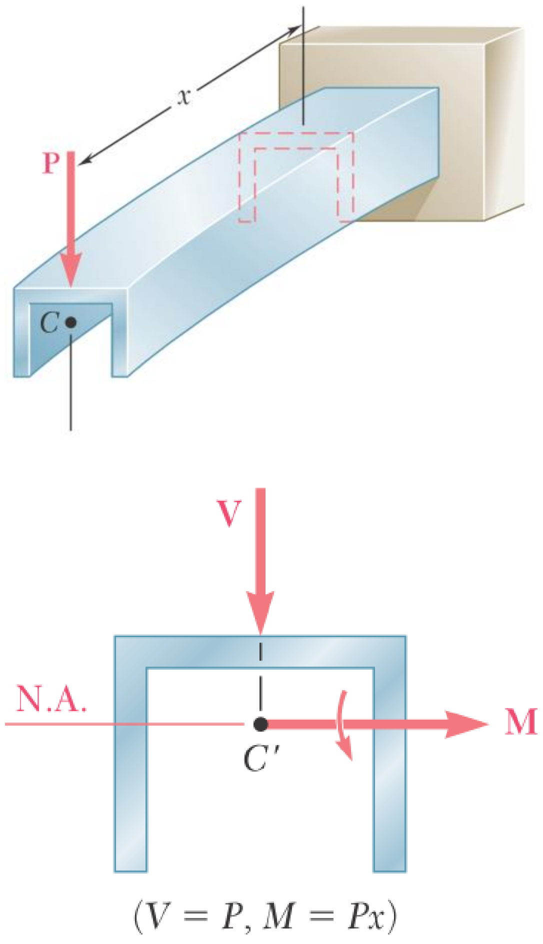


Knowing that the vertical shear is 50 kips in a W10 X 68 rolled-steel beam, determine the horizontal shearing stress in the top flange at a point a located 4.31 in. from the edge of the beam. The dimensions and other geometric data of the rolled-steel section are given in
Appendix C.

$$Q = (4.31\ in.)(0.770\ in.)(4.815\ in.) = 15.98\ in^3$$

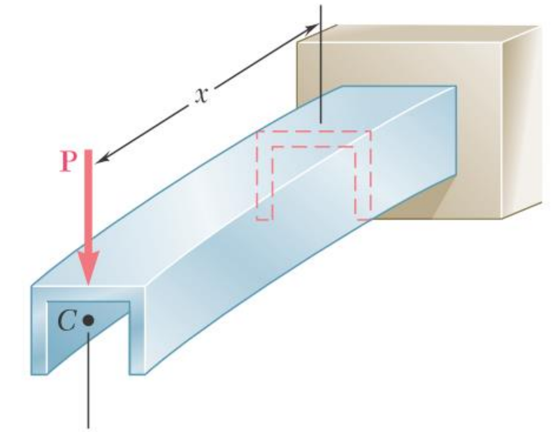
$$\tau = \frac{VQ}{It} = \frac{(50\ \text{kips})(15.98\ \text{in}^3)}{(394\ in^3)(0.770\ in.)} \rightarrow \tau = 2.63\ ksi$$

Unsymmetric Loading Of Thin-walled Members and Shear Center



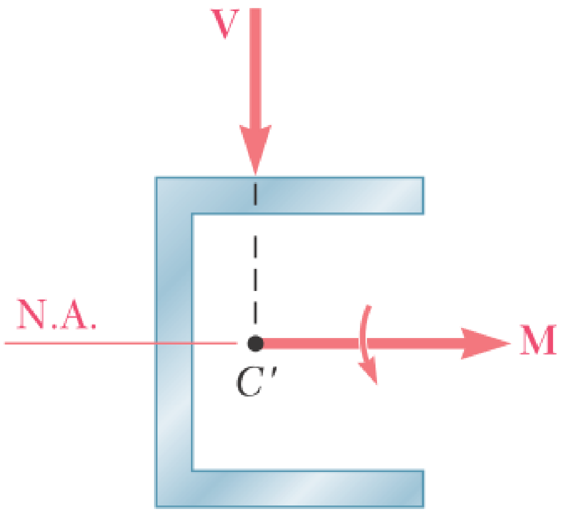
- Our analysis of the effects of transverse loadings has been limited to members possessing a vertical plane of symmetry and to loads applied in that plane.
- The members were observed to bend in the plane of loading, and in any given cross section, the bending couple M and the shear V were found to result in normal and shearing stresses:

$$\sigma_x = -\frac{My}{I} \text{ and } \tau_{ave} = \frac{VQ}{It}$$



- In this section, the effects of transverse loads on thin-walled members that do not possess a vertical plane of symmetry are examined.

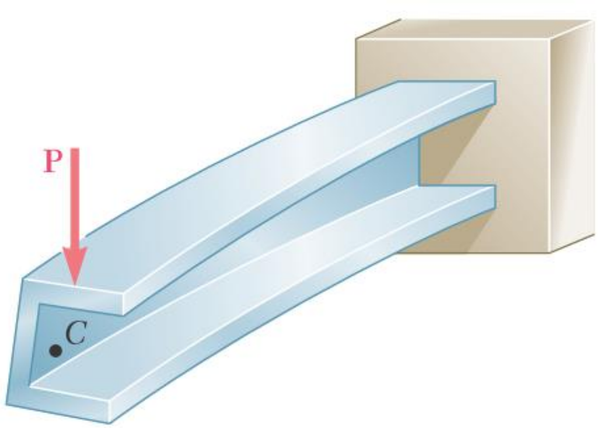
- Assume that the channel member has been rotated through 90° and that the line of action of P still passes through the centroid of the end section.



$(V = P, M = Px)$

- The couple vector M representing the bending moment in a given cross section is still directed along a principal axis of the section, and the neutral axis will coincide with that axis.

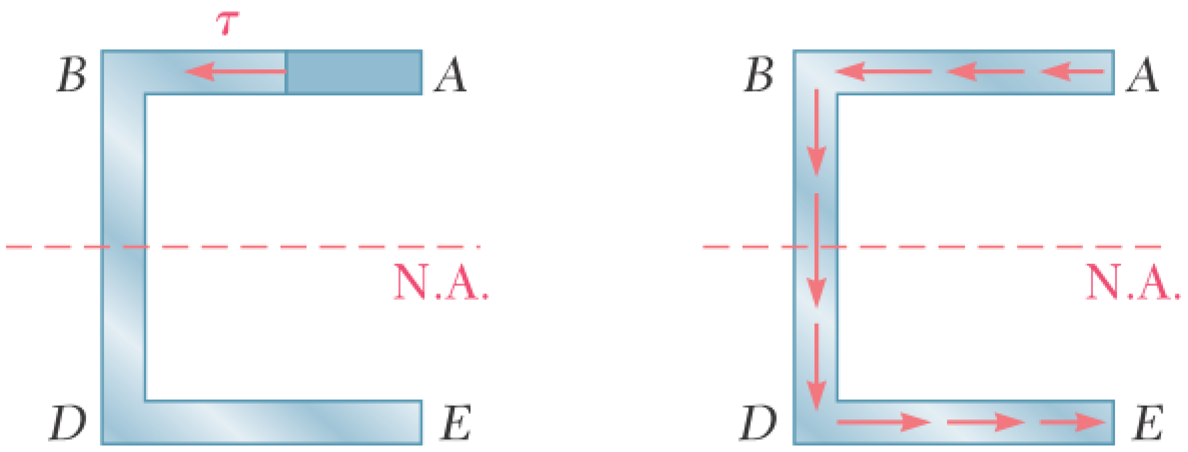
- $\sigma_x = -\frac{My}{I}$ can be used to compute the normal stresses in the section. However, $\tau_{ave} = \frac{VQ}{It}$ cannot be used to determine the shearing stresses, since this equation was derived for a member possessing a vertical plane of symmetry.



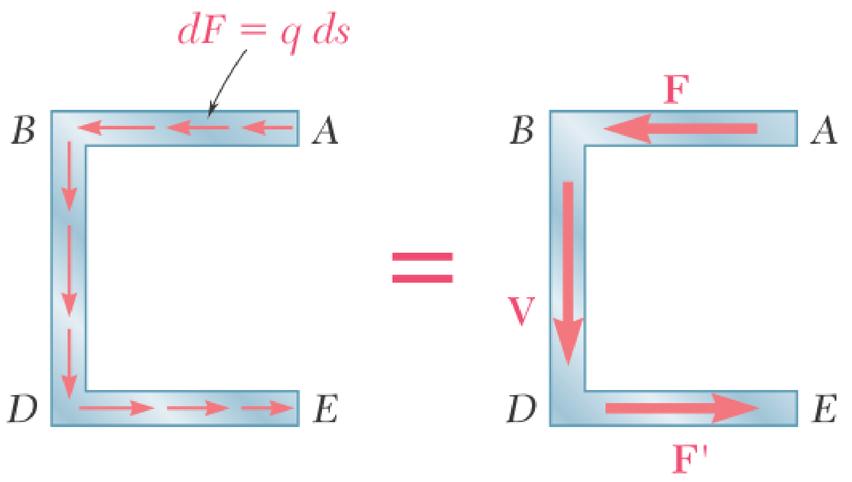
- Actually, the member will be observed to bend and twist under the applied load, and the resulting distribution of shearing stresses will be quite different from that given by $\tau_{ave} = \frac{VQ}{It}$.

- **Is it possible to apply the vertical load P so that the channel member of figure will bend without twisting?**

- **If so, where should the load P be applied?**



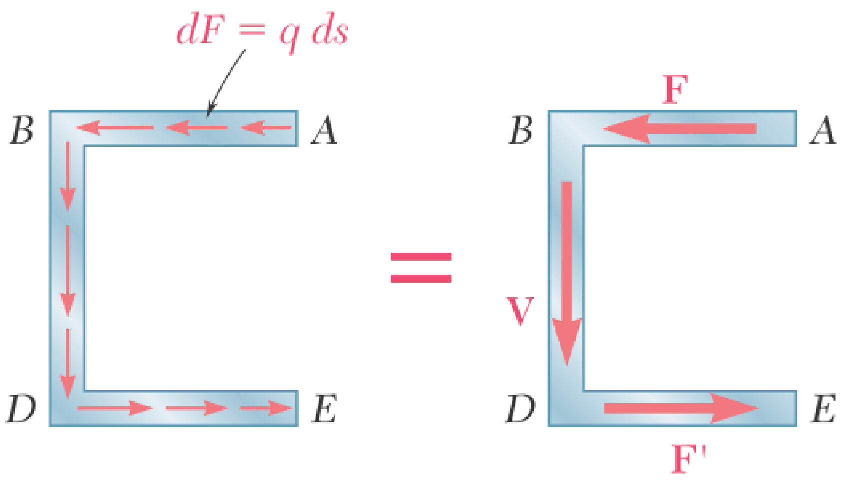
- If the member bends without twisting, the shearing stress at any point of a given cross section can be obtained from $\tau_{ave} = \frac{VQ}{It}$, where Q is the first moment of the shaded area with respect to the neutral axis and the distribution of stresses is as shown with $\tau = 0$ at both A and E.



- The shearing force exerted on a small element of cross-sectional area $dA = tds$ is $dF = \tau dA = \tau tds$ or $dF = qds$, where q is the shear flow $q = \tau t = {}^{VQ}/_{I}$. The resultant of the shearing forces exerted on the elements of the upper flange AB of the channel is a horizontal force F of magnitude

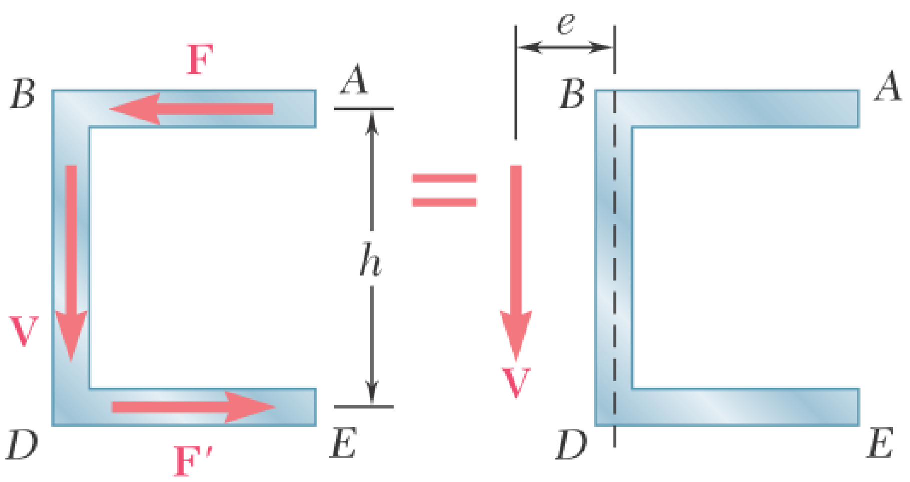
$$F = \int_{A}^{B} qds$$

- Because of the symmetry of the channel section about its neutral axis, the resultant of the shearing forces exerted on the lower flange DE is a force F' of the same magnitude as F but of opposite sense.



- The resultant of the shearing forces exerted on the web BD must be equal to the vertical shear V in the section:

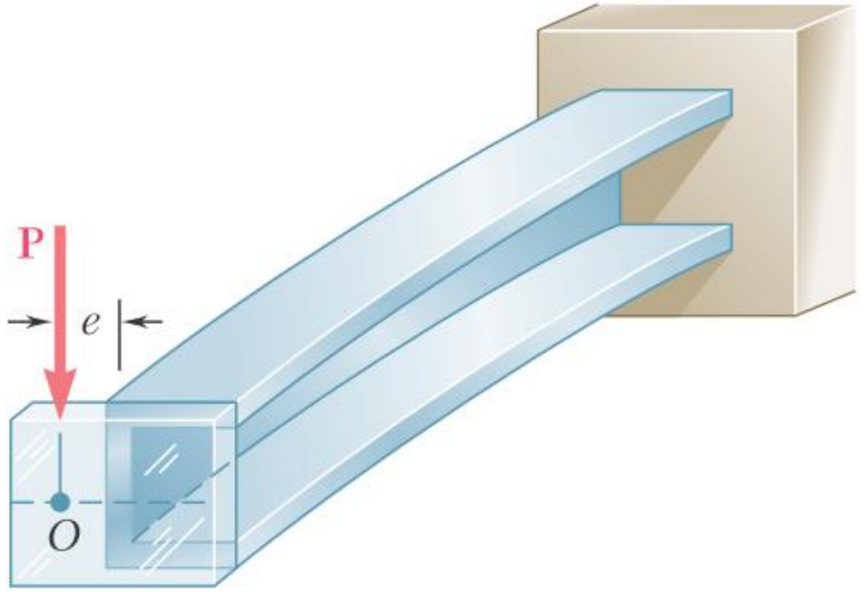
$$V = \int_B^D q ds$$



- The forces F and F' form a couple of moment $F \times h$, where h is the distance between the center lines of the flanges AB and DE. This couple can be eliminated if the vertical shear V is moved to the left through a distance e so the moment of V about B is equal to $F \times h$. Thus, $V \times e = F \times h$ or

$$e = \frac{Fh}{V}$$

- When the force P is applied at a distance e to the left of the center line of the web BD, the member bends in a vertical plane without twisting.
- The point O where the line of action of P intersects the axis of symmetry of the end section is the shear center of that section.



- In the case of an oblique load P, the member will also be free of twist if the load P is applied at the shear center of the section.
- The load P then can be resolved into two components P_z and P_y neither of which causes the member to twist.