ME 224 STRENGTH OF MATERIALS

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Course Description

STRENGTH OF MATERIALS ME 224

Credit Strucutre: (3-1)4

Course Objectives:

The main objective of the course is to develop in the second year mechanical engineering students a clear understanding of relationship between the loads applied to elastic bodies, and the resultant stresses, strains and deformations.

Prerequisities:

ME 201

Textbook(s):

F.P. Beer and E.R. Johnston, "Mechanics of Materials", McGraw-Hill Int Book Company

Reference:

R.C. Hibbeler, "Mechanics of materials", Prentice Hall Inc, 1997

Instructors: Prof. Dr. Ömer Yavuz BOZKURT, Asst. Prof. Dr. Nurettin Furkan DOĞAN

Exams: 1. Midterm 30% ; 2. Midterm 30% ; Final: 40% **Attendance:** The attendance of 70% is mandatory.

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2023-2024 SPRING SEMESTER ME 224 GENERAL CONTENT

INTRODUCTION	13.02.2024	EXAMS		
Ch 1. Normal, Shear stress	13.02.2024	??-02-2024 First Tutorial		
Ch 1. Oblique Plane, Factor of Safety	16.02.2024	??-03-2024 Second Tutorial		
Ch 1. Pr.Hr.	20.02.2024	??-03-2024 Third Tutorial		
STRESS AND STRAIN	00.00.0004	??-04-2024 Fourth Tutorial		
Ch 2. Stress-Strain- Deformation	23.02.2024	??-04-2024 Fifth Tutorial		
Ch 2. Pr.Hr.	27.02.2024			
Ch 2. Statically Indeterminate Pr.	01.03.2024	05-04-2024 First Examination		
Ch 2. Poissons' Raito and so on	05.03.2024	31-05-2024 Second Examination		
Ch 2. Pr.Hr.	08.03.2024			
TORSION	10 02 2024	Textbook		
Ch 3. Torsion	12.03.2024	Mechanics of Materials		
Ch 3. Pr.Hr.	15.03.2024	By Ferdinand P. Beer and E. R.		
Ch 3. Power Transmission	19.03.2024	Johnston		
PURE BENDING	22 02 2024			
Ch 4. Pure Bending	22.03.2024	Auxiliary Books		
Ch 4. Pr.Hr.	26.03.2024	Mechanics of Materials		
Ch 4. Eccentic Axial Loading	29.03.2024	By R.C. Hibbeler		
Ch 4. Pr.Hr.	02.04.2024			
FIRST FYAM	05 04 2024	Many books related with the		
FINGT EAAM	03.04.2024	Strength in the Library		

2023-2024 SPRING SEMESTER ME 224 GENERAL CONTENT

SHEARING STRESS	16 04 2024	EXAMS		
Ch 6. in Beams	10.04.2024	??-02-2024 First Tutorial		
Ch 6. Pr.Hr.	19.04.2024	??-03-2024 Second Tutorial		
Ch 6. in Thin walled members	23.04.2024	??-03-2024 Third Tutorial		
Ch 6. Pr.Hr.	26.04.2024	??-04-2024 Fourth Tutorial		
PRINCIPAL STRESSES UNDER A GIVEN		??-04-2024 Fifth Tutorial		
LOADING	30.04.2024			
Ch 8. Combined Loadings		05-04-2024 First Examination		
Ch 8. Pr.Hr.	03.05.2024	31-05-2024 Second Examination		
TRANSFORMATION OF STRESS				
Ch 7. Transformation of Plane Stress and	07.05.2024	Textbook		
Mohr's Circle		Mechanics of Materials		
Ch 7. Pressure Vessels and Yield Criteria's	10.05.2024	By Ferdinand P. Beer and E. R.		
Ch 7. Pr.Hr.	14.05.2024	Johnston		
DEFLECTION OF BEAMS	17.05.2024			
Ch 9. Integration (Deflection of Beams)	17.05.2024	Auxiliary Books		
Ch 9. Pr. Hr.	21.05.2024	Mechanics of Materials		
Ch 9. Singularity Function(Deflection of	24 05 2024	By R.C. Hibbeler		
Beams)	24.03.2024			
Ch 9. Pr. Hr.	28.05.2024	Many books related with the Strength		
SECOND EXAM	31.05.2024	in the Library		

Chapter 1- Introduction Concept of Stress

Mechanics In Science

• Mechanics is one the essential branch in physics. It can be examining in solid mechanics and fluid mechanics.



Main Objective of Mechanics of Materials

• The main objective of the study of mechanics of materials is to provide the future engineer with the means of analyzing and designing tips various machines, machine elements and structures.

 Both the analysis and design of a given structure involve the determination of stresses and deformations.

Review of Statics

Statics has two main goals:

- 1. Introduction to basic concepts of force, couples and moments in two and three dimensions structures.
- 2. Developments of analytical skills relevant to the areas of mentioned in above.

Example in Statics



- The structure consists of a boom (50mm x 30 mm) and rod (d=20mm) joined by pins at the junctions and supports
- Determine the internal force in each structural member and the reaction forces at the supports

Structure Free-Body Diagram



- Structure is detached from supports and the external loads and reaction forces are indicated
- Conditions for static equilibrium:

 $\sum M_C = 0 = A_x (0.6 \text{ m}) - (30 \text{ kN})(0.8 \text{ m})$ $A_x = 40 \text{ kN}$ $\sum F_x = 0 = A_x + C_x$ $C_x = -A_x = -40 \text{ kN}$ $\sum F_y = 0 = A_y + C_y - 30 \text{ kN} = 0$ $A_y + C_y = 30 \text{ kN}$

• A_y and C_y can not be determined from these equations

Component Free-Body Diagram



- In addition to the complete structure, each component and each point must satisfy the conditions for static equilibrium
- Consider a free-body diagram for the boom: $\sum M_B = 0 = -A_y(0.8 \text{ m})$

 $A_{y} = 0$

substitute into the structure equilibrium equation

$$C_y = 30 \,\mathrm{kN}$$

• Results:

$$A = 40 \,\mathrm{kN} \rightarrow C_x = 40 \,\mathrm{kN} \leftarrow C_y = 30 \,\mathrm{kN}$$

Reaction forces are directed along boom and rod

Method of Joints



Fig. 1.5 Free-body diagrams of two-force members *AB* and *BC*.



Alternative Solution Of Using Two Force Members Concept

- The boom and rod are 2-force members, i.e., the members are subjected to only two forces which are applied at member ends
- For equilibrium, the forces must be parallel to to an axis between the force application points, equal in magnitude, and in opposite directions
- Joints (points) must satisfy the conditions for static equilibrium which may be expressed in the form of a force triangle:

$$\sum \vec{F}_B = 0$$

$$\frac{F_{AB}}{4} = \frac{F_{BC}}{5} = \frac{30 \text{ kN}}{3}$$

$$F_{AB} = 40 \text{ kN} \qquad F_{BC} = 50 \text{ kN}$$

Axial Loading: Normal Stress



- Consider an axially loaded member. The resultant of the internal forces for an axially loaded member is *normal* to a section cut perpendicular to the member axis.
- The force intensity (force per unit area) on that section is defined as the normal stress, σ (sigma).

$$\sigma = \frac{P}{A}$$

Average Normal stress in a member under axial loading

A positive sign indicates a **tensile stress** (member in tension), and a negative sign indicates a **compressive stress** (member in compression).





Normal Stress (σ) in Axially Loaded Memb

To define the stress at a given point Q of the cross section, consider a small ٠ area ΔA $= \lim_{\Delta A \to 0} \frac{\Delta F}{\Lambda A}$

 σ



• The normal stress at a particular point may not be equal to the average stress but the resultant of the stress distribution must satisfy

$$P = \sigma_{ave} A = \int dF = \int_{A} \sigma \, dA$$



• The detailed distribution of stress is statically indeterminate, i.e., can not be found from statics alone. 1 - 15

Centric & Eccentric Loading

C

P'

- A uniform distribution of stress in a section infers that the line of action for the resultant of the internal forces passes through the centroid of the section.
- A uniform distribution of stress is only possible if the concentrated loads on the end sections of two-force members are applied at the section centroids. This is referred to as *centric loading*.

- If a two-force member is *eccentrically loaded*, then the resultant of the stress distribution in a section must yield an axial force and a moment.
- The stress distributions in eccentrically loaded members cannot be uniform or symmetric.

Stress Analysis

1.b. Stress analysis for the BC member (except connection region)



Can this structure safely support the 30 kN load?

• From a statics analysis

 F_{AB} = 40 kN (compression)

- F_{BC} = 50 kN (tension)
- At any section through member BC (except connection region), the internal reaction force is 50 kN with a force intensity or stress of

$$\sigma_{BC} = \frac{P}{A} = \frac{50 \times 10^3 \,\mathrm{N}}{314 \times 10^{-6} \,\mathrm{m}^2} = 159 \,\mathrm{MPa}$$



• From the material properties for steel, the allowable stress is

 $\sigma_{\rm all} = 165 \, {\rm MPa}$

• Conclusion: the strength of member *BC* is adequate

Design

1.c. Design for the BC member (Changing material from steel to aluminum)



For reasons based on cost, weight, availability, etc., the choice is made to construct the rod from aluminum (σ_{all} = 100 MPa). What is an appropriate choice for the rod diameter?

$$\sigma_{all} = \frac{P}{A} \qquad A = \frac{P}{\sigma_{all}} = \frac{50 \times 10^3 \,\text{N}}{100 \times 10^6 \,\text{Pa}} = 500 \times 10^{-6} \,\text{m}^2$$
$$A = \pi \frac{d^2}{4}$$
$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(500 \times 10^{-6} \,\text{m}^2)}{\pi}} = 2.52 \times 10^{-2} \,\text{m} = 25.2 \,\text{mm}$$

An aluminum rod 26 mm or more in diameter is adequate

Shearing Stress (τ)



- Force *P* is applied transversely to the member.
- Corresponding internal forces act in the plane of section A and are called *shearing* forces.



- The resultant of the internal shear force distribution is defined as the *shear* of the section and is equal to the load *P*.
- The corresponding average shear stress is,

$$\tau_{\rm ave} = \frac{P}{A}$$

Fig. 1.15 This shows the resulting internal shear force on a section between transverse forces.

Shearing Stress Examples







Shear Loading from Hole Punch



Shearing Stress Examples



(a) Diagram of bolt in single shear; (b) section E-E' of the bolt.



(a) Diagram of bolt in double shear; (b) section K-K' and L-L' of the bolt.

Bearing Stress in Connections



Fig. 1.20 Equal and opposite forces between plate and bolt, exerted over bearing surfaces.



Fig. 1.21 Dimensions for calculating bearing stress area.

• Bolts, rivets, and pins create stresses on the points of contact or *bearing surfaces* of the members they connect.

• The resultant of the force distribution on the surface is equal and opposite to the force exerted on the pin.

• Corresponding average force intensity is called the *bearing stress*,





Linkage in Tension and Compression



Force is affecting the hatched area under tension. For this case, circular part is also checked.



Force is affecting the hatched area under compression

Stress Analysis & Design Example

1.d. A complete Stress analysis for the AC and BC member (including connection region)



Fig. 1.22 Components of boom used to support 30 kN load.

Would like to determine the stresses in the members and connections of the structure shown.

From a statics analysis: F_{AB} = 40 kN (compression) F_{BC} = 50 kN (tension)

Must consider maximum normal stresses in *AB* and *BC*, and the shearing stress and bearing stress at each pinned connection

Rod & Boom Normal Stresses



- The rod is in tension with an axial force of 50 kN.
- At the rod center, the average normal stress in the circular cross-section ($A = 314 \times 10^{-6} \text{m}^2$) is $\sigma_{BC} = +159 \text{ MPa}$.
- At the flattened rod ends, the smallest cross-sectional area occurs at the pin centerline,

 $A = (20 \text{ mm})(40 \text{ mm} - 25 \text{ mm}) = 300 \times 10^{-6} \text{ m}^2$

$$\sigma_{BC,end} = \frac{P}{A} = \frac{50 \times 10^3 N}{300 \times 10^{-6} m^2} = 167 \text{ MPa}$$

- The boom is in compression with an axial force of 40 kN and average normal stress of -26.7 MPa.
- The minimum area sections at the boom ends are unstressed since the boom is in compression.

Pin Shearing Stresses



• The cross-sectional area for pins at A, B, and C,

$$A = \pi r^2 = \pi \left(\frac{25\,\mathrm{mm}}{2}\right)^2 = 491 \times 10^{-6}\,\mathrm{m}^2$$

• The force on the pin at C is equal to the force exerted by the rod BC,

$$\tau_{C,ave} = \frac{P}{A} = \frac{50 \times 10^3 \,\mathrm{N}}{491 \times 10^{-6} \,\mathrm{m}^2} = 102 \,\mathrm{MPa}$$

• The pin at *A* is in double shear with a total force equal to the force exerted by the boom *AB*,

$$\tau_{A,ave} = \frac{P}{A} = \frac{20 \,\mathrm{kN}}{491 \times 10^{-6} \,\mathrm{m}^2} = 40.7 \,\mathrm{MPa}$$

Pin Shearing Stresses



• Divide the pin at *B* into sections to determine the section with the largest shear force,

 $P_E = 15 \,\mathrm{kN}$ $P_G = 25 \,\mathrm{kN}$ (largest)

• Evaluate the corresponding average shearing stress,

$$\tau_{B,ave} = \frac{P_G}{A} = \frac{25 \,\mathrm{kN}}{491 \times 10^{-6} \,\mathrm{m}^2} = 50.9 \,\mathrm{MPa}$$

Pin Bearing Stresses



To determine the bearing stress at A in the boom AB, we have t = 30 mm and d = 25 mm,

$$\sigma_b = \frac{P}{td} = \frac{40 \,\text{kN}}{(30 \,\text{mm})(25 \,\text{mm})} = 53.3 \,\text{MPa}$$

To determine the bearing stress at A in the bracket, we have t = 2(25 mm) = 50 mm and d = 25 mm,



$$\sigma_b = \frac{P}{td} = \frac{40 \,\text{kN}}{(50 \,\text{mm})(25 \,\text{mm})} = 32.0 \,\text{MPa}$$

Stress in Two Force Members



• Axial forces on a two-force member result in only normal stresses on a plane cut perpendicular to the member axis.

• Transverse forces on bolts and pins result in only shear stresses on the plane perpendicular to bolt or pin axis.

• We will show that either axial or transverse forces may produce both normal and shear stresses with respect to a plane other than one cut perpendicular to the member axis.

Stress on an Inclined Plane



- Pass a section through the member forming an angle θ with the normal plane.
- From equilibrium conditions, the distributed forces (stresses) on the plane must be equivalent to the force *P*.
- Resolve *P* into components normal and tangential to the inclined section,

 $F = P\cos\theta$ $V = P\sin\theta$

• The average normal and shear stresses on the inclined plane are

$$\sigma = \frac{F}{A_{\theta}} = \frac{P\cos\theta}{A_0/\cos\theta} = \frac{P}{A_0}\cos^2\theta$$
$$\tau = \frac{V}{A_{\theta}} = \frac{P\sin\theta}{A_0/\cos\theta} = \frac{P}{A_0}\sin\theta\cos\theta$$

Maximum Stresses



• Normal and shearing stresses on an inclined plane

$$\sigma = \frac{P}{A_0} \cos^2 \theta \quad \tau = \frac{P}{A_0} \sin \theta \cos \theta$$

• The maximum normal stress occurs when the reference plane is perpendicular to the member axis,

$$\sigma_{\rm m} = \frac{P}{A_0} \quad \tau' = 0$$

• The maximum shear stress occurs for a plane at <u>+</u> 45° with respect to the axis,

$$\tau_m = \frac{P}{A_0} \sin 45 \cos 45 = \frac{P}{2A_0} = \sigma'$$

Stress Under General Loadings



- A member subjected to a general combination of loads is cut into two segments by a plane passing through *Q*
 - The distribution of internal stress components may be defined as,

$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F^x}{\Delta A}$$

$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta V_y^x}{\Delta A} \quad \tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta V_z^x}{\Delta A}$$

• For equilibrium, an equal and opposite internal force and stress distribution must be exerted on the other segment of the member.

State of Stress



Fig. 1.35 Positive resultant forces on a small element at point *Q* resulting from a state of general stress.

• Consider the moments about the *z* axis:

$$\sum M_{z} = 0 = (\tau_{xy} \Delta A)a - (\tau_{yx} \Delta A)a$$

$$\tau_{xy} = \tau_{yx}$$

similarly, $\tau_{yz} = \tau_{zy}$ and $\tau_{yz} = \tau_{zy}$

• It follows that only 6 components of stress are required to define the complete state of stress

- Stress components are defined for the planes cut parallel to the *x*, *y* and *z* axes. For equilibrium, equal and opposite stresses are exerted on the hidden planes.
- The combination of forces generated by the stresses must satisfy the conditions for equilibrium:

$$\sum F_x = \sum F_y = \sum F_z = 0$$

$$\sum M_x = \sum M_y = \sum M_z = 0$$





Stress Matrix =
$$|\sigma| = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{bmatrix}$$



Normal stress is indicated with this symbol



Shearing stress is indicated with this symbol



These indices show the **axis parallel to the normal of the surface on which the stress acts**.



These indices show the **axis to which the <u>stress</u> <u>is parallel.</u>**

Allowable Load and Allowable Stress: Factor of Safety

The maximum load that a structural member or a machine component will be allowed to carry under normal conditions is considerably smaller than the ultimate load. This smaller load is the *allowable load* (sometimes called the *working* or *design load*).

The ratio of the ultimate load to the allowable load is used to define the *factor of safety*:



Factor of safety = $F.S. = \frac{\text{ultimate load}}{\text{allowable load}}$

Factor of safety = $F.S. = \frac{\text{ultimate stress}}{\text{allowable stress}}$

Structural members or machines must be designed such that the working stresses are less than the yield strength of the material.





1. Using free-body diagram of bracket:

+ $\gamma \Sigma M_C = 0$:	<i>P</i> (0.6 m) - (50 kN)(0.3 m) -	(15 kN)(0.6 m) = 0	P = 40 kN
$\Sigma F_x = 0$:	$C_x = 40 \text{ kN}$		
$\Sigma F_y = 0$:	$C_y = 65 \text{ kN}$	$C = \sqrt{C_x^2 + C_y^2} =$	76.3 kN



Free-body diagram of bracket.



Fig. 2 Free-body diagram of pin at point *C*.



Fig. 3 Bearing loads at bracket support at point *C*.

a. Control Rod *AB*. Since the factor of safety is 3.3, the allowable stress is

$$\sigma_{\rm all} = \frac{\sigma_U}{F.S.} = \frac{600 \text{ MPa}}{3.3} = 181.8 \text{ MPa}$$

For P = 40 kN, the cross-sectional area required is

$$A_{\rm req} = \frac{P}{\sigma_{\rm all}} = \frac{40 \,\rm kN}{181.8 \,\rm MPa} = 220 \times 10^{-6} \,\rm m^2$$
$$A_{\rm req} = \frac{\pi}{4} \,d_{AB}^2 = 220 \times 10^{-6} \,\rm m^2 \qquad d_{ab} = 16.74 \,\rm mm$$

b. Shear in Pin C. For a factor of safety of 3.3, we have

$$\tau_{\rm all} = \frac{\tau_U}{F.S.} = \frac{350 \text{ MPa}}{3.3} = 106.1 \text{ MPa}$$

As shown in Fig. 2 the pin is in double shear. We write

$$A_{\rm req} = \frac{C/2}{\tau_{\rm all}} = \frac{(76.3 \text{ kN})/2}{106.1 \text{ MPa}} = 360 \text{ mm}^2$$
$$A_{\rm req} = \frac{\pi}{4} d_C^2 = 360 \text{ mm}^2 \qquad d_C = 21.4 \text{ mm} \qquad \text{Use: } d_C = 22 \text{ mm}$$

c. Bearing at C. Using d = 22 mm, the nominal bearing area of each bracket is 22t. From Fig. 3 the force carried by each bracket is C/2 and the allowable bearing stress is 300 MPa. We write

$$A_{\rm req} = \frac{C/2}{\sigma_{\rm all}} = \frac{(76.3 \text{ kN})/2}{300 \text{ MPa}} = 127.2 \text{ mm}^2$$

Thus, $22t = 127.2$ $t = 5.78 \text{ mm}$ **Use:** $t = 6 \text{ mm}$



EXERCISE 1.

The 1.4-kip load **P** is supported by two wooden members of uniform cross section that are joined by the simple glued scarf splice shown. Determine the normal and shearing stresses in the glued splice.

EXERCISE 2.

Two wooden members of uniform cross section are joined by the simple scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 75 psi, determine (a) the largest load **P** that can be safely supported, (b) the corresponding shearing stress in the splice.

A 600 lb/ft 35° B C D 5 kips 1.4 ft 1.4 ft 1.4 ft

Link *AB* is to be made of a steel for which the ultimate normal stress is 65 ksi. Determine the cross-sectional area of *AB* for which the factor of safety will be 3.20. Assume that the link will be adequately reinforced around the pins at *A* and *B*.

EXERCISE 4.

Link *BC* is 6 mm thick, has a width w = 25 mm, and is made of a steel with a 480-MPa ultimate strength in tension. What is the factor of safety used if the structure shown was designed to support a 16-kN load **P**?

EXERCISE 3.

