ELECTROSTATIC GENERATOR

Electrostatic generators convert mechanical energy into the electrical energy directly. The electric charges are moved against the force of electric fields, thereby higher potential energy is gained at the cost of mechanical energy.

\[
\begin{align*}
\text{An insulated belt is moving with uniform velocity } v \text{ in an electric field of strength } E(x). \text{ The width of the belt is } b \text{ and the charge density } \sigma. \text{ Consider a length } dx \text{ of the belt, the charge } dq = \sigma b dx. \\
\text{The force experienced by this charge } \\
F = \int \sigma b E dx \quad \Rightarrow \quad F = \sigma b E v \\
\text{The power required to move the belt } \\
= \text{Force } \times \text{ Velocity} \\
= F \cdot v = \sigma b E v
\end{align*}
\]
\[ \text{Current} \rightarrow I = \frac{dq}{dt} - \sigma_b \frac{dx}{dt} = \sigma b V \]

The power required to move the belt?

\[ P = Fv = \sigma b Vv = V \cdot I. \]

Assuming no losses, the power output is also equal to VI.

Van de Graaf Generator

- Upper pulley (made of metal, insulated from earth)
- Motor-driven pulley
- Rubber belt
- Metal point
- Lower pulley (made of insulating material such as plastic or resin)
- Ground

Generally used in physics laboratories for particle acceleration and other processes in research.
He developed Van de Graaff Generator in 1931.

An insulating belt is run over pulleys.

The belt width → a few cm → metres

Belt speed → up to 30 m/sec. by a motor connected to lower pulley.

→ The belt near the lower pulley is charged electrostatically by an excitation arrangement.

→ The charge is conveyed to the upper end where it is collected from the belt by discharging points connected to the inside of an insulated metal electrode through which the belt passes.

→ The entire equipment is enclosed in an coated metal tank filled with SF6.

→ The potential of metal sphere can be increased up to a few million volts (for example 6 MV)

Range: 20 million volts.

The purpose of metal sphere is to make electric field uniform to reduce corona discharges.

→ When the upper metal point collects charges its potential rises. The potential at any instant is given as $V = \frac{q}{C}$ where $q →$ charge

$C →$ capacitance
If \( E_r = 1 \) (air) and \( E_0 = 8.85 \times 10^{-12} \text{ F/m} \),
\[
\vec{E} = D = E_0 E = \left(8.85 \times 10^{-12}\right) (30,000,000) \quad E = 30 \text{ kV/m}
\]

\[
\sigma = \text{surface flux charge density}
\]

\[
\sigma = 26.562 \times 10^{-6} \text{ C/m}^2
\]

If \( b = 1 \text{ meter} \) and belt velocity \( v = 10 \text{ m/sec} \),

we can calculate the current supplied by the generator \( I \)

\[
I = \sigma b v \quad \text{(previously derived)}
\]

\[
I = 26.562 \times 10^{-6} \times 1 \times 10 \quad A
\]

\[
= 0.265 \text{ mA}
\]

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**The Advantages of Vande Graaf Generator**

1. Very high voltages, even in few million volts, can be easily generated.
2. Ripple free output.
3. Precision and flexibility in the control.

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**The Disadvantages of Vande Graaf Generator**

1. Low current output (upto few mA).
2. Physical limitation on belt velocity due to its tendency for vibration.
A Vande Graaf generator is given with the following parameters:

- Belt width → \( b = 0.9 \text{ m} \)
- Belt speed → \( v = 12 \text{ m/s} \)

(a) Calculate the surface charge density \( \sigma \) in \( \text{C/m}^2 \) if the generated field on the metal sphere is 23 \( \text{kV/cm} \).

(b) Calculate the current supplied by the generator.

**Solution**

\[ \sigma = \epsilon_0 E = \left( 8.85 \times 10^{-12} \text{ F/m} \right) \left( \frac{2300000 \text{ V}}{\text{m}} \right) \]

\[ \sigma = \frac{2.03642 \times 10^{-5} \text{ C/m}^2}{\text{surface charge density}} \]

Since \( I = \frac{dQ}{dt} \text{ Coulomb/second} \),

\[ I = \sigma b v = \left( 2.03642 \times 10^{-5} \text{ C/m}^2 \right) \left( \frac{0.9}{5} \right) \left( \frac{12}{5} \right) \]

\[ = 2.193336 \times 10^{-4} \text{ A} \]

or

\[ = 2.193336 \mu \text{A} \]

(Vande Graaf videos!)
GENERATION OF HIGH AC VOLTAGES

→ Generally single-phase transformer operating at power frequency (50 or 60 Hz) is the most common for testing HV AC equipment.

→ Test transformer normally has low power rating with high voltage ratings, used for generally short-time tests.

→ The currents required for the test on various equipment are shown below:

  - Insulator, CB, bushing, instrument transformer → 0.1 – 0.5 A
  - Power transformer, HV capacitor → 0.5 – 1.0 A
  - Cables → 1 A and above.

→ The design of a test transformer is similar to a potential transformer, but with low flux density to avoid saturation of the core that produces large harmonics.

[CASCaded TRANSformERS]

→ For voltages higher than 600 kV, it is desired to cascade two or more transformers. Because of modularity and easy transport and low cost.

(Thin iron insulating expenses for very-high voltages)
The cost of single-phase 800 kV transformer > The cost of single-phase 400 kV transformer + The cost of single-phase 1600 kV transformer

The following figure shows a basic scheme for cascading three transformers.

The main disadvantage of cascading the transformers is that the lower stages of the transformers are loaded more as compared with the upper stages.
p: primary
s: secondary
t: tertiary
x: short-circuit reactance

Equivalent circuit of 3-stage transformer:

After simplification:

\[ v_1 = \left(3N_s / N_p \right) v_1 \text{(actual)} \]

A simplified equivalent circuit:

\[ x_{res} = \sum_{i=1}^{n} \left( \frac{n-i+1}{2} \right) x_{pi}^2 + x_{s1}^2 + (i-1) x_{t1}^2 \]

SC: shunt reactance

Stage of primary, secondary, tertiary

Relevant stage of primary, secondary, tertiary

Relevant reactance number of the \( i \)-th transformer

Relevant reactance number of the \( i \)-th transformer

Relevant reactance number of the \( i \)-th transformer

Relevant reactance number of the \( i \)-th transformer
Assume the output is connected to a load.
If losses are ignored then:

\[ P_{in} = P_{out} \]

(using equivalent oct 3-stage transformer)

If losses are not ignored then:

\[ P_{in} = P_{out} + P_{losses} \]

\[ 3V_1I_n = I_{V_2} \]

\[ \cos \theta_2 \]

If load is generally capacitive:

If heating of the tested component is ignored:

By referring to simplified eq. 1.66:
Assuming the output is connected to the load:

\[ V_1 - jX_{res}I = V_2 = 0 \]

or

\[ V_1' = V_2 + jX_{res}I \]

Since

\[ V_1' = \frac{3N_S}{N_P} V_1 \]

Actual output voltage

\[ V_1 = \left( \frac{N_P}{3N_S} \right) \left[ V_2 + jX_{res}I \right] \]

Actual input voltage

\[ V_1 = \frac{N_P}{3N_S} \]

Number of sec. times.
The reactive power produced by the load (tested component) should be compensated by means of the inductors or the tap-changing settings of the transformers to avoid excess amounts of voltage because of leading test current.

The reactive power generated by the tested component is

\[ \mathcal{Q}_{\text{GEN}} = \frac{V_2^2}{X_c} \]

or

\[ \mathcal{Q}_{\text{GEN}} = (I_1)^2 X_c \]

Current of the tested component.