# EEE270
# Introduction to Electrical Energy Systems

**Lecture 6 – Single-line Diagrams and Per-Unit Calculations**

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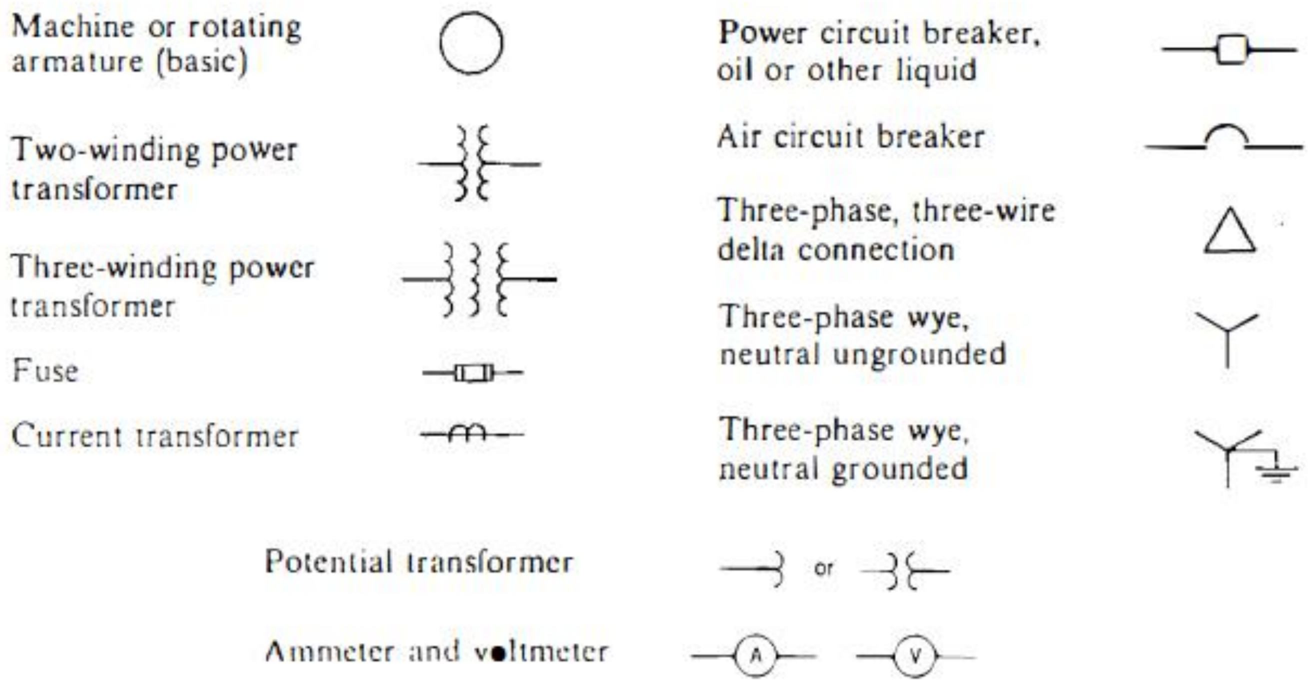
# SINGLE-LINE (ONE-LINE DIAGRAMS)

- Since a balanced three-phase system is always solved as a single-phase or per-phase equivalent circuit composed of one of the three lines and a neutral return, it is seldom necessary to show more than one phase and the neutral return when drawing a diagram of the circuit.
- Often the diagram is simplified further *by omitting the completed circuit* through the neutral and by indicating the component parts by standard symbols rather than by their equivalent circuits.
- Circuit parameters are not shown, and **a transmission line is represented by a single line between its two ends**. Such a simplified diagram of an electric system is called a **single-line** *or* **one-line diagram** .
- Single-line diagram is indicated by a **single line** and **standard symbols and** show how the transmission lines and associated apparatus of the electric system are connected together.
- The purpose of the one-line diagram is to supply in concise form the significant information about the system.
- The importance of different features of a system varies with the problem under consideration, and the amount of information included on the diagram depends on the purpose for which the diagram is intended.

# SINGLE-LINE (ONE-LINE DIAGRAMS)

- The **American National Standards Institute (ANSI)** and the **Institute of Electrical and Electronics Engineers (IEEE)** have published a set of standard symbols for single-line diagrams as shown below:




Source: Power System analysis by William Grainger, John Stevenson

# SINGLE-LINE (ONE-LINE DIAGRAMS)

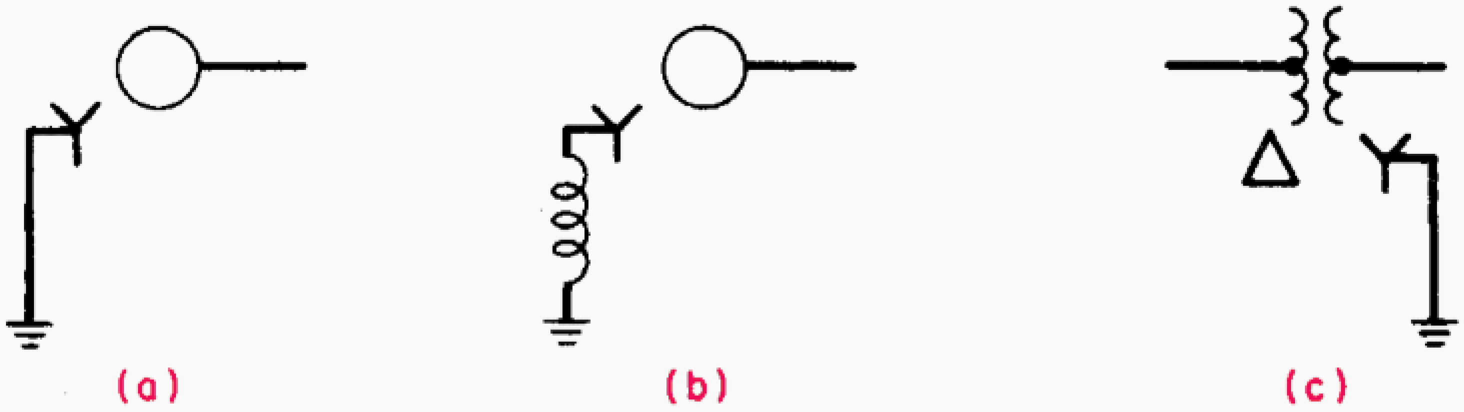
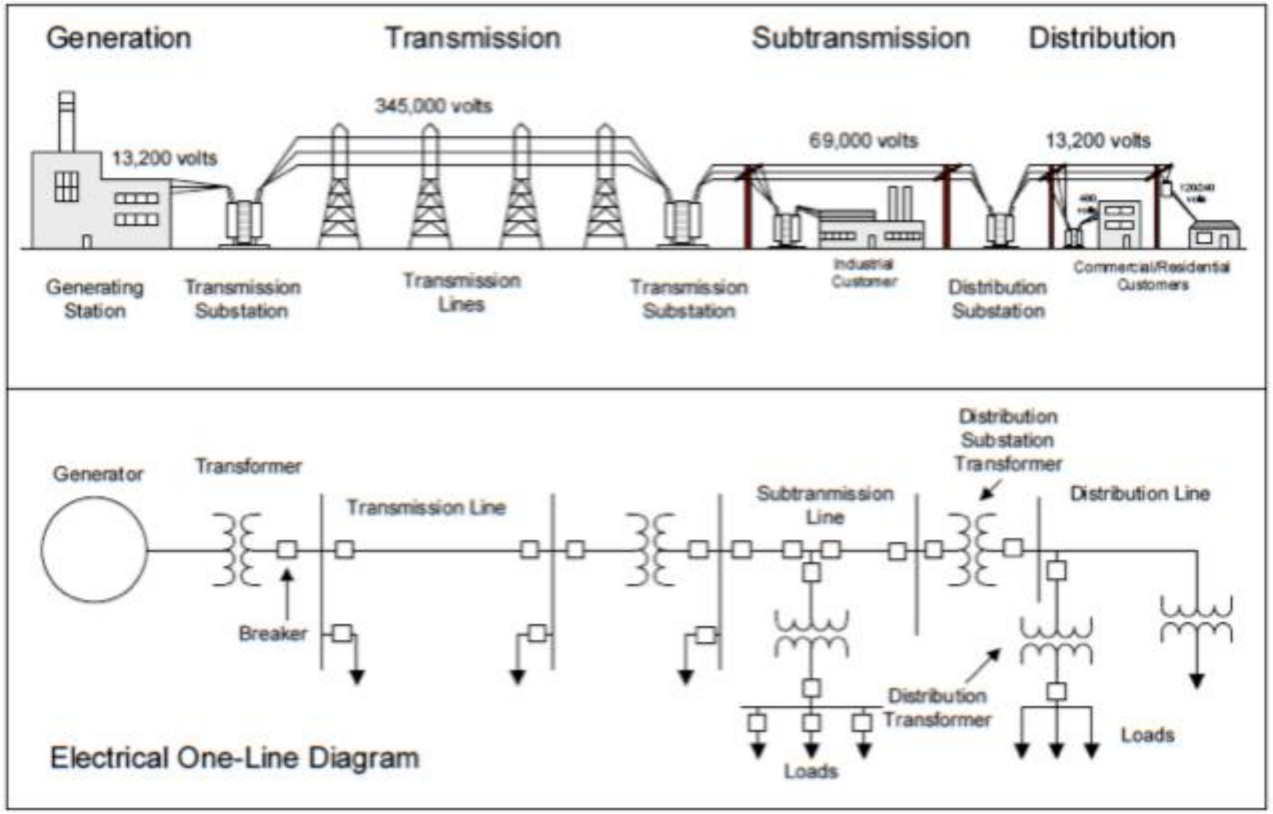



Figure 3 – Identification for wye-connected generator or motor. (a) Solidly grounded. (b) Grounded through an inductance. (c) The transformer is identified as being delta-wye, with the wye side solidly grounded.

Source: https://electrical-engineering-portal.com/calculate-draw-single-line-diagram-power-system

# SINGLE-LINE (ONE-LINE DIAGRAMS)




A single-line diagram of a typical electric power system

Source: Keller, J. & Kroposki, Benjamin. Understanding Fault Characteristics of Inverter-Based Distributed Energy Resources. 10.2172/971441.

# IMPEDANCE AND REACTANCE DIAGRAMS


FIGURE 1.25
Single-line diagram of an electrical power system.

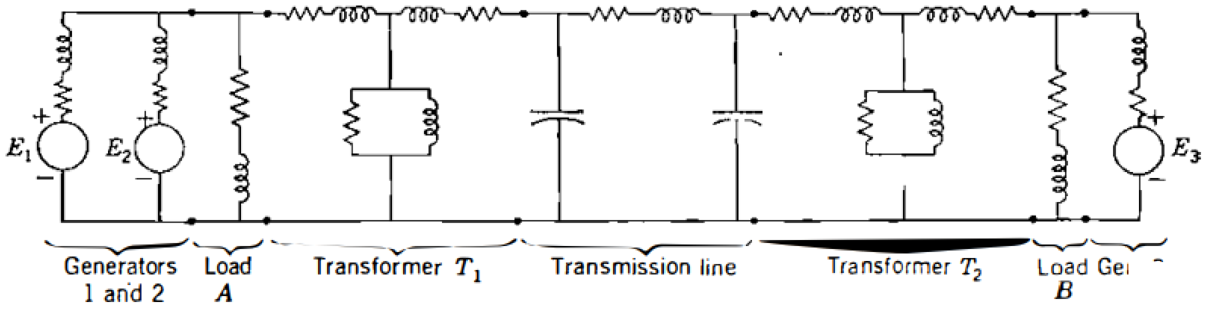



FIGURE 1.26
The per-phase impedance diagram corresponding to the single-line diagram of Fig. 1.25.

To calculate current and voltage of the components in steady-state, **the per-phase impedance diagram** is generally used.

**Reactance diagram** is generally used for **fault-calculations**; the resistance is ignored.

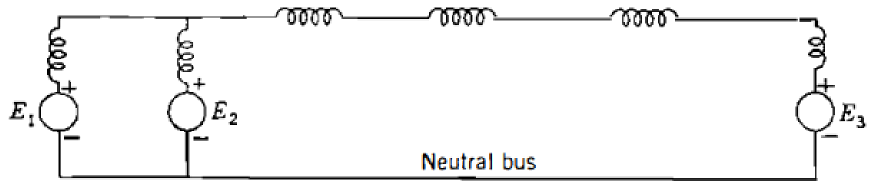



FIGURE 1.27
Per-phase reactance diagram adapted from Fig. 1.26 by omitting all loads, resistances, and shunt admittances.

Source: Power System analysis by William Grainger, John Stevenson

# Per-unit Calculations

- Power transmission lines are operated at voltage levels where the **kilovolt (kV)** is the most convenient unit to express voltage.
- Due to the large amount of power transmitted, terms like **kilowatts (kW), megawatts (MW), kilovolt-amperes, and megavolt-amperes (MVA)** are commonly used.
- These quantities, along with amperes and ohms, are often expressed as a **percent** or **per unit of a base** or reference value specified for each case.
- Per unit value of any quantity is defined as "*the ratio of actual value to the chosen base value in the same unit*".

$$Perunit\ value = \frac{Actual\ value\ in\ any\ unit}{Base\ value\ in\ the\ same\ unit}$$

- For example, if a **base voltage** of **120 kV** is chosen, the voltages of **108 kV, 120 kV, and 126 kV** become **0.90, 1.00, and 1.05 per unit**, respectively (*or* 90%, 100%, and 105%).

# Advantages of per-unit system

- Per-unit data representation yields valuable relative magnitude information.
- Circuit analysis of systems containing transformers of various transformation ratios is greatly simplified.
- The per-unit systems are ideal for the computerized analysis and simulation of complex power system problems.
- Manufacturers usually specify the impedance values of equivalent in per-unit of the equipment rating. If the any data is not available, it is easier to assume its per-unit value than its numerical value. For example, the reactance of a typical three-phase transformer is around 10-15% or 0.1-0.15 pu.
- The ohmic values of impedances are referred to secondary is different from the value as referee to primary. However, if base values are selected properly, the per-unit impedance is the same on the two sides of the transformer.
- The circuit laws are valid in per-unit systems, and the power and voltages equations are simplified since the factors of $\sqrt{3}$ and 3 are eliminated.
- The per-unit calculation is simpler and often more informative than using actual amperes, ohms, and volts directly.

- The equations to calculate the ***base quantities*** are shown below:

$$\text{Base current, A} = \frac{\text{base kVA}_{1\phi}}{\text{base voltage, kV}_{LN}} \tag{1.44}$$

$$\text{Base impedance, }\Omega = \frac{\text{base voltage, V}_{LN}}{\text{base current, A}} \tag{1.45}$$

$$\text{Base impedance, }\Omega = \frac{(\text{base voltage, kV}_{LN})^2 \times 1000}{\text{base kVA}_{1\phi}} \tag{1.46}$$

$$\text{Base impedance, }\Omega = \frac{(\text{base voltage, kV}_{LN})^2}{\text{MVA}_{1\phi}} \tag{1.47}$$

$$\text{Base power, kW}_{1\phi} = \text{base kVA}_{1\phi} \tag{1.48}$$

$$\text{Base power, MW}_{1\phi} = \text{base MVA}_{1\phi} \tag{1.49}$$

$$\text{Per-unit impedance of an element} = \frac{\text{actual impedance, }\Omega}{\text{base impedance, }\Omega} \tag{1.50}$$

# Per-unit Calculations

A numerical example clarifies the relationships. For instance, if

$$\text{Base kVA}_{3\phi} = 30{,}000 \text{ kVA}$$

and

$$\text{Base kV}_{LL} = 120 \text{ kV}$$

where subscripts $_{3\phi}$ and $_{LL}$ mean "three-phase" and "line to line," respectively,

$$\text{Base kVA}_{1\phi} = \frac{30{,}000}{3} = 10{,}000 \text{ kVA}$$

and

$$\text{Base kV}_{LN} = \frac{120}{\sqrt{3}} = 69.2 \text{ kV}$$

For an actual line-to-line voltage of 108 kV in a balanced three-phase set the line-to-neutral voltage is $108/\sqrt{3} = 62.3$ kV, and

$$\text{Per-unit voltage} = \frac{108}{120} = \frac{62.3}{69.2} = 0.90$$

For total three-phase power of 18,000 kW the power per phase is 6000 kW, and

$$\text{Per-unit power} = \frac{18{,}000}{30{,}000} = \frac{6{,}000}{10{,}000} = 0.6$$

# CHANGING THE BASE OF PER-UNIT QUANTITIES

- Sometimes the per-unit impedance of a component of a system is expressed on a base other than the one selected as base for the part of the system in which the component is located.
- Since all impedances in any one part of a system must be expressed on the same impedance base when making computations, it is necessary to have a means of converting per-unit impedances from one base to another.
- To change from *per-unit impedance on a given base* to *per-unit impedance on a new base*, the following equation is used:

$$\text{Per-unit } Z_{new} = \text{per-unit } Z_{given} \left( \frac{\text{base kV}_{given}}{\text{base kV}_{new}} \right)^2 \left( \frac{\text{base kVA}_{new}}{\text{base kVA}_{given}} \right)$$

# CHANGING THE BASE OF PER-UNIT QUANTITIES

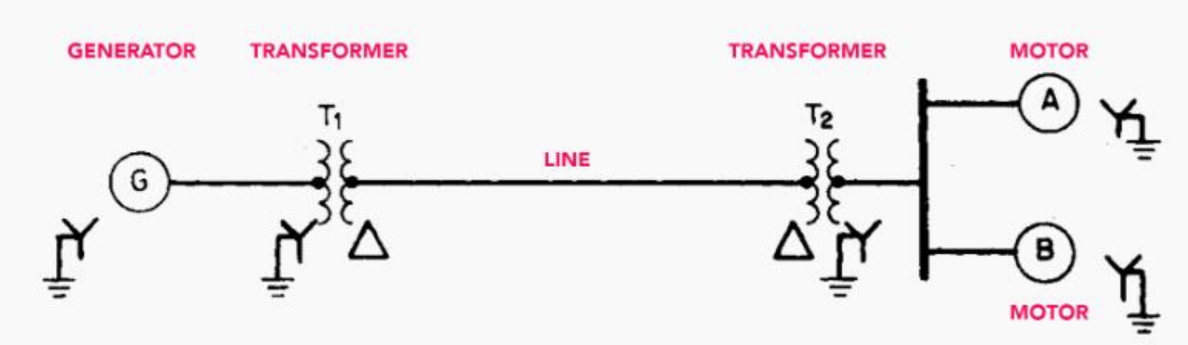
**Example 1.5.** The reactance of a generator designated $X''$ is given as 0.25 per unit based on the generator's nameplate rating of 18 kV, 500 MVA. The base for calculations is 20 kV, 100 MVA. Find $X''$ on the new base.

***Solution.*** By Eq. (1.56)

$$X'' = 0.25\left(\frac{18}{20}\right)^2\left(\frac{100}{500}\right) = 0.0405 \text{ per unit}$$

# CHANGING THE BASE OF PER-UNIT QUANTITIES

**EXAMPLE:** For the power system shown in the figure, draw the **pu reactance diagram**, with all reactances marked in per-unit (p.u.) values. Choose **25000 kVA** as the base S, and as the base voltage at generator side. **13.8 kV.**




| Generator | Transformers (each) | Motor A | Motor B | Transmission Line |
|---|---|---|---|---|
| 13.8kV | 25,000 kVA | 15,000 kVA | 10,000 kVA | – |
| 25,000 kVA 3-phase | 13.2/69 kV | 13.0 kV | 13.0 kV | – |
| X" = 15 percent | $X_L$ = 11 percent | X" = 15 percent | X" = 15 percent | X = 65 Ω |

Source: https://electrical-engineering-portal.com/calculate-draw-single-line-diagram-power-system

## Solution:

### 1. Establish Base Voltage through the System

By observation of the magnitude of the components in the system, a base value of **apparent power S** is chosen. It should be of the general magnitude of the components, and the choice is arbitrary. In this problem, **25,000 kVA is chosen as the base S**, and simultaneously, at the generator end **13.8 kV is selected as a base voltage $V_{base}$**.

The base voltage of the transmission line is then determined by the turns ratio of the connecting transformer:
(13.8 kV)(69 kV / 13.2 kV) = **72.136 kV**

**The base voltage of the motors is determined likewise but with the 72.136 kV value, thus:**
(72.136 kV)(13.2 kV / 69 kV) = **13.8 kV**

The selected base S value remains constant throughout the system, **but the base voltage is 13.8 kV at the generator and at the motors, and 72.136 kV on the transmission line**.

## 2. Calculate the Generator Reactance

No calculation is necessary for correcting the value of the generator reactance because it is given as **0.15 p.u. (15 percent)**, based on **25,000 kVA** and **13.8 kV**. If a different **S base** were used in this problem, then a correction would be necessary as shown for the transmission line, electric motors, and power transformers.

## 3. Calculate the Transformer Reactance

It is necessary to make a correction when the transformer nameplate reactance is used because the calculated operation is at a different voltage, 13.8 kV / 72.136 kV instead of 13.2 kV / 69 kV.

**Use the equation for correction: per-unit reactance:**

(nameplate per-unit reactance) (base kVA/nameplate kVA) (nameplate kV/base kV)$^2$ = (0.11) (25,000/25,000) (13.2/13.8)$^2$ = **0.101 p.u**.

**This applies to each transformer.**

## 4. Calculate the Transmission-Line Reactance

Use the equation:

- $X_{per\ unit}$ = (ohms reactance)(base kVA)/(1000)(base kV)$^2$ =
- $\mathbf{X_{per\ unit}}$ **=** (65) (25,000)/(1000)(72.1)$^2$ = **0.313 p.u.**

## 5. Calculate the Reactance of the Motors

Corrections need to be made in the nameplate ratings of both motors because of differences of ratings in kVA and kV as compared with those selected for calculations in this problem. Use the correcting equation from Step 3, above.

**For motor A:**

$X''_A$ = (0.15 p.u.) (25,000 kVA / 15,000 kVA) (13.0 kV / 13.8 kV)$^2$ = **0.222 p.u.**

**For motor B:**

$X''_B$ = (0.15 p.u.)(25,000 kVA /10,000 kVA)(13.0 kV / 13.8 kV)$^2$ = **0.333 p.u.**

## 6. Draw the Reactance Diagram

The completed reactance diagram is shown in Figure 5:

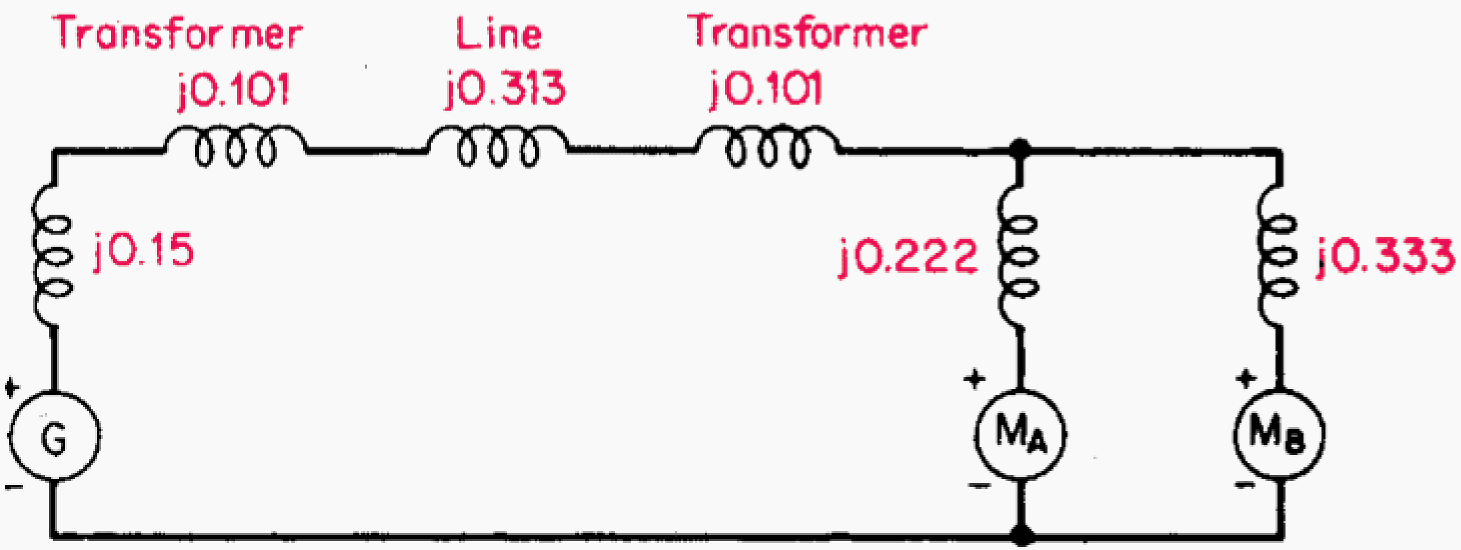



Figure 5 – Single line reactance circuit diagram (reactances shown on a per-unit basis)

# CHANGING THE BASE OF PER-UNIT QUANTITIES

**EXAMPLE:** Find the generator terminal voltage *in the previous example* assuming both motors operating at **12 kV**, **three-quarters load**, and **unity power factor**.

## 7. Calculate Operating Conditions of the Motors

If the motors are operating at 12 kV, this represents **12 kV/13.8 kV = 0.87 per-unit voltage**. At unity power factor, the load is given as three-quarters or 0.75 p.u.

> Thus, expressed in per unit, the combined motor current is obtained by using the equation:
> $I_{per\ unit}$ = per-unit power/per-unit voltage = 0.75/0.87 = **0.862 ∠0° p.u.**

## 8. Calculate the Generator Terminal Voltage

**The voltage at the generator terminals is:**

- $V_G = V_{motor}$ + the voltage drop through transformers and transmission line
- $V_G = 0.87\angle 0° + 0.862\ \angle 0°(j0.101 + j0.313 + j0.101)$
- $V_G = 0.87 + j0.444 =$ **0.977 ∠27.03° p.u.**

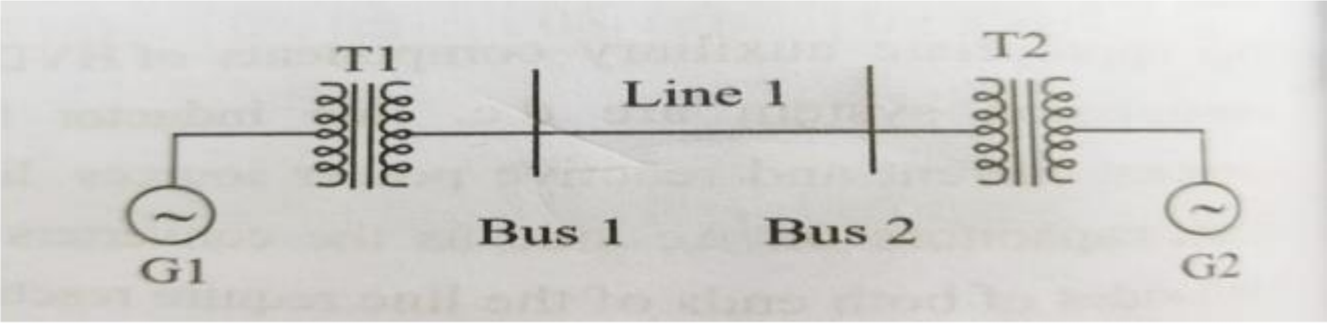
In order to obtain the actual voltage, multiply the per-unit voltage by the base voltage at the generator. Thus,

- $V_G = (0.977\ \angle 27.03°)\ (13.8\ kV) =$ **13.48 ∠27.03° kV**

# CHANGING THE BASE OF PER-UNIT QUANTITIES

SELF-STUDY: For the following power system network shown in the figure, find the new pu reactance for each element based on the new base values: **25 kV** as the base voltage at the generator G1 and **200 MVA** as the MVA base.

The specifications of the component are as follows:
G1: 25 kV, 100 MVA, X=9%
G2: 25 kV, 100 MVA, X=9%
T1: 25/220 kV, 90 MVA, X=12%
T2: 220/25 kV, 90 MVA, X=12%
Transmission Line: 220 kV, X=150 ohms.



**Solution:**

$$X_{G1} = j0.09 * \left(\frac{200}{100}\right) * \left(\frac{25}{25}\right)^2 = j0.18\, p.u.$$

$$X_{T1} = j0.12 * \left(\frac{200}{90}\right) * \left(\frac{25}{25}\right)^2 = j0.27\, p.u.$$

$$X_1 = X_{\Omega} * \frac{MVA_{base}}{(kV_b)^2}$$

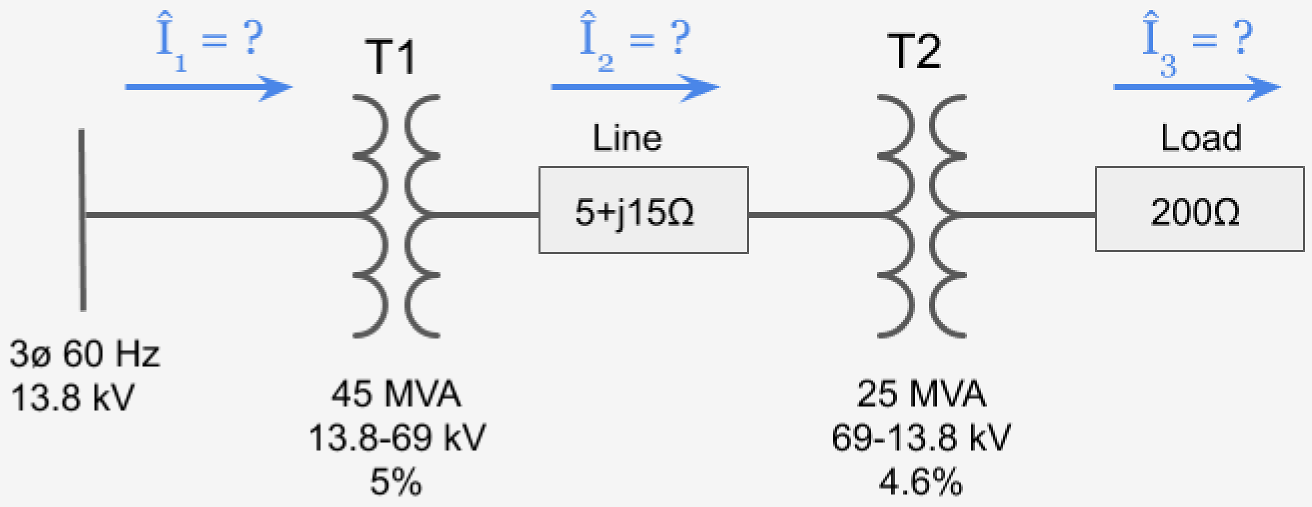
$$X_1 = j150 * \frac{200}{(220)^2} = j0.62\, p.u.$$

$$X_{T2} = X_{T1} = j0.27\, p.u.$$

$$X_{G2} = j0.09 * \left(\frac{200}{100}\right) * \left(\frac{25}{25}\right)^2 = j0.18\, p.u.$$

**SELF-STUDY:**

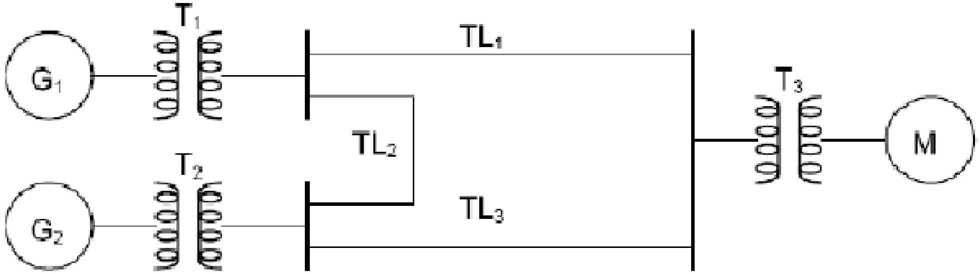
Using the Per Unit system and taking into account the transformer percent impedances, solve for the current in each part of the three-phase system shown below. Assume both transformers are either delta – delta or wye – wye connected and that there is no phase shift between primary and secondary current and voltage.




Take **V=13.8 kV** and **S=80 MVA** as new base values for the primary side of transformer T1.

## SELF-STUDY:

A one-line diagram of a three-phase power system is shown below. Draw the impedance diagram of the power system, and mark all impedances in per unit. Use a base of **100 MVA** and **138 kV** for the transmission lines. All transformers are connected to step up the voltage of the generators to the transmission line voltages. Calculate the terminal voltage of G2 (in pu) if G1 is out of service and the motor draws 50 MW at unity power factor with 1 pu voltage at its terminals.




| item | MVA | kV | $X_{pu}$ | item | MVA | kV | |
|---|---|---|---|---|---|---|---|
| $G_1$ | 45 | 13.2 | 0.15 | $T_3$ | 70 | 138 / 11.6 | 0.10 |
| $G_2$ | 55 | 18 | 0.12 | Line 1 | $Z_{TL} = j40\ \Omega$ | | |
| Motor | 75 | 11.6 | 0.23 | Line 2 | $Z_{TL} = j20\ \Omega$ | | |
| $T_1$ | 50 | 13.8 / 138 | 0.10 | Line 3 | $Z_{TL} = j15\ \Omega$ | | |
| $T_2$ | 60 | 19.05 / 138 | 0.10 | | | | |

**END OF THE LECTURE**

Any questions ?