Example on page: 83, Chapter 4:

Example: (magnetic field induction): A three-phase untransposed transmission line and a telephone line are supported on the same towers as shown in the figure. The power line carries 60 Hz balanced current of 200 A per phase. The telephone line is located directly below phase b. Assuming balanced three-phase currents in the power line, find the voltage per kilometer induced in the telephone line.

Solution:

\[ \lambda_{12}(I_a) = 0.2I_a \ln \frac{D_{a2}}{D_{a1}} \text{ mWb/km} \]

Since \( D_{b1} = D_{b2} \), \( \lambda_{12} \) due to \( I_b \) is zero. The flux linkage between conductors 1 and 2 due to current \( I_c \) is

\[ \lambda_{12}(I_c) = 0.2I_c \ln \frac{D_{c2}}{D_{c1}} \text{ mWb/km} \]
Total flux linkage between conductors 1 and 2 due to all currents is

$$\lambda_{12} = 0.2I_a \ln \frac{D_{a2}}{D_{a1}} + 0.2I_c \ln \frac{D_{c2}}{D_{c1}} \text{ mWb/km}$$

For positive phase sequence, with $I_a$ as reference, $I_c = I_a/\angle -240^\circ$ and we have

$$\lambda_{12} = 0.2I_a \left( \ln \frac{D_{a2}}{D_{a1}} + 1\angle -240^\circ \ln \frac{D_{c2}}{D_{c1}} \right) \text{ mH/km}$$

With $I_a$ as reference, the instantaneous flux linkage is

$$\lambda_{12}(t) = \sqrt{2} |\lambda_{12}| \cos(\omega t + \alpha)$$

Thus, the induced voltage in the telephone line per kilometer length is

$$v = \frac{d\lambda_{12}(t)}{dt} = \sqrt{2} \omega |\lambda_{12}| \cos(\omega t + \alpha + 90^\circ)$$

The rms voltage induced in the telephone line per kilometer is

$$V = \omega |\lambda_{12}| / \angle 90^\circ = j\omega \lambda_{12}$$

From the circuits geometry

$$D_{a1} = D_{c2} = (3^2 + 4^2)^{\frac{1}{2}} = 5 \text{ m}$$
$$D_{a2} = D_{c1} = (4.2^2 + 4^2)^{\frac{1}{2}} = 5.8 \text{ m}$$

The total flux linkage is

$$\lambda_{12} = 0.2 \times 200 \angle 0^\circ \ln \frac{5.8}{5} + 0.2 \times 200 \angle -240^\circ \ln \frac{5}{5.8}$$
$$= 10.283 \angle -30^\circ \text{ mWb/km}$$

The voltage induced in the telephone line per kilometer is

$$V = j\omega \lambda_{12} = j2\pi 60 (10.283 \angle -30^\circ)(10^{-3}) = 3.88/60^\circ \text{ V/km}$$