EEE270
Introduction to Electrical Energy Systems

Lecture 2 - SINUSOIDAL STEADY-STATE ANALYSIS

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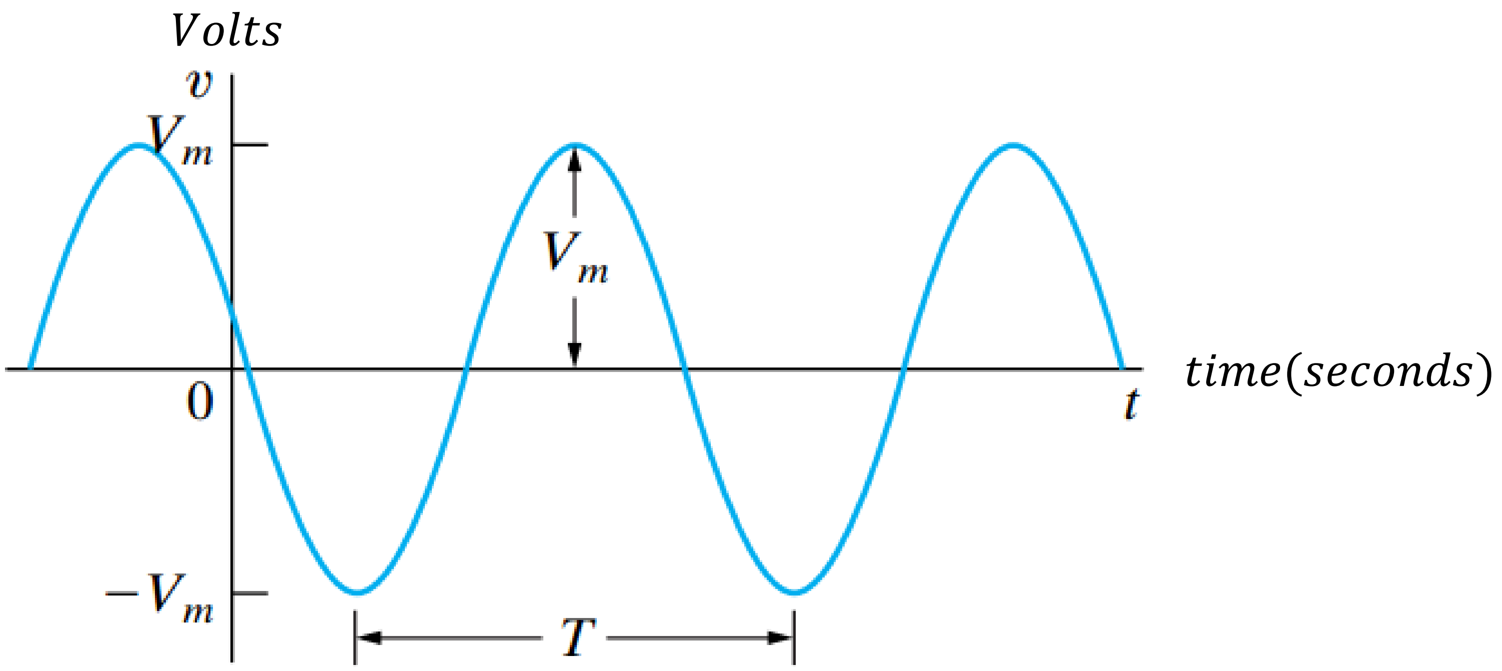
The Sinusoidal Source

- A sinusoidal voltage source (independent or dependent) produces a voltage that varies sinusoidally with time.
- A sinusoidally varying function (*voltage or current*) can be expressed with either the *sine function* or the *cosine function*.
- We will use ***cosine function*** throughout our discussion.
- A sinusoidal voltage waveform can be expressed as

$$v(t) = V_m \cos(\omega t + \varphi)$$

The Sinusoidal Source

$$v = V_m \cos(\omega t + \varphi)$$



$$\varphi = \frac{3\pi}{2} \text{ or } 270°$$

The Sinusoidal Source

$v = V_m \cos(\omega t + \varphi)$

$V_m \rightarrow maximum\ amplitude\ (peak)$

$\omega \rightarrow angular\ frequency\ (\frac{radians}{seconds})$

$\omega = 2\pi f$ where $f = \frac{1}{T}$

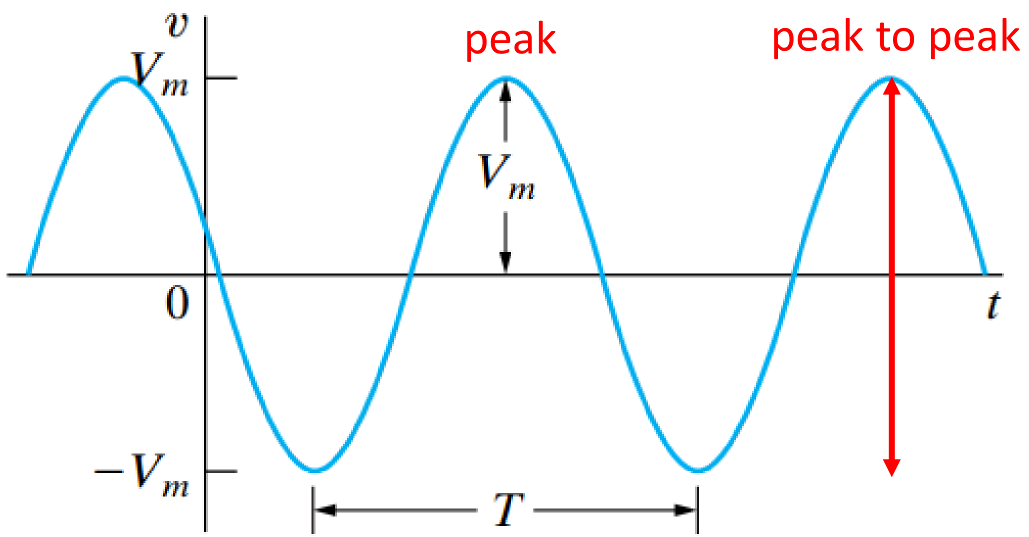
f$\rightarrow frequency(Hertz, Hz)$

$T \rightarrow period\ or\ cycle\ (seconds)$

$\varphi \rightarrow phase\ angle\ (radians)$

$2V_m \rightarrow peak\ to\ peak$

$V_{rms} = V_{peak}/\sqrt{2} \rightarrow$ **root mean square (RMS) value**



Example:

A sinusoidal current has a maximum amplitude of 20 A. The current passes through one complete cycle in 1 ms. The magnitude of the current at zero time is 10 A.

Find,

a) The frequency of the current waveform.
b) The angular frequency of the current waveform.
c) The time-domain expression for this current waveform.
d) The rms value of the current waveform.

Solution:

a) From the statement of the problem, T=1ms, hence
$f = 1/T = 1000Hz$

b) $w = 2\pi f = 2000\pi$ rad/s

c) We have $i(t) = I_m \cos(\omega t + \varphi)$= $20\cos(2000\boldsymbol{\pi} t + \varphi)$ but $i(0) = 10A$.
Therefore $10 = 20\cos(\varphi)$ and $\varphi = 60^{\circ}$. Thus the expression becomes

$i(t) = 20cos(2000\pi t + 60^{\circ})$

d) The rms value of a sinusoidal current is ${}^{I_m}/_{\sqrt{2}}$. Therefore the rms value is ${}^{20}/_{\sqrt{2}}$ A or 14.14 A.

Example:

A sinusoidal voltage is given by the expression
$v(t) = 300\cos(120\boldsymbol{\pi}t + 30)$

a) What is the period of the voltage in milliseconds?
b) What is the frequency of the voltage in Hertz?
c) What is the value of the voltage at t = 2.778 ms?
d) What is the rms value of the voltage?

Solution:

a) From the expression for v, w= $120\boldsymbol{\pi}$ rad/s.

Since, $w = 2\boldsymbol{\pi} f = 2\boldsymbol{\pi}/T, T = 2\boldsymbol{\pi}/w = 2\boldsymbol{\pi}/120\boldsymbol{\pi}$= $\frac{1}{60} s$ or 16.667ms.

b) The frequency is $\mathrm{f} = 1/T$, which is 60 Hz.

c) $w = \frac{2\boldsymbol{\pi}}{T} = \frac{2\boldsymbol{\pi}}{16.667}$; at t= 2.778 ms, wt is nearly $\frac{2\boldsymbol{\pi}}{16.667} * 2.778 = 1.047 rad$ or $60°$

Therefore,

$$v(2.778ms) = 300\cos(60 + 30) = 0V$$

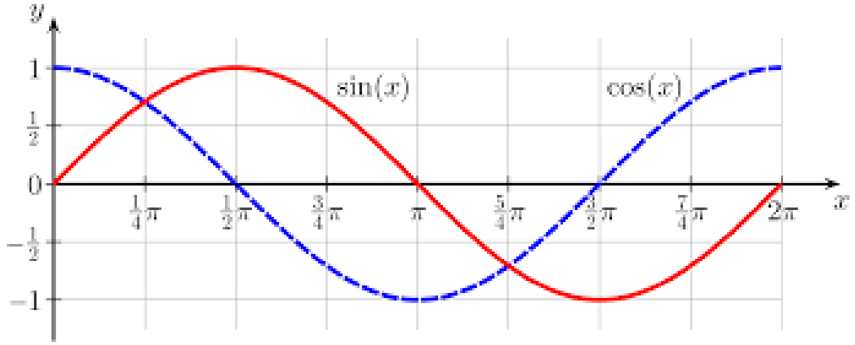
d) The rms value of the voltage is $V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 212.13V$.

Converting SIN to COS

cosine function can be converted into ***sine function***, and vice versa.

$$\cos(\omega t + \phi) = \sin\left(\omega t + \phi + 90^\circ\right)$$

$$\sin(\omega t + \phi) = \cos\left(\omega t + \phi - 90^\circ\right)$$



Examples for SIN->COS conversion

$i(t)=5\sin(100\pi t)$ A = $5\cos(100\pi t-90°)$ A

$i(t)=5\sin(100\pi t+30°)$ A = $5\cos(100\pi t+30°-90°)$ = $5\cos(100\pi t-60°)$ A

$i(t)=5\sin(100\pi t-30°)$ A = $5\cos(100\pi t-30°-90°)$ = $5\cos(100\pi t-120°)$ A

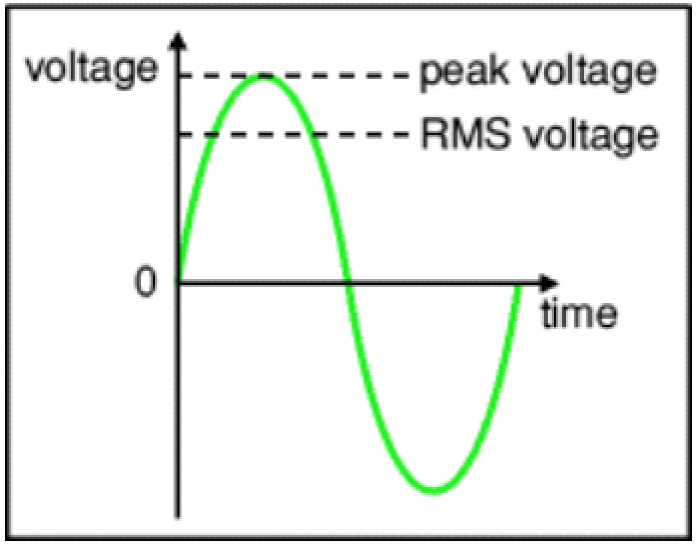
RMS Value (Root Mean Square)

$$v = V_m \cos(\omega t + \varphi)$$

rms value of v is calculated as

$$V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} V_m^2 cos^2(\omega t + \varphi)dt}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$



Difference between peak and rms voltage

IMPORTANT: In everyday use, AC voltages and currents are always given as RMS values, if otherwise is not stated.

For example, a **220V** AC supply means **220V RMS** with the peak voltage of **311.126 V**.

Example:

Suppose $v = 565.5440\cos(2\pi 50t + 30^{\circ})$, Then the rms value of the voltage waveform is given as

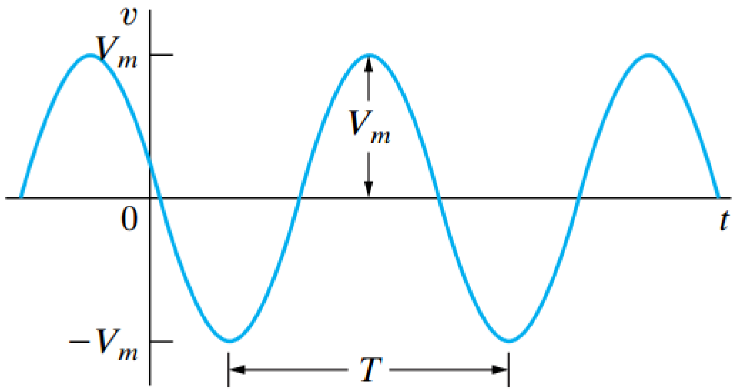
$$V_m = 565.5440V$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{565.5440}{\sqrt{2}} = \mathbf{399.9V}$$

IMPORTANT: The **multimeter** always shows the **RMS value** of the AC voltage or AC current.

DC Value (Average, Mean)

$v(t) = V_m \cos(\omega t + \varphi)$



DC value of v(t) is calculated as

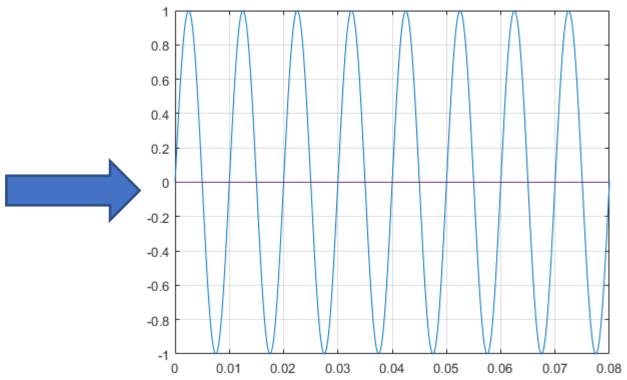
$$V_{dc} = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)dt$$

$$V_{dc} = \frac{1}{T}\int_{t_0}^{t_0+T} V_m cos(\omega t + \varphi)dt$$

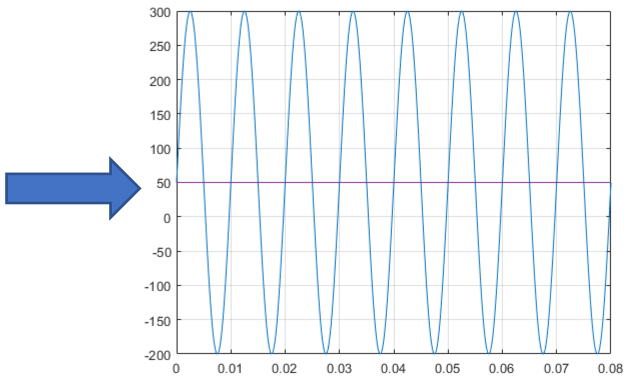
$$V_{dc} = 0$$

DC value of the sinusoidal voltage having NO DC COMPONENT is equal to **zero**.

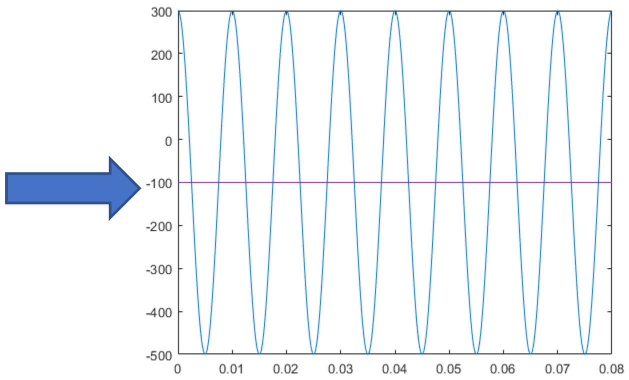
DC Value (Average, Mean)



DC value = 0

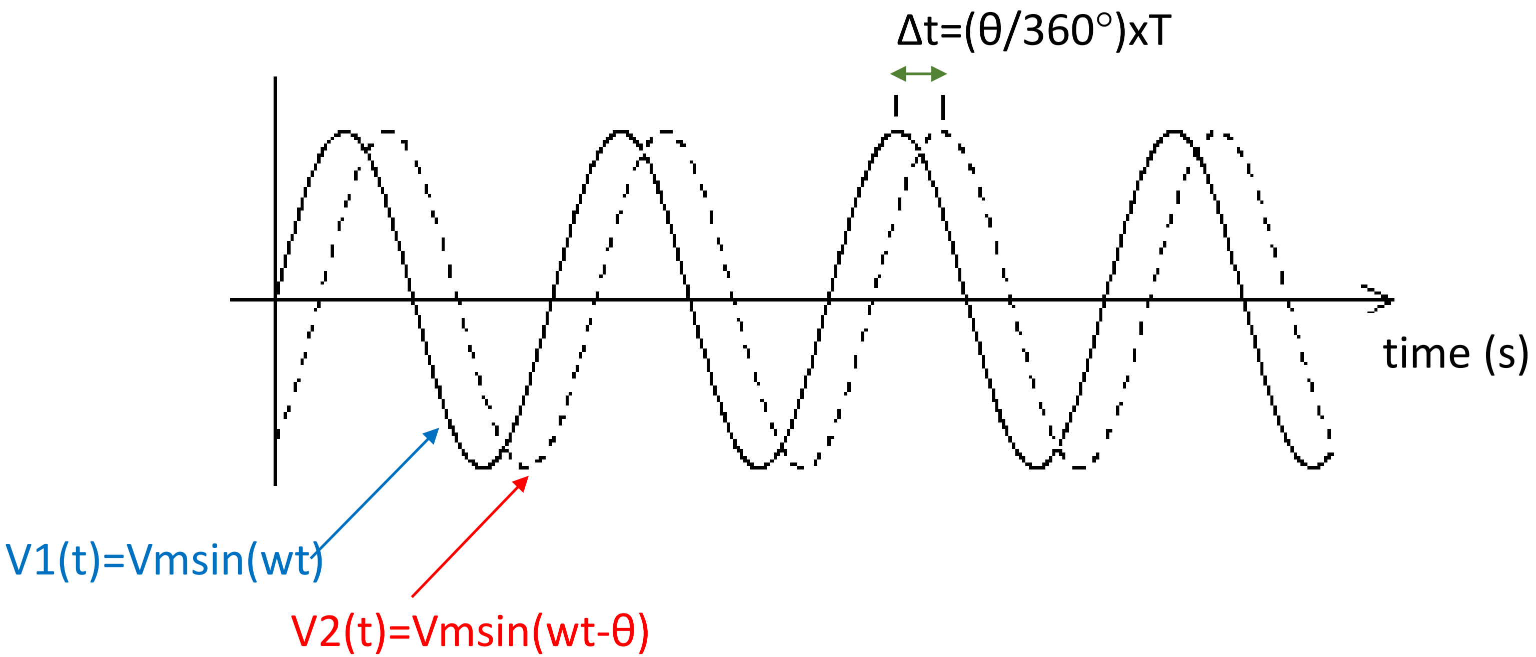


DC value = 50



DC value = -100

The concept of *Lagging* and *Leading*

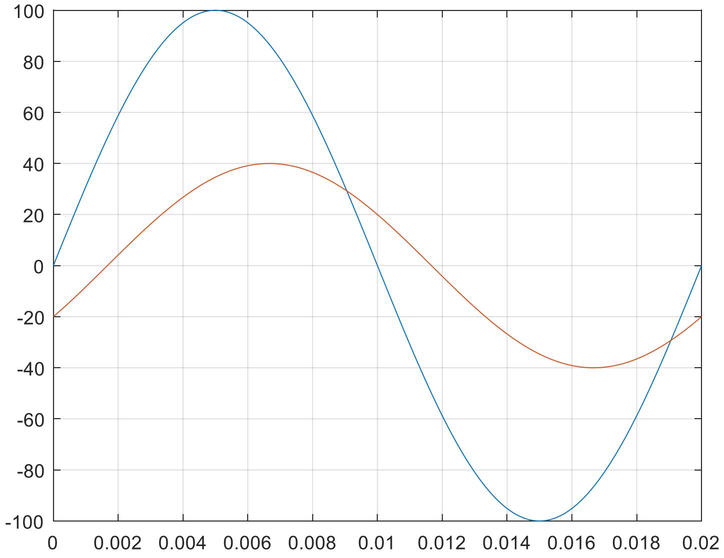


- V2 is lagging V1 by the angle θ
- V1 is leading V2 by the angle θ
- Angle θ is called "***phase (shift) angle***"
- In general, if θ=0° then V1 and V2 are said to be "***in-phase***"
- In general, if θ ≠ 0° then V1 and V2 are said to be " **θ degrees *out-of-phase***"
- To compare two waveforms, their ***frequency*** should be ***same***

$v(t)=100\sin(2\pi 50t)$

$i(t)=40\sin(2\pi 50t-30°)$

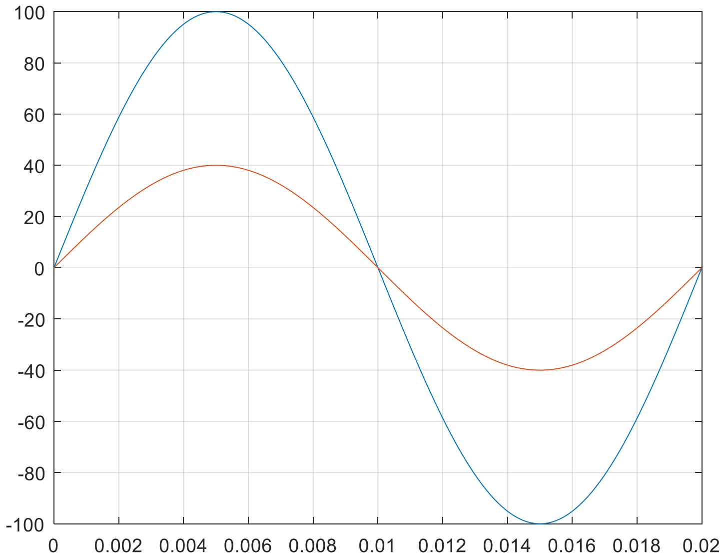
i lags v by 30°



$v(t)=100\sin(2\pi 50t)$

$i(t)=40\sin(2\pi 50t)$

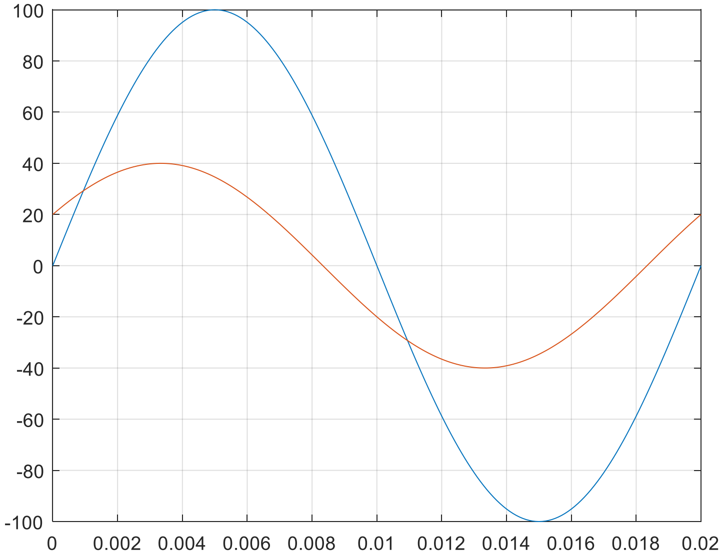
v and i are in-phase



$v(t)=100\sin(2\pi 50t)$

$i(t)=40\sin(2\pi 50t+30°)$

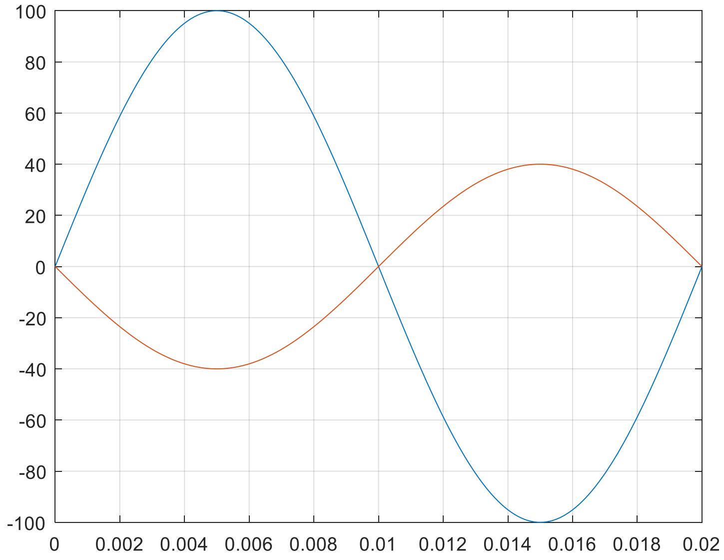
i leads v by 30°



$v(t)=100\sin(2\pi 50t)$

$i(t)=40\sin(2\pi 50t\pm 180°)$

v and i are 180° out-of-phase



The concept of *Lagging* and *Leading*

Two cars are going on autobahn

LAGGING CAR

LEADING CAR

The concept of leading and lagging of AC waveforms are **opposite** of this autobahn example !

Example:

Find the angle by which i1 lags v1 if

$v1(t) = 120\cos(120\pi t - 40°)$ volts

and

$i1(t) = 2.5\sin(120\pi t + 20°)$ amps

Solution:

$v1(t) = 120\cos(120\pi t - 40°)$ volts

$i1(t) = 2.5\cos(120\pi t + 20° - 90°) = 2.5\cos(120\pi t - 70°)$

Answer: i1 lags v1 by 30°

Hint: The waveform which has **more negative phase angle lags** the other one

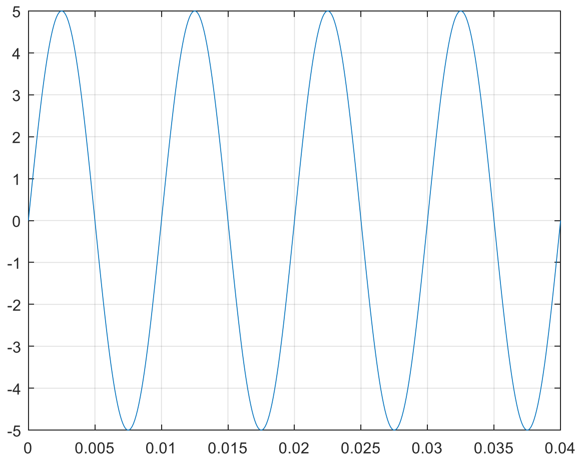
Self-study

Sketch the following waveforms for 2 cycles (periods)

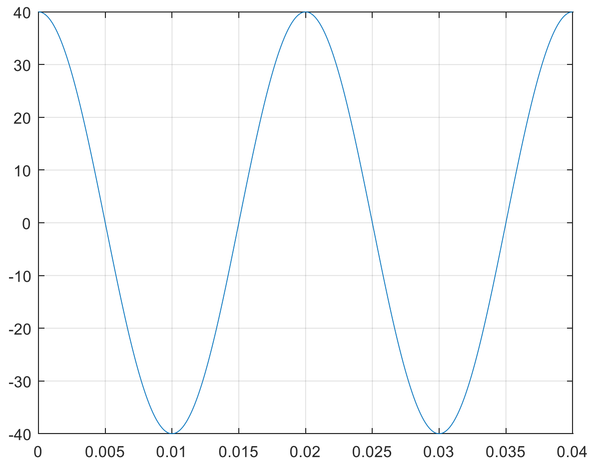
- ❑ $v(t)=50\sin(2000t+30°)$ volts
- ❑ $i(t)=0.04\sin(100t-45°)$ amps
- ❑ $v(t)=34500\sin(2\pi 50t-11°)$ volts
- ❑ $i(t)=37\sin(2\pi 60t+7°)$ amps

Self-study

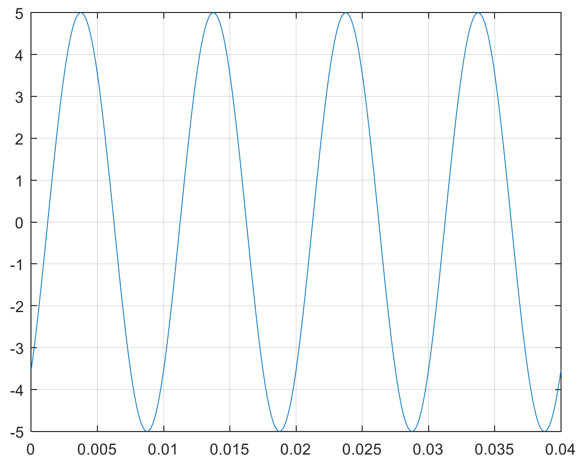
Write the expression for each waveform using both ***sin*** and ***cos*** functions.



a)



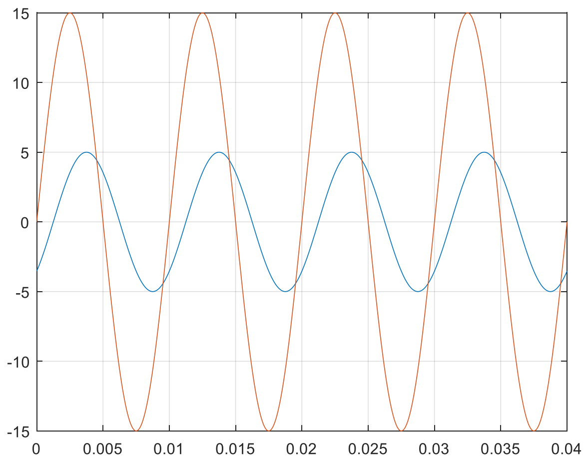
b)



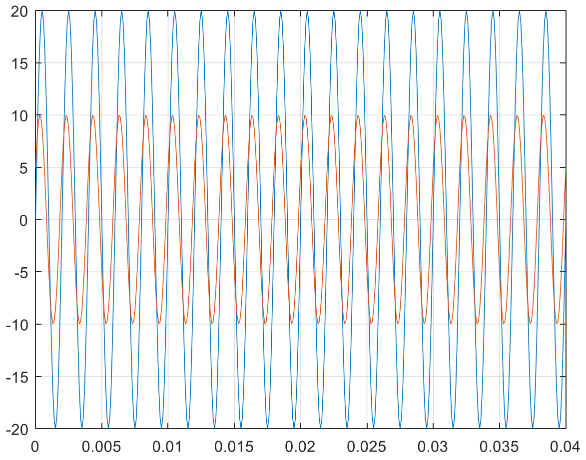
c)

Self-study

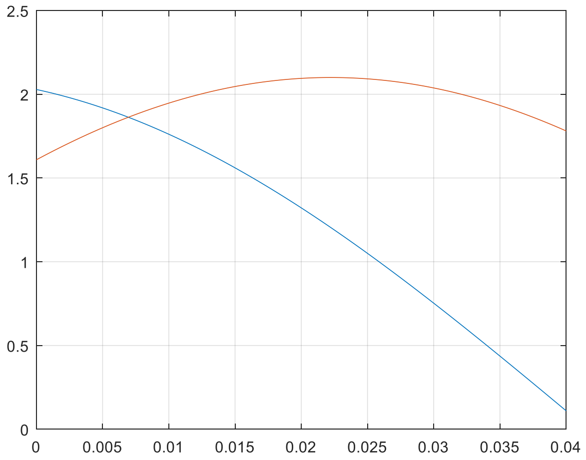
Specify which waveform lags the other one in each figure ?



a)



b)



c)

The Phasor

The phasor is a complex number that carries the **amplitude** and **phase angle** information of a sinusoidal function.

The relationship between exponential function to the trigonometric function is given as; (*Euler Identity*)

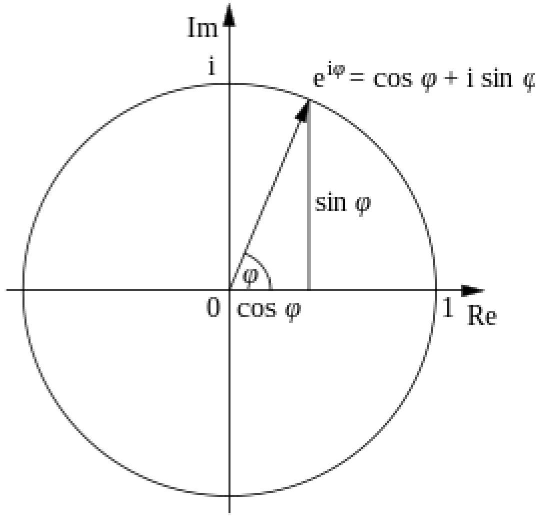
$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

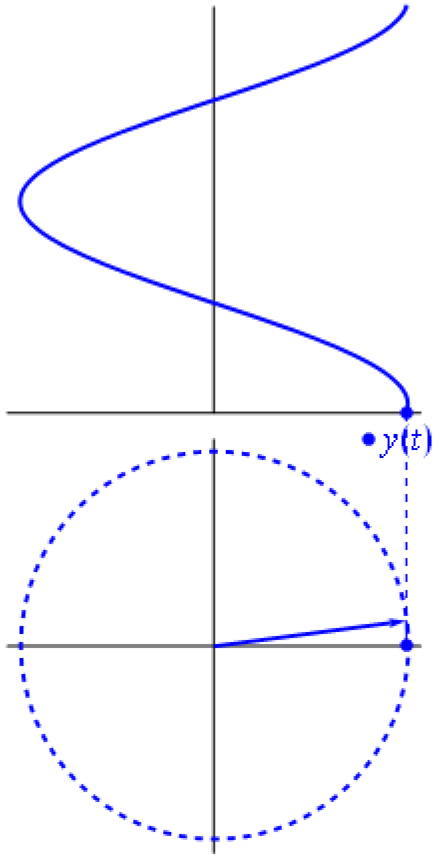
cosine function is the **real part** of the exponential function and sine function is the **imaginary part** of the exponential function

$$\cos\theta = \Re\{e^{j\theta}\} \qquad \sin\theta = \Im\{e^{j\theta}\}$$

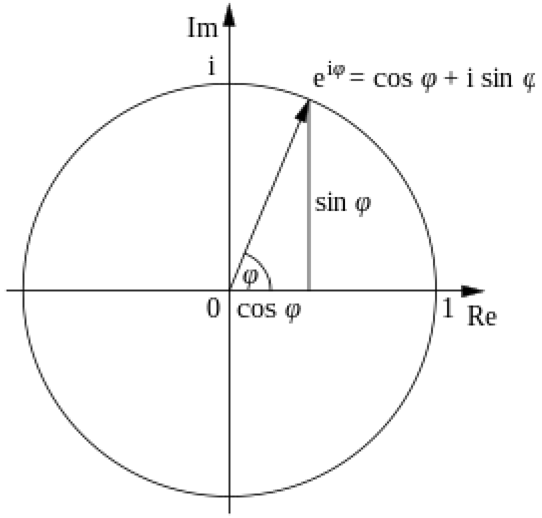
$\Re$ denotes the "real part of" and

$\Im$ denotes the "imaginary part of"





The Phasor



- Since we are interested in **cosine function** in analyzing,

$$v = V_m \cos(\omega t + \phi)$$

$$= V_m \Re\{e^{j(\omega t + \phi)}\}$$

$$= V_m \Re\{e^{j\omega t} e^{j\phi}\}.$$

or we can write

$$v = \Re\{V_m e^{j\phi} e^{j\omega t}\}$$

The Phasor

$$v = \Re\{V_m e^{j\phi} e^{j\omega t}\}$$

The quantity $V_m e^{j\phi}$ is a complex number that carries the **amplitude** and **phase angle** of the given sinusoidal function.

This complex number is by definition the **phasor representation**, or **phasor transform**, of the given sinusoidal function. Thus,

$$\mathbf{V} = V_m e^{j\phi} = \mathcal{P}\{V_m \cos(\omega t + \phi)\},$$

Phasor Transform

$$\mathbf{V} = V_m \cos\phi + jV_m \sin\phi.$$

The Phasor

$$\mathbf{V} = V_m e^{j\phi} = \mathcal{P}\{V_m \cos(\omega t + \phi)\},$$ **Phasor Transform**

$$\mathbf{V} = V_m \cos\phi + jV_m \sin\phi.$$

The phasor transform transfers the sinusoidal function from the ***time domain*** to the ***complex-number domain***, which is also called the **frequency domain.**

$$\mathbf{V} = V_m \cos\phi + jV_m \sin\phi.$$ **Phasor in rectangular form**

$$1\angle\phi^\circ \equiv 1e^{j\phi}.$$

$$V = V_m \angle\phi$$ **Phasor in polar form**

Example:

Obtain the phasor transform of $v(t) = 100\cos(\omega t + 30^{\circ})$

$$P\left(v(t)\right) = P\left(100\cos\left(\omega t + 30^{\circ}\right)\right) = 100e^{j30^{\circ}} = 100\angle 30^{\circ}$$

Example:

Transform time-domain voltage **v(t) = 100cos(400t-30°) volts** into phasor domain.

V = 100/__-30° volts   at   w=400 rad/s   or   f = w/2π = 63.66 Hz.

Generally written in **bold** and/or *Italics*

Phasor sign

Phase angle

Example:

Transform time-domain current **i(t) = 1.0sin(t+19°) A** into phasor domain.

First we have to convert **sin** to **cos** as follows:

$i(t) = 1.0\sin(t+19^\circ)$ A

$i(t) = 1.0\cos(t+19^\circ-90^\circ)$ A

$i(t) = 1.0\cos(t-71^\circ)$ A

Then we can make the transformation as follows:

$$I = 1\angle -71^\circ \text{ A}$$

Current phasor

Phasor magnitude

phase angle in degrees

Inverse Phasor Transform

The step of going from the phasor transform to the time-domain expression is referred to as finding the inverse phasor transform and is formalized by the following equation,

$$\mathcal{P}^{-1}\{V_m e^{j\phi}\} = \Re\{V_m e^{j\phi} e^{j\omega t}\}$$

Example:

Obtain the inverse phasor transform of $\mathbf{V} = 130\angle 45^{\circ}$

$$P^{-1}(\mathbf{V}) = P^{-1}\left(130\angle 45^{\circ}\right) = 130\cos(\omega t + 45^{\circ})$$

Conclusions – Phasor Transform

- The phasor transform is useful in circuit analysis because it reduces the task of finding the maximum amplitude and phase angle of the steady state sinusoidal response to the algebra of complex numbers.
- The phasor transform, along with the inverse phasor transform, allows you to go back and forth between the time domain and the frequency domain. Therefore, when you obtain a solution, you are either in the time domain or the frequency domain. You cannot be in both domains simultaneously.

Conclusions – Phasor Transform

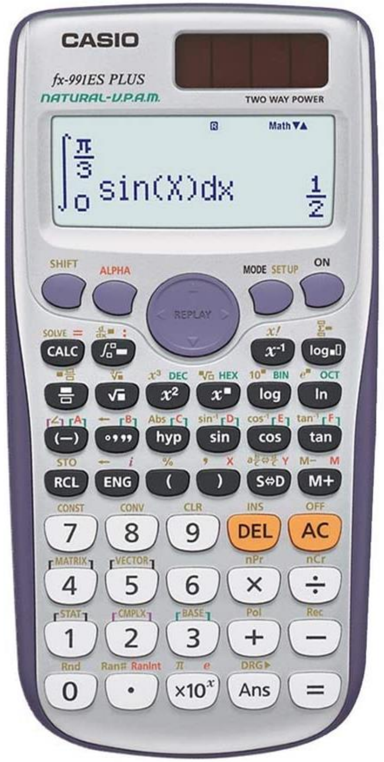
- The phasor transform is also useful in circuit analysis because it applies directly to the sum of sinusoidal functions. Circuit analysis involves summing currents and voltages, so the importance of this observation is obvious.

$$v = v_1 + v_2 + \cdots + v_n$$ ← Time domain

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n.$$ ← Frequency domain

Example: Adding Cosines Using Phasors

If $y_1 = 20\cos(\omega t - 30^{\circ})$ and $y_2 = 40\cos(\omega t + 60^{\circ})$, express $y_1 + y_2$ as a single sinusoid function.
Taking the phasor transform of y_1 and y_2, we get



$$\mathbf{Y}_1 = P(y_1) = P\left(20\cos(\omega t - 30^{\circ})\right) = 20e^{-j30^{\circ}} = 20\angle -30^{\circ}$$

$$\mathbf{Y}_2 = P(y_2) = P\left(40\cos(\omega t + 60^{\circ})\right) = 40e^{j60^{\circ}} = 40\angle 60^{\circ}$$

$$\mathbf{Y} = \mathbf{Y}_1 + \mathbf{Y}_2 = 20\angle -30^{\circ} + 40\angle 60^{\circ} =$$

$$= 20\left(\cos(-30^{\circ}) + jsin(-30^{\circ})\right) + 40\left(\cos(60^{\circ}) + jsin(-60^{\circ})\right)$$

$= (17.32 - j10) + (20 + j34.64) = 37.32 + j24.64 = 44.72\angle 33.43^{\circ}$ Then take inverse phasor transform to obtain

$$y = P^{-1}(\mathbf{Y}) = P^{-1}\left(44.72\angle 33.43^{\circ}\right) = 44.72\, cos(\omega t + 33.43^{\circ})$$

Homework

9.1 Find the phasor transform of each trigonometric function:

a) $v = 170\cos(377t - 40^\circ)$ V.

b) $i = 10\sin(1000t + 20^\circ)$ A.

c) $i = [5\cos(\omega t + 36.87^\circ) + 10\cos(\omega t - 53.13^\circ)]$ A.

d) $v = [300\cos(20{,}000\pi t + 45^\circ) - 100\sin(20{,}000\pi t + 30^\circ)]$ mV.

9.2 Find the time-domain expression corresponding to each phasor:

a) $\mathbf{V} = 18.6\angle{-54^\circ}$ V.

b) $\mathbf{I} = (20\angle{45^\circ} - 50\angle{-30^\circ})$ mA.

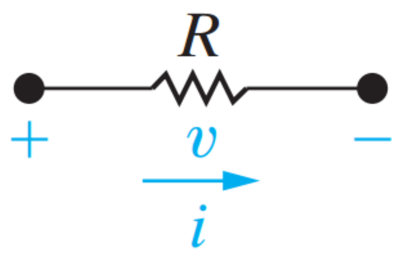
c) $\mathbf{V} = (20 + j80 - 30\angle{15^\circ})$ V.

The Passive Circuit Elements in the Frequency Domain

The systematic application of the phasor transform in circuit analysis requires two steps.

- First, we must establish the relationship between the phasor current and the phasor voltage at the terminals of the passive circuit elements.
- Second, we must develop the phasor-domain version of Kirchhoff's laws.

The V-I Relationship for a Resistor



- Suppose

$$i = I_m \cos(\omega t + \theta_i)$$

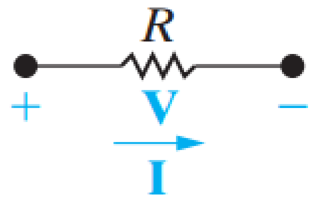
- Then using Ohm's law

$$v = R[I_m \cos(\omega t + \theta_i)]$$

$$= RI_m[\cos(\omega t + \theta_i)],$$

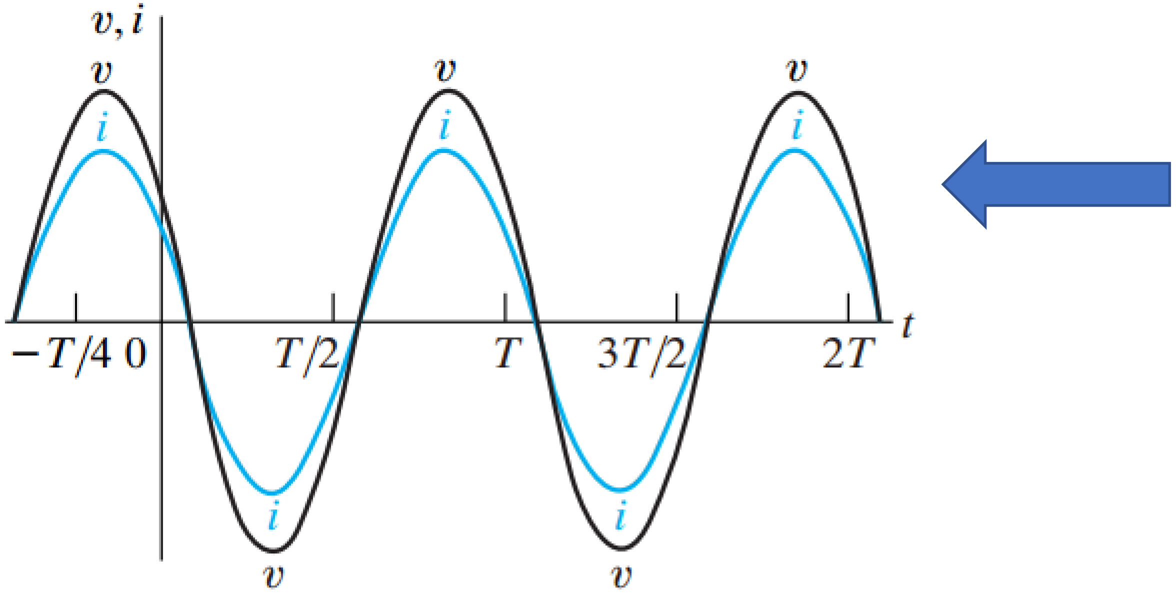
- The phasor transform of this voltage is $\mathbf{V} = RI_m e^{j\theta_i} = RI_m\angle\theta_i.$
- Since $I_m\angle\theta_i$ is the phasor representation of sinusoidal current we have

$$\mathbf{V} = RI_m e^{j\theta_i} = RI_m\angle\theta_i. \Rightarrow \mathbf{V} = R\mathbf{I},$$



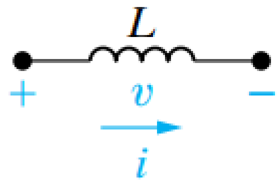
The V-I Relationship for a Resistor

At the terminals of a resistor, there is no phase shift between the current and voltage. The following figure shows this phase relationship, where the phase angle of both the voltage and the current waveforms is **same**. These signals are said to be **in phase** because they both reach corresponding values on their respective curves at the same time (for example, they are at their positive maxima at the same instant).



The V-I Relationship for a Resistor

The V-I Relationship for an Inductor



- Suppose $i = I_m \cos(\omega t + \theta_i)$

$$v = L\frac{di}{dt} = -\omega L I_m \sin(\omega t + \theta_i).$$

Writing this expression in cosine form (sine is 90 degree shifted version of cosine)

$$v = -\omega L I_m \cos(\omega t + \theta_i - 90^\circ)$$

The phasor representation of the voltage is given as;

$$\mathbf{V} = -\omega L I_m e^{j(\theta_i - 90^\circ)}$$

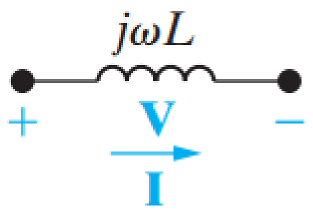
$$= -\omega L I_m e^{j\theta_i} e^{-j90^\circ} = j\omega L I_m e^{j\theta_i} = j\omega L \mathbf{I}$$

$$e^{-j90^\circ} = \cos 90^\circ - j \sin 90^\circ = -j$$

The V-I Relationship for an Inductor

- Therefore, for an inductor

$$\mathbf{V} = j\omega L \mathbf{I}$$



$$\mathbf{V} = (\omega L \angle 90^\circ) I_m \angle \theta_i$$

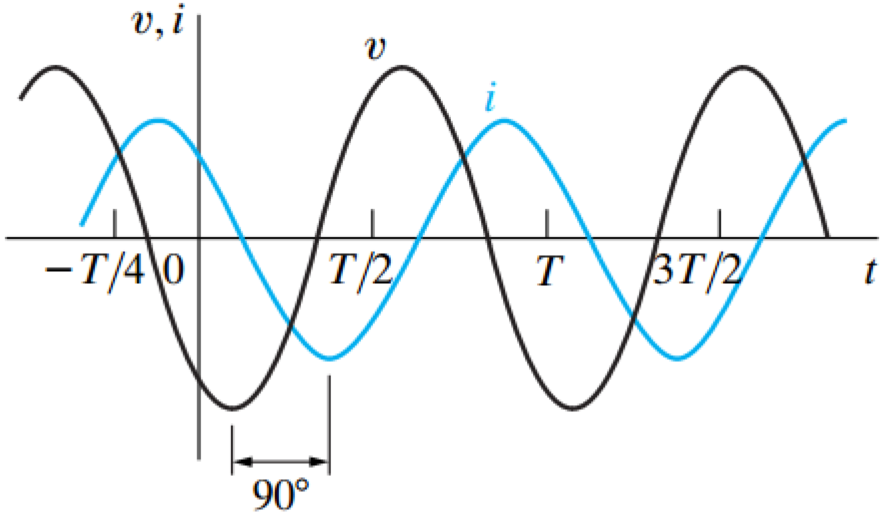
$$= \omega L I_m \angle (\theta_i + 90)^\circ,$$

The voltage **leads** the current by 90 degree or equivalently, the current **lags** behind the voltage by 90 degree.

The V-I Relationship for an Inductor

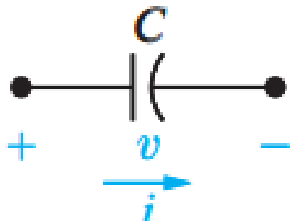
$$\mathbf{V} = (\omega L \angle 90^\circ) I_m \angle \theta_i$$

$$= \omega L I_m \angle (\theta_i + 90)^\circ,$$



The V-I Relationship for a Capacitor

- Suppose $v = V_m \cos(\omega t + \theta_v)$



$$i = C\frac{dv}{dt}$$

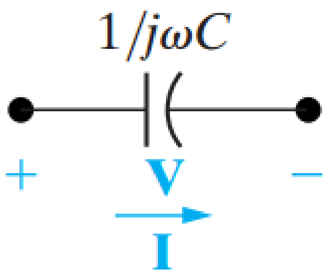
$$\mathbf{I} = j\omega C\mathbf{V}$$

If we solve for $\mathbf{V}$ we get

$$\mathbf{V} = \frac{1}{j\omega C}\mathbf{I}$$

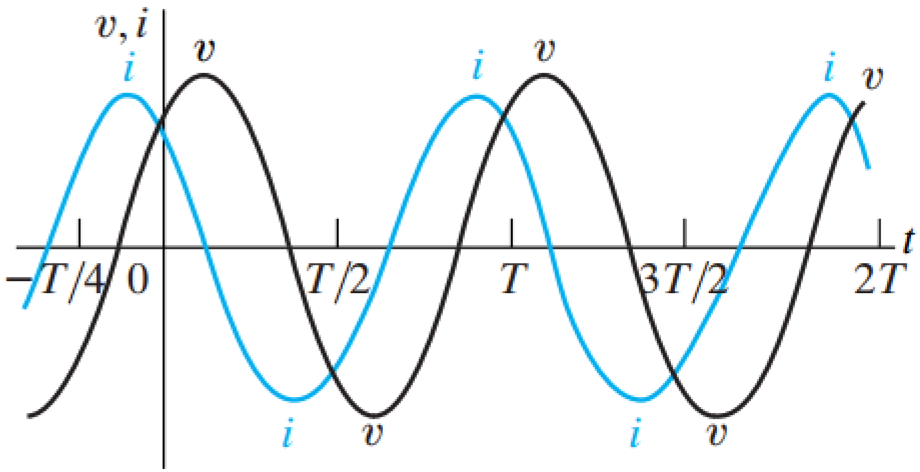
The V-I Relationship for a Capacitor

$$\mathbf{V} = \frac{\mathbf{1}}{\mathbf{j\omega C}}\mathbf{I}$$



$$\mathbf{V} = \frac{1}{\omega C}\angle{-90^\circ} I_m \angle{\theta_i^\circ}$$

$$= \frac{I_m}{\omega C}\angle{(\theta_i - 90)^\circ}.$$



The voltage across the terminals of a capacitor **lags** behind the current by **exactly 90 degrees.**

Impedance

- **Impedance** is defined as the ratio of voltage-phasor to the current-phasor related to any equipment or component.
- **Impedance** is a complex number, i.e., it has a *real part* and a *reactive part*.
- The imaginary part of the impedance is called as **reactance**.
- The unit of impedance is **ohm** (Ω)
- We generally use letter **z** or **Z** to represent the **impedance.**

$$\mathbf{Z = V/I}$$

Impedance

- The **impedance** of a **resistance** is equal to

$Z_R = V/I = RI/I = \mathbf{R = R/\_\_0° \ \Omega}$

- The **impedance** of an **inductor** is equal to

$Z_L = V/I = j\omega LI/I = \mathbf{j\omega L = \omega L\ /\_\_90° \ \Omega}$

- The **impedance** of a **capacitor** is equal to

$Z_C = V/I = V/Vj\omega C = \mathbf{1/j\omega C = j/j^2\omega C = -j/\omega C = \omega C/\_\_-90° \ \Omega}$

$\mathbf{j^2 = -1}$

Impedance and reactance of various circuit elements

Circuit Element	Impedance	Reactance
Resistor	R	—
Inductor	$j\omega L$	ωL
Capacitor	$j(-1/\omega C)$	$-1/\omega C$

Impedance

- $\mathbf{Z_R}$ is not function of frequency, since $\mathbf{Z_R}$ = R
- $\mathbf{Z_L}$ is a function of frequency, since $\mathbf{Z_L}$ **= jwL**

  at w=0 ➔ $\mathbf{Z_L}$ = 0

  at w-> ∞ ➔ $\mathbf{Z_L}$ -> ∞

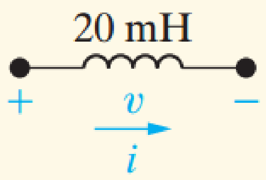
- $\mathbf{Z_C}$ is a function of frequency, since $\mathbf{Z_C}$ = 1/jwC

  at w=0 ➔ $\mathbf{Z_L}$ -> ∞

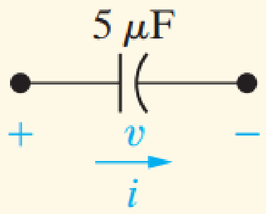
  at w-> ∞ ➔ $\mathbf{Z_L}$ -> 0

Homework

9.3 The current in the 20 mH inductor is $10 \cos(10{,}000t + 30^\circ)$ mA. Calculate (a) the inductive reactance; (b) the impedance of the inductor; (c) the phasor voltage $\mathbf{V}$; and (d) the steady-state expression for $v(t)$.



9.4 The voltage across the terminals of the 5 μF capacitor is $30 \cos(4000t + 25^\circ)$ V. Calculate (a) the capacitive reactance; (b) the impedance of the capacitor; (c) the phasor current $\mathbf{I}$; and (d) the steady-state expression for $i(t)$.

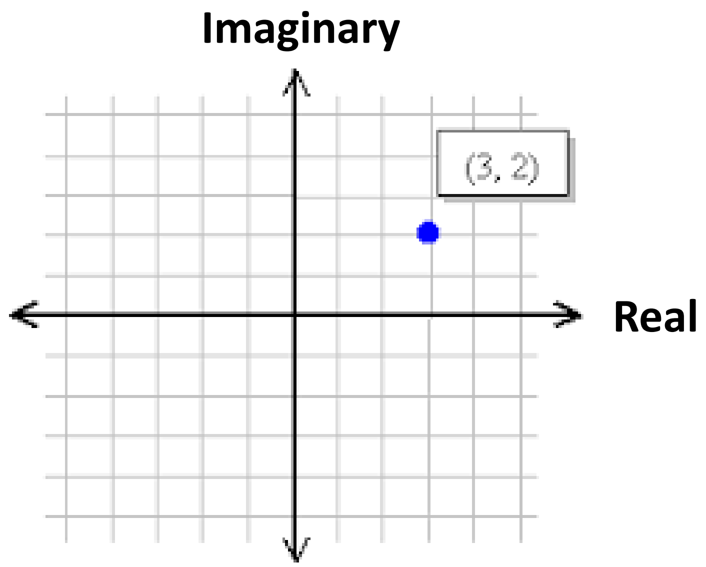


Review on operations with complex numbers

1) Representation of complex numbers:

a) In rectangular coordinates:

complex number = real part + **j** imaginary part



For example ➔ a = 3+j2 or 3+2i

(**j** or **i** can be interchangeably used)

Review on operations with complex numbers

1) Representation of complex numbers:

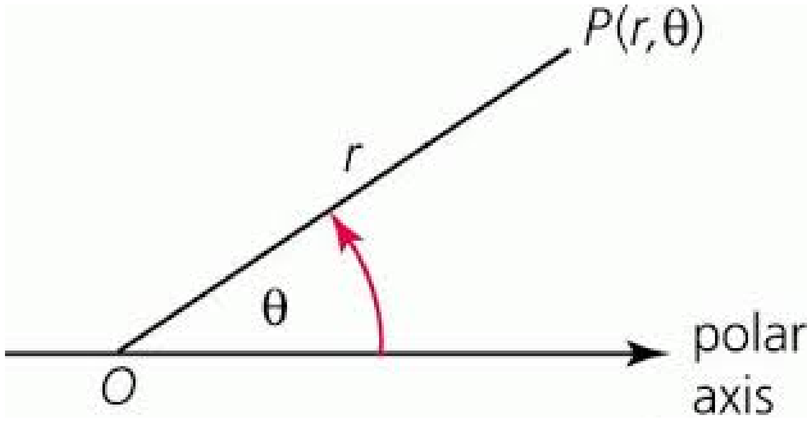
b) In polar coordinates:

complex number = magnitude /__ phase angle

For example ➔ a = 5/_28°

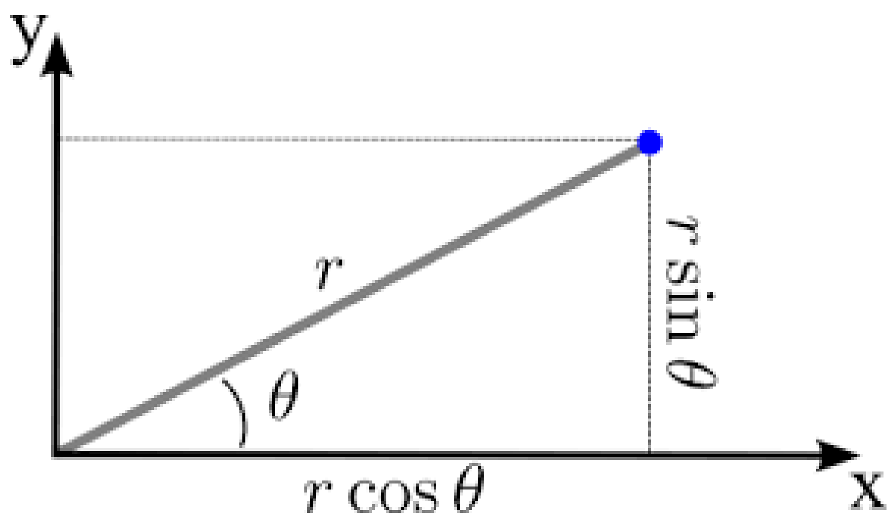
Magnitude r

phase angle θ



Review on operations with complex numbers

2) Transformation from polar to rectangular (cartesian) coordinates

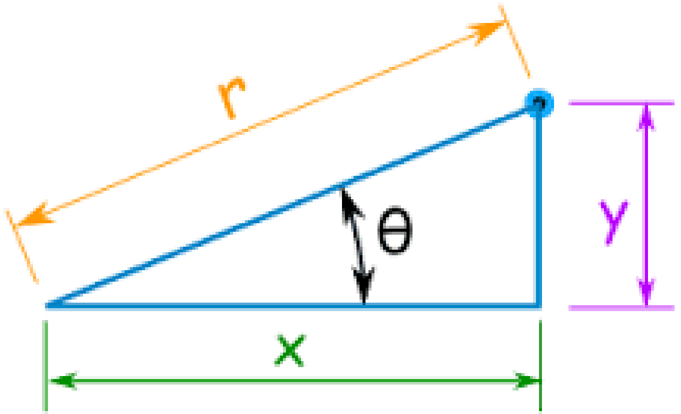


x = rcosθ
y = rsinθ

r/_θ = x + jy

Review on operations with complex numbers

3) Transformation from rectangular to polar coordinates



$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

x+jy = r/_θ

Review on operations with complex numbers

4) Addition and subtraction in polar coordinates

Let

$z_1 = A\angle B$ and $z_2 = C\angle D$

$z_1 \pm z_2 = (A\cos B \pm C\cos D) + j(A\sin B \pm C\sin D)$

Review on operations with complex numbers

5) Multiplication and division in polar coordinates

Let

z_1 = A/_B and z_2 = C/_D

$z_1.z_2$ = AC/_(B+D)

z_1/z_2 = A/C/_(B-D)

NOTE: Multiplication and division are rather **simple** in polar coordinates

Review on operations with complex numbers

6) Addition and subtraction in rectangular coordinates

Let

z_1 = A+jB and z_2 = C+jD

$z_1 \pm z_2 = (A \pm C) + j(B \pm D)$

NOTE: Addition and subtraction are rather **simple** in rectangular coordinates

Review on operations with complex numbers

7) Multiplication and division in rectangular coordinates

Let

$z_1 = A+jB$ and $z_2 = C+jD$

$z_1 \cdot z_2 = (A+jB)(C+jD)$

$z_1 \cdot z_2 = AC+jAD+jBC-BD$

$z_1 \cdot z_2 = (AC-BD)+j(AD+BC)$

$j^2 = -1$

$z_1 / z_2 = (A+jB)/(C+jD)$

$$z_1 / z_2 = \frac{(A+jB)}{(C+jD)}\frac{(C-jD)}{(C-jD)} = \frac{(A+jB)(C-jD)}{(C^2+D^2)}$$

$z_1 / z_2 = [(AC+BD)+j(BC-AD)]/(C^2+D^2)$

Review on operations with complex numbers

7) Multiplication with -1

Let

z_1 = A+jB and z_2 = C/_D

z_1 = A+jB ➔ -1(z_1) = -A-jB

z_2 = C/_D ➔ -1(z_2) = C/_(D±180°)

Review on operations with complex numbers

7) Complex conjugate

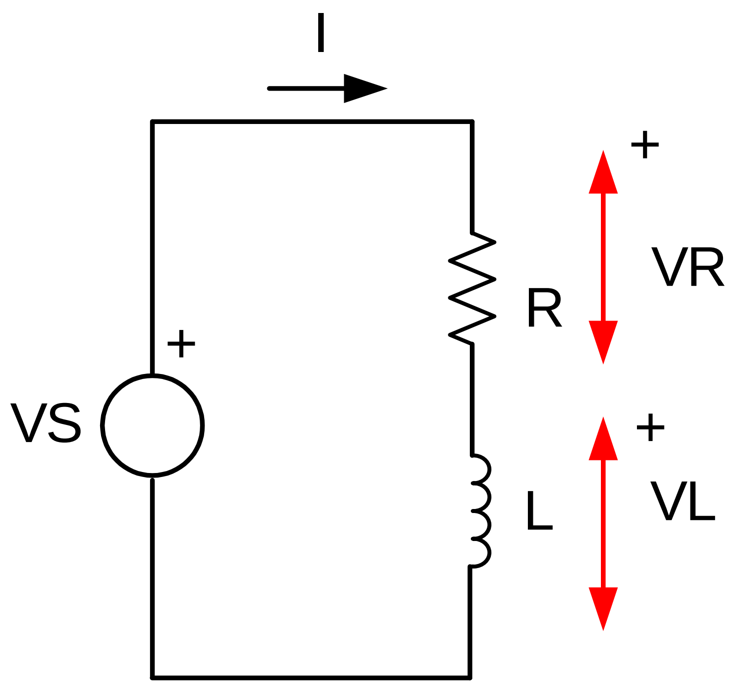
Let

z_1 = A+jB and z_2 = C/_D

z_1^* = (A+jB)* = (A-jB)

z_2^* = (C/_D)* = C/_-D

AC steady-state analysis of RL circuits



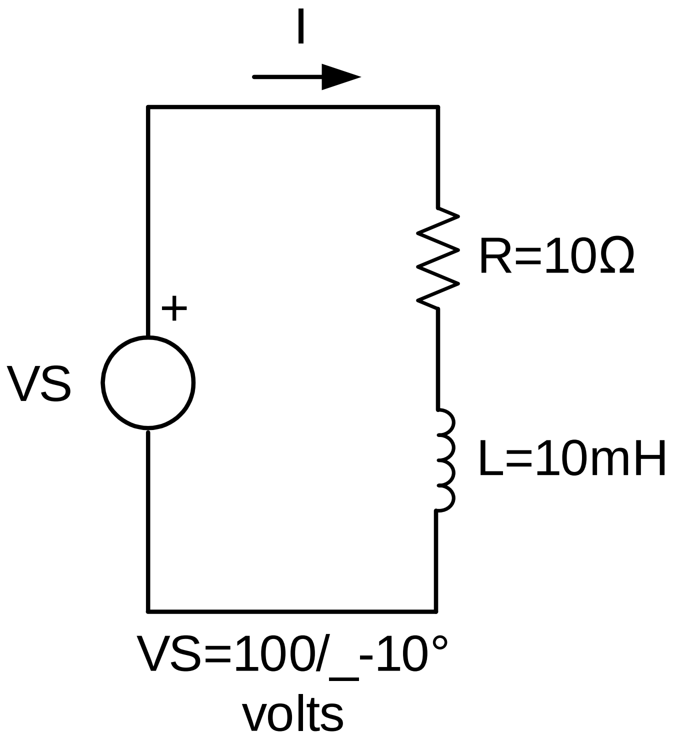
Applying Kirchhoff's Voltage Law (**KVL**)

VS = VR + VL
VS = RI + jwLI
VS = I(R+jwL)
I = VS / (R+jwL)
R+jwL is called "*impedance of series RL circuit* "

RL circuits are important in power systems, because

- Most of the electrical loads are in this form, for example electrical motors
- Generators and transformers are practically modelled in this form
- Cables and transmission lines are usually modelled in this form

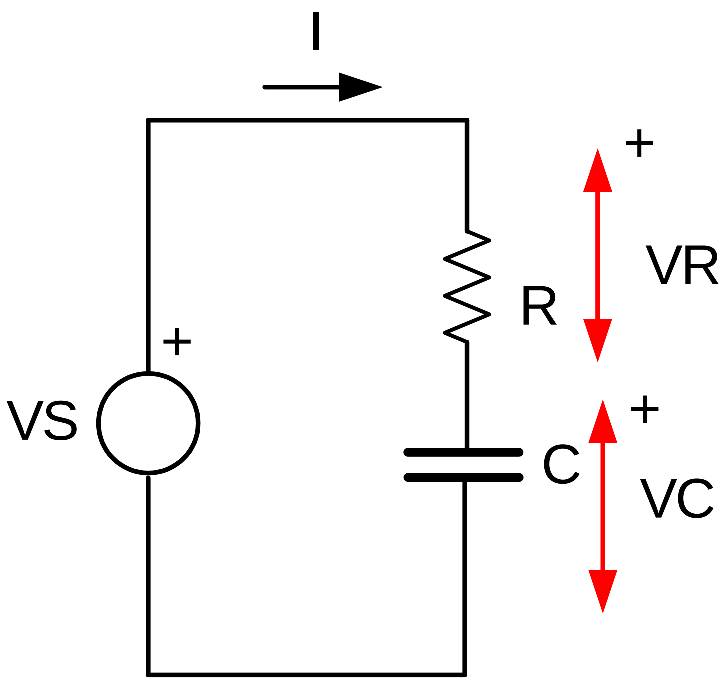
Example:



Find

a) The impedance of the RL circuit at f=1kHz
b) Time-domain current i(t) and express it

AC steady-state analysis of RC circuits



Applying **KVL**

VS = VR + VC

VS = RI + I/jwC

VS = I(R+1/jwC)

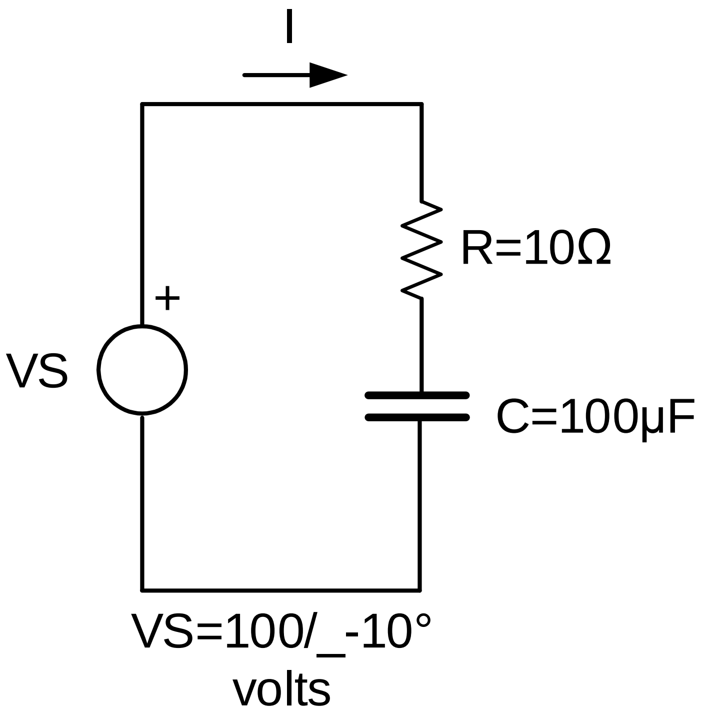
I = VS / (R+1/jwC)

R+1/jwC is called “*impedance of series RC circuit* “

or

R-j/wC

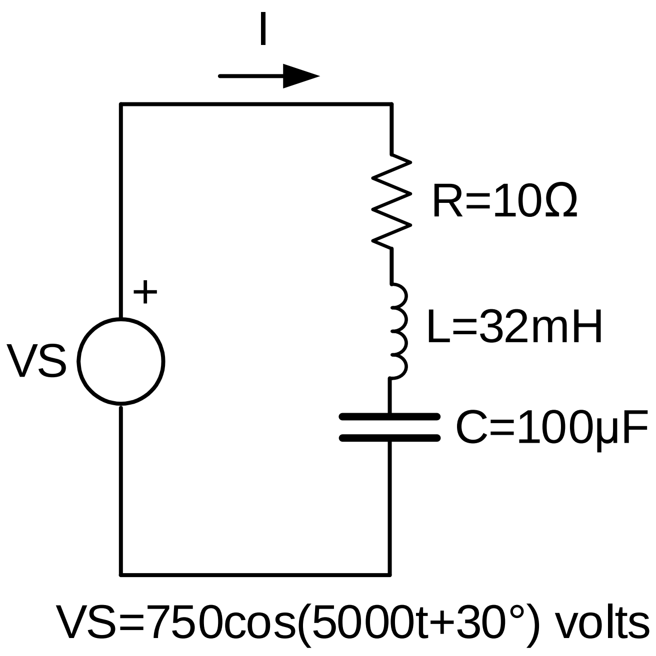
Example:



Find

a) The impedance of the RC circuit at f=1kHz
b) Time-domain current i(t) and express it

Example:



Answer the following questions

a) Construct the phasor-domain equivalent circuit
b) Find the time-domain current i(t)

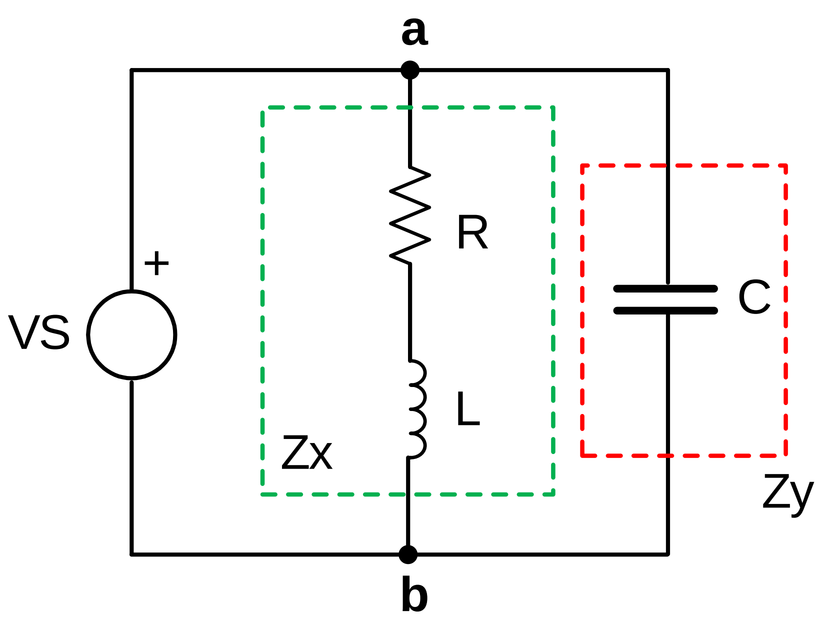
Admittance

- **Admittance** is the ratio of the current-phasor to the voltage-phasor related to any equipment or component
- The unit of **admittance** is **Siemens** (S) or **ohm**$^{-1}$ or **mho** or **Ω**$^{-1}$
- Since **impedance** is a complex number, **admittance** is also a complex number
- We generally use letter **y** or **Y** to represent **admittance**

$$\mathbf{Y = I/V}$$

$$\mathbf{Y = Z^{-1}}$$

Example:



Calculate the impedance and the admittance between points **ab**

If

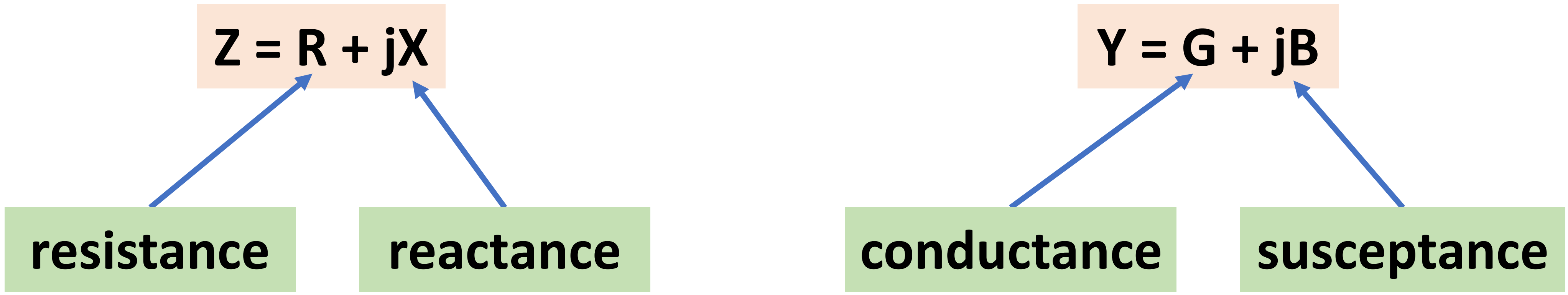
R = 85 Ω

L = 10 mH

C = 10 µF

w = 10000 rad/s (angular frequency of the source, VS)

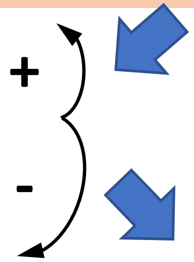
Resistance, conductance, reactance, susceptance



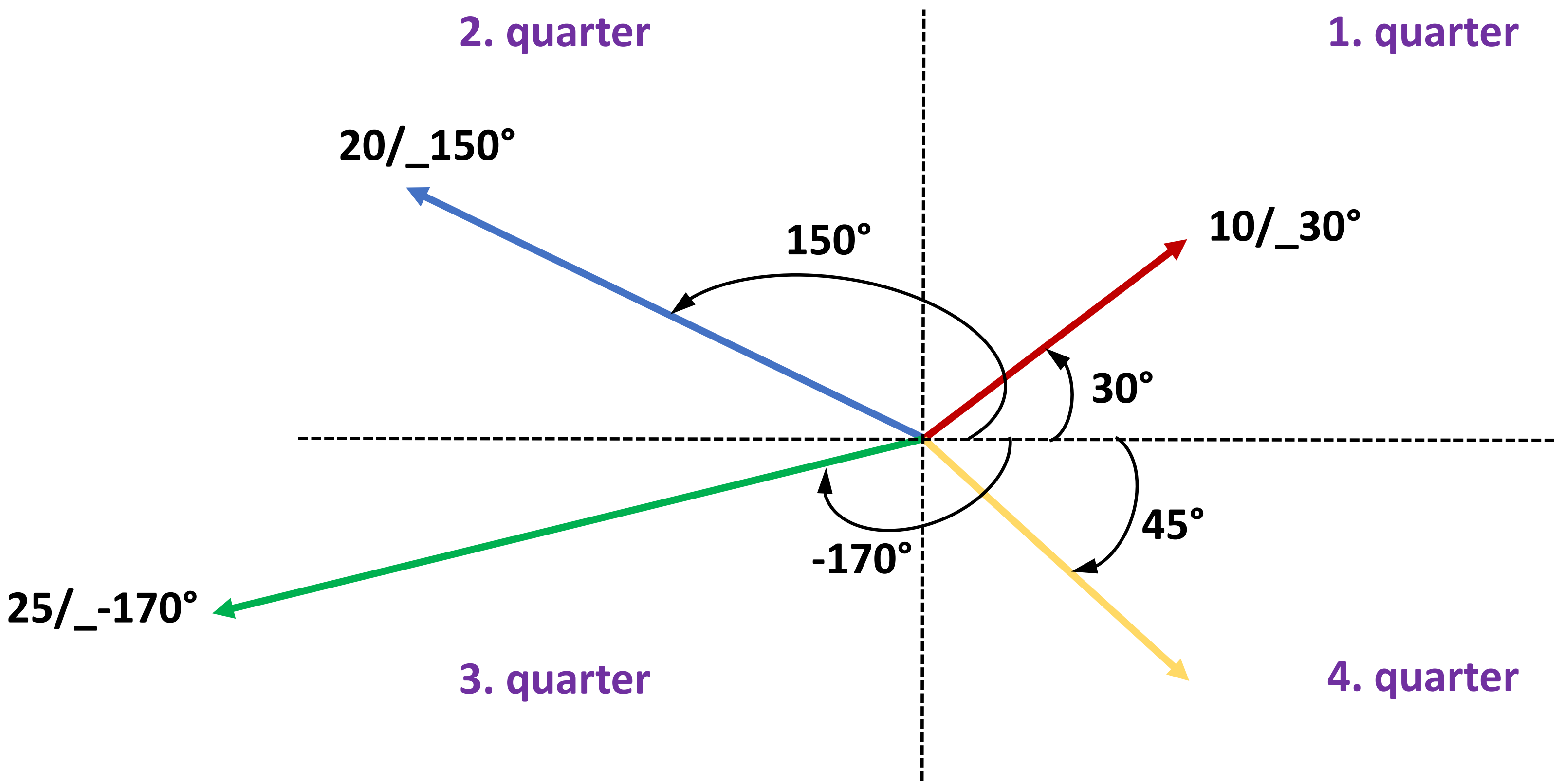
Phasor Diagrams

- **Phasor diagram** is a graphical view of either phasor current or voltage on the two-dimensional coordinate system (rectangular or polar coordinates)

Counter clock wise direction (**CCW**) positive direction

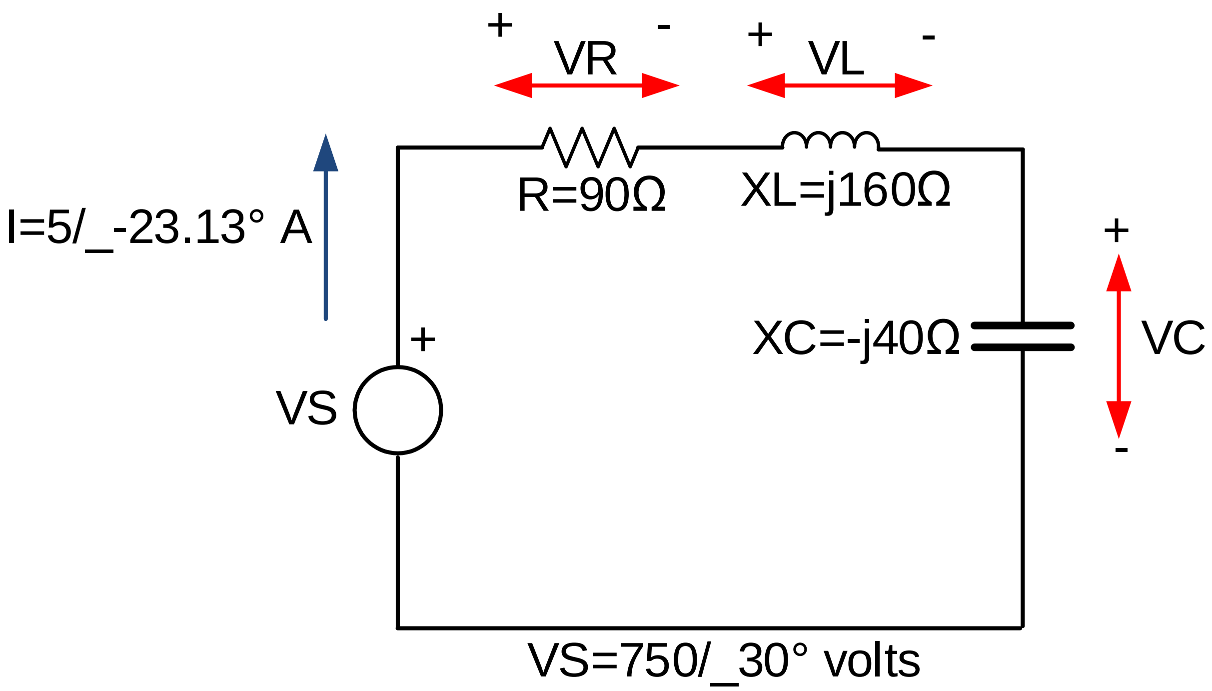


Clock wise direction (**CW**) **negative** direction



Example:

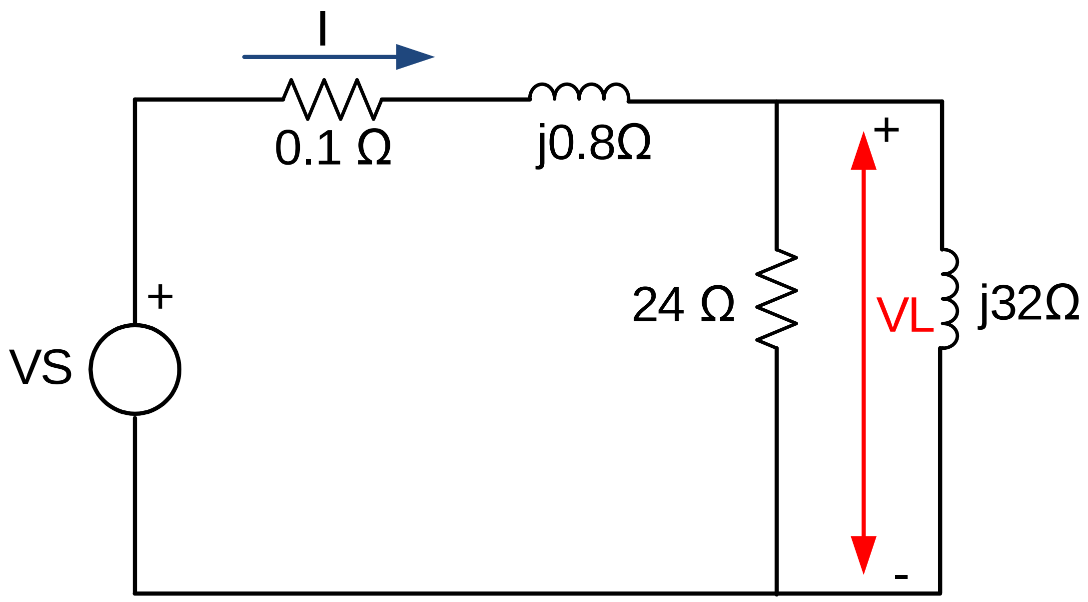
Analyze voltage phasors of the following circuit



Self-study

For the given single-phase low voltage system answer the following questions:

a) Calculate voltage phasor VS=?
b) Calculate the **capacitive reactance** that is connected in shunt with 24Ω resistance to make the current **I maximum**
c) Calculate current **I** under the conditions in part (b)



END OF THE LECTURE

Any questions ?