EEE 321

Electromechanical Energy Conversion – I

By

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CHAPTER 2

TRANSFORMERS

Introduction

- **Transformer** is an electrical machine that converts ac electrical energy/power at **one voltage level** to ac electrical energy/power at **another voltage level** through the action of magnetic field.

AC ELECTRICAL ENERGY AT ONE VOLTAGE LEVEL →  → AC ELECTRICAL ENERGY AT ANOTHER VOLTAGE LEVEL

Introduction

Who invented the transformer? Ottó Bláthy, Miksa Déri, Károly Zipernowsky of the Austro-Hungarian Empire First designed and used the transformer in both experimental, and commercial systems. Later on Lucien Gaulard, Sebstian Ferranti, and William Stanley perfected the design.

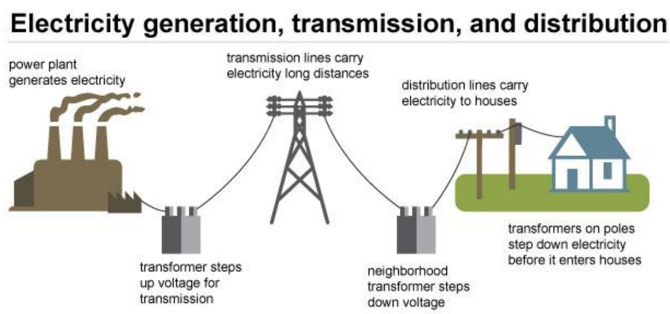
The original 1885 Stanley prototype transformer at the Berkshire Museum.

Source: http://edisontechcenter.org/Transformers.html

Introduction

Why are transformers important ?

- Transformers make possible electric generation at the most economical generator voltage, power transfer at the most economical transmission voltage, and power utilization at the most suitable voltage for the particular utilization device.

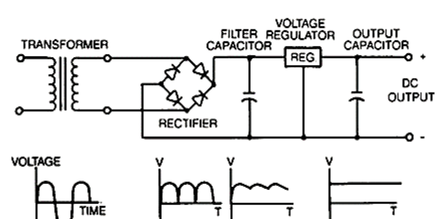


Source: Adapted from National Energy Education Development Project (public domain)

Introduction

Why are transformers important ?

- Transformer is also widely used in low-power, low-current electronic and control circuits for performing such functions:
  - Matching the impedances of a source and its load for maximum power transfer
  - Isolating one circuit from another
  - Isolating direct current while maintaining ac continuity between two circuits

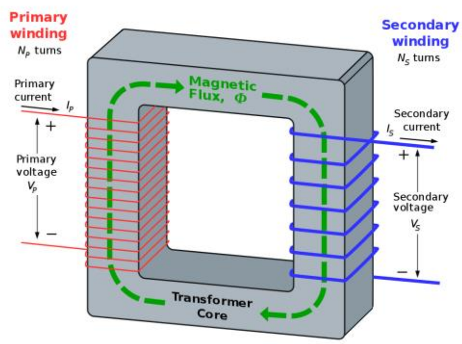


Source: https://www.quora.com

Introduction

- Transformer consists of **two or more coils** of wire wrapped around a **common ferromagnetic core**.
- These coils are (*usually*) **not directly (*electrically*) connected**.
- The only connection between the coils is the **common magnetic flux** present within the core.

The winding connected to the power source is called «***primary winding***» or «***input winding***»



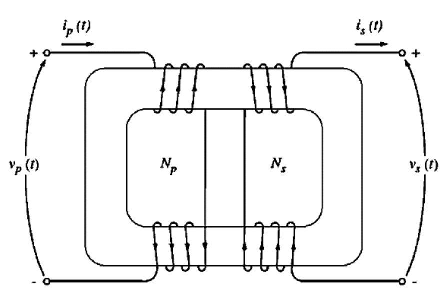
2-winding single-phase transformer

The winding connected to the loads is called «***secondary winding***» or «***output winding***».

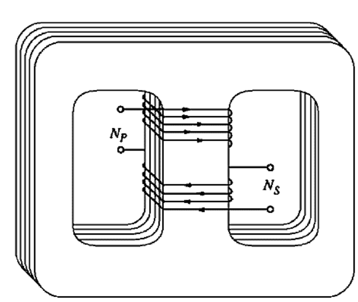
I f there is a ***third winding*** on the transformer, it is called «***tertiary winding***»

Construction Of Transformers

- Power transformers are constructed by **two types of cores**.
- These are «**core form**» and «**shell form**».
- **In core form**, there is a simple rectangular laminated piece of steel with the transformer windings wrapped around two sides of the rectangle.
- **In shell form**, there is a three-legged laminated core with the windings wrapped around the center leg.
- In either case, the core is constructed of **thin laminations** electrically isolated from each other in order to **minimize eddy currents**.

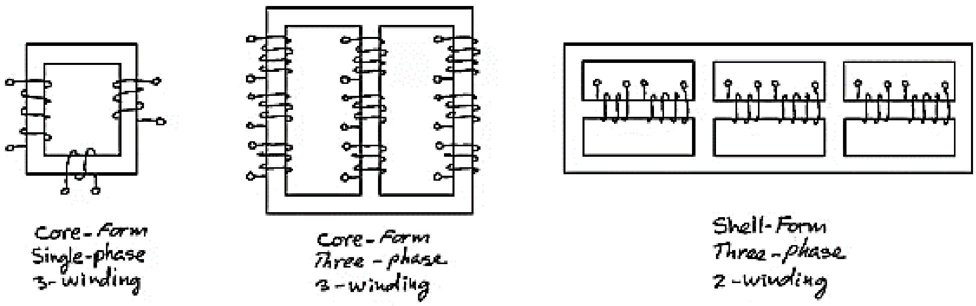


core form



shell form

Construction Of Transformers



Source: https://www.quora.com

Construction Of Transformers

- The primary and secondary windings in a physical transformer are wrapped one on top of the other with the **low-voltage winding innermost**. Such an arrangement serves two purposes:

- It simplifies the problem of insulating the high-voltage winding from the core.
- It results in much less leakage flux than would be the case if the two windings were separated by a distance on the core.

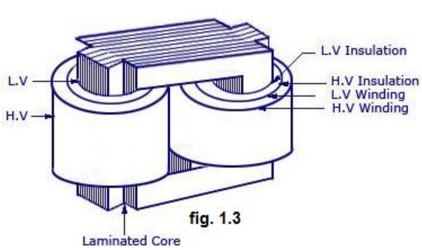
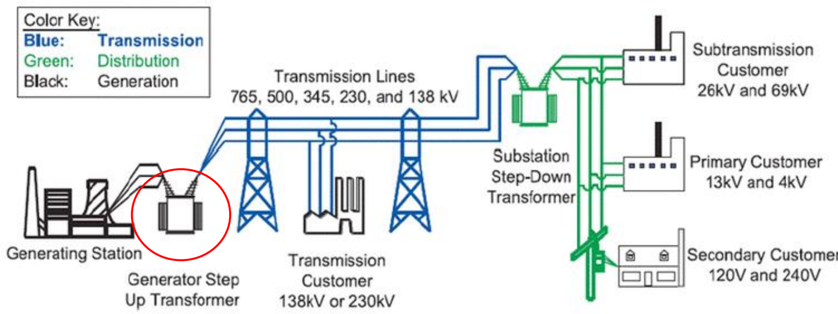


fig. 1.3

Source: https://www.top-ee.com

Transformer Types

- **Unit transformer:** This transformer is a **step-up** transformer and connected to the output of a generator and used to step the voltage from (***10-36 kV***) up to transmission levels (***110 kV or more***).



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250 MVA,110 kV three phase unit transformer

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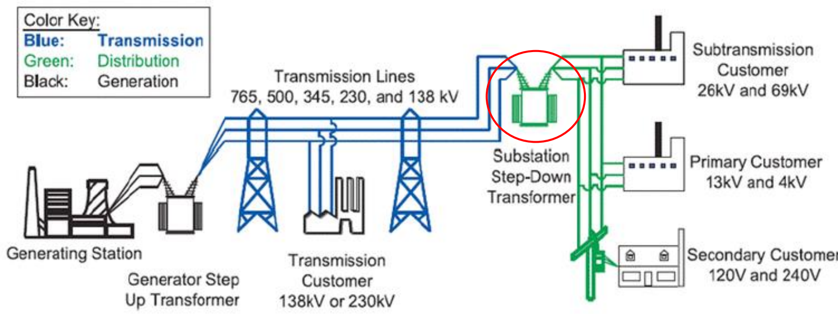
Transformer Types

- **Substation transformer:** This transformer is a **step-down** transformer and connected to the end of a transmission line in a substation and used to step the transmission voltage from (***110 kV or more***) down to distribution levels (***36 kV or less***).



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50/60 MVA ONAN/ONAF 230kV/13.8kV substation power transformer

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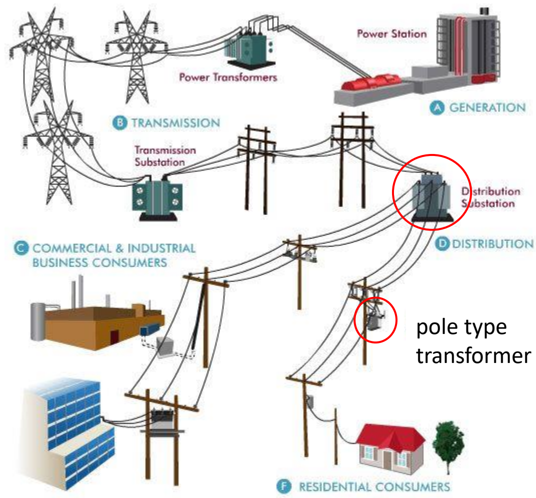
Transformer Types

- **<u>Distribution transformer:</u>** This transformer is a **<u>step-down</u>** transformer and connected to the end of a distribution line and used to step the distribution voltage from (***36 kV or less***) down to the low voltage level (***1 kV or less***).



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Transformer Types

Some examples to distribution transformers

200KVA 11/0.415KV Distribution Transformer

50KVA 33/0.415KV Distribution Transformer

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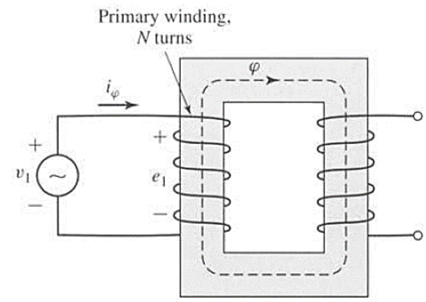
Transformer Analysis: No-load Condition

- The figure shows schematically a transformer with its **secondary** side **open-circuited**.
- Then a **alternating voltage v1** is applied to the **primary terminals**.
- Now, a **small steady-state current** $i_\emptyset$, called the «**exciting current**», flows in the primary.
- This **exciting current** establishes an **alternating flux** in the magnetic circuit (*Ampere's Law*).
- This **flux** induces a **counter emf (e1)** in the primary side as follows: (*Faraday's Law*)

$$e_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\varphi}{dt}$$

where

λ_1 = flux linkage of the primary winding
φ = flux in the core linking both windings
N_1 = number of turns in the primary winding



The **exciting current** of a typical power transformer is about **1 to 2 percent of full-load current**

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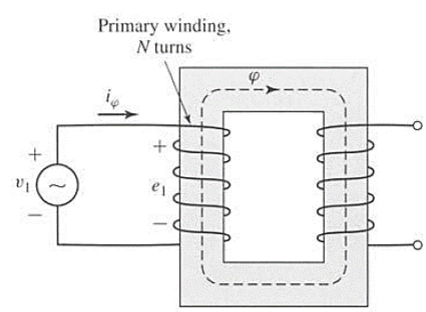
Transformer Analysis: No-load Condition

- If **R1** is the **resistance of the primary winding**, this resistance causes a **small voltage drop** and the following voltage equation can be written:

$$v_1 = R_1 i_\varphi + e_1$$

Under **no-load conditions**:

- ❖ we can neglect the effects of **primary leakage flux**
- ❖ Resistive voltage drop is **too small**
- ❖ The waveforms of **voltage** and **flux** are **very nearly sinusoidal**



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Transformer Analysis: No-load Condition

- If the instantaneous flux in the core is

$$\varphi = \phi_{\max} \sin \omega t$$

- The induced voltage can be written as

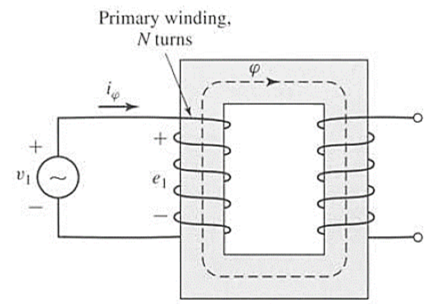
$$e_1 = N_1 \frac{d\varphi}{dt} = \omega N_1 \phi_{\max} \cos \omega t$$

- The **rms value** of the induced voltage is

$$E_1 = \frac{2\pi}{\sqrt{2}} f N_1 \phi_{\max} = \sqrt{2}\pi f N_1 \phi_{\max}$$

- The peak flux is then determined as

$$\emptyset_{max} = \frac{E_1}{\sqrt{2}\pi f N_1}$$

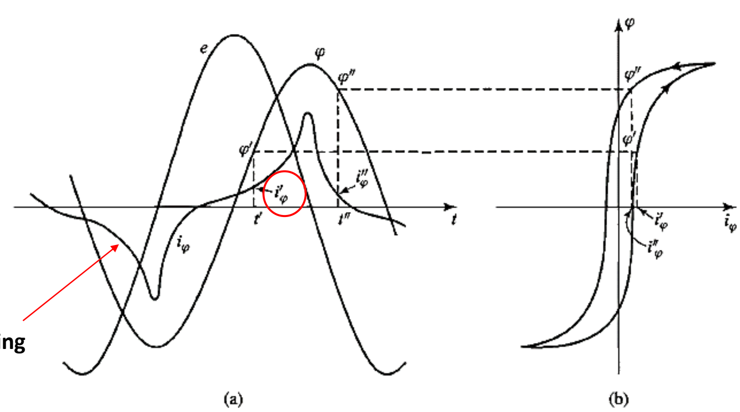


- if R1 is ignored

$$v_1 = R_1 i_\varphi + e_1 \Rightarrow v_1 \approx e_1 \Rightarrow V_1 \approx E_1$$

$$\boxed{\emptyset_{max} \approx \frac{V_1}{\sqrt{2}\pi f N_1}}$$

Transformer Analysis: No-load Condition

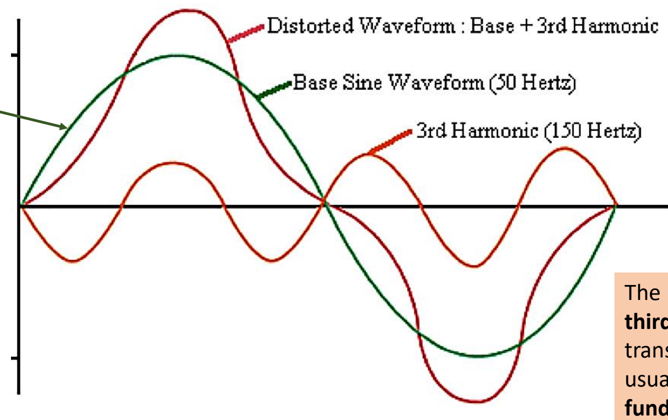


Non-sinusoidal exciting current having odd-harmonics

Excitation phenomena. (a) voltage, flux, and exciting current (b) corresponding hysteresis loop

Transformer Analysis: No-load Condition

Fundamental component



Source: http://www.electricalc.ro

The principal harmonic is the **third**. For typical power transformers, the **third harmonic** usually is **about 40 percent** of the **fundamental component of exciting current.**

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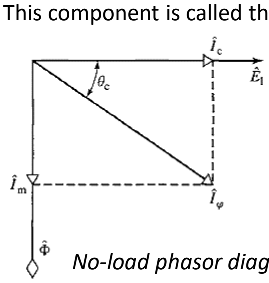
Transformer Analysis: No-load Condition

- Since exciting current **is about 1-2 % of rated current** and the third harmonic component is about **40% of the fundamental component of exciting current**, the effects of harmonics are **very small** under no-load condition and **can be neglected** in the analysis.
- So the fundamental component is approximately the same as the exciting current itself.
- The **fundamental component** of exciting current can be resolved into **two components**:
  - One is **in-phase** with the counter emf
    - This component supplies the power absorbed by **hysteresis** and **eddy-current losses** in the core.
    - It is referred to as **core-loss component** of the exciting current.
  - The other one is **lagging** the counter emf by **90°**
    - This component is called the "**magnetizing current**".



No-load phasor diagram

Ic is the **in-phase** component
Im is the **lagging** component and called «**magnetizing current**»

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Transformer Analysis: No-load Condition

Core loss calculation:

- The **core loss (*Pc*)** (***hysteresis + eddy current losses***) is calculated as follows:

$$P_c = E_1 I_\varphi \cos\theta_c$$

where

P_c is the core loss of the transformer (Watt)
E_1 is the rms value of the induced voltage (V)
θ_c is the power factor angle of exciting current

Pc must be **multiplied by 3** if the transformer is a **three-phase transformer**.

Ideal Transformer

We assume the followings for an ideal transformer;

- Winding resistances (R1,R2) are negligible
- All the flux is confined to the core and links both windings (*leakage flux is negligible*)
- There are no losses in the core
- Permeability of the core is so high

- The flux in the core is generated as follows:

$$v_1 = \cancel{R_1} i_\varphi + e_1$$

0

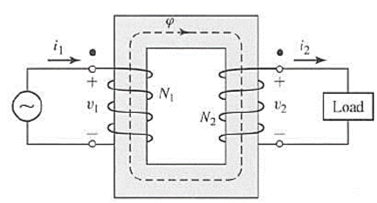
$$e_1 = N_1 \frac{d\varphi}{dt}$$

$$v_1 = e_1 = N_1 \frac{d\varphi}{dt}$$

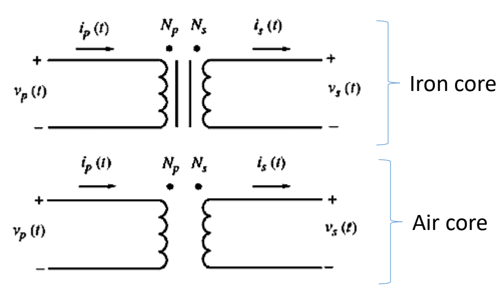
- Since, the same flux also links the secondary :

$$v_2 = e_2 = N_2 \frac{d\varphi}{dt}$$

$$\boxed{\frac{v_1}{v_2} = \frac{N_1}{N_2}}$$



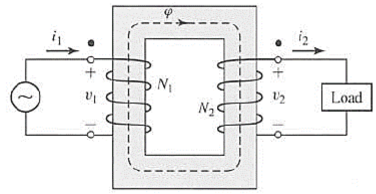
Ideal transformer and load



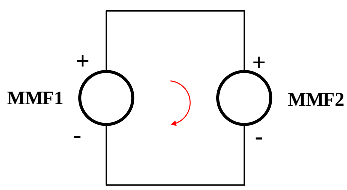
Schematic symbols of a transformer

Ideal Transformer

- If a load is connected to the secondary side, load current i2 flows through secondary winding as shown.
- This current generates a **MMF = N2i2** at the secondary side.
- If we think that **MMF** is like **EMF** (*voltage source*), the following equation can be written:



Ideal transformer and load



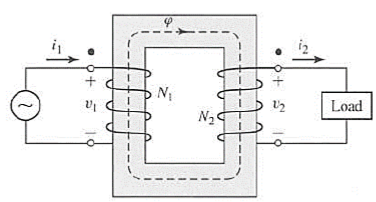
MMF1 - MMF2 = 0

$$\underbrace{N_1 i_1}_{\text{MMF1}} - \underbrace{N_2 i_2}_{\text{MMF2}} = 0 \qquad \boxed{\frac{i_1}{i_2} = \frac{N_2}{N_1}}$$

Ideal Transformer

- The "**turns ratio**" of a transformer is defined as

$$a = \frac{N_1}{N_2} = \frac{V_1}{V_2} \qquad a = \frac{N_1}{N_2} = \frac{I_2}{I_1}$$



Ideal transformer and load

- Since "***a***" is a **real number** for ideal transformer:
  - ❑ The **phase angles** of V_1 and V_2 are **same**.
  - ❑ The **phase angles** of I_1 and I_2 are **same**.

- For a **step-up** transformer ➔ **a < 1**
- For a **step-down** transformer ➔ **a > 1**
- For an **insulating** transformer ➔ **a = 1**

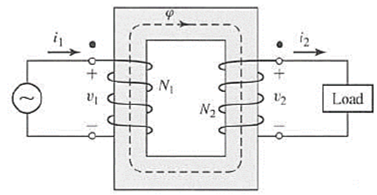
- For an ideal transformer, **instantaneous power** at both sides are **same:**

$$\frac{V_1}{V_2} = a \qquad \frac{I_2}{I_1} = a \qquad v_1 i_1 = a v_2 \left(\frac{1}{a}\right) i_2 = v_2 i_2$$

Because ideal transformer is an ideal device having **no loss**

Ideal Transformer

- Since **instantaneous power** is **same** at both sides, **active** and **reactive power** as well as **complex** and **apparent power** at both sides are also **same**



Ideal transformer and load

$$P_{pri} = V_1 I_1 \cos(\theta)$$

$$P_{sec} = V_2 I_2 \cos(\theta)$$

$$P_{sec} = \frac{V_1}{\cancel{a}} \cancel{a} I_1 \cos(\theta) = V_1 I_1 \cos(\theta) = P_{pri}$$

Active power is same

$$Q_{pri} = V_1 I_1 sin(\theta)$$

$$Q_{sec} = V_2 I_2 sin(\theta)$$

$$Q_{sec} = \frac{V_1}{\cancel{a}} \cancel{a} I_1 \, sin(\theta) = V_1 I_1 \, sin(\theta) = Q_{pri}$$

Reactive power is same

$S_{pri} = S_{sec}$ **Complex power is same**

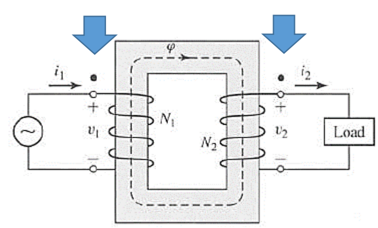
$|S_{pri}| = |S_{sec}|$ **Apparent power is same**

Ideal Transformer

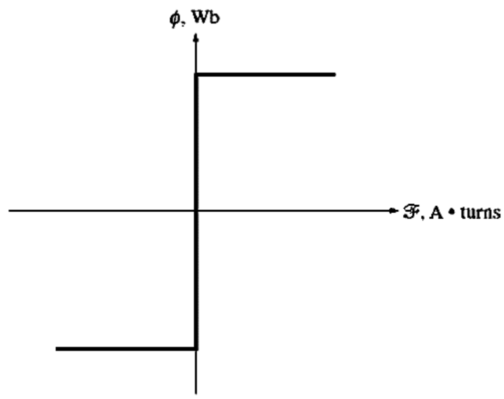
Dot Convention:

1) If the primary voltage is **positive** at the **dotted end** of the winding with respect to the **undotted end**, then the secondary voltage will be **positive** at the **dotted end** also.

1) If the primary current of the transformer **flows into** the **dotted end** of the **primary winding**, the secondary current will **flow out** of the **dotted end** of the **secondary winding**.



Ideal Transformer



The magnetization curve of an ideal transformer

Impedance Transformation in a Transformer

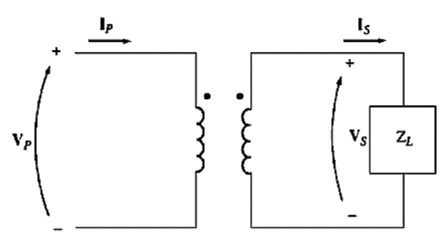
- An **impedance (ZL)** at the **secondary side** of a transformer can be transferred to the **primary side** or vice-versa:

$$Z_L = \frac{V_S}{I_S} = \frac{V_P/a}{aI_P} = \frac{1}{a^2}\frac{V_P}{I_P}$$

- If we call $Z_L' = V_P/I_P$

$Z_L' = a^2 Z_L$ (*Referred impedance to the primary side*)

- In reality, there is **no actual movement** of this impedance. This is just **in theory** which simplifies our calculations.



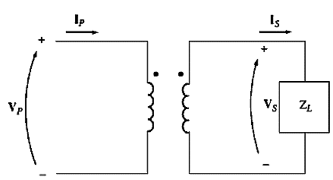
Impedance Transformation in a Transformer

- **Current** and **voltage** can also be transferred from one side to another:

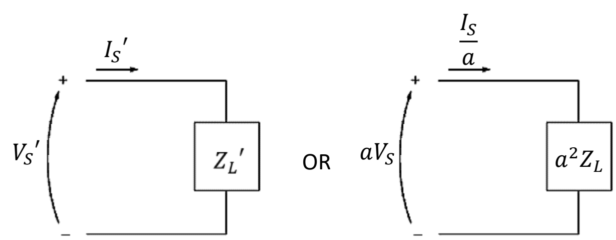
By making transformations, the transformer symbol is **eliminated** from the circuit.

$$a = \frac{I_S}{I_P} \Rightarrow I_P = \frac{I_S}{a} = I_S' \quad (\textit{Referred current to the primary side})$$

$$a = \frac{V_P}{V_S} \Rightarrow V_P = aV_S = V_S' \quad (\textit{Referred voltage to the primary side})$$



Before transformation



After transformation

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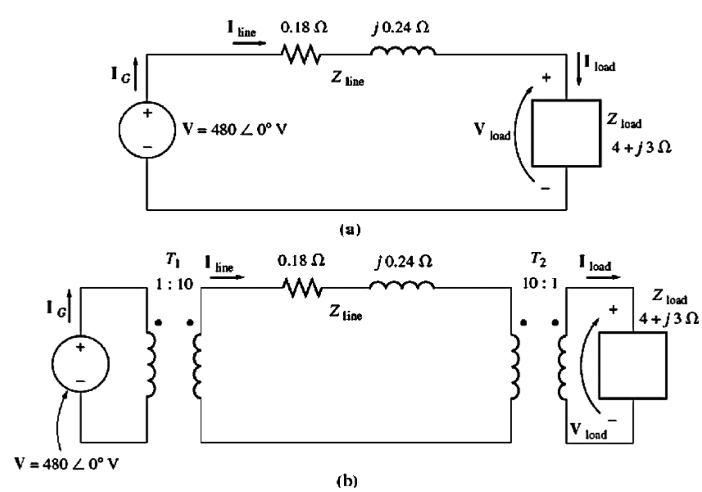
Ideal Transformer

Example:

Compare the **transmission losses** of both simple power systems.



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Theory of Real Single-Phase Transformers

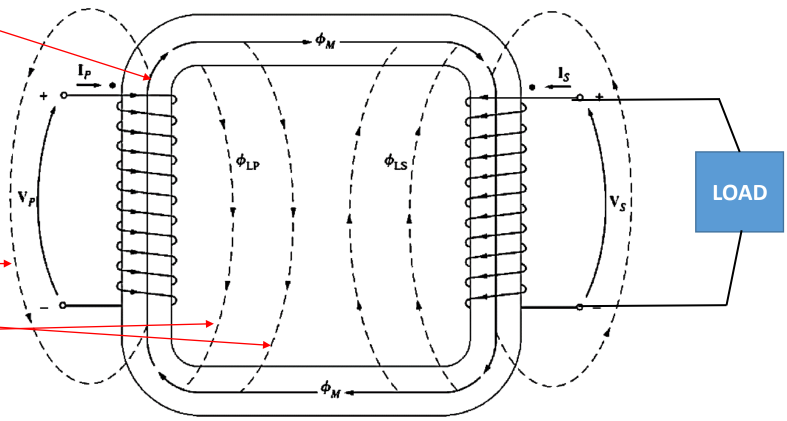
- The **total flux ($\emptyset_P$)** linking the **primary winding** is divided into two components:

  - **Mutual Flux ($\emptyset_M$)**
  - **Primary Leakage Flux** ($\emptyset_{LP}$)

$$\emptyset_P = \emptyset_M + \emptyset_{LP}$$

Mutual flux ($\emptyset_M$) is confined to the iron core and produced by the combined effect of the primary and secondary currents

Primary leakage flux ($\emptyset_{LP}$) is the flux which links only the primary winding and occurs due to the finite magnitude of core permeability



Theory of Real Single-Phase Transformers

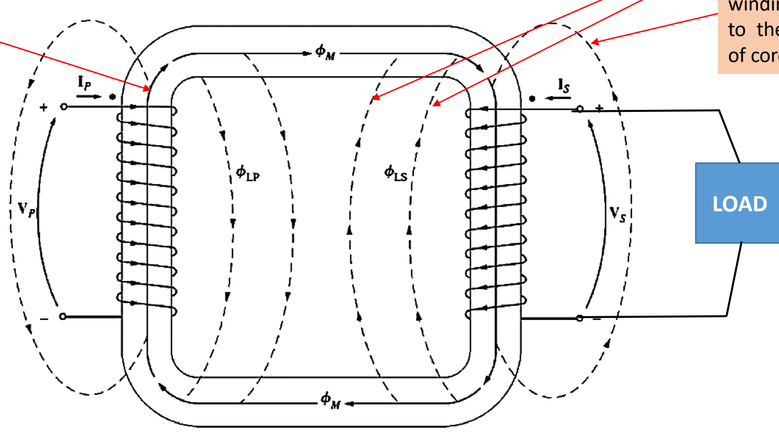
- Similarly, the **total flux ($\emptyset_S$)** linking the **secondary winding** is divided into two components:

  - **Mutual Flux ($\emptyset_M$)**
  - **Secondary Leakage Flux** ($\emptyset_{LS}$)

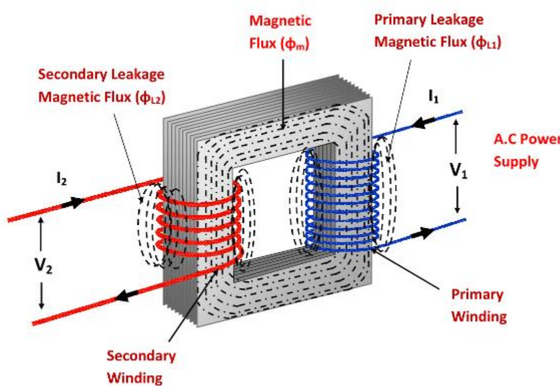
$$\emptyset_S = \emptyset_M + \emptyset_{LS}$$

Secondary leakage flux ($\emptyset_{LS}$) is the flux which links only the secondary winding and occurs due to the finite magnitude of core permeability

Mutual flux ($\emptyset_M$) is confined to the iron core and produced by the combined effect of the primary and secondary currents



Theory of Real Single-Phase Transformers



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Theory of Real Single-Phase Transformers

- Applying **Faraday's Law** to the **primary side**:

$$v_P(t) = N_P \frac{d\overline{\phi}_P}{dt}$$

$$= \underbrace{N_P \frac{d\phi_M}{dt}}_{e_P(t)} + \underbrace{N_P \frac{d\phi_{LP}}{dt}}_{e_{LP}(t)}$$

$e_P(t)$: Primary voltage component caused by the mutual flux

$e_{LP}(t)$: Voltage component caused by the primary leakage flux

$$\boxed{v_P(t) = e_P(t) + e_{LP}(t)}$$

- Applying **Faraday's Law** to the **secondary side**:

$$v_S(t) = N_S \frac{d\overline{\phi}_S}{dt}$$

$$= \underbrace{N_S \frac{d\phi_M}{dt}}_{e_S(t)} + \underbrace{N_S \frac{d\phi_{LS}}{dt}}_{e_{LS}(t)}$$

$e_S(t)$: Secondary voltage component caused by the mutual flux

$e_{LS}(t)$: Voltage component caused by the secondary leakage flux

$$\boxed{v_S(t) = e_S(t) + e_{LS}(t)}$$

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Theory of Real Single-Phase Transformers

Turns Ratio:

$$e_P(t) = N_P \frac{d\phi_M}{dt}$$

$$e_S(t) = N_S \frac{d\phi_M}{dt}$$

$$\boxed{\frac{e_P(t)}{e_S(t)} = \frac{N_P}{N_S} = a}$$

- The above equation means that the ratio of the primary voltage caused by the mutual flux to the secondary voltage caused by the mutual flux is equal to the turns ratio of the transformer.

- In a well-designed transformer,

$$\phi_M >> \phi_{LP}$$

$$\phi_M >> \phi_{LS},$$

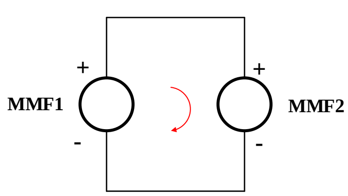
$$\frac{v_P(t)}{v_S(t)} \cong \frac{N_P}{N_S} = a$$

Theory of Real Single-Phase Transformers

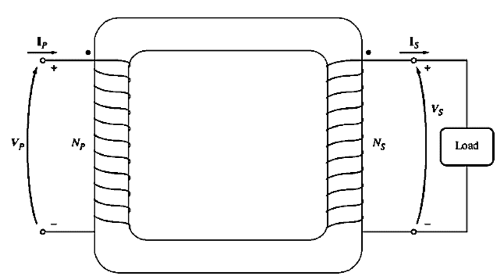
- If a load is connected to the secondary side, load current i2 flows through secondary winding as shown.

- This current generates a **MMF = NSiS** at the secondary side.

- If we think that **MMF** is like **EMF** (*voltage source*), the following equation can be written:



MMF1 - MMF2 = 0



A real transformer with a load connected to its secondary

$$\mathscr{F}_{net} = \underbrace{N_P i_P}_{\text{MMF1}} - \underbrace{N_S i_S}_{\text{MMF2}} = \phi \mathscr{R}$$

In a well-designed transformer, core reluctance is **very small**

$$\mathcal{R} \approx 0$$

$$\boxed{\frac{i_S}{i_P} \approx \frac{N_P}{N_S}}$$

The Exact Equivalent Circuit of a Transformer

We have to consider the following points when modelling a real transformer:

1) **Copper losses:** Copper losses are the resistive heating losses in the primary and secondary windings of the transformer. They are proportional to the square of the current in the windings.

Core Losses

2) **Eddy current losses:** Eddy current losses are resistive heating losses in the core of the transformer. They are proportional to the square of the voltage applied to the transformer.

3) **Hysteresis losses:** Hysteresis losses are associated with the rearrangement of the magnetic domains in the core during each half-cycle, as explained before. These losses are proportional to the applied voltage.

4) **Leakage flux:** These fluxes escape the core and pass through only one of the windings. Leakage fluxes produce a self-inductance in the primary and secondary coils, and the effects of this inductance must be considered.

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The Equivalent Circuit of a Transformer

Leakage reactance of primary side

$$X_P = 2\pi f L_{LP}$$

How to model leakage flux in the primary side ?

- Voltage component caused by the primary leakage flux

$$e_{LP}(t) = N_P \frac{d\phi_{LP}}{dt}$$

- The above equation can be written in terms of flux linkage:

$$\lambda_{LP} = N_P \phi_{LP} \Rightarrow e_{LP}(t) = \frac{d}{dt}\lambda_{LP}$$

- Since **flux linkage** is equal to the **inductance x current**:

$$\lambda_{LP} = L_{LP} i_P \Rightarrow e_{LP}(t) = \frac{d}{dt} L_{LP} i_P \Rightarrow e_{LP}(t) = L_{LP}\frac{d}{dt} i_P$$

Well-known voltage/current equation for an **inductor**

L_{LP}

We call this inductor as «**self-inductance of the primary winding**» and use it to model **leakage flux of the primary** in the equivalent circuit of the transformer

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The Equivalent Circuit of a Transformer

Leakage reactance of secondary side

$X_S = 2\pi f L_{LS}$

How to model leakage flux in the secondary side ?

- Voltage component caused by the secondary leakage flux

$$e_{LS}(t) = N_S \frac{d\phi_{LS}}{dt}$$

- The above equation can be written in terms of flux linkage:

$$\lambda_{LS} = N_S \phi_{LS} \Rightarrow e_{LS}(t) = \frac{d}{dt}\lambda_{LS}$$

- Since **flux linkage** is equal to the **inductance x current**:

$$\lambda_{LS} = L_{LS} i_S \Rightarrow e_{LS}(t) = \frac{d}{dt} L_{LS} i_S \Rightarrow e_{LS}(t) = L_{LS} \frac{d}{dt} i_S$$

Well-known voltage/current equation for an **inductor**

L_{LS}

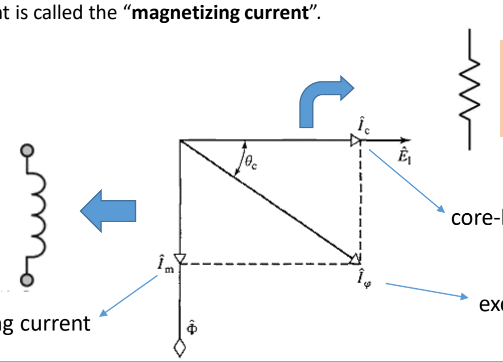
We call this inductor as «**self-inductance of the secondary winding**» and use it to model **leakage flux of the secondary** in the equivalent circuit of the transformer

The Equivalent Circuit of a Transformer

How to model core losses ?

- The exciting current can be resolved into **two components**:
  - One is **in-phase** with the counter emf
    - This component supplies the power absorbed by **hysteresis** and **eddy-current losses** in the core.
    - It is referred to as **core-loss component** of the exciting current.
  - The other one is **lagging** the counter emf by **90°**
    - This component is called the "**magnetizing current**".

A **shunt resistor in the excitation branch** can be used to model **core losses**



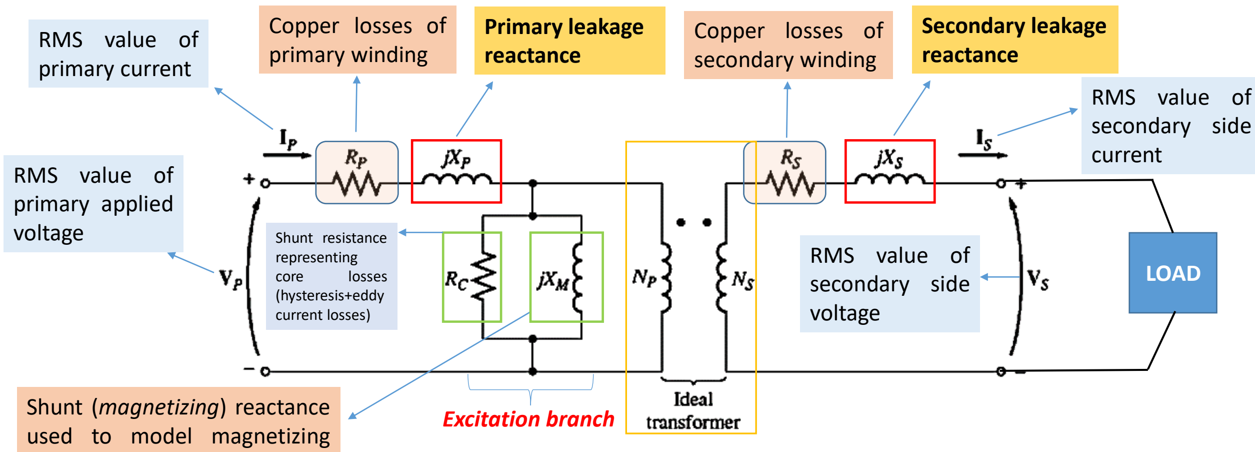
An **shunt inductor in the excitation branch** can be used to model **magnetizing current**

core-loss component of exciting current

exciting current

magnetizing current

The Equivalent Circuit of a Transformer



The exact circuit model of a single-phase real transformer

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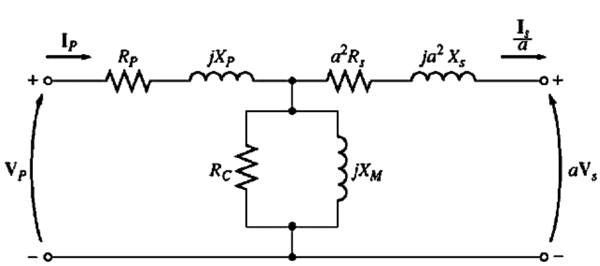
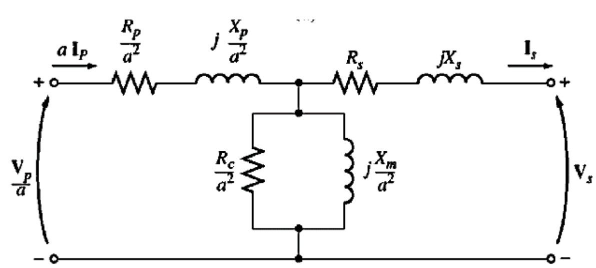
The Equivalent Circuit of a Transformer

- The **ideal transformer** symbol **can be removed** from the equivalent circuit by **refereeing** one side into another as shown below:



Equivalent circuit of single-phase transformer referrred to ***PRIMARY*** side



Equivalent circuit of single-phase transformer referrred to ***SECONDARY*** side

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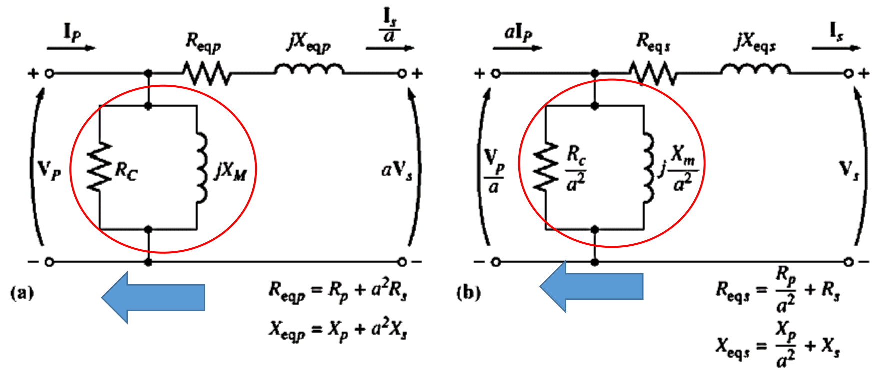
Approximate Equivalent Circuits

- Approximate equivalent circuit can be obtained by **shifting the excitation branch to the left side**.



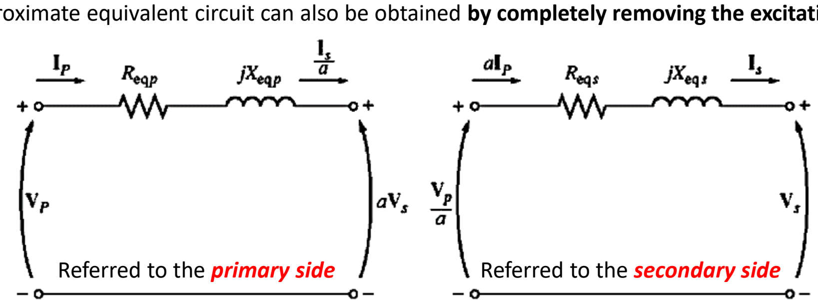
Referred to the ***primary side*** Referred to the ***secondary side***

Excitation branch is moved to <u>LEFT</u>

<u>FOUNDATION:</u> Under normal operation, the **voltage drop in Rp and Xp** is **<u>very smal</u>**l and **<u>can be neglected</u>**.

Approximate Equivalent Circuits

- A further approximate equivalent circuit can also be obtained **by completely removing the excitation branch**.



Excitation branch is <u>REMOVED</u>

<u>FOUNDATION:</u> Especially, in power system analysis, **excitation branch can be <u>removed</u>** in which the results are **<u>not so much effected</u>** from this action. Because excitation branch current is **<u>much less</u>** than the primary current of the transformer under rated conditions.

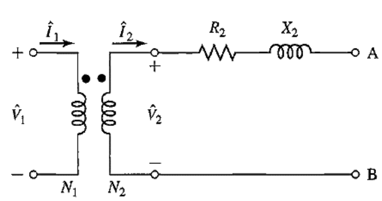
Approximate Equivalent Circuits

Example: A **50-kVA 2400:240-V 60-Hz** distribution transformer has a leakage impedance of **0.72 + j0.92 ohm** in the high-voltage winding and **0.0070 + j0.0090 ohm** in the low-voltage winding. At rated voltage and frequency, the impedance of the shunt branch (*equal to the impedance of Rc and jXm in parallel*) accounting for the exciting current is **6.32 + j43.7 ohm** when viewed from the low-voltage side.

Draw the equivalent circuit referred to
(a) the high-voltage side
(b) the low-voltage side, and label the impedances numerically.

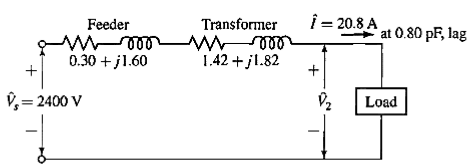
Self-study

The equivalent circuit shows an ideal transformer with an impedance of (**R2+jX2**) equals to **1 + j4 ohm** connected in series with the secondary. The turns ratio **N1/N2 = 5:1**. (a) Draw an equivalent circuit with the series impedance referred to the primary side. (b) For a primary voltage of **120 V rms** and a short connected across the terminals A-B, calculate the primary current and the current flowing in the short.

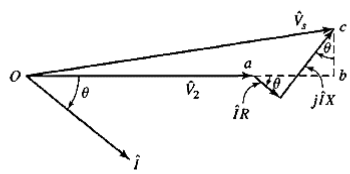


Self Study

The **50-kVA 2400:240-V 60-Hz** single-phase distribution transformer has a leakage impedance of **0.72 + j0.92 ohm** in the high-voltage winding and **0.0070 + j0.0090** ohm in the low-voltage winding. This transformer is used to step down the voltage at the load end of a feeder whose impedance is **0.30 + j 1.60 ohm**. The voltage Vs at the sending end of the feeder is **2400 V**. Find the voltage at the secondary terminals of the transformer when the load connected to its secondary draws rated current from the transformer and the power factor of the load is **0.80 lagging**. *Neglect excitation branch*.



One-line diagram of the system referred to HV side



Phasor diagram of the system

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Determining Equivalent Circuit Parameters

- It is possible to experimentally determine the values of the inductances and resistances in the transformer approximate model (*slide no:46*).
- An adequate approximation of these values can be obtained by **two tests**:
  - **Open-circuit test**
  - **Short-circuit test**
- In the **open-circuit test**, the secondary side is **open-circuited** and **excitation branch parameters** are approximayely found.
- In the **short-circuit test**, the secondary side is **short-circuited** and **series equivalent impedance** is approximayely found.

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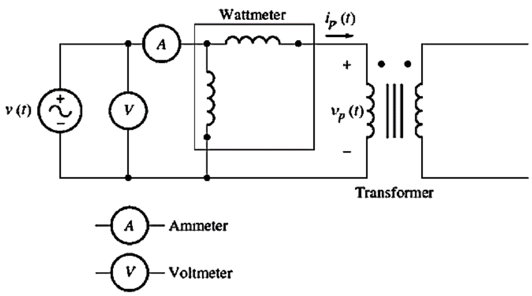
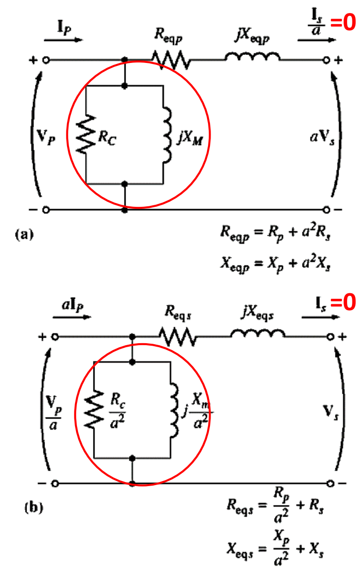
Determining Equivalent Circuit Parameters

Open Circuit Test:



Connection of measurement devices at the ***PRIMARY*** *side in the open circuit test*



Consider this circuit if measurement devices are replaced at **PRIMARY** side

Consider this circuit if measurement devices are replaced at **SECONDARY** side

Determining Equivalent Circuit Parameters

Steps of Open Circuit Test:

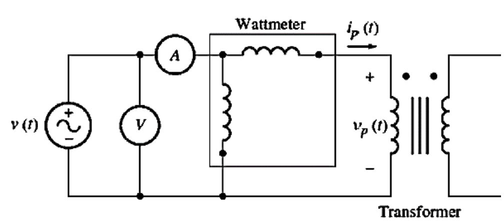
- The **full** (***rated***) line voltage is applied to the primary of the transformer.
- The **input voltage**, **input current**, and **input real power** to the transformer are measured by the **voltmeter**, **ammeter**, and **wattmeter**, respectively as shown in the figure.
- From this information, it is possible to determine the **excitation impedance** using the equations below:

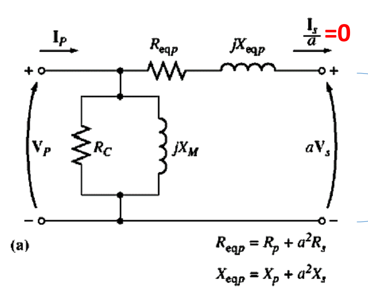
- The magnitude of the **excitation admittance** (*referred to the primary side*) can be found as

$$|Y_E| = \frac{I_{OC}}{V_{OC}}$$

I_{OC} → Ammeter reading

V_{OC} → Voltmeter reading





Open circuit

Determining Equivalent Circuit Parameters

Steps of Open Circuit Test:

- The **power factor angle** of the **excitation branch** is calculated as

$$\mathrm{PF} = \cos\theta = \frac{P_{OC}}{V_{OC} I_{OC}}$$

Wattmeter reading

Voltmeter reading Ammeter reading

$$\theta = \cos^{-1}\frac{P_{OC}}{V_{OC} I_{OC}}$$ (***Power factor angle***)

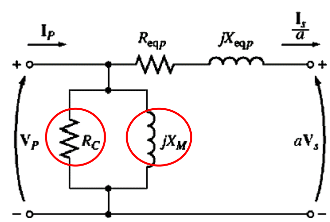
- From the information of $|Y_E|$ and $\boldsymbol{\theta}$, it is possible to calculate R_C and X_M, as follows:

$$Y_E = \frac{I_{OC}}{V_{OC}}\angle -\theta$$

$|Y_E|$

Phase angle is always **LAGGING**

$$Y_E = G_C - jB_M = \frac{1}{R_C} - j\frac{1}{X_M}$$



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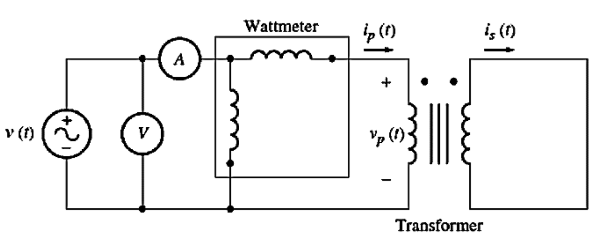
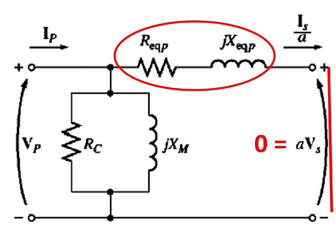
Determining Equivalent Circuit Parameters

Short Circuit Test:



Connection of measurement devices at the ***PRIMARY*** *side in short-circuit test*

(a)

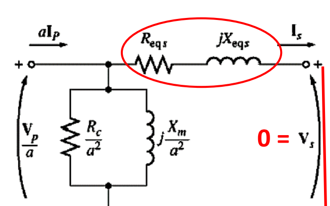


$R_{eqp} = R_p + a^2 R_s$

$X_{eqp} = X_p + a^2 X_s$

Consider this circuit if measurement devices are replaced at **PRIMARY** side

(b)



$R_{eqs} = \frac{R_p}{a^2} + R_s$

$X_{eqs} = \frac{X_p}{a^2} + X_s$

Consider this circuit if measurement devices are replaced at **SECONDARY** side

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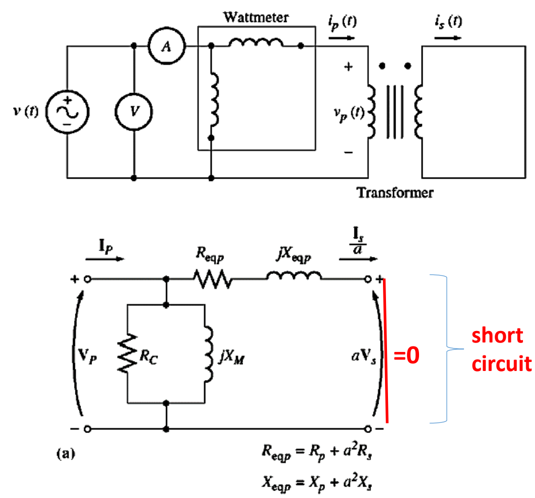
Determining Equivalent Circuit Parameters

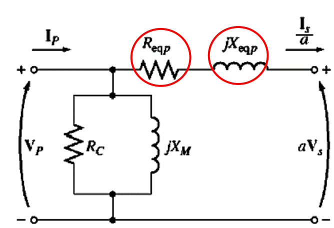
Steps of Short Circuit Test:

- The secondary terminals of the transformer are short circuited.
- The primary terminals are connected to a fairly low-voltage source.
- The input voltage is adjusted until the current in the short-circuited windings is equal to its rated value. (*Be sure to keep the primary voltage at a safe level. It would not be a good idea to burn out the transformer's windings while trying to test it.*)
- The **input voltage**, **input current**, and **input real power** are again measured by the **voltmeter**, **ammeter**, and **wattmeter**, respectively.
- From this information, it is possible to determine the **series impedance** using the following equations:



Determining Equivalent Circuit Parameters

Steps of Short Circuit Test:



- The magnitude of the **series impedance** (*referred to the primary side*) can be found as

$$|Z_{SE}| = \frac{V_{SC} \longrightarrow \text{Voltmeter reading}}{I_{SC} \longrightarrow \text{Ammeter reading}}$$

- The **power factor angle** of the **series impedance** is calculated as

$$\text{PF} = \cos\theta = \frac{P_{SC}}{V_{SC} I_{SC}}$$

P_{SC} → Wattmeter reading; V_{SC} → Voltmeter reading; I_{SC} → Ammeter reading

$$\Rightarrow \theta = \cos^{-1}\frac{P_{SC}}{V_{SC} I_{SC}} \quad (\textit{Power factor angle})$$

- From the information of $|Z_E|$ and θ, it is possible to calculate R_{eq} and X_{eq}, as follows:

$$\Rightarrow Z_{SE} = \underbrace{\frac{V_{SC}}{I_{SC}}}_{|Z_{SE}|}\angle\theta^\circ \Rightarrow Z_{SE} = R_{eq} + jX_{eq} = (R_P + a^2R_S) + j(X_P + a^2X_S)$$

(also named as "**short-circuit impedance**")

Self-study

The equivalent circuit impedances of a **20-kVA**, **8000/240-V**, **60-Hz** transformer are to be determined. The open-circuit test and the short-circuit test were performed on the primary side of the transformer, and the following data were taken:

Open-circuit test (on primary)	Short-circuit test (on primary)
$V_{OC} = 8000\ \text{V}$	$V_{SC} = 489\ \text{V}$
$I_{OC} = 0.214\ \text{A}$	$I_{SC} = 2.5\ \text{A}$
$V_{OC} = 400\ \text{W}$	$P_{SC} = 240\ \text{W}$

Find the impedances of the approximate equivalent circuit (*slide no:46, figure b*) referred to the primary side, and sketch that circuit.

Voltage Regulation of Transformers

- A practical transformer has always a **series impedance**.
- As a result, the **secondary (*output*) voltage** of a transformer **varies** with **the load** even if the **primary (*input*) voltage** remains **constant**.
- **Voltage regulation** is the percentage of voltage difference between **no load** and **full load (*rated*)** voltages of a transformer with respect to its **full load voltage**.
- Voltage regulation is a performance parameter of the transformers which is used for comparison of transformers.

$$\boxed{\text{VR} = \frac{V_{S,\text{nl}} - V_{S,\text{fl}}}{V_{S,\text{fl}}} \times 100\%}$$

where
VR is the voltage regulation in % percentage
$V_{s,nl}$ is the **no-load** voltage of secondary side
$V_{s,fl}$ is the **full-load** (*rated load*) voltage of secondary side

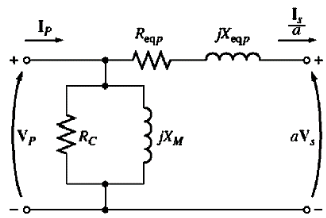
A well-designed transformer will normally has **low** voltage regulation which is mostly less than **10%**.

An **ideal transformer** will have **zero** voltage regulation

Voltage Regulation of Transformers

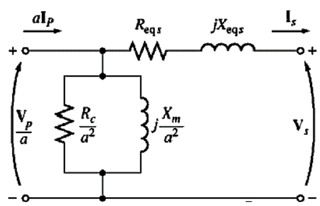
- Voltage regulation equation can be in different forms:

$$V_R = \frac{V_p - aV_s}{aV_s} x100\%$$



Equivalent circuit of single-phase transformer referrred to ***PRIMARY*** side

$$V_R = \frac{V_p/a - V_s}{V_s} x100\%$$



Equivalent circuit of single-phase transformer referrred to ***SECONDARY*** side

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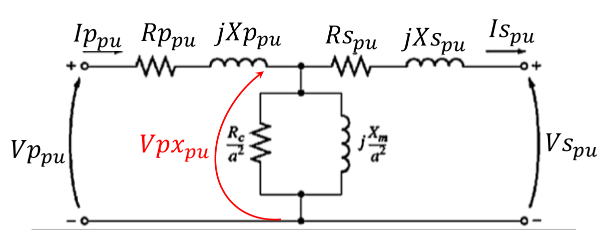
Voltage Regulation of Transformers

- Voltage regulation equation can be in different forms:

$$V_R = \frac{Vpx_{pu} - Vs_{pu}}{Vs_{pu}} x100\%$$



Per-unit equivalent circuit of single-phase transformer

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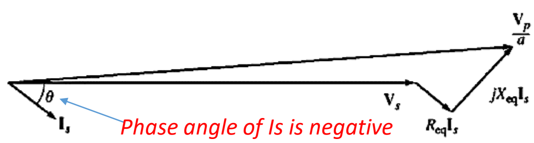
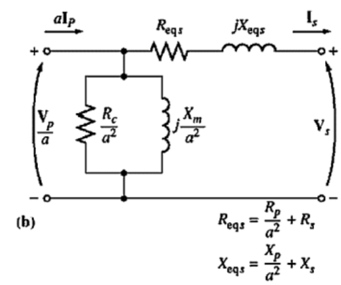
Transformer Phasor Diagram

Case-1: Lagging PF Load:



Phasor diagram of a transformer operating at a lagging power factor

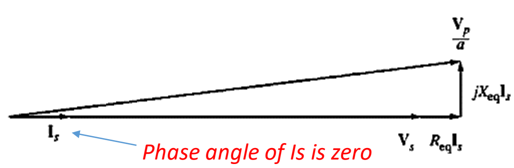


$V_R = \frac{V_p/a - V_s}{V_s}$ Since $|V_p/a| > |V_s|$ $V_R > 0$

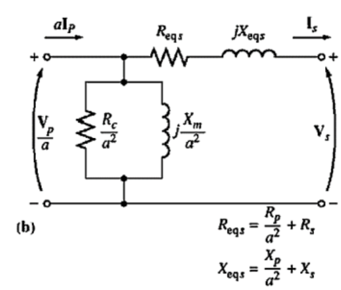
(Voltage regulation is always **positive** for **lagging PF** loads)

Transformer Phasor Diagram

Case-2: Unity PF Load:



Phasor diagram of a transformer operating at unity power factor

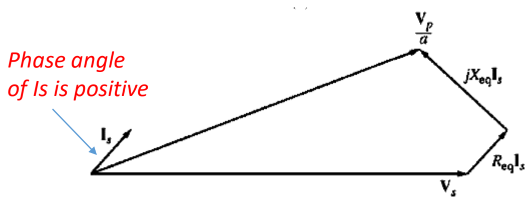


$V_R = \frac{V_p/a - V_s}{V_s}$ Since $|V_p/a| > |V_s|$ $V_R > 0$

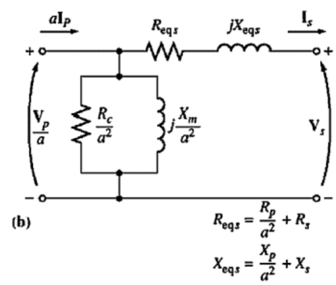
(Voltage regulation is always **positive** for **unity PF** loads)

Transformer Phasor Diagram

Case-2: Leading PF Load:



Phasor diagram of a transformer operating at leading factor



$V_R = \frac{V_p/a - V_s}{V_s}$ ➡ Since $|V_p/a| < |V_s|$ ➡ $V_R < 0$

(Voltage regulation is always **negative** for **leading PF** loads)

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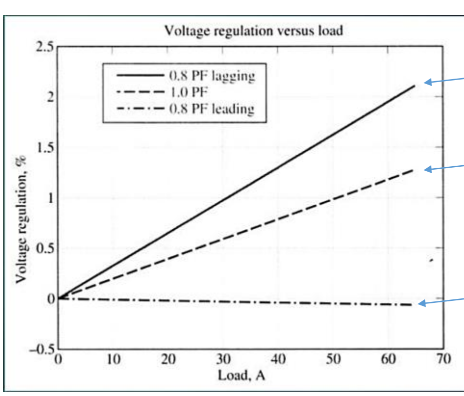
Voltage Regulation of Transformers

- So far, we have seen that voltage regulation can be either **positive** or **negative** depending on the **load power factor type**.
- Also, voltage regulation **<u>changes</u>** as load **<u>changes</u>** as seen below:



$V_R > 0$ if for **LAGGING PF** load

$V_R > 0$ if for **UNITY PF** load

$V_R < 0$ if for **LEADING PF** load

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Transformer Efficiency

- The **efficiency** of a transformer is defined as the ratio of its output real (*active*) power ($\boldsymbol{P_{out}}$) to its input real (*active*) power ($\boldsymbol{P_{in}}$), usually expressed in % percentage.
- The efficiency of an ideal transformer is 100%.
- For practical transformers, the efficiency is close but less than 100%.
- The efficiency is a performance parameter like "*voltage regulation*", used to compare transformers.
- The efficiency of a transformer can be calculated by the following equation:

$$\boxed{\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%}$$

$$P_{out} = \text{Vs}.\text{Is}.\cos(\theta_s)$$

$$P_{in} = \text{Vp}.\text{Ip}.\cos(\theta_p)$$

Vs is the rms voltage of secondary side

Is is the rms current of secondary side

θ_s is the phase angle between Vs and Is

Vp is the rms voltage of primary side

Ip is the rms current of primary side

θ_p is the phase angle between Vp and Ip

Transformer Efficiency

Typical efficiencies for some power transformers

Single Phase		Three Phase	
kVA	Efficiency	kVA	Efficiency
15	97.7	15	97.0
25	98.0	30	97.5
37.5	98.2	45	97.7
50	98.3	75	98.0
75	98.5	112.5	98.2
100	98.6	150	98.3
167	98.7	225	98.5
250	98.8	300	98.6
333	98.9	500	98.7
-	-	750	98.8
Source: MEMA TP-1-2002		1000	98.9

Source: https://library.automationdirect.com

Transformer Efficiency

- The **output real power** of a transformer is always **less** than its **input real power,** because of the **real losses**.

$P_{out} < P_{in}$

$P_{in} = P_{out} + P_{losses}$

- Because of this fact, the efficiency is **always less** than 100%.

$$\boxed{\eta = \frac{P_{out}}{P_{in}} \times 100\%} < 100\%$$

- The **real losses** are due to the
  - **Copper losses** of the primary and secondary windings
  - **Hysteresis losses** of the core
  - **Eddy current losses** of the core

CORE LOSSES (*iron losses*)

$$P_{losses} = P_{copper} + P_{core}$$

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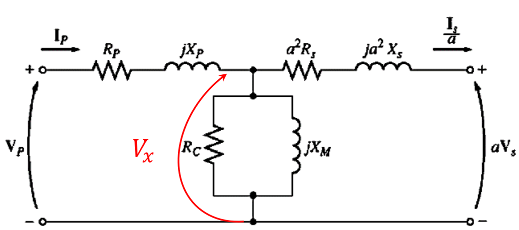
Transformer Loss Calculation

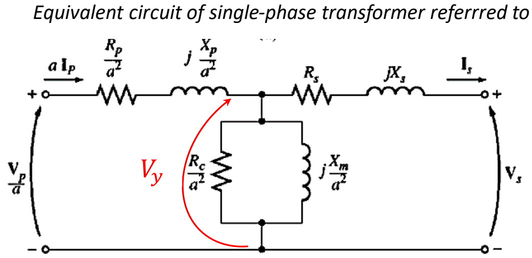
- The **real losses** can be calculated according to the type of the equivalent circuit of the transformer:



$$P_{copper} = R_p {I_p}^2 + a^2 R_s \left(\frac{I_s}{a}\right)^2$$

$$P_{core} = \frac{Vx^2}{R_C}$$

*Equivalent circuit of single-phase transformer referrred to **PRIMARY** side*



$$P_{copper} = \left(\frac{R_p}{a^2}\right)(aI_p)^2 + R_s {I_s}^2$$

$$P_{core} = \frac{{V_y}^2}{\left(\frac{R_C}{a^2}\right)}$$

*Equivalent circuit of single-phase transformer referrred to **SECONDARY** side*

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Transformer Loss Calculation

- There are also **reactive losses** of the transformers, besides their real losses.
- Reactive losses are due to the
  - Leakage reactance of the primary winding
  - Leakage reactance of the secondary winding
  - The reactance of the core

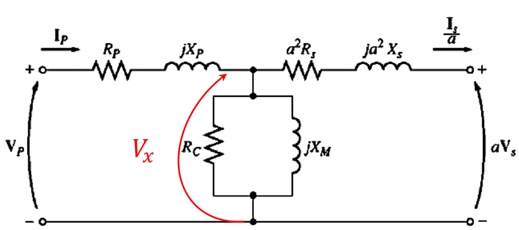
$$Q_{losses} = Q_{windings} + Q_{core}$$

Rated power (kVA)	Reactive power (kvar) to be compensated	
	No load	Full load
100	2.5	6.1
160	3.7	9.6
250	5.3	14.7
315	6.3	18.4
400	7.6	22.9
500	9.5	28.7
630	11.3	35.7
800	20	54.5
1000	23.9	72.4
1250	27.4	94.5
1600	31.9	126
2000	37.8	176

Reactive power consumption of some distribution transformers with 20 kV primary windings

Transformer Loss Calculation

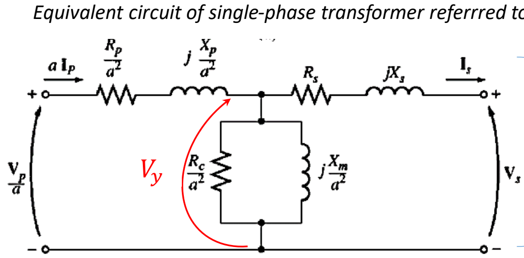
- The **reactive losses** can be calculated according to the type of the equivalent circuit of the transformer:



$$Q_{windings} = X_p {I_p}^2 + a^2 X_s \left(\frac{I_s}{a}\right)^2$$ Reactive losses of both windings

$$Q_{core} = \frac{{Vx}^2}{X_M}$$ Reactive losses of the core

Equivalent circuit of single-phase transformer referrred to ***PRIMARY*** *side*



$$Q_{windings} = \left(\frac{X_p}{a^2}\right)(aI_p)^2 + X_s {I_s}^2$$

$$Q_{core} = \frac{{V_y}^2}{\left(\frac{X_M}{a^2}\right)}$$

$$Q_{losses} = Q_{windings} + Q_{core}$$

Equivalent circuit of single-phase transformer referrred to ***SECONDARY*** *side*

Self-study

- Solve the Example 2.5 at page 103 from Chapman

Self-Study

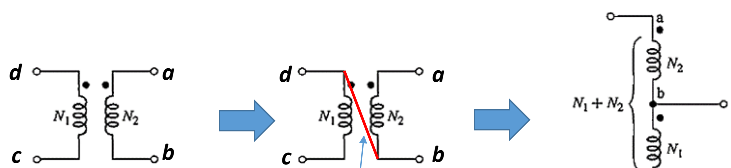
An open circuit and short-circuit tests are performed on a 50-kVA 2400:240-V single-phase transformer and the following data are obtained. Find at first, the equivalent circuit parameters of the transformer and determine the efficiency and the voltage regulation at full load for a power factor of

a) 0.95 lagging
b) 0.75 leading
c) Unity

Open circuit test (on the HV side)	Short circuit test (on the HV side)
Voltmeter reading = 240 V	Voltmeter reading = 48 V
Ammeter reading = 5.41 A	Ammeter reading = 20.8 A
Wattmeter reading = 186 W	Wattmeter reading = 617 W

Autotransformers

- If a there is an **electrical connection** between the **primary** and **secondary** circuits, then this type of transformer is called "***autotransformer***".
- In a conventional **two-winding transformer**, the **primary** and **secondary** windings are **electrically isolated**.
- In **autotransformer**, the windings are both **magnetically coupled** as well as **electrically connected**
- This is the **major difference** between **autotransformer** and **two-winding transformer**.
- In **autotransformer**, the voltage ratio is **close to 1:1**.
- **Autotransformers** have some **advantages** such as ***lower leakage reactance***, ***lower losses***, ***smaller exciting current***, and ***lower cost*** when compared to two-winding transformers.
- The **disadvantage** of **autotransformers** is that, unlike ordinary transformers, there is a **direct physical connection** between the **primary** and the **secondary** circuits, so **the electrical isolation of the two sides is lost**.



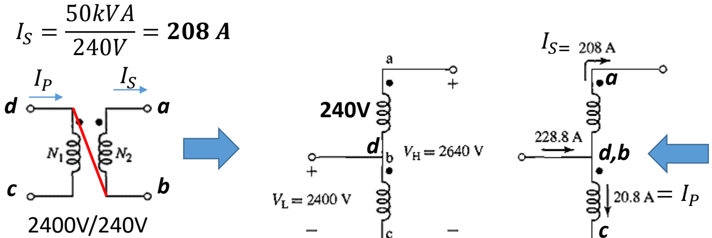
Two-winding transformer — *Electrical connection* — *Autotransformer*

Autotransformers

Example: The **50-kVA, 2400:240-V** transformer is connected as an autotransformer, as shown in the figure. The winding ***ab*** is the **240-V** winding, while the winding ***dc*** is the **2400-V** winding.
(*It is assumed that the 240-V winding has enough insulation to withstand a voltage of 2640 V to ground*.)
a) Compute the voltage and current ratings of high-voltage and low-voltage sides, respectively.
b) Compute the kVA rating of the autotransformer.

$$I_P = \frac{50kVA}{2400V} = \mathbf{20.8\ A}$$

$$I_S = \frac{50kVA}{240V} = \mathbf{208\ A}$$



2400V/240V
50-kVA

Input KVA rating:

$$|S_{in}| = V_P I_P = (2400V)(228.8A) = \mathbf{549.12kVA}$$

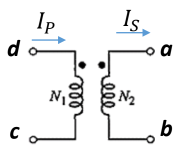
Output KVA rating:

$$|S_{out}| = V_S I_S = (2640V)(208A) = \mathbf{549.12kVA}$$

KCL at point ***d***: 208 A+20.8 A=**228.8 A**

Autotransformers

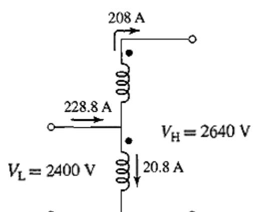
Example: If the losses of the transformer (**50-kVA, 2400:240-V**) given in the previous example is **800 W**, calculate the **efficiency of this transformer** under rated conditions for a **power factor of 0.8 lagging**, if it were connected as an autotransformer.



50 kVA, 2400V/240V

Efficiency = $\frac{P_{out}}{P_{in}} x100\% = \frac{50kVA*0.8}{50kVA*0.8+800W} = \mathbf{98.03}\%$

Losses **do not change** since primary voltage is ***constant*** and each winding has ***same current rating***



549.12 kVA, 2400V/2640V

Efficiency = $\frac{P_{out}}{P_{in}} x100\% = \frac{549.12kVA*0.8}{549.12kVA*0.8+800W} = \mathbf{99.85}\%$ **Better!**

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Autotransformers

0.5 kVA, 110V/0-130V Variac (Variable Autotransformer)

200 MVA, Distribution Auto-transformer / Oil-immersed

Source: http://www.directindustry.com

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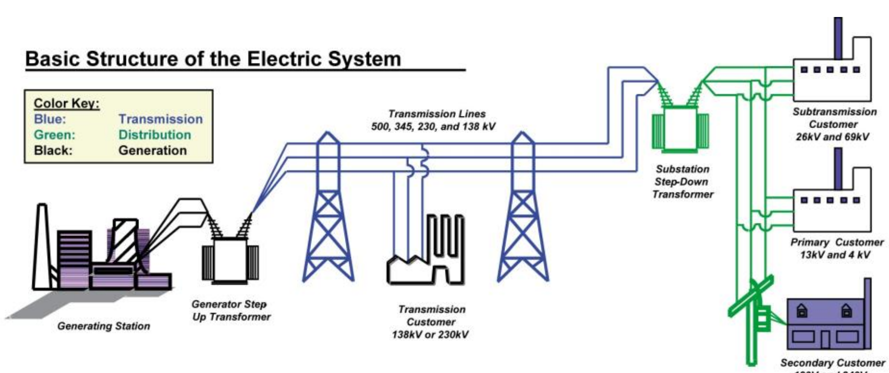
Self-study

- Solve Example 2.7 at page 113 from Chapman
- Solve Example 2.8 at page 116 from Chapman

Three-phase Transformers

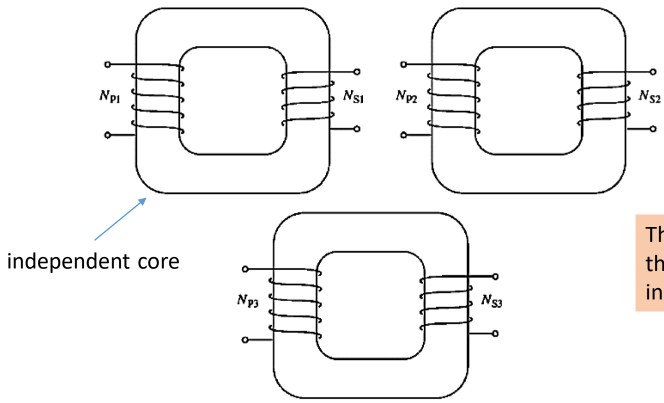
- Almost all the major power generation and distribution systems in the world today are **three-phase AC systems**.
- **Three-phase transformers** are used to interconnect these systems having **different voltage levels**.



Source: http://www2.econ.iastate.edu/classes/econ458/tesfatsion/Home458Team.htm

Three-phase Transformers

- Transformers for three-phase circuits can be constructed in **two different ways**:
  - The **first approach** is simply to take **three single-phase transformers** and connect them as a **three-phase bank**.



The **advantage** is that each unit in the bank could be replaced individually in case of failure.

A three-phase transformer bank composed of independent transformers

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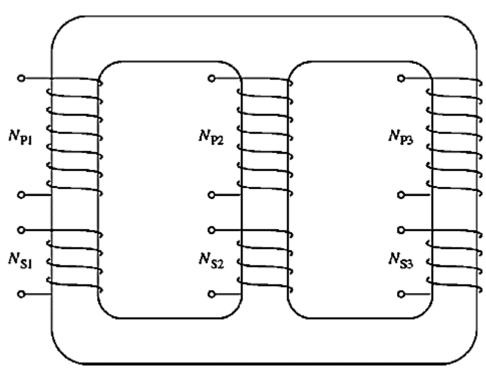
Three-phase Transformers

- Transformers for three-phase circuits can be constructed in **two different ways**:
  - The **second approach** is to make a three-phase transformer consisting of **three sets of windings** wrapped on a **common core**.
  - This option is **mostly preferred** in practice today.

The **advantages** are
- ❑ It is lighter
- ❑ It is smaller
- ❑ It is cheaper
- ❑ It is more efficient



A three-phase transformer wound on a single three-legged core.

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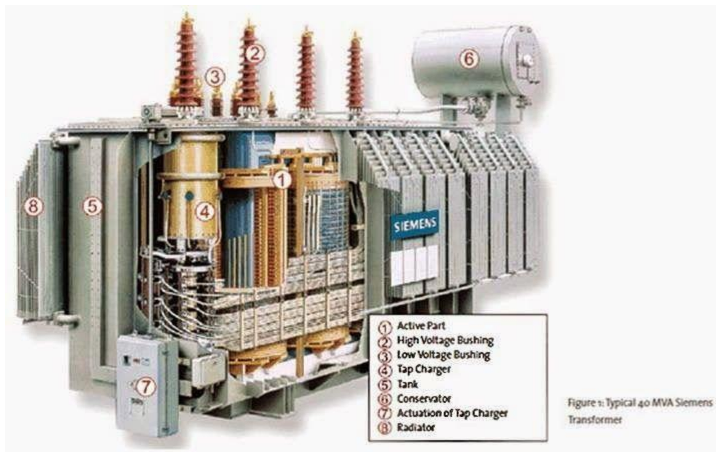
Three-phase Transformers



Source: http://www.siemens.com

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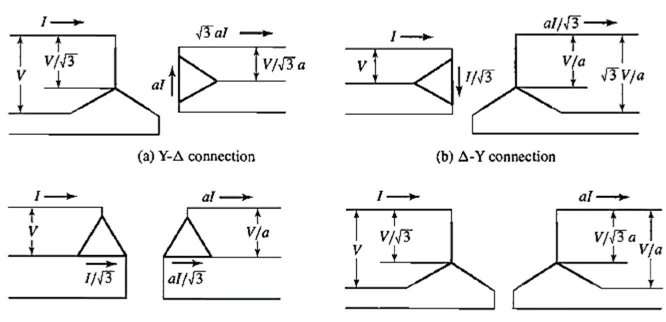
Three-phase Transformers

- The **primaries** and **secondaries** of any **three-phase transformer** can be independently connected in either **Wye (Y)** or **Delta (D)**.
- This gives a total of **<u>four possible connections</u>** for **three-phase transformers**:

1) Wye-Wye (Y-Y) or Star-Star
2) Wye-Delta (Y-D) or Star-Delta
3) Delta-Wye (D-Y) or Delta-Star
4) Delta-Delta (D-D)



Common three-phase transformer connections. The transformer windings are indicated by the heavy lines

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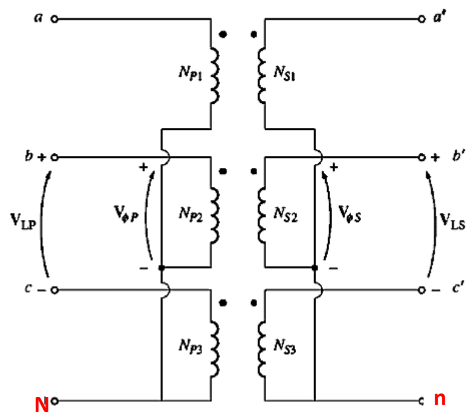
Three-phase Transformers

1) Wye-Wye (Y-Y) Connection:

- In this connection type, both the **primary** and **secondary** side are connected in **Wye (Y)** as shown below.



Y-Y wiring diagram

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Three-phase Transformers

1) Wye-Wye (Y-Y) Connection:

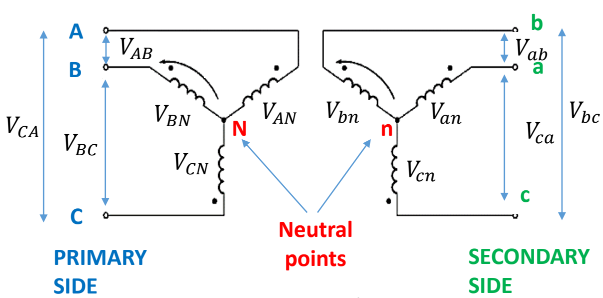
- In this connection type, both the **primary** and **secondary** side are connected in **Wye (Y)** as shown below.

$$V_{AB} = V_{AN} - V_{BN}$$
$$V_{BC} = V_{BN} - V_{CN}$$
$$V_{CA} = V_{CN} - V_{AN}$$

LL voltages

$$V_{ab} = V_{an} - V_{bn}$$
$$V_{bc} = V_{bn} - V_{cn}$$
$$V_{ca} = V_{cn} - V_{an}$$

LL voltages



Y-Y connection diagram

$|V_{AB}| = |V_{BC}| = |V_{CA}| = V_{LL}$ **(*pri. side line-to-line voltages*)**

$|V_{AN}| = |V_{BN}| = |V_{CN}| = V_{LN}$ **(*pri. side line-to-neutral voltages*)**

$$\boxed{V_{LL} = \sqrt{3}V_{LN}}$$

$|V_{ab}| = |V_{bc}| = |V_{ca}| = V_{ll}$ **(*sec. side line-to-line voltages*)**

$|V_{an}| = |V_{bn}| = |V_{cn}| = V_{ln}$ **(*sec. side line-to-neutral voltages*)**

$$\boxed{V_{ll} = \sqrt{3}V_{ln}}$$

$$\boxed{a = \frac{V_{LN}}{V_{ln}} = \frac{V_{LL}/\sqrt{3}}{V_{ll}/\sqrt{3}} = \frac{V_{LL}}{V_{ll}}}$$ **(*turns ratio*)**

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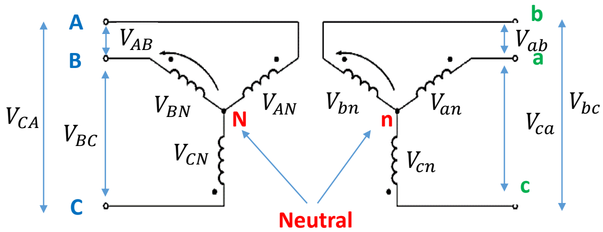
Three-phase Transformers

1) Wye-Wye (Y-Y) Connection:

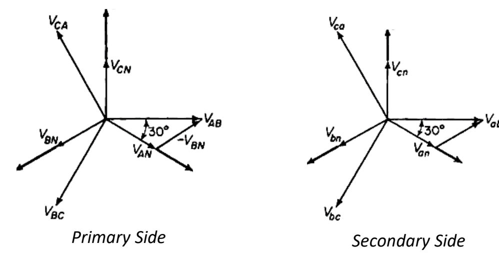
- In this connection type, both the **primary** and **secondary** side are connected in **Wye (Y)** as shown below.

$$\left.\begin{aligned} V_{AB} &= V_{AN} - V_{BN} \\ V_{BC} &= V_{BN} - V_{CN} \\ V_{CA} &= V_{CN} - V_{AN} \end{aligned}\right\} \text{LL voltages}$$

$$\left.\begin{aligned} V_{ab} &= V_{an} - V_{bn} \\ V_{bc} &= V_{bn} - V_{cn} \\ V_{ca} &= V_{cn} - V_{an} \end{aligned}\right\} \text{LL voltages}$$



Y-Y connection diagram



Phasor diagrams of Y-Y connection

Source: http://scooplocal.co/transformer-phasor-diagrams-yy.html

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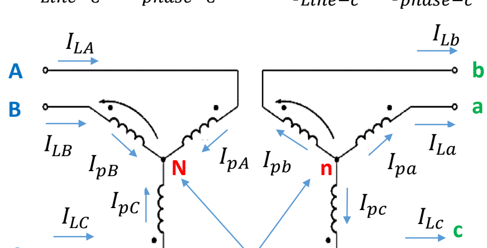
Three-phase Transformers

1) Wye-Wye (Y-Y) Connection:

$$I_{Line-A} = I_{phase-A} \qquad I_{Line-a} = I_{phase-a}$$
$$I_{Line-B} = I_{phase-B} \qquad I_{Line-b} = I_{phase-b}$$
$$I_{Line-C} = I_{phase-C} \qquad I_{Line-c} = I_{phase-c}$$



Y-Y connection diagram

$|I_{LA}| = |I_{LB}| = |I_{LC}| = I_{L-pri}$ **(pri. side line currents)**

$|I_{pA}| = |I_{pB}| = |I_{pC}| = I_{p-pri}$ **(pri. side phase currents)**

$$\boxed{I_{L-pri} = I_{p-pri}}$$

$|I_{La}| = |I_{Lb}| = |I_{Lc}| = I_{L-sec}$ **(sec. side line currents)**

$|I_{pa}| = |I_{pb}| = |I_{pc}| = I_{p-sec}$ **(sec. side phase currents)**

$$\boxed{I_{L-sec} = I_{p-sec}}$$

$$\boxed{a = \frac{I_{p-sec}}{I_{p-pri}} = \frac{I_{L-sec}}{I_{L-pri}}}$$ ***(turns ratio)***

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Three-phase Transformers

The Attributes of Wye-Wye (Y-Y) Connection:

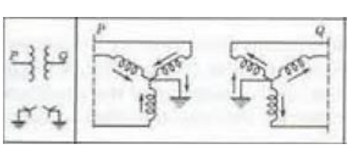
- The **Y-Y** connection is **seldom used** because:
  - If loads connected to the transformer are **unbalanced**, then the **LL** voltages of the transformer **can become severely unbalanced** due to **unequal voltage drops on phase impedances** of the transformer.
  - **Third-harmonic voltages** can be large because of the **nonlinearity** of the **transformer core.**
- Both problems **negatively affect** the **power quality** of the power system in which this type of transformer is connected.
- The **unbalance problem** and the **third-harmonic problem** can be solved using one of two techniques:

Three-phase Transformers

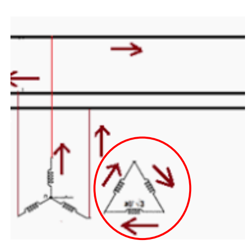
The Attributes of Wye-Wye (Y-Y) Connection:

Both the **unbalance problem** and the **third-harmonic problem** can be solved using one of two techniques:

1) Solidly ground the **neutral points (*N* and *n*)** of the **Y-Y** transformer: By this way, the neutral provides a return path for any current imbalances in the load. Moreover, by this way, the third-harmonic currents flow in the neutral instead of building up unwanted large third-harmonic voltages.



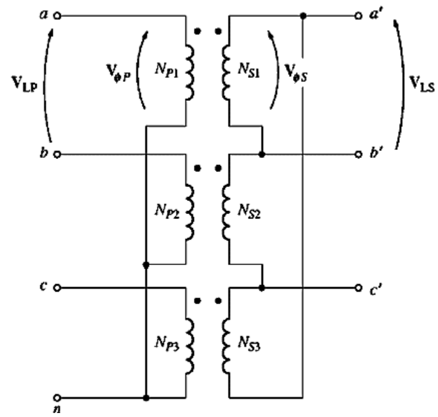
2) Add a third (tertiary) winding to the transformer as shown below. By this way, the third-harmonic voltages cause a circulating current flow within this winding and harmonics will not be reflected outside of the windings.



Three-phase Transformers

2) Wye-Delta (Y-D) Connection:

- In this connection type, the **primary** is connected as **Wye (Y)** and **secondary** is connected as **Delta (D)** as shown below.



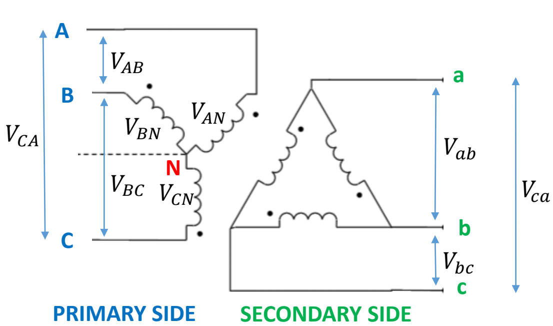
Y-D wiring diagram

Three-phase Transformers

2) Wye-Delta (Y-D) Connection:

- In this connection type, the **primary** is connected as **Wye (Y)** and **secondary** is connected as **Delta (D)** as shown below.

$$\left.\begin{aligned} V_{AB} &= V_{AN} - V_{BN} \\ V_{BC} &= V_{BN} - V_{CN} \\ V_{CA} &= V_{CN} - V_{AN} \end{aligned}\right\}$$ **LL voltages at PRIMARY SIDE**



Y-D connection diagram

$|V_{AB}| = |V_{BC}| = |V_{CA}| = V_{LL}$ ***(pri. side line-to-line voltages)***

$|V_{AN}| = |V_{BN}| = |V_{CN}| = V_{LN}$ ***(pri. side line-to-neutral voltages)***

$$\boxed{V_{LL} = \sqrt{3}V_{LN}}$$

$|V_{ab}| = |V_{bc}| = |V_{ca}| = V_{ll}$ ***(sec. side line-to-line voltages)***

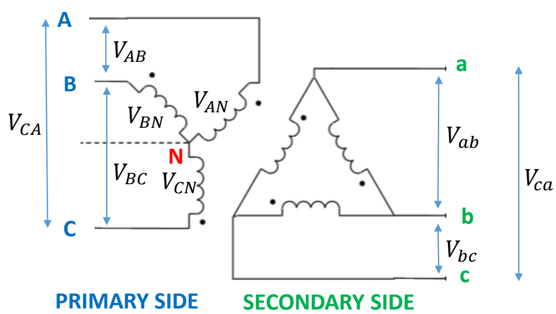
$$\boxed{a = \frac{V_{LN}}{V_{ll}} = \frac{V_{LL}/\sqrt{3}}{V_{ll}}}$$ ***(turns ratio)***

Three-phase Transformers

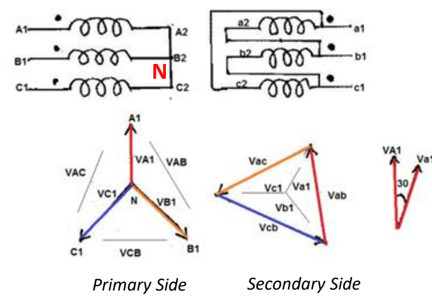
2) Wye-Delta (Y-D) Connection:

- In this connection type, the **primary** is connected as **Wye (Y)** and **secondary** is connected as **Delta (D)** as shown below.

$$\left.\begin{aligned} V_{AB} &= V_{AN} - V_{BN} \\ V_{BC} &= V_{BN} - V_{CN} \\ V_{CA} &= V_{CN} - V_{AN} \end{aligned}\right\} \text{LL voltages at PRIMARY SIDE}$$



Y-D connection diagram



Phasor diagrams of Y-D connection

Source: https://electricalnotes.wordpress.com

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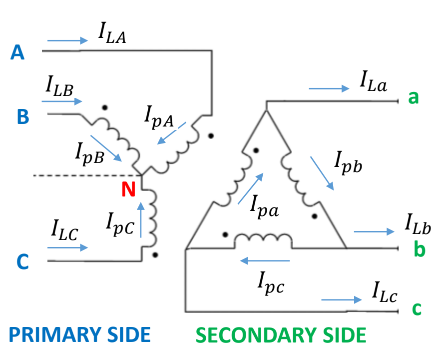
Three-phase Transformers

2) Wye-Delta (Y-D) Connection:

$$I_{Line-A} = I_{phase-A}$$
$$I_{Line-B} = I_{phase-B}$$
$$I_{Line-C} = I_{phase-C}$$



Y-D connection diagram

$|I_{LA}| = |I_{LB}| = |I_{LC}| = I_{L-pri}$ **(pri. side line currents)**

$|I_{pA}| = |I_{pB}| = |I_{pC}| = I_{p-pri}$ **(pri. side phase currents)**

$$\boxed{I_{L-pri} = I_{p-pri}}$$

$|I_{La}| = |I_{Lb}| = |I_{Lc}| = I_{L-sec}$ **(sec. side line currents)**

$|I_{pa}| = |I_{pb}| = |I_{pc}| = I_{p-sec}$ **(sec. side phase currents)**

$$\boxed{I_{L-sec} = \sqrt{3}.I_{p-sec}}$$

$$\boxed{a = \frac{I_{p-sec}}{I_{p-pri}} = \frac{I_{L-sec}/\sqrt{3}}{I_{L-pri}}}$$ ***(turns ratio)***

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Three-phase Transformers

The Attributes of Wye-Delta (Y-D) Connection:

- This connection has **no problem** with **third-harmonic components** in its voltages, since they are consumed in a circulating current on the **delta side**.
- This connection is also **more stable** with respect to **unbalanced loads**, since the **delta side** partially **redistributes any imbalance** that occurs.
- This arrangement has a property that the **primary side LL voltages** are leading the **secondary side LL voltages** by **30° degrees**.
- This **natural phase shift** can cause problems during **paralleling different types of transformers**. For example when paralleling a **Y-Y** transformer and a **Y-D** transformer.
- Since, the **phase angles of the secondary voltages must be same** if two transformers are **connected in parallel** for **load sharing**.

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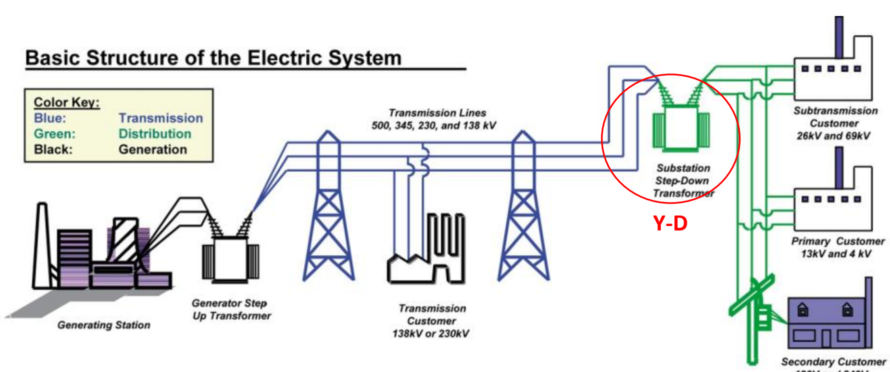
Three-phase Transformers

The Attributes of Wye-Delta (Y-D) Connection:

- The **Y-D** connection is commonly used in **stepping down** from a **high voltage** to a **medium or low voltage**.
- One reason is that a **neutral** is provided **for grounding on the high-voltage side** that is used to **protect** the transmission line from **very high fault currents** and to **reduce insulation costs of the transmission side**.



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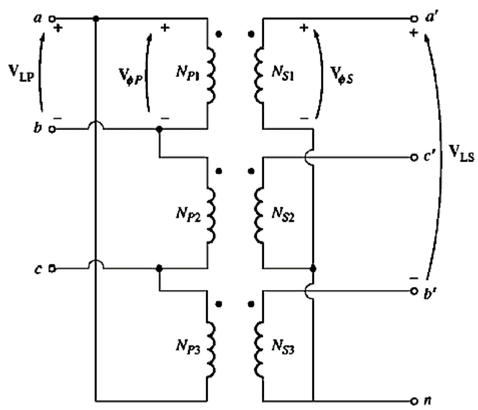
Three-phase Transformers

3) Delta-Wye (D-Y) Connection:

- In this connection type, the **primary** is connected as **Delta (D)** and **secondary** is connected as **Wye (Y)** as shown below.



D-Y wiring diagram

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Three-phase Transformers

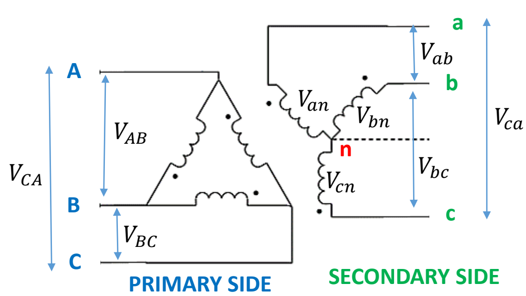
3) Delta-Wye (D-Y) Connection:

- In this connection type, the **primary** is connected as **Delta (D)** and **secondary** is connected as **Wye (Y)** as shown below.

$$V_{ab} = V_{an} - V_{bn}$$
$$V_{bc} = V_{bn} - V_{cn}$$
$$V_{ca} = V_{cn} - V_{an}$$

LL voltages at SECONDARY SIDE

$|V_{AB}| = |V_{BC}| = |V_{CA}| = V_{LL}$ ***(pri. side line-to-line voltages)***



D-Y connection diagram

$|V_{ab}| = |V_{bc}| = |V_{ca}| = V_{ll}$ ***(sec. side line-to-line voltages)***

$|V_{an}| = |V_{bn}| = |V_{cn}| = V_{ln}$ ***(sec. side line-to-neutral voltages)***

$$\boxed{V_{ll} = \sqrt{3}V_{ln}}$$

$$\boxed{a = \frac{V_{LL}}{V_{ln}} = \frac{V_{LL}}{V_{ll}/\sqrt{3}}}$$ ***(turns ratio)***

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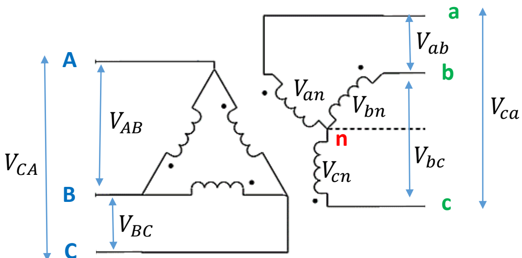
Three-phase Transformers

3) Delta-Wye (D-Y) Connection:

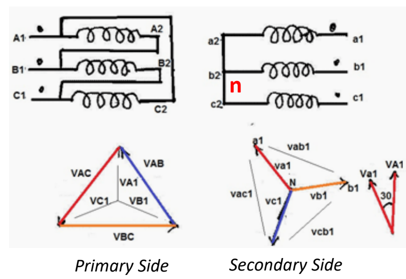
- In this connection type, the **primary** is connected as **Delta (D)** and **secondary** is connected as **Wye (Y)** as shown below.

$$V_{ab} = V_{an} - V_{bn}$$
$$V_{bc} = V_{bn} - V_{cn}$$
$$V_{ca} = V_{cn} - V_{an}$$

LL voltages at SECONDARY SIDE



D-Y connection diagram



Phasor diagrams of D-Y connection

Source: https://electricalnotes.wordpress.com

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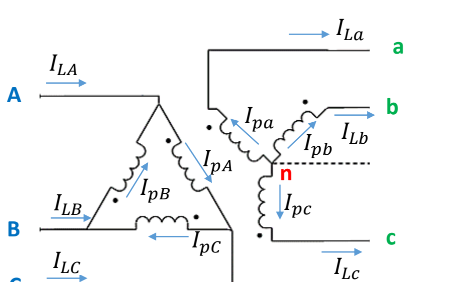
Three-phase Transformers

3) Delta-Wye (D-Y) Connection:

$$I_{Line-a} = I_{phase-a}$$
$$I_{Line-b} = I_{phase-b}$$
$$I_{Line-c} = I_{phase-c}$$



D-Y connection diagram

$|I_{LA}| = |I_{LB}| = |I_{LC}| = I_{L-pri}$ ***(pri. side line currents)***

$|I_{pA}| = |I_{pB}| = |I_{pC}| = I_{p-pri}$ ***(pri. side phase currents)***

$$\boxed{I_{L-pri} = \sqrt{3}.I_{p-pri}}$$

$|I_{La}| = |I_{Lb}| = |I_{Lb}| = I_{L-sec}$ ***(sec. side line currents)***

$|I_{pa}| = |I_{pb}| = |I_{pc}| = I_{p-sec}$ ***(sec. side phase currents)***

$$\boxed{I_{L-sec} = I_{p-sec}}$$

$$\boxed{a = \frac{I_{p-sec}}{I_{p-pri}} = \frac{I_{L-sec}}{I_{L-pri}/\sqrt{3}}}$$ ***(turns ratio)***

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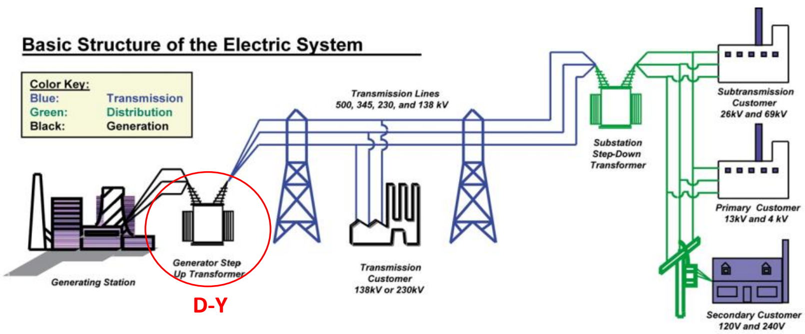
Three-phase Transformers

The Attributes of Delta-Wye (D-Y) Connection:

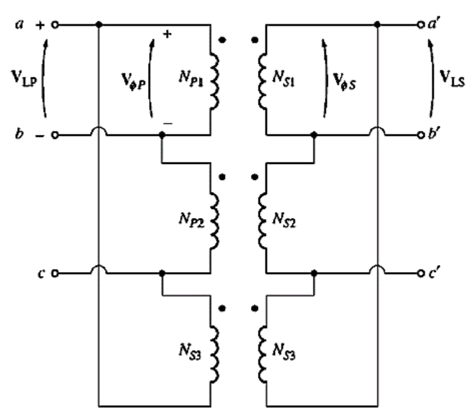
- The **D-Y** connection has the **same advantages** and the **same phase shift** as the **Y-D** transformer.
- The **D-Y** connection is commonly used in **stepping up** from **medium or low voltage** to a **high voltage**.
- One reason is that a **neutral** is provided **for grounding on the high-voltage side** that is used **to protect the transmission line** from **very high fault currents** and **to reduce insulation costs of the transmission side**.



Three-phase Transformers

4) Delta-Delta (D-D) Connection:

- In this connection type, the **primary** and **secondary** side are connected as **Delta (D)** as shown below.

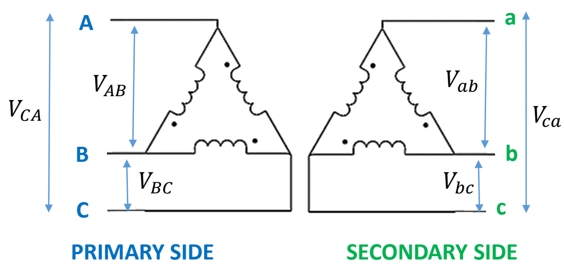


D-D wiring diagram

Three-phase Transformers

4) Delta-Delta (D-D) Connection:

- In this connection type, the **primary** and **secondary** side are connected as **Delta (D)** as shown below.



D-D connection diagram

$|V_{AB}| = |V_{BC}| = |V_{CA}| = V_{LL}$ ***(pri. side line-to-line voltages)***

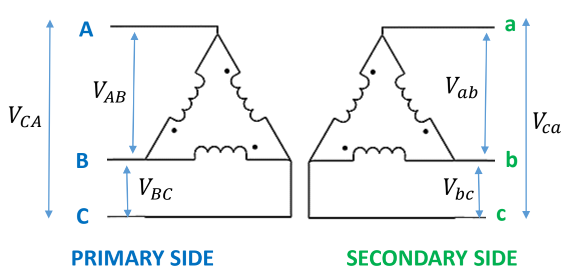
$|V_{ab}| = |V_{bc}| = |V_{ca}| = V_{ll}$ ***(sec. side line-to-line voltages)***

$\boxed{a = \frac{V_{LL}}{V_{ll}}}$ ***(turns ratio)***

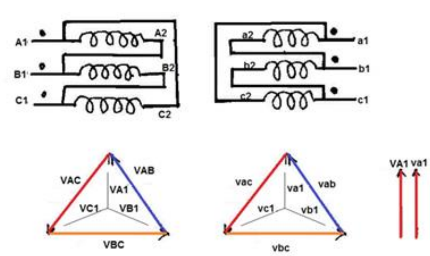
Three-phase Transformers

4) Delta-Delta (D-D) Connection:

- In this connection type, the **primary** and **secondary** side are connected as **Delta (D)** as shown below.



D-D connection diagram

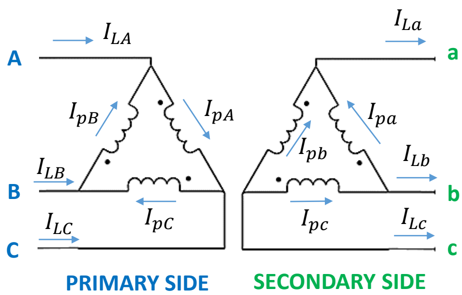


Phasor diagrams of D-D connection

Source: https://electricalnotes.wordpress.com

Three-phase Transformers

4) Delta-Delta (D-D) Connection:



D-D connection diagram

$|I_{LA}| = |I_{LB}| = |I_{LC}| = I_{L-pri}$ **(*pri. side line currents*)**

$|I_{pA}| = |I_{pB}| = |I_{pC}| = I_{p-pri}$ **(*pri. side phase currents*)**

$$\boxed{I_{L-pri} = \sqrt{3} I_{p-pri}}$$

$|I_{La}| = |I_{Lb}| = |I_{Lb}| = I_{L-sec}$ **(*sec. side line currents*)**

$|I_{pa}| = |I_{pb}| = |I_{pc}| = I_{p-sec}$ **(*sec. side phase currents*)**

$$\boxed{I_{L-sec} = \sqrt{3} I_{p-sec}}$$

$$\boxed{a = \frac{I_{p-sec}}{I_{p-pri}} = \frac{I_{L-sec}/\sqrt{3}}{I_{L-pri}/\sqrt{3}}}$$ **(*turns ratio*)**

Derating of a Transformer

- If a **60-Hz transformer** is to be operated on **50-Hz power system**, its applied voltage (*primary*) must also be **reduced** by a factor of **1/6**.
- If this is not done, the transformer core can go to the **saturation region** and the peak flux in the core will be **too high**.
- This reduction in applied voltage together with the frequency is called «***derating***».

$v(t) = V_M \sin \omega t$ (*primary applied voltage*)

$\phi(t) = \frac{1}{N_P} \int v(t)\, dt$ (*flux in the core is calculated by Faraday's Law*)

$= \frac{1}{N_P} \int V_M \sin \omega t \, dt$

$$\boxed{\phi(t) = \frac{V_M}{\omega N_P} \cos \omega t}$$

$\frac{V_M}{w}$ → This ratio must be held **constant**

- By considering the same equation, a **50-Hz transformer** may be operated at a **20 % higher voltage** on **60-Hz power system** if this action does **not cause insulation problems**.

Inrush Current Problem

- We know that the **flux** in the core is determined by the **primary voltage** of the transformers.

$$\phi(t) = \frac{1}{N_P} \int v(t)\, dt$$

- The **phase angle of primary voltage** will effect the result of this integral, for example if

$$v(t) = V_M \sin(\omega t + 90°) \quad \Rightarrow \quad \phi_{max} = \frac{V_{max}}{\omega N_P}$$

- Or;

$$v(t) = V_M \sin \omega t \quad \Rightarrow \quad \boxed{\phi_{max} = \frac{2V_{max}}{\omega N_P}}$$

- As seen above, depending on the **phase angle**, the **maximum flux** can be **different**
- We know that the **magnetization current** and the **maximum flux** are related with each other.

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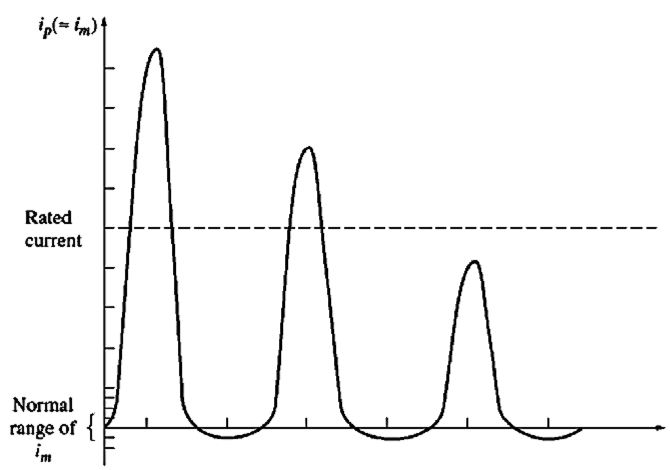
Inrush Current Problem

- The applied phase angle of the voltage is not normally controlled on **starting**, so there can be **huge inrush currents** during the first several cycles after the transformer is connected to the line.

- The transformer and the power system to which it is connected must be able to withstand these currents.



The inrush current due to a transformer's magnetization current on starting

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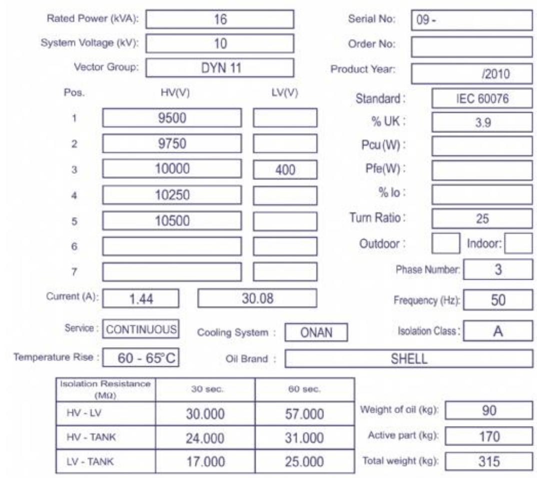
The Transformer Nameplate



Rated Power (kVA):	16	
System Voltage (kV):	10	
Vector Group:	DYN 11	

Pos.	HV(V)	LV(V)
1	9500	
2	9750	
3	10000	400
4	10250	
5	10500	
6		
7		
Current (A):	1.44	30.08

Serial No:	09 -
Order No:	
Product Year:	/2010
Standard:	IEC 60076
% UK:	3.9
Pcu (W):	
Pfe (W):	
% Io:	
Turn Ratio:	25
Outdoor:	
Indoor:	
Phase Number:	3
Frequency (Hz):	50
Isolation Class:	A
Service:	CONTINUOUS
Cooling System:	ONAN
Temperature Rise:	60 - 65°C
Oil Brand:	SHELL

Isolation Resistance (MΩ)	30 sec.	60 sec.
HV - LV	30.000	57.000
HV - TANK	24.000	31.000
LV - TANK	17.000	25.000

Weight of oil (kg):	90
Active part (kg):	170
Total weight (kg):	315

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Self-study

- Solve **<u>all problems</u>** of Chapter 2 from Chapman

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Review Questions

1) Is the turns ratio of a transformer the same as the ratio of voltages across the transformer? Why or why not?
2) Why does the magnetization current impose an upper limit on the voltage applied to a transformer core?
3) What components compose the excitation current of a transformer? How are they modeled in the transformer 's equivalent circuit?
4) What is the leakage flux in a transformer? Why is it modeled in a transformer equivalent circuit as an inductor?
5) List and describe the types of losses that occur in a transformer.
6) Why does the power factor of a load affect the voltage regulation of a transformer?
7) Why does the short-circuit test essentially show only I^2R losses and not excitation losses in a transformer?
8) Why does the open-circuit test essentially show only excitation losses and not I^2R losses?
9) Why can autotransformers handle more power than conventional transformers of the same size?
10) What are the problems associated with the Y - Y three-phase transformer connection?
11) What types of connections can be used? What are their advantages and disadvantages?
12) Can a 60-Hz transformer be operated on a 50-Hz system? What actions are necessary to enable this operation?
13) What happens to a transformer when it is first connected to a power line? Can anything be done to mitigate this problem?
14) A distribution transformer is rated at 18 kVA, 20,000/480 V, and 60 Hz. Can this transformer safely supply 15 kVA to a 415-V load at 50 Hz? Why or why not?
15) Why does one hear a hum when standing near a large power transformer?

END OF CHAPTER 2

TRANSFORMERS