

EEE270 Introduction to Electrical Energy Systems

Chapter 7 – Voltage Drop and Power Loss Calculations

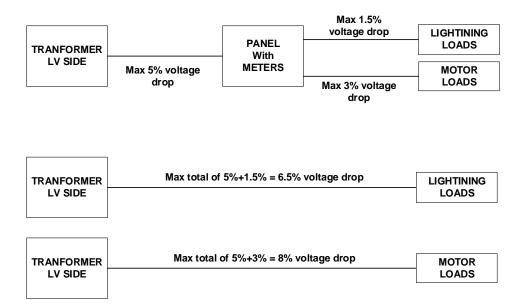
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Assist. Prof. Dr. Ali Osman ARSLAN aoarslan@gantep.edu.tr

CABLE SIZE SELECTION

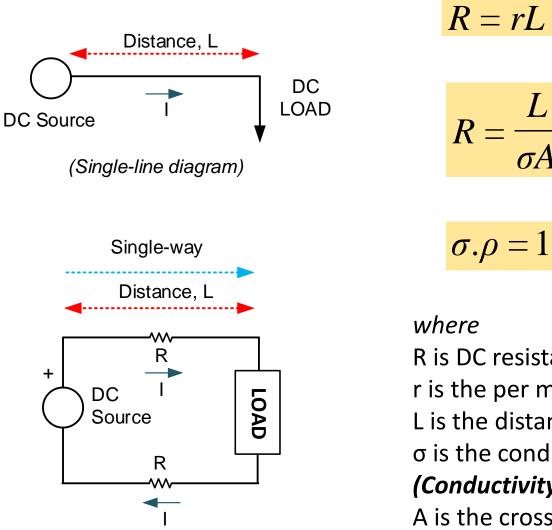
The proper cable size is found for a given load by the following analysis:

- The current rating (ampacity) of the cable should be sufficient to carry the required amount of current safely under the installation conditions
- There appears a voltage drop on the conductor due to the resistance and inductive reactance. The conductor size should satisfy the voltage drop criteria as follows for LV systems in general



If voltage drop criteria is not clear or not given, then the choice of cable size is done according to the magnitude of load current. This is known as "Isi Tahkiki (thermal investigation)" in electrical projects.

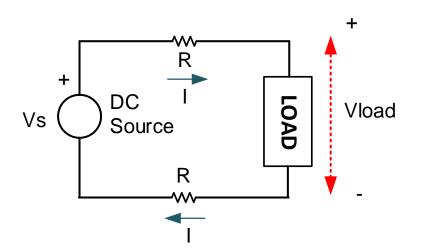
Voltage Drop Calculation in DC Distribution Systems



R = rL

where

R is DC resistance of the overhead-line or cable (single-way) r is the per meter DC resistance of the overhead-line or cable (Ω/m) L is the distance between source and load (m) σ is the conductivity of the conductor (m/ Ω .mm²) (Conductivity is generally represented with letter "k" in Turkish sources) A is the cross-sectional area of the conductor (mm²) ρ is the resistivity of the conductor (Ω .mm²/m) 3



From KVL:

Vs - RI - Vload - RI = 0 Vs - 2RI - Vload = 0 $\Delta V = Vs - Vload$ $\Delta V = 2RI (voltage drop on the line, 2-way)$

Pin = Vs.I (input power, W)Pout = Pload = Vload.I (output power, W) $Ploss = 2RI^2$ (power loss, W)(R is one-way resistanace)Pin = Pout + Ploss

 $\Delta V = 2RI \quad \text{(voltage drop)}$

$$\% e = \frac{\Delta V}{V_n} x100\%$$
 (percentage voltage drop)

where V_n is the nominal system voltage

- ✓ V_n can be equal to Vs or
- ✓ V_n can be equal to Vload or
- ✓ V_n can be equal to a voltage, close to Vs or Vload

$$\eta = \frac{Pout}{Pin} \times 100\% \text{ (percentage efficiency)}$$

$$\% Ploss = \frac{2RI^2}{Pload} \times 100\%$$
(percentage power loss)

Let's find a useful equation for **percentage voltage drop**;

%
$$e = \frac{\Delta V}{V_n} x100\%$$
 (percentage voltage drop)

$$\% e = \frac{\Delta V}{V_n} x 100\% = \frac{2RI}{Vn} x 100\%$$

Since I = Pload / Vload

 $\% e = \frac{2RPload}{Vn.Vload} x100\%$

If we assume Vn = Vload

$$\% e = \frac{2RPload}{Vload^2} x100\%$$

$$\% e = \frac{2RPload}{Vload^2} x100\%$$

Since
$$\Rightarrow R = \frac{L}{\sigma A}$$

$$\% e = \frac{2.Pload.L}{Vload^2.\sigma.A} x100\%$$

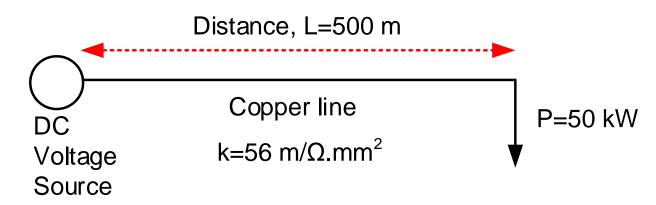
Example: Single-line diagram of a DC distribution line is shown in the figure. If copper conductor is used, load power is 50 kW, and load voltage is 600 V (DC), answer the following questions:

a) Find the cross-section of the copper conductor so that maximum percentage voltage drop becomes 5 % ($e \le 5\%$)

b) Find the voltage drop on the line

c) Find the voltage of the DC source

d) Find the power loss and the efficiency of this distribution line

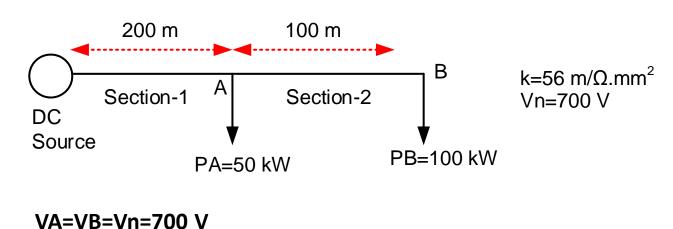


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mm ²	İletken	Sigorta	İletken	Sigorta	İletken	Sigorta
	А	A	А	A	A	A
0.75	-	-	12	6	15	10
1	11	6	15	10	19	10
1.5	15	10	18	10	24	20
2.5	20	16	26	20	32	25
4	25	20	34	25	42	35
6	33	25	44	35	54	50
10	45	35	61	50	73	63
16	61	50	82	63	98	80
25	83	63	108	80	129	100
35	103	80	135	100	158	125
50	132	100	168	125	198	160
70	165	125	207	160	245	200
95	197	160	250	200	292	250
120	235	200	292	250	344	315
150	-		335	250	391	315
185	-	-	382	315	448	400
240	12	100	453	400	528	400

Example: A 300 meter DC underground feeder has two sections feeding two loads, as shown in the figure. Answer the following questions:

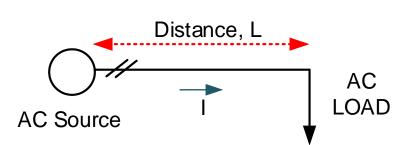
a) For a maximum of **10** % power loss in the feeder, what should be the value of the cross-sectional area of the copper underground conductor ?

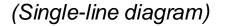
b) Is the cross-section found in (a) suitable for this condition, %e ≤ 5% ? If not then choose a suitable one.

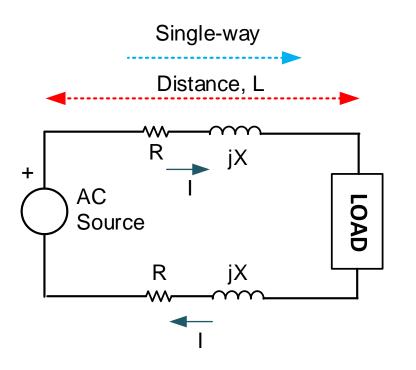


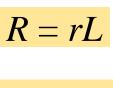
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çok damar	lı hatlar			and the inference	sequinent ye	i i un nun co	
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mm ²	İletken	Sigorta	İletken	Sigorta	İletken	Sigorta	
	A	A	A	A	A	A	
0.75	-	-	12	6	15	10	
1	11	6	15	10	19	10	
1.5	15	10	18	10	24	20	
2.5	20	16	26	20	32	25	
4	25	20	34	25	42	35	
6	33	25	44	35	54	50	
10	45	35	61	50	73	63	
16	61	50	82	63	98	80	
25	83	63	108	80	129	100	
35	103	80	135	100	158	125	
50	132	100	168	125	198	160	
70	165	125	207	160	245	200	
95	197	160	250	200	292	250	
120	235	200	292	250	344	315	
150	-	-	335	250	391	315	
185	2	-	382	315	448	400	
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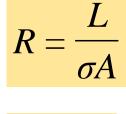
Voltage Drop Calculation in Single-phase Systems











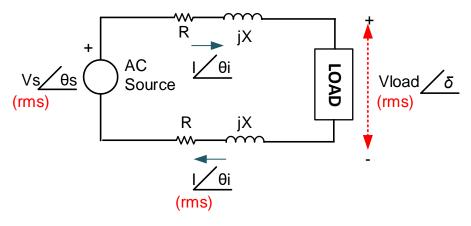
 $\sigma.
ho=1$

X = xL Z = R + jXIn distribution systems, we generally <u>ignore</u> the <u>capacitive effect</u> of the lines

For **underground cables** <u>capacitive effect may not be</u> <u>ignored</u>

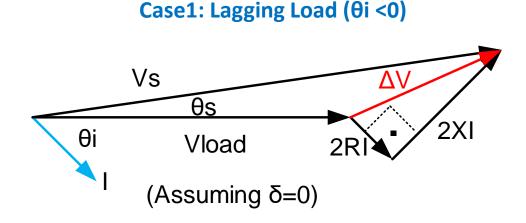
where

R is AC resistance of the overhead-line or cable (single-way) X is the reactance of the overhead-line or cable (single-way) r is the per meter AC resistance of the overhead-line or cable (Ω/m) x is the per meter reactance of the overhead-line or cable (Ω/m) Z is the impedance of the overhead-line or cable (single-way) L is the distance between source and load (m) σ is the conductivity of the conductor ($m/\Omega.mm^2$) A is the cross-sectional area of the conductor (mm^2) ρ is the resistivity of the conductor ($\Omega.mm^2/m$)

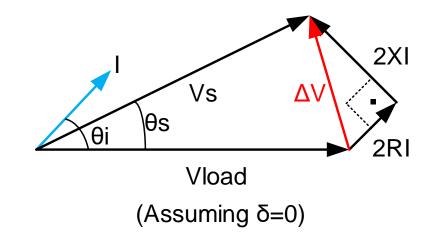


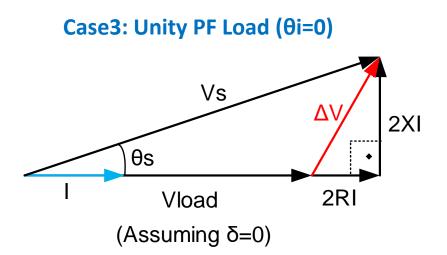
From KVL:

Vs/_ θ s - (I/_ θ i)(R+jX) - (Vload/_ δ) - (I/_ θ i)(R+jX) = 0 Vs /_ θ s - 2(I/_ θ i)(R+jX) - (Vload/_ δ) = 0 $\Delta V = Vs/_{\theta}s - Vload/_{\delta}$ $\Delta V = 2(I/_ <math>\theta$ i)(R+jX) (voltage drop on the line, 2-way) $\Delta V = 2(I/_ {\theta}i)Z$ (voltage drop on the line, 2-way)



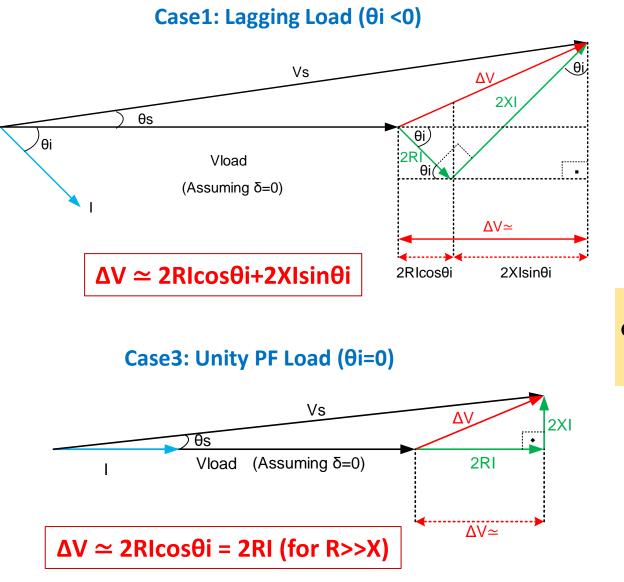
Case2: Leading Load (θi >0)

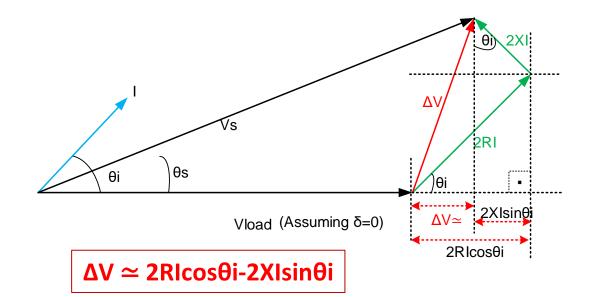




Approximate Voltage Drop Equations

Case2: Leading Load (θi >0)





 $\% e = \frac{\Delta V}{V_n} x100\%$ (percentage voltage drop)

where

 V_n is the nominal (rated) **rms** system voltage

- ✓ V_n can be equal to Vs or
- ✓ V_n can be equal to Vload or
- ✓ V_n can be equal to a voltage, close to Vs or V¹⁰_nad

Let's find a useful equation for percentage voltage drop in **single-phase systems**;

$$\% e = \frac{\Delta V}{V_n} x100\%$$
 (percentage voltage drop)

$$\% e = \frac{\Delta V}{V_n} x 100\% = \frac{2RI\cos\theta_i \pm 2XI\sin\theta_i}{Vn} x 100\%$$

Since I = Pload / (Vload x cos(δ - θ i)), (assuming δ =0)

$$\% e = \frac{2(R\cos\theta_i \pm X\sin\theta_i)Pload}{Vn.Vload.\cos\theta_i} x100\%$$

If we assume Vn = Vload

$$\% e = \frac{2(R\cos\theta_i \pm X\sin\theta_i)Pload}{Vload^2 \cdot \cos\theta_i} x100\%$$

$$\% e = \frac{2(R\cos\theta_i \pm X\sin\theta_i)Pload}{Vload^2 \cdot \cos\theta_i} x100\%$$

 $\% e = \frac{2.(R \pm X \tan \theta_i) Pload}{X^2} \times 100\%$

 $Vload^{2}$

$$\%e = \frac{2.R.Pload}{Vload^2} \cdot f(\varphi)x100\%$$

Since
$$\Rightarrow R = \frac{L}{\sigma A}$$

$$\%e = \frac{2.Pload.L}{Vload^2.\sigma.A} xf(\varphi) x100\%$$

Or

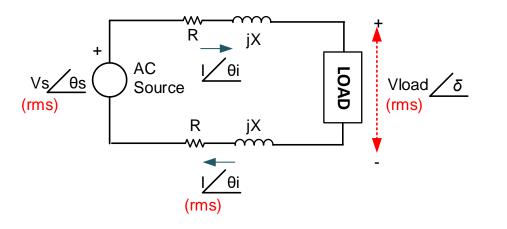
Or

$$\% e = \frac{(1 \pm \frac{X}{R} \tan \theta_i).2.R.Pload}{Vload^2} \times 100\%$$

Let Inductive (lagging PF)

$$f(\varphi) = (1 \pm \frac{X}{R} \tan \theta_i)$$

Capacitive (leading PF)



 $Pin = Vs.I.\cos(\theta_s - \theta_i)$ (input active power, W)

$$Qin = Vs.I.\sin(\theta_s - \theta_i)$$
 (input reactive power, VAR)

$$Qout = Qload = Vload.I.sin(\delta - \theta i)$$

(output reactive power, VAR)

 $Qloss = 2XI^{2}$ (reactive power loss, VAR) (X is one-way reactance)

Qin = Qout + Qloss

 $Pout = Pload = Vload.I.\cos(\delta - \theta i)$ (output active power, W)

 $\frac{Ploss = 2RI^{2}}{(R is one-way resistance)}$

Pin = Pout + Ploss

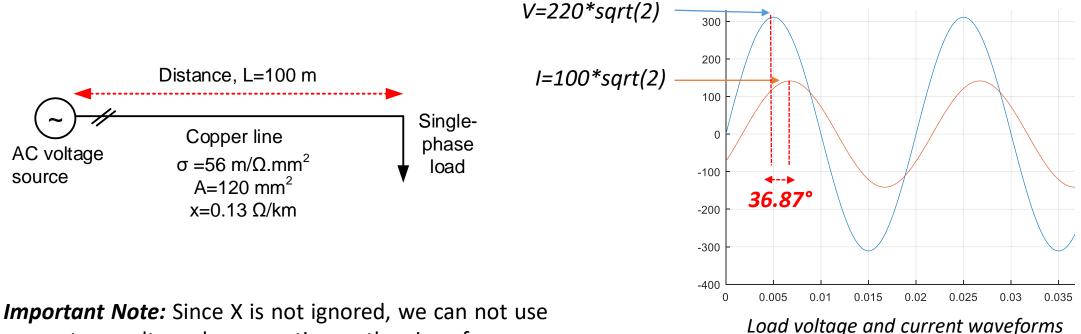
$$\eta = \frac{Pout}{Pin} \times 100\%$$
 (percentage efficiency)

$$\% Ploss = \frac{2RI^2}{Pload} x100\%$$

(percentage (active) power loss)

Example: A 100 meter single-phase line feeds a load as shown in the figure. Answer the following questions:
a) For a maximum of 3 % voltage drop on the line, is given value for A=120 mm² suitable or not ?
b) Find the voltage drop on the line.

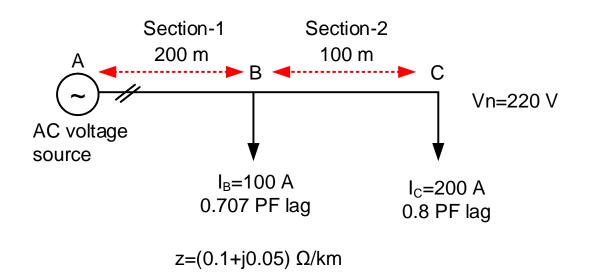
c) Find the value of the AC voltage source, active-reactive power loss, and input active power.



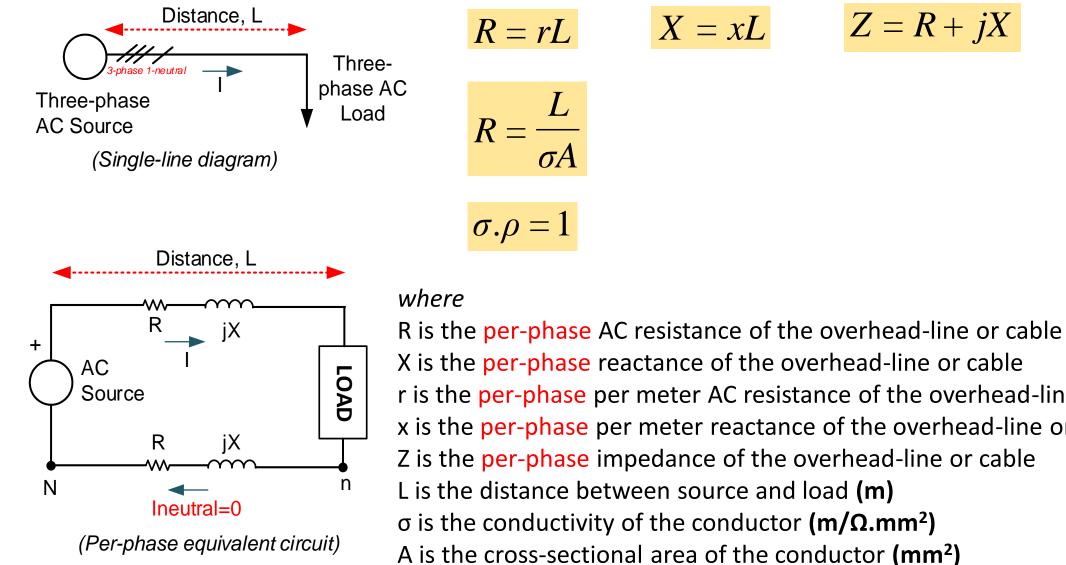
percentage voltage drop equation as the given form

Example: For the given single-phase radial distribution line, answer the following questions:

- a) Find the current in each section.
- b) Find the voltage drop on each section and on overall line (A-C).
- c) Find the percentage voltage drop on each section and on overall line (A-C).
- d) Find the voltage of the AC voltage source.

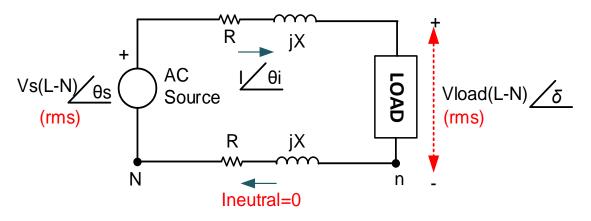


Voltage Drop Calculation in Three-phase Systems



(Assuming system is **Y-connected**)

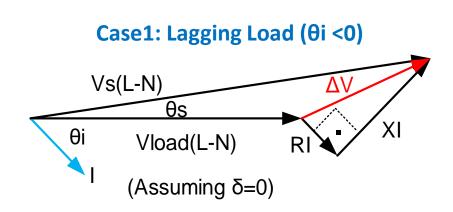
X is the per-phase reactance of the overhead-line or cable r is the per-phase per meter AC resistance of the overhead-line or cable (Ω/m) x is the per-phase per meter reactance of the overhead-line or cable (Ω/m) Z is the per-phase impedance of the overhead-line or cable A is the cross-sectional area of the conductor (mm²) ρ is the resistivity of the conductor (Ω .mm²/m) 16



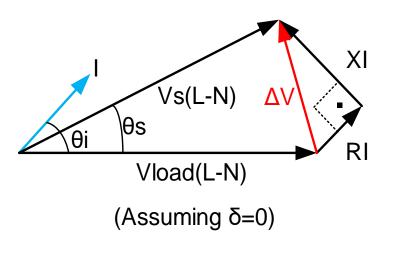
From KVL:

Vs(L-N)/_ θ s - (I/_ θ i)(R+jX) - (Vload(L-N)/_ δ) = 0 $\Delta V = Vs(L-N)/_{\theta}s - Vload(L-N)/_{\delta}$ $\Delta V = (I/_ <math>\theta$ i)(R+jX) (per phase voltage drop on the line) $\Delta V = (I/_ {\theta}i)Z$ (per phase voltage drop on the line)

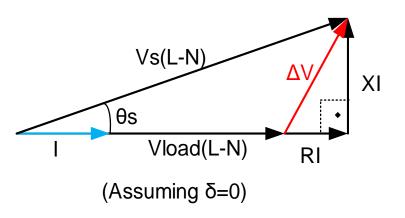
Since for balanced three-phase systems, <u>neutral current is zero</u>, multiplication factor **2** disappears from voltage drop equation



Case2: Leading Load (θi >0)

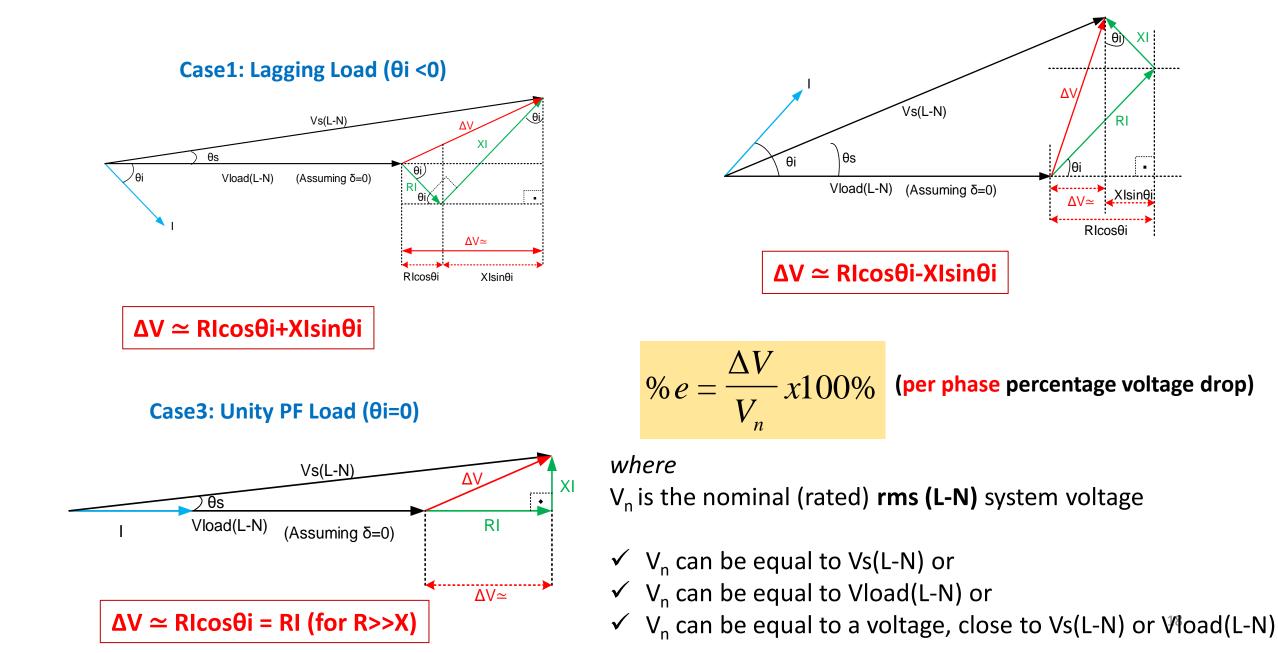






Approximate Voltage Drop Equations

Case2: Leading Load (θi >0)



Let's find a useful equation for percentage per-phase voltage drop in three-phase systems;

$$\% e = \frac{\Delta V}{V_n} x100\% \text{ (per-phase percentage voltage drop)}$$
For Y-connection
$$\% e = \frac{\Delta V}{V_n} x100\% = \frac{RI \cos \theta_i \pm XI \sin \theta_i}{V_n} x100\%$$
Since $\Rightarrow I = \frac{Pload}{3.Vload(LN).\cos(\delta - \theta_i)} \text{ (assuming } \delta = 0\text{)}$

$$\% e = \frac{(R \cos \theta_i \pm X \sin \theta_i) Pload}{3.V_n Vload(LN).\cos \theta_i} x100\%$$

If we assume Vn = Vload(LN)

$$\% e = \frac{(R\cos\theta_i \pm X\sin\theta_i)Pload}{3.Vload(LN)^2.\cos\theta_i} x100\%$$

$$\% e = \frac{(R\cos\theta_i \pm X\sin\theta_i)Pload}{3.Vload(LN)^2.\cos\theta_i} x100\%$$

 $\%e = \frac{R.Pload}{3.Vload(LN)^2} \cdot f(\varphi) x 100\%$

or

$$\% e = \frac{(R \pm X \tan \theta_i) Pload}{3.V load (LN)^2} x100\%$$

or

$$\% e = \frac{(1 \pm \frac{X}{R} \tan \theta_i) . R. Pload}{3. Vload (LN)^2} x100\%$$

Let Inductive (lagging PF)

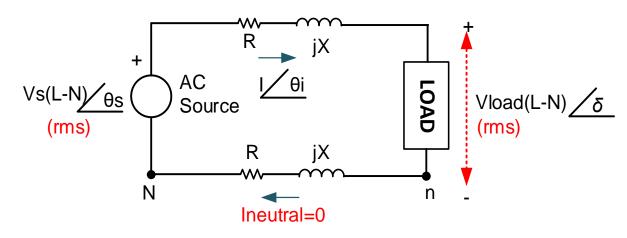
$$f(\varphi) = (1 \pm \frac{X}{R} \tan \theta_i)$$

Capacitive (leading PF)

Since
$$\Rightarrow R = \frac{L}{\sigma A}$$

 $\% e = \frac{Pload. L. f(\varphi)}{3. Vload (LN)^2. \sigma. A} x100\%$
Since $\Rightarrow Vload (LN) = \frac{VLoad (LL)}{\sqrt{3}}$
 $\% e = \frac{Pload. L. f(\varphi)}{Vload (LL)^2. \sigma. A} x100\%$

For Y-connected load



 $Qin = 3.Vs(LN).I.sin(\theta_s - \theta_i)$

 $Qin = \sqrt{3.Vs(LL).I.sin(\theta_s - \theta_i)}$ (input reactive power, VAR)

$$Qout = Qload = 3.Vload(LN).I.\sin(\delta - \theta i)$$
$$Qout = Qload = \sqrt{3.Vload(LL)}.I.\sin(\delta - \theta i)$$

(output reactive power, VAR)

 $Pin = 3.Vs(LN).I.\cos(\theta_s - \theta_i)$ (input active power, W) $Qloss = 3XI^2$ (reactive power loss, VAR) (X is per phase reactance) $Pin = \sqrt{3}.Vs(LL).I.\cos(\theta_s - \theta_i)$

 $Pout = Pload = 3.Vload(LN).I.\cos(\delta - \theta i)$ $Pout = Pload = \sqrt{3}.Vload(LL).I.\cos(\delta - \theta i)$ (output active power, W)

(active power loss, W) $Ploss = 3RI^2$ (R is per-phase resistance)

Pin = Pout + Ploss

Qin = Qout + Qloss

 $\eta = \frac{Pout}{Pin} x100\%$ (percentage efficiency)

$$\% Ploss = \frac{3RI^2}{Pload} x100\%$$

(percentage (active) power loss)

For Δ -connected load

$$Pin = 3.Vs(LL).\frac{I}{\sqrt{3}}.\cos(\theta_s - \theta_i)$$

 $Pin = \sqrt{3.Vs(LL).I.\cos(\theta_s - \theta_i)}$

$$Pout = Pload = 3.Vload(LL).\frac{I}{\sqrt{3}}.\cos(\delta - \theta i)$$

 $Pout = Pload = \sqrt{3.Vload(LL).I.\cos(\delta - \theta i)}$ (output active power, W)

 $\frac{Ploss = 3RI^{2}}{(R is per-phase resistance)}$

Pin = *Pout* + *Ploss*

$$Qin = 3.Vs(LL) \cdot \frac{I}{\sqrt{3}} \cdot \sin(\theta_s - \theta_i)$$

 $Qin = \sqrt{3.Vs(LL).I.sin(\theta_s - \theta_i)}$ (input reactive power, VAR)

$$Qout = Qload = 3.Vload(LL).\frac{I}{\sqrt{3}}.\sin(\delta - \theta i)$$

$$Qout = Qload = \sqrt{3}.Vload(LL).I.\sin(\delta - \theta i)$$

(output reactive power, VAR)

$$Qloss = 3XI^{2}$$
 (reactive power loss, VAR)
(X is per-phase reactance)
$$Qin = Qout + Qloss$$

$$\eta = \frac{Pout}{Pin} x100\%$$
 (percentage efficiency)
% $Ploss = \frac{3RI^2}{Pload} x100\%$

(percentage (active) power loss)

Example: For %e \leq 5, calculate the maximum length of the Rose conductor when it carries its maximum apparent power in a three-phase system. Take power factor as 0.8 lagging, Vn = 220 V (L-N), and σ = 35 m/ Ω .mm²

$$\% e = \frac{Pload.L}{3.Vload(LN)^2.\sigma.A}.f(\varphi)x100\%$$

Substituting the parameters into the equation:

$$5 = \frac{(58.08x10^3).L}{3.220^2.35.(21.14)}.(1.191)x100\%$$

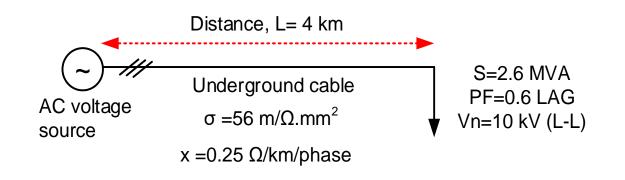
L(max)=**77.655 m**

K.Stan- dardı Amma	Kesit	Çap.	Kopma Kuvve- ti	Ağır- lık	DC Di- renci 20°Çda	End. Reakt. f=50Hz	f (4) Fakt. (0, %) end	y f(y)/3kqV x10 ⁻⁶	Yükl. Akımı (Tasıma
Adı 🧭	2	1111	kgk	kgk/m	Ω/km.f	fi/km.f		1/А.ш	Kopari 871
Rose	21,44	-5,88	415	58	1,3510	0,345	(1,191)	2,44	110
Lily	26 .66	6.61	515	. 73	1,0720	0,337	1,735	2,01	125
tris :	33,65	7.42	640	. 92 .	0,8498	0,330	1,291	1,66	143
Pansy	42,37	8,33	775	116	0,6739	0,322	1,358	1,39	-165
	53,49	9,36	940	146	0,5341	0,315	1.412	1,17	193
Poppy Aster	67.45	10,51	1185	184	0,4236	0,307	1,543	0,99	225
Phlox	84 .99	11,80	1435	232	0,3360	0,300	1,669	0,85	262
Oxlip	107,30	13,25	1810	293 -	0,2664	0,293	1,824	0,74	306

Alçak Gerilimli Hava Hatlarında Kullanılan Alüminyum İletkenlar ve Özellikleri

Example: For the given three-phase underground distribution, answer the following questions:

- a) For a maximum of 6 % Ploss, what should be the value for cross-section of the conductor, A=?
- b) Find the real and the imaginary parts of the line current.
- c) Find the per-phase voltage drop on the cable.
- d) Find the percentage per-phase voltage drop on the cable.
- e) Find L-L voltage, input active power, and power factor of the three-phase voltage source.



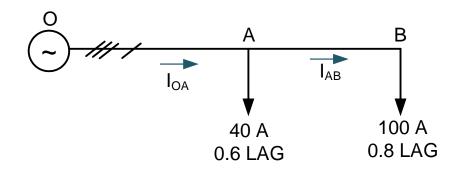
P.S1: For medium voltage distribution, there is **no neutral line**

P.S2: All voltages in medium voltage systems (>1kV) are Line-to-Line (L-L) voltages if otherwise is not specified

P.S3: For medium voltage distribution, conductor cross-sections are starting from **50 mm²**

P.S4: If there is more than one condition to choose a cross-section, always choose the biggest one who satisfies all conditions

Example: For the given low-voltage three-phase radial distribution line, find the current in each section and choose suitable conductors from the given table according to the current carrying capacities. Note that required percentage voltage drop and the distances <u>are not given</u>.

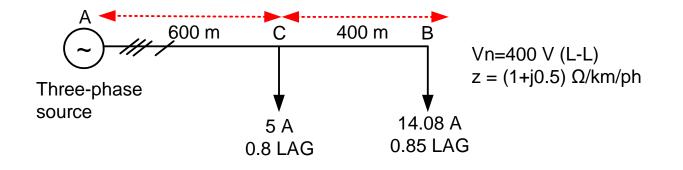


Alçak Gerilimli	Hava	Hatlarinda	Kullanilan	Alüminyum	fletkenler	¥6	Özellikleri	

K.Stan- dardı Annas Adı	.Kesit	Çap .	Kopma Kuvve- ti kgk	Ağır- lık kgk/m	DC Di- renci 20°Çda Ω/km.f	End. Reakt. f=50Hz fl/km.f	f (y) Fakt (0, %) end 1	y f(φ)/3kq¥ x10 ^{−6} 1/A.m	Yükl Akımı (Tasıma Kopoci (2011)
Rose	21,44	5,88	415	58	1,3510	0,345	1,191	2,44	110
Lily	26,66	6,61	515	73	1,0720	0,337	1,235	2,01	125
Iris	33,65	7,42	640	92	0,8498	0,330	1,291	1,66	143
Pansy	42,37	8,33	775	116	0,6739	0,322	1,358	1,39	165
Poppy	53,49	9,36	940	146	0,5341	0,315	1,442	1,17	193
Aster	67,45	10,51	1185	184	0,4236	0,307	1,543	0,99	225
Phlox	84,99	11,80	1435	232	0,3360	0,300	1,669	0,85	262
Oxlip	107,30	13,25	1810	293	0,2664	0,293	1,824	0,74	306

Example: For the three-phase low voltage distribution, answer the following questions:

- a) Find the current distributions on the line.
- b) Find the rms (L-L) voltage of the points A and C, if Vn=400 V (L-L) at point B.
- c) Does the available cross-section satisfy the condition: $\%e \le 5$?
- d) Does the available cross-section satisfy the condition: $%Ploss \le 6$?



END OF THE LECTURE

Any questions ?