SERIES RESONANT CIRCUIT

- Equivalent circuit of a single stage loaded transformer -

L1 = Inductance of the voltage regulator and transformer primary
L = The exciting inductance of the transformer
L2 = Inductance of the transformer secondary
C = Capacity of the load (Tested Component)

Normally L >> L1 and L >> L2 hence L is neglected.

This variable in the impedance of C equals to the impedance of L1 and L2 then series resonance occurs and the load voltage may go up 20 to 40 times the direct input voltage.

Single transformer/reactor series resonance circuit.
For certain settings of the reactor, the inductive reactance would be equal to the capacitive reactance of the circuit, hence resonant will take place. Thus reactive power requirement of the supply becomes zero and it has to supply only the circuit losses.

The inductors are designed for high quality factors:

\[ Q = \frac{wL}{R}. \]

Therefore the feed transformer injects only the circuit losses.

\[ R \rightarrow \text{generally low value} \]

**Series resonant circuit with variable reactors**

After the resonant condition is achieved, the output voltage can be increased by increasing the input voltage.
Under resonance, the supply voltage
\[ V_o = \frac{V}{R} \cdot \frac{1}{\omega C} \rightarrow \text{capacitance of the tested component} \]
\[ \text{output voltage} \]
\[ \text{supply angular frequency} \]
\[ \text{resistance showing all the losses of the system} \]
(output voltage of \( V \))

Since at resonance
\[ \omega L = \frac{1}{\omega C} \]

Then
\[ V_o = \frac{V \cdot \omega L}{R} = V \cdot Q \]

quality factor of the inductor
40 \( \leq \) \( Q \)

if \( Q = 40 \) this means that \( V_o = 40V \), the output voltage is 40 times the supply voltage.

Advantages of series resonance circuit

1. Low power requirement
2. The near sinusoidal wave may be obtained. Hence harmonics are greatly eliminated.
   It is used for measurement of partial discharge and measurement of capacitance of the tested component.
3. Series or parallel connection of several units is not a problem.
   However, in a cascaded transformer structure, transformer impedance is a great problem.
A 100 kVA 250 V/200 kV feed transformer has a resistance and reactance of 1% and 5% respectively. This transformer is used to test a cable at 400 kV at 50 Hz. The cable takes a charging current of 0.5 A at 400 kV. Determine the series inductance required. Assume 1% resistance of the inductor. Also determine input voltage to the transformer. Neglect dielectric loss of the cable.

**Solution:**
The resistance and reactance of the transformer are
\[
\frac{1}{100} \times 200^2 = 4000 \text{ ohm} = 4 \text{ k}\Omega
\]
\[
\frac{2}{100} \times 200^2 = 20 \text{000 ohm} = 20 \text{k}\Omega
\]

The resistance of the inductor is also 20 k\Omega.

The capacitive reactance of capacitor (Test specimen)
\[
\text{Impedance} = \frac{400 \times 1000 \text{ V}}{0.5 \text{ cm}} = 800 \text{ k}\Omega
\]

For resonance \(X_L = X_C\)
Inductive reactance of the transformer is 20 k\Omega. Therefore, an additional inductive reactance required will be
\[
800 - 20 = 780 \text{ k}\Omega
\]

The inductance required = \[\frac{780 \times 1000}{314}\] \text{ H} = 248.4 H

The total resistance of the circuit = \(8 \text{ k}\Omega\), \(4\text{ k}\Omega + 4\text{ k}\Omega = 8\text{ k}\Omega\)

Under resonance condition the supply voltage
\[
(\text{Secondary voltage}) = 20 = 0.5 \times 8 = 4 \text{kV}
\]

Therefore, the primary voltage = \[\frac{4 \times 250}{200}\] \text{ V} = 5 \text{ volts}
**Chapter 6: Generation of Impulse Voltages and Currents**

**Impulse Voltage:**

It is a unidirectional voltage which, without oscillations, rises rapidly to a maximum value and falls more or less rapidly to zero.

![Graph showing impulse voltage](image)

**Full Impulse Voltage:**

A full impulse voltage is characterized by its peak value and its two time intervals, the wave front and wave tail time.

1. **The wave front time** = \(1.25 (t_2 - t_1)\)
   - Usually it is difficult to measure.
   - Time for voltage to reach 50% of peak value.

2. **The wave tail time** = \((t_3 - t_2)\)
   - Time at which the voltage is 50% of peak value.
   - Time for voltage to reach 10% of peak value.

**Nominal Steepness of the Wavefront:**

The nominal steepness of the wavefront is the average rate of rise of voltage between the points on the wavefront where the voltage is 10% and 90% of the peak value respectively.
The standards are:

- Wave front time $\rightarrow 1 \mu\text{s}$ (1 microsecond) $\pm 50\%$ of wave front time (max. $120\%$ for half-value of wave tail time).
- Wave tail time $\rightarrow 50 \mu\text{s}$ (50 microseconds).
- Peak value $\rightarrow 100 \text{kV}.$

Or

- Wave front time $\rightarrow 1.5 \mu\text{s}$
- Wave tail time $\rightarrow 40 \mu\text{s}$
- Peak value $\rightarrow 700 \text{kV}.$

**Impulse Generator Circuits**

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DC Supply

$\text{L}_1 \rightarrow \text{Inductance of the generator and the leads connecting the generator.}$

$\text{R}_4 \rightarrow \text{Resistance representing the resistances of the capacitances + resistance of the leads connecting the generator.}$

$\text{L}_3 \rightarrow \text{External elements for waveform control.}$
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```
\text{TEST SPEC}
\text{VEN}
```
R₁ and R₄ control the duration of the wave. C₂ and C₄ represent the capacitances to earth of high voltage components and the leads.

C₄ also includes the capacitance of the test object.

L₄ represents the inductance of the test object.

To make analysis simple, the simplified equivalent circuits are shown below:

\[ C_i \overset{V_i \sin}{\rightarrow} i(t) \]

\[ \begin{align*}
R_2 & \rightarrow \text{wave tail control resistance} \\
\text{The capacitor } C_1 \text{ is charged through a charging resistance (not shown) to a dc voltage and then discharged by flashing over the switching gap with a pulse of suitable value. The desired impulse voltage appears across the load capacitance } C_2. 
\end{align*} \]
Analysis of Circuit (a):

After the gap spark over, $i(t)$ flows in the circuit. Using Laplace transform, the impedance of the circuit is

$$Z(s) = R_1 + \frac{1}{C_1 s} + \frac{R_2}{R_2 C_2 s + 1}$$

$$I(s) = \frac{V_0}{Z(s)} = \frac{V_0 C_1 (R_2 C_2 s + 1)}{R_1 R_2 C_2 s^2 + (R_1 + R_2 + R_2 C_2) s + 1}$$

and

$$V(s) = I(s) \frac{R_2}{R_2 C_2 s + 1}$$

$$V(s) = \frac{V_0 R_2 C_1}{R_1 R_2 C_2 s^2 + (R_1 + R_2 + R_2 C_2) s + 1}$$

If we write in that format:

$$V(s) = \frac{1}{(s + \alpha - \beta)(s + \alpha + \beta)} \frac{V_0}{R_1 C_2}$$

where

$$\alpha = \frac{R_1 + R_2 C_2 + R_2 C_1}{2 R_1 R_2 C_2}$$

$$\beta = \frac{1}{2} \sqrt{\left(\frac{R_1 + R_2 C_2 + R_2 C_1}{R_1 R_2 C_2}\right)^2 - \frac{4}{R_1 R_2 C_2}}$$
Taking inverse Laplace Transform of the output voltage:

\[ v(t) = \frac{V_0}{2\beta RC_2} \left[ e^{-(\alpha-\beta)t} - e^{-(\alpha+\beta)t} \right] \]

Let \( V_n = \frac{V_0}{2\beta RC_2} \)

\[ v(t) = V_n \left[ e^{-(\alpha-\beta)t} - e^{-(\alpha+\beta)t} \right] \]

Let \( t_1 \rightarrow \) wave front time

\( t_2 \rightarrow \) wave tail time

At time \( t_1 \), the slope of the wave is zero (\( \frac{dv}{dt} = 0 \))

\[ \frac{dV(t)}{dt} = V_n \left[ -(\alpha-\beta)e^{-(\alpha-\beta)t} + (\alpha+\beta)e^{-(\alpha+\beta)t} \right] = 0 \]

or

\[ \frac{\alpha+\beta}{\alpha-\beta} = e^{2\beta t_1} \]

or

\[ 2\beta t_1 = \ln \frac{\alpha+\beta}{\alpha-\beta} \]

or

\[ t_1 = \frac{1}{2\beta} \ln \frac{\alpha+\beta}{\alpha-\beta} \]
And the peak value of the voltage is given by

\[ V(t = t_1) = V_0 \left[ e^{-(\alpha - \beta)t_1} - e^{-(\alpha + \beta)t_1} \right] \]

To obtain \( t_2 \), substitute \( t = t_2 \) and the voltage is half of what it is when \( t = t_1 \) in the equation:

\[ V(t = t_2) = \frac{V_0}{2} \left[ e^{-(\alpha - \beta)t_1} - e^{-(\alpha + \beta)t_1} \right] \]

Let \( t_2 = Kt_1 \) where \( K \) is a constant, then:

\[ V(t = Kt_1) = \frac{V_0}{2} \left[ e^{-(\alpha - \beta)t_1} - e^{-(\alpha + \beta)t_1} \right] \]

much small therefore neglected

\[ e^{-(\alpha - \beta)t_1} = \frac{1}{2} \left[ e^{-(\alpha - \beta)t_1} - e^{-(\alpha + \beta)t_1} \right] \]

or

\[ 2e^{-(\alpha - \beta)(K-1)t_1} = 1 - e^{-2\beta t_1} \]

or

\[ 2 - \beta = \frac{2}{t_1(K-1)} \ln \left[ \frac{2}{1-e^{-2\beta t_1}} \right] \]

If \( 2\beta t_1 > 4 \) the above equation reduces to

\[ \alpha - \beta = \frac{0.47}{t_1(K-1)} \]

with this assumption the error is within 2%.
<table>
<thead>
<tr>
<th>Wave</th>
<th>α</th>
<th>β</th>
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<tbody>
<tr>
<td>0.5/5</td>
<td>4.080</td>
<td>3.922</td>
</tr>
<tr>
<td>1/5</td>
<td>1.557</td>
<td>1.366</td>
</tr>
<tr>
<td>1/10</td>
<td>2.049</td>
<td>1.961</td>
</tr>
<tr>
<td>1.5/40</td>
<td>1.716</td>
<td>1.757</td>
</tr>
<tr>
<td>1/50</td>
<td>3.044</td>
<td>3.029</td>
</tr>
</tbody>
</table>

Values of α and β for typical waveform.

→ Design of a specific impulse voltage with α and β (?)

Approximate capacitance of some equipment:

<table>
<thead>
<tr>
<th>Equipment</th>
<th>Capacitance</th>
</tr>
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<tbody>
<tr>
<td>Insulator</td>
<td>25 pF</td>
</tr>
<tr>
<td>Bushing</td>
<td>150-400 pF</td>
</tr>
<tr>
<td>Cable</td>
<td>2.500 pF</td>
</tr>
<tr>
<td>HV Capacitor</td>
<td>1000 pF</td>
</tr>
</tbody>
</table>