CHAPTER 5: Generation of High DC and AC Voltages

Application area of high dc voltage:
→ HVDC transmission
→ Testing of HVDC cables with long lengths (because of large capacitance.
→ Electrostatic painting
→ Cement industry
→ In physics, for particle acceleration
→ Medical equipment (X-Rays)

The value of a direct test voltage is defined by

\[
V_{dc} = \frac{1}{T} \int_{0}^{T} V(t) \, dt
\]

\( T \rightarrow \) period of \( V(t) \)
\( f = 1/T \rightarrow \) frequency of \( V(t) \)

Test dc voltages always have ripple defined by

\[
SV = \frac{1}{2} [V_{\text{max}} - V_{\text{min}}]
\]

The ripple factor is defined as

\[
\frac{SV}{V_{dc}}
\]

Test dc voltages should not have ripple factor more than 5%, or specified in a standard depending on the application.
HALF-WAVE RECTIFIER CIRCUIT

→ The simplest circuit for generating high direct voltage.
→ RL : Load resistance
→ C : Capacitor to smooth dc output voltage.
→ Assume ideal transformer (no losses)
→ Diode has negligible or small internal resistance during conduction period.

\[ V_{max} \quad \text{V}_{in} \quad \uparrow \quad 2.5V \]

\[ \text{Output voltage with} \quad C \]
Assuming the charge supplied by the transformer to the load during the conduction period $T$, which is very small to be negligible, the charge supplied by the transformer to the capacitor during conduction equals the charge supplied by the capacitor to the load. Note that $i(t) = i_L(t)$, during one period $T = \frac{1}{f}$ of the AC voltage, a charge $Q$ is transferred to the load $R_L$ and is given by

$$Q = \int i_L(t) \, dt = \int \frac{V_R(t)}{R_L} \, dt = IT = \frac{I}{f}$$

where $I \rightarrow$ mean value of the output current $i_L(t)$. This charge is supplied by the capacitor over the period $T$ when the voltage changes from $U_{max}$ to $U_{min}$ over period $T$.

The charge delivered by the capacitor is

$$\int dB = \int_{U_{min}}^{U_{max}} C \, dU =$$

or

$$Q = C \left[ U_{max} - U_{min} \right] \quad \text{(The magnitude of the charge)}$$
Since \( V = \frac{1}{2} [v_{\text{max}} - v_{\text{min}}] \)

\[ V = C \cdot 2 \Delta V = \frac{I}{f} = \frac{I}{f_c} \]

or

\[ V = \frac{I}{2c} = \frac{I}{2f_c} \]

- The higher the frequency of supply and the larger the value of the filter capacitor, the smaller will be the ripple in the dc output.

The disadvantages of the half-wave rectifier:

1. The circuit size is very large if high and pure dc output voltages are desired.

2. The transformer may get saturated if the amplitude of the dc current is comparable with the nominal value of the ac current of the transformer.

- Half-wave rectifier circuits are not suitable for HVDC transmission applications since they supply very small dc currents to the load.
Greinacher voltage doubler circuit.

Suppose B is more positive with respect to A and the diode D1 conducts thus charging the capacitor C1 to \( V_{\text{max}} \) with the polarity as shown in the figure.

During the next half cycle terminal A of the capacitor C1 rises to \( V_{\text{max}} \) and hence terminal M attains a potential of \( 2V_{\text{max}} \). Thus, the capacitor C2 is charged to \( 2V_{\text{max}} \) through D2. Normally the voltage across the load will be less than \( 2V_{\text{max}} \) depending upon the time constant of the circuit \( C_2 R_L \).

Matlab simulation: [greinacher.m]

\[ Z = R_L \cdot C_2 \]
In 1932, Cockroft and Walton improved the circuit of Gérald, for producing high DC voltages. The following figure shows a multi-stage single-phase cascade circuit of the Cockroft-Walton type.
No Load Operation: The sector ABM'M'A is exactly identical to Grenado voltage doubler circuit. The voltage across C1 becomes 2Vmax when m attains a voltage 2Vmax.

OK

→ The voltage across all capacitors is 2Vmax except for C1 where it is only Vmax.

→ The total output voltage is $2nV_{max}$ where n is the number of stages (at no load).

→ Since equal stress of the elements (both capacitors + diodes) is very helpful and promotes a modular design of such generators.

Generator Loaded: When the circuit (generator) is loaded i.e. the output voltage will be always less than $2nV_{max}$. The output voltage will have ripples.

For n-stage circuit the ripple will be

$$SV = \frac{n}{2f} \left[ \frac{1}{C_1} + \frac{2}{C_{n-1}} + \frac{3}{C_{n-2}} + \ldots + \frac{n}{C_1} \right]$$
\[ \Delta V = \frac{I}{2 fc} \frac{n(n+1)}{2} = \frac{In(n+1)}{4 fc} \]

- \( I \rightarrow \) Load current
- \( n \rightarrow \) Stage number
- \( f \rightarrow \) Frequency of supply
- \( C \rightarrow \) Capacitance of the right column (for each)

Output voltage ripple.

\[ (After \ some \ tedious \ derivation \ from \ on \ the \ book) \]

\[ \Delta V = \frac{I}{fc} \left[ \frac{2 \pi}{3} \frac{n^2}{6} \right] \]

If \( n > 4 \) we find that the equation can be approximated as:

\[ \Delta V \approx \frac{I}{fc} \frac{2 \pi}{3} \]
After some derivations

\[ n_{opt} = \sqrt{\frac{U_{max} f C}{I}} \]

\( n_{opt} \rightarrow \text{optimal number of stages} \)

\( U_{max} \rightarrow \text{maximum output voltage under load} \)

\( f \rightarrow \text{freq. of supply} \)

\( C \rightarrow \text{capacitor} \)

\( I \rightarrow \text{load current} \)

Ex: Determine the optimum number of stages \( n_{opt} \) under these conditions:

\( f = 500 \text{ Hz} \)

\( C = 7 \mu F \)

\( I = 500 \text{mA} \)

\( U_{max} = 100 \text{kV} \)

\[ n_{opt} = \sqrt{\frac{100 \times 10^3 \times 500 \times 7 \times 10^{-6}}{500 \times 10^{-3}}} = 26 \]

\[ \rightarrow \text{It is more economical to use high frequency and smaller value of capacitance to reduce ripples.} \]
Voltage Multiplier with Cascaded Transformers

The main limitation of Cockcroft-Walton type circuit is its limited DC power output. This disadvantage can be overcome if single- or full-wave rectifier systems, each having its own AC power source, are connected in series at the DC output only.

\[
\text{NOTE:}
\]

Every HV usually feeds two half-wave rectifiers.

At fundamental frequency (50 or 60 Hz), this circuit provides an economical DC power supply for HV testing purposes with moderate ripple factors and high power capabilities.

DC cascade circuit with cascaded transformers.
Ex: A ten stage Cockcroft-Walton circuit has all capacitors of 0.06 μF. The secondary voltage of the supply transformer is 100 kV at a freq of 150 Hz. If the load current is 1 mA, determine

(i) Voltage regulation
(ii) The ripple.
(iii) The optimum number of stages for max. output voltage.
(iv) The maximum output voltage at no load.

Solutions

\[ C = 0.06 \, \mu F, \, I = 1 \, mA, \, f = 150 \, Hz. \]

\[ n = 10 \]

Voltage drop = \[ \frac{I}{fC} \left[ \frac{2}{3} n^3 - \frac{n}{6} \right] \]

\[ \text{Voltage drop} = \frac{1 \times 10^{-3}}{150 \times 0.06 \times 10^{-6}} \left[ \frac{2}{3} \times 10^3 - \frac{10}{6} \right] \]

\[ = 73.889 \, V \, \text{or} \, 73.889 \, kV \]

or using approximated equation

\[ \text{Voltage drop} \approx \frac{I}{fC} \frac{2n^3}{3} = \frac{1 \times 10^{-3}}{150 \times 0.06 \times 10^{-6}} \left[ \frac{2}{3} \times 10^3 \right] \]

\[ = 74.074 \, V \, \text{or} \, 74.074 \, kV \]

Therefore, voltage regulation in percentage is

\[ \text{VR} = \frac{74.074}{2 \times 10^4} \times 100 \% = 3.7 \% \]
the ripple \[ \Delta v = \frac{I_{n}(n+1)}{4\pi c} = \frac{4 \times 10^{-3} \times 10 \times 10}{(10) (150) (0.06 \times 10^{-6})} \]
\[ = 3.056 \text{ kV} \]
\[ \text{max. output voltage at no load} = 2\Delta v_{\text{max}} = (2)(10)(100 \text{ kV}) = 2000 \text{ kV} \]
\[ \text{2 mV (DC)} \]