EEE270
Introduction to Electrical Energy Systems

Lecture 4 - BALANCED THREE-PHASE SYSTEMS

Prof. Dr. A. Mete VURAL
mvural@gantep.edu.tr

Assist. Prof. Dr. Ali Osman ARSLAN
aoarslan@gantep.edu.tr

Balanced Three-phase Systems

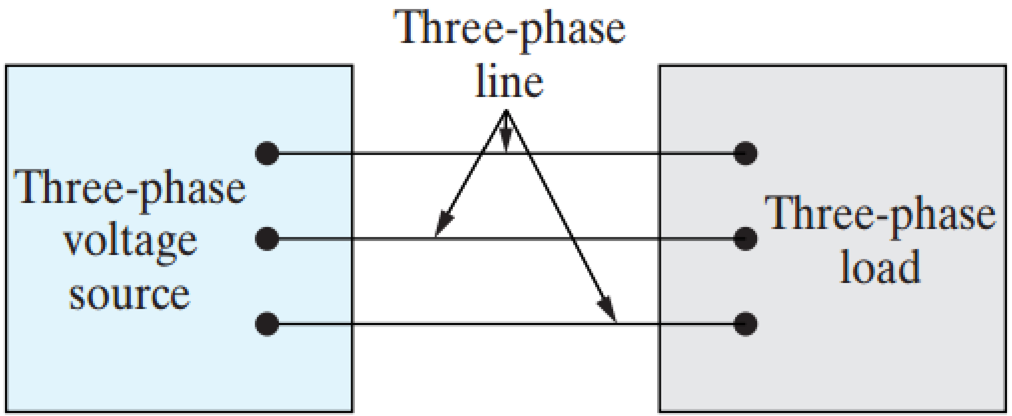
- Balanced Three-Phase Voltages
- Three-Phase Voltage Sources
- Analysis of the Wye-Wye Circuit
- Analysis of the Wye-Delta Circuit
- Power Calculations in Balanced Three-Phase Circuits

Balanced Three-Phase Circuits

- Generating, transmitting, distributing, and using large blocks of electrical power is accomplished with three-phase circuits.
- Three-phase system was independently invented by Galileo Ferraris, Mikhail Dolivo-Dobrovolsky, and Nikola Tesla in the late 1880s.
- Three-phase systems are used to deliver power to large motors and heavy loads. Because three-phase currents naturally produce rotating magnetic fields.
- A three-phase system is more economical than an equivalent single-phase or two-phase system at the same voltage. Because it uses less conductor to transmit the same amount of electrical power.

Balanced Three-Phase Circuits

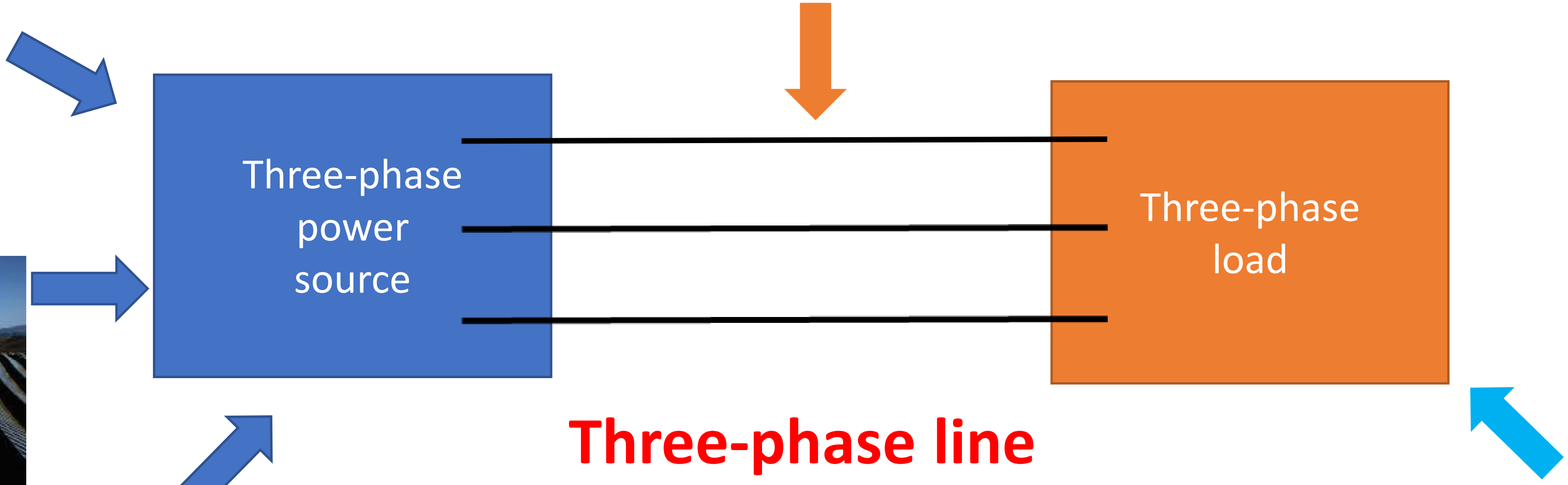
- The basic structure of a three-phase system consists of voltage sources connected to loads by means of transformers and transmission lines.
- To analyze a three-phase circuit, we can reduce it to a **three-phase voltage source** connected to a **three-phase load** via a **three-phase line**, as shown below:



Wind Turbine

Solar PV

Dam



Three-phase load

Main components of a three-phase power system

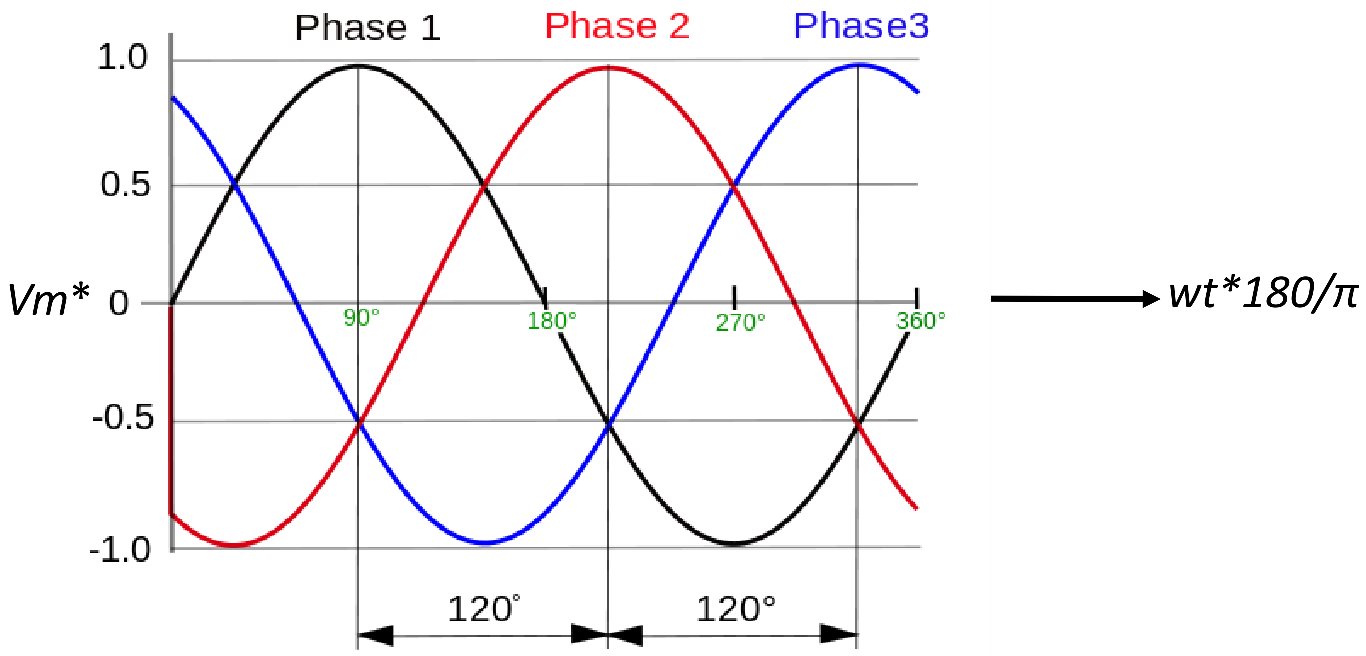
Balanced Three-Phase Voltages

- A set of balanced three-phase voltages consists of three sinusoidal voltages that have identical amplitudes and frequencies but are out of phase with each other by exactly 120°.

- Standard practice is to refer to the three phases as a, b, and c, and to use the a-phase as the reference phase. The three voltages are referred to as the **a-phase voltage**, the **b-phase voltage**, and the **c-phase voltage**.

Balanced Three-phase Voltages

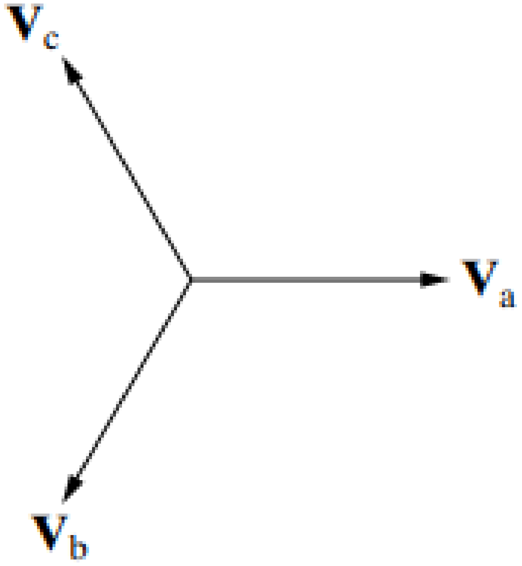
- They have equal amplitudes.
- They have same frequency (50 or 60 Hz).
- They are **out of phase** with each other by exactly **120°**
- Phase sequence is important for paralell operation of three-phase circuits.



variation of instantaneous three-phase voltages wrt time (***positive sequence***)

ABC Phase Sequence

Only two possible phase relationships can exist between the a-phase voltage and the b- and c-phase voltages.
One possibility is for the b-phase voltage to lag the a-phase voltage by 120°, in which case the c-phase voltage must lead the a-phase voltage by 120 °.
This phase relationship is known as the abc (or positive) phase sequence



$$\mathbf{V}_a = V_m \angle 0^\circ,$$

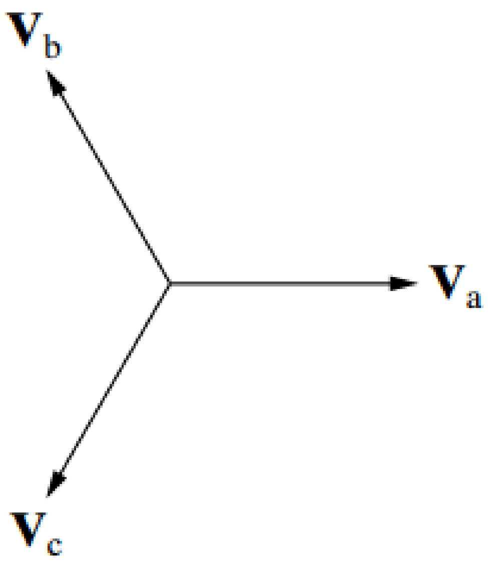
$$\mathbf{V}_b = V_m \angle -120^\circ,$$

$$\mathbf{V}_c = V_m \angle +120^\circ,$$

→ **abc** phase sequence or **positive phase sequence**

ACB Phase Sequence

The only other possibility is for the b-phase voltage to lead the a-phase voltage by 120°, in which case the c-phase voltage must lag the a-phase voltage by 120°. This phase relationship is known as the acb (or negative) phase sequence



$$\mathbf{V}_a = V_m \angle 0^\circ,$$
$$\mathbf{V}_b = V_m \angle +120^\circ,$$
$$\mathbf{V}_c = V_m \angle -120^\circ.$$

→ **acb** phase sequence or **negative phase sequence**

Example: What is the sequence of each of the following set of voltages?

a) $V_a(t) = 208\cos(wt+76°)$ V
$V_b(t) = 208\cos(wt+316°)$ V
$V_c(t) = 208\cos(wt-164°)$ V

⇨

$V_a(t) = 208\cos(wt+76°-76°)$
$V_b(t) = 208\cos(wt+316-76°°)$
$V_c(t) = 208\cos(wt-164°-76°)$

⇨

$V_a(t) = 208\cos(wt+0°)$ V
$V_b(t) = 208\cos(wt+240°)$ V
$V_c(t) = 208\cos(wt-240°)$ V

positive sequence

b) $V_a(t) = 120\cos(wt-49°)$ V
$V_b(t) = 120\cos(wt-289°)$ V
$V_c(t) = 120\cos(wt+191°)$ V

⇨

$V_a(t) = 120\cos(wt-49°+49°)$
$V_b(t) = 120\cos(wt-289°+49°)$
$V_c(t) = 120\cos(wt+191°+49°)$

⇨

$V_a(t) = 120\cos(wt+0°)$ V
$V_b(t) = 120\cos(wt-240°)$ V
$V_c(t) = 120\cos(wt+240°)$ V

negative sequence

Balanced Three-Phase Voltages

Another important characteristic of a set of balanced three-phase voltages is that the sum of the voltages is zero.

$$\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c = 0. \quad \Rightarrow \quad V_m \angle 0^\circ + V_m \angle -120^\circ + V_m \angle +120^\circ = 0$$

$$\mathbf{V}_a = V_m \angle 0^\circ,$$

$$\mathbf{V}_b = V_m \angle -120^\circ,$$

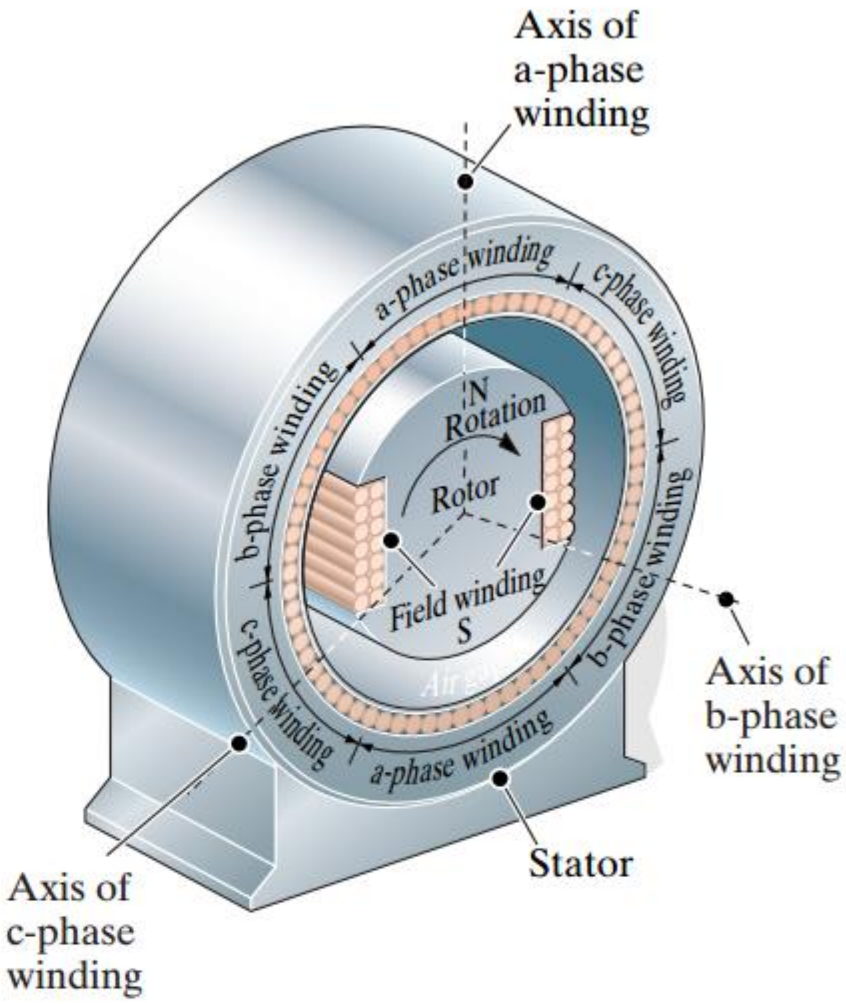
$$\mathbf{V}_c = V_m \angle +120^\circ,$$

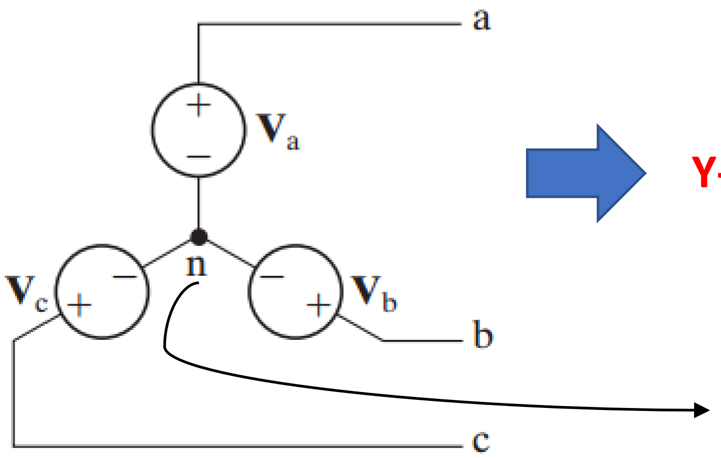
$$v_a + v_b + v_c = 0. \quad \Rightarrow$$ the sum of the instantaneous voltages also is zero

Three-Phase Voltage Sources

- A three-phase voltage source is a generator with three separate windings distributed around the periphery of the stator.
- Each winding comprises one phase of the generator. The rotor of the generator is an electromagnet driven at synchronous speed by a prime mover, such as a steam or gas turbine.
- Rotation of the electromagnet induces a sinusoidal voltage in each winding. The phase windings are designed so that the sinusoidal voltages induced in them are equal in amplitude and out of phase with each other by 120 degree.
- The phase windings are stationary with respect to the rotating electromagnet, so the frequency of the voltage induced in each winding is the same.

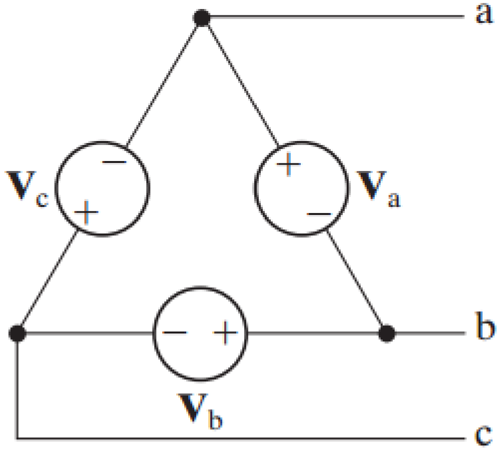
Three-Phase Voltage Sources





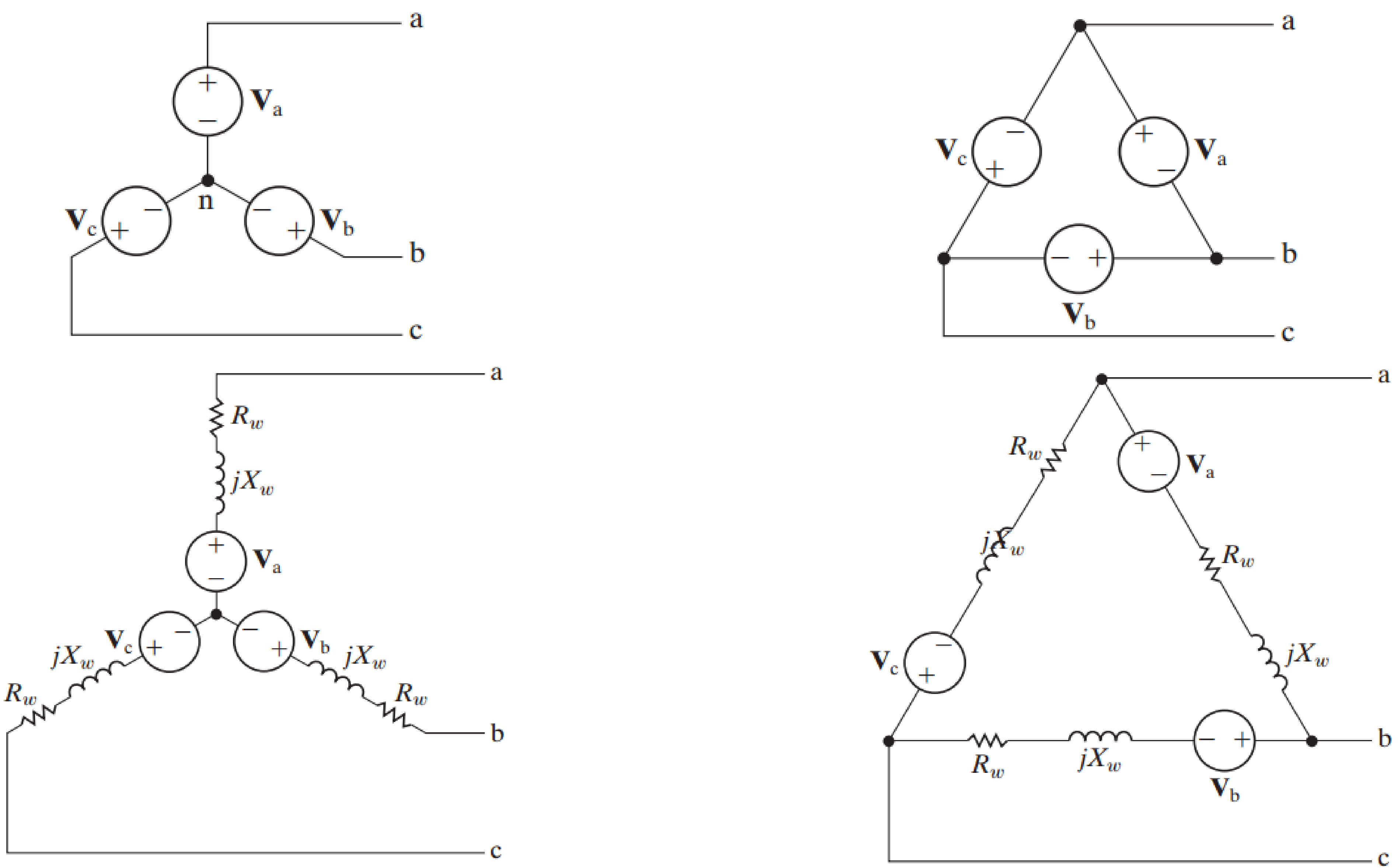
Y-connected Source

Neutral terminal

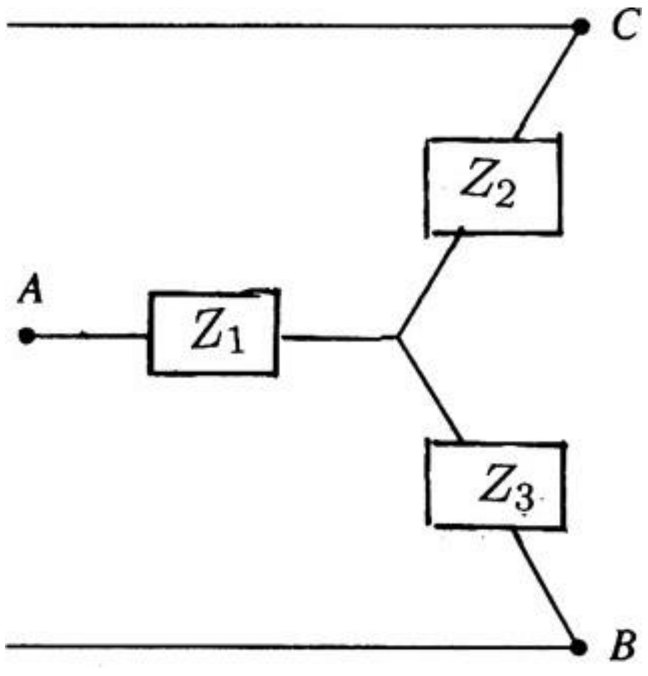


Δ-connected Source

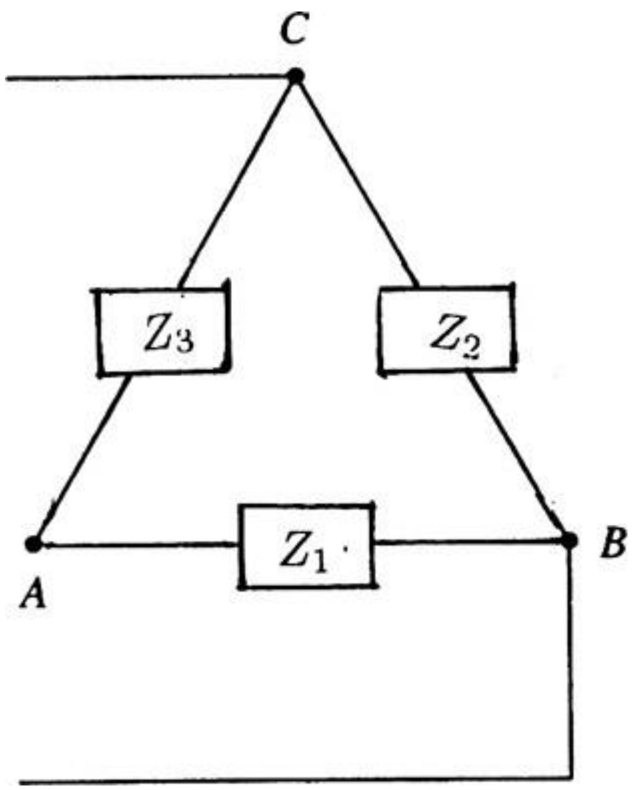
A model of a three-phase source with winding impedance



Types of three-phase loads



Wye (Y)-connected load

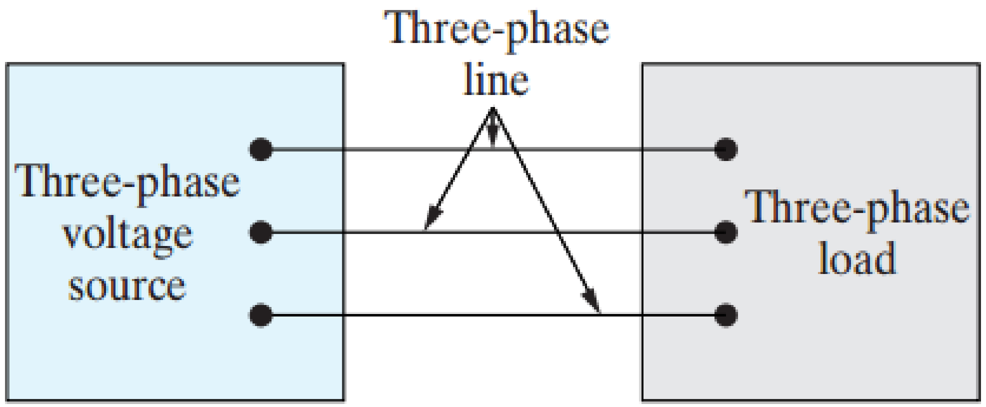


Delta (Δ)-connected load

If Z1 = Z2 = Z3 => balanced load

Possible Connections of Three-Phase Systems

Three-phase sources and loads can be either Y-connected or Δ-connected, the basic circuit in Fig. represents four different configurations:



Source	Load
Y	Y
Y	Δ
Δ	Y
Δ	Δ

Analysis of the Wye-Wye Circuit

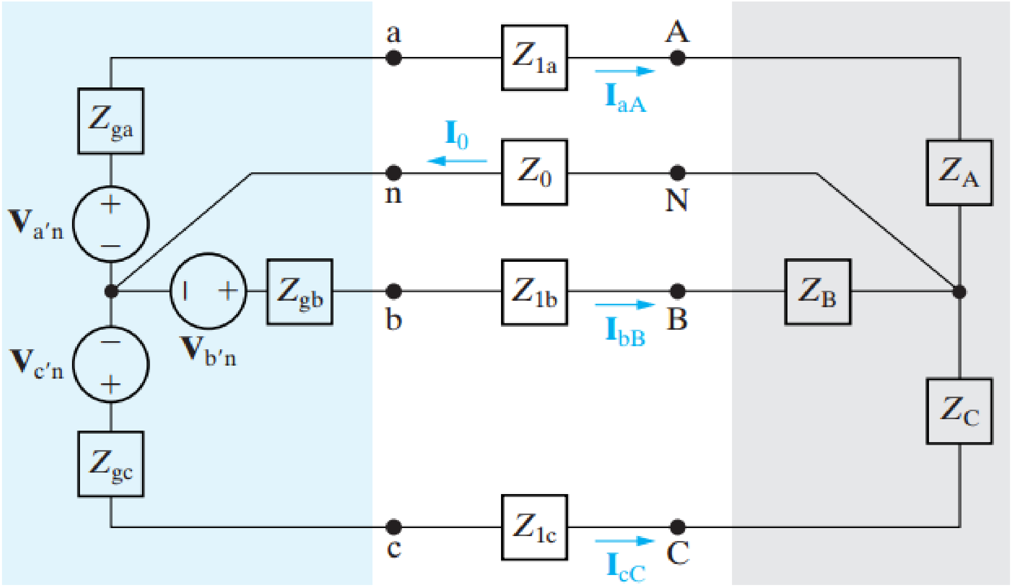


Figure 11.6 ▲ A three-phase Y-Y system.

1. The voltage sources form a set of balanced three-phase voltages. In Fig. 11.6, this means that $\mathbf{V}_{a'n}$, $\mathbf{V}_{b'n}$, and $\mathbf{V}_{c'n}$ are a set of balanced three-phase voltages.
2. The impedance of each phase of the voltage source is the same. In Fig. 11.6, this means that $Z_{ga} = Z_{gb} = Z_{gc}$.
3. The impedance of each line (or phase) conductor is the same. In Fig. 11.6, this means that $Z_{1a} = Z_{1b} = Z_{1c}$.
4. The impedance of each phase of the load is the same. In Fig. 11.6, this means that $Z_A = Z_B = Z_C$.

Using the source neutral as the reference node and letting $\mathbf{V}_N$ denote the node voltage between the nodes **N** and **n**

$$\frac{\mathbf{V}_N}{Z_0} + \frac{\mathbf{V}_N - \mathbf{V}_{a'n}}{Z_A + Z_{1a} + Z_{ga}} + \frac{\mathbf{V}_N - \mathbf{V}_{b'n}}{Z_B + Z_{1b} + Z_{gb}} + \frac{\mathbf{V}_N - \mathbf{V}_{c'n}}{Z_C + Z_{1c} + Z_{gc}} = 0.$$

Analysis of the Wye-Wye Circuit

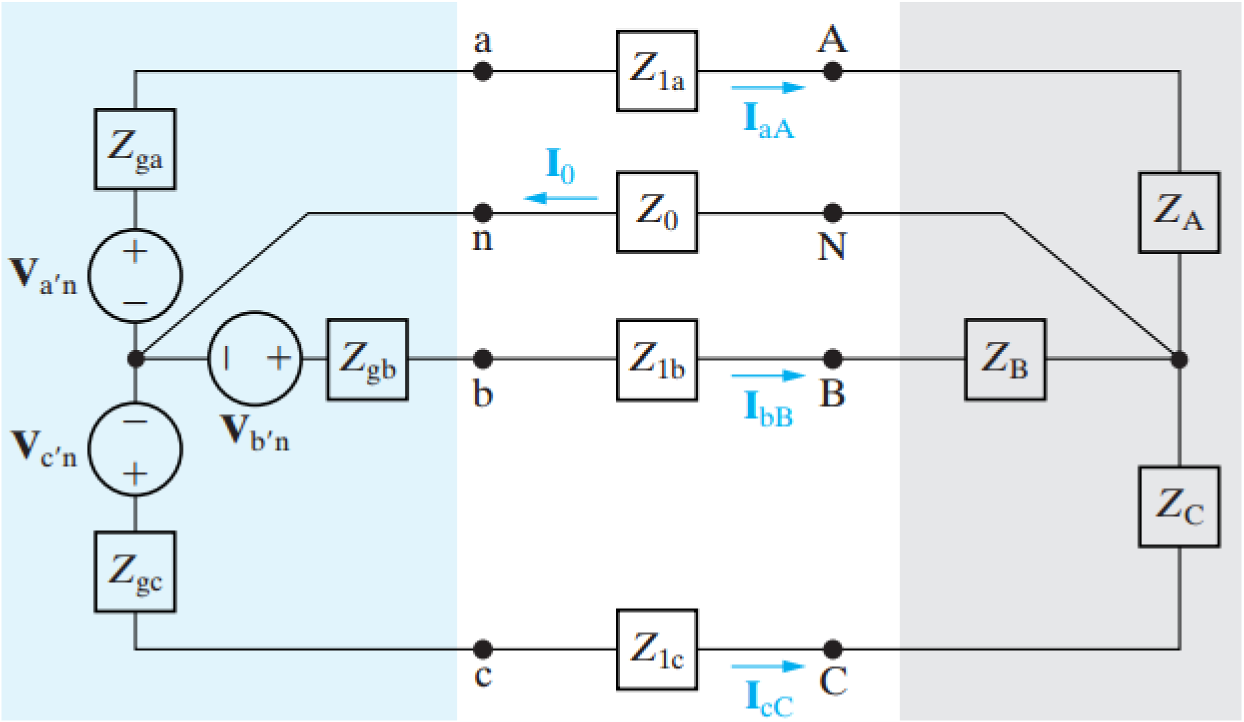
$$\frac{\mathbf{V}_N}{Z_0} + \frac{\mathbf{V}_N - \mathbf{V}_{a'n}}{Z_A + Z_{1a} + Z_{ga}} + \frac{\mathbf{V}_N - \mathbf{V}_{b'n}}{Z_B + Z_{1b} + Z_{gb}} + \frac{\mathbf{V}_N - \mathbf{V}_{c'n}}{Z_C + Z_{1c} + Z_{gc}} = 0.$$

$$Z_\phi = Z_A + Z_{1a} + Z_{ga} = Z_B + Z_{1b} + Z_{gb} = Z_C + Z_{1c} + Z_{gc}$$

$$\mathbf{V}_N\left(\frac{1}{Z_0} + \frac{3}{Z_\phi}\right) = \frac{\mathbf{V}_{a'n} + \mathbf{V}_{b'n} + \mathbf{V}_{c'n}}{Z_\phi}$$

$$\mathbf{V}_N = 0$$

Analysis of the Wye-Wye Circuit



$$\mathbf{V}_N = 0$$

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{a'n} - \mathbf{V}_N}{Z_A + Z_{1a} + Z_{ga}} = \frac{\mathbf{V}_{a'n}}{Z_\phi},$$

$$\mathbf{I}_{bB} = \frac{\mathbf{V}_{b'n} - \mathbf{V}_N}{Z_B + Z_{1b} + Z_{gb}} = \frac{\mathbf{V}_{b'n}}{Z_\phi},$$

$$\mathbf{I}_{cC} = \frac{\mathbf{V}_{c'n} - \mathbf{V}_N}{Z_C + Z_{1c} + Z_{gc}} = \frac{\mathbf{V}_{c'n}}{Z_\phi}.$$

A single-phase equivalent circuit

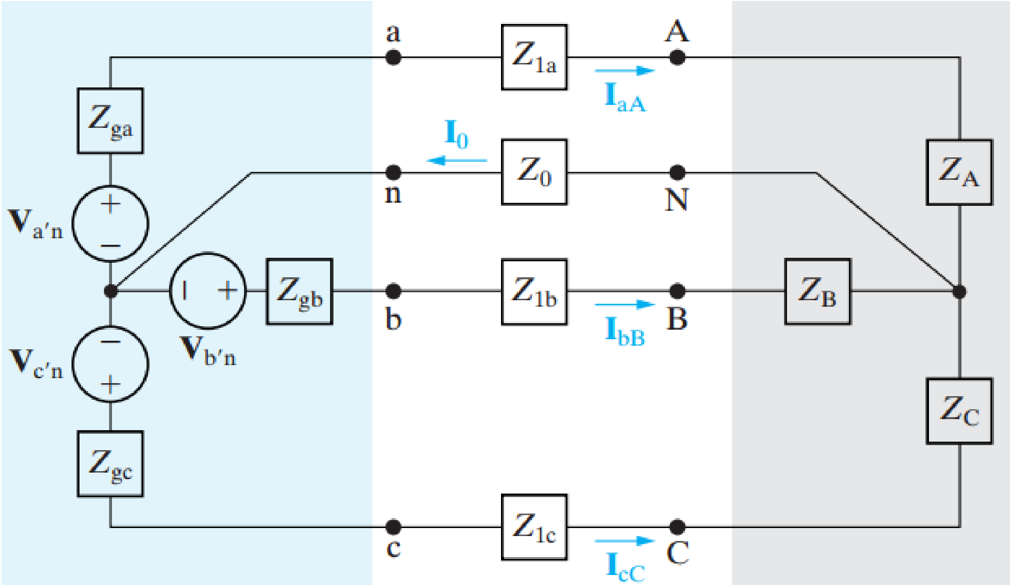
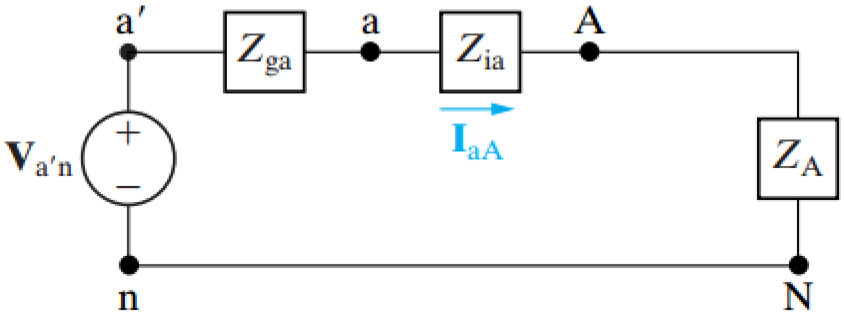
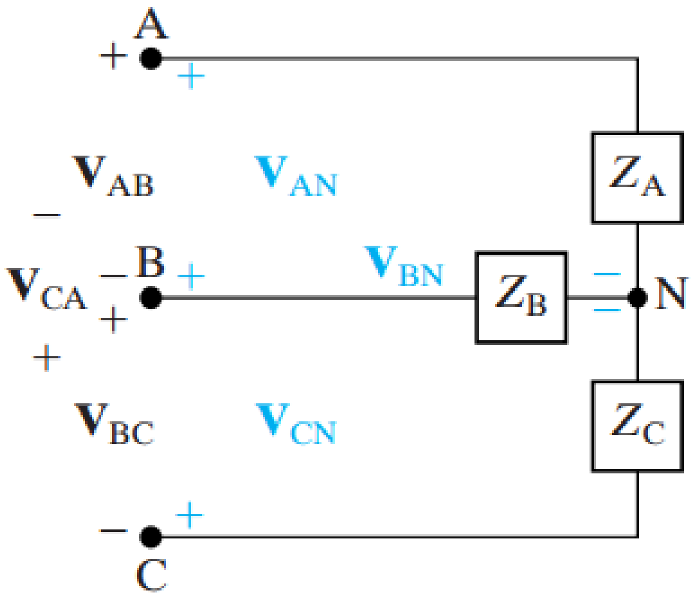


Figure 11.6 ▲ A three-phase Y-Y system.



$$I_o = I_{aA} + I_{bB} + I_{cC}.$$

The voltages of Wye-connected load



$\mathbf{V}_{AB}$, $\mathbf{V}_{BC}$, $\mathbf{V}_{CA}$ — line-to-line voltages

$\mathbf{V}_{AN}$, $\mathbf{V}_{BN}$, $\mathbf{V}_{CN}$ — line-to-neutral voltages

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN},$$

$$\mathbf{V}_{BC} = \mathbf{V}_{BN} - \mathbf{V}_{CN},$$

$$\mathbf{V}_{CA} = \mathbf{V}_{CN} - \mathbf{V}_{AN}.$$

The voltages of Wye-connected load for positive phase sequence

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN},$$

$$\mathbf{V}_{BC} = \mathbf{V}_{BN} - \mathbf{V}_{CN},$$

$$\mathbf{V}_{CA} = \mathbf{V}_{CN} - \mathbf{V}_{AN}.$$

$$\mathbf{V}_{AB} = V_\phi \angle 0^\circ - V_\phi \angle -120^\circ = \sqrt{3} V_\phi \angle 30^\circ,$$

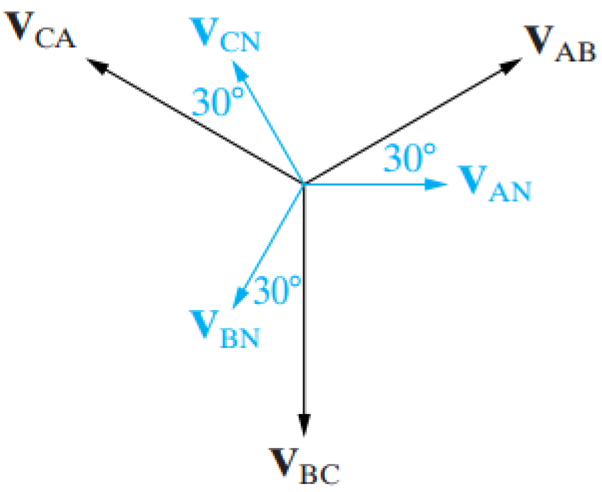
$$\mathbf{V}_{BC} = V_\phi \angle -120^\circ - V_\phi \angle 120^\circ = \sqrt{3} V_\phi \angle -90^\circ,$$

$$\mathbf{V}_{CA} = V_\phi \angle 120^\circ - V_\phi \angle 0^\circ = \sqrt{3} V_\phi \angle 150^\circ.$$

$$\mathbf{V}_{AN} = V_\phi \angle 0^\circ,$$

$$\mathbf{V}_{BN} = V_\phi \angle -120^\circ,$$

$$\mathbf{V}_{CN} = V_\phi \angle +120^\circ,$$



1. The magnitude of the **line-to-line** voltage is $\sqrt{3}$ times the magnitude of the line-to-neutral voltage.
2. The line-to-line voltages **form** a balanced three-phase set of voltages.
3. The set of **line-to-line** voltages **leads** the set of **line-to-neutral** voltages by 30° for **ABC phase sequence**

The voltages of Wye-connected load for negative phase sequence

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN},$$

$$\mathbf{V}_{BC} = \mathbf{V}_{BN} - \mathbf{V}_{CN},$$

$$\mathbf{V}_{CA} = \mathbf{V}_{CN} - \mathbf{V}_{AN}.$$

$$\mathbf{V}_{AB} = V_\phi \angle 0^\circ - V_\phi \angle 120^\circ = \sqrt{3} V_\phi \angle -30^\circ$$

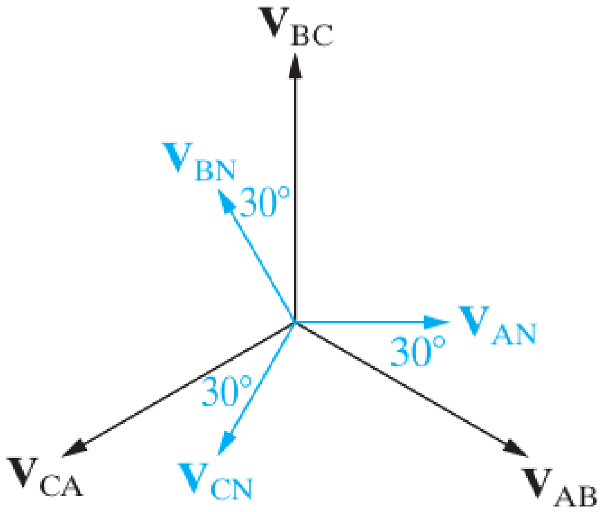
$$\mathbf{V}_{BC} = V_\phi \angle 120^\circ - V_\phi \angle -120^\circ = \sqrt{3} V_\phi \angle 90^\circ$$

$$\mathbf{V}_{CA} = V_\phi \angle -120^\circ - V_\phi \angle 0^\circ = \sqrt{3} V_\phi \angle -150^\circ$$

$$\mathbf{V}_{AN} = V_\phi \angle 0^\circ,$$

$$\mathbf{V}_{BN} = V_\phi \angle +120^\circ,$$

$$\mathbf{V}_{CN} = V_\phi \angle -120^\circ,$$



1. The magnitude of the **line-to-line** voltage is $\sqrt{3}$ times the magnitude of the line-to-neutral voltage.
2. The line-to-line voltages **form** a balanced three-phase set of voltages.
3. The set of **line-to-line** voltages lags the set of **line-to-neutral** voltages by 30° for **ACB phase sequence**

Summary

- Line voltage refers to the voltage across any pair of lines.
- phase voltage refers to the voltage across a single phase.
- Line current refers to the current in a single line.
- phase current refers to current in a single phase.

Example

A balanced three-phase Y-connected generator with positive sequence has an impedance of $0.2 + j0.5\ \Omega/\phi$ and an internal voltage of $120\ V/\phi$. The generator feeds a balanced three-phase Y-connected load having an impedance of $39 + j28\ \Omega/\phi$. The impedance of the line connecting the generator to the load is $0.8 + j1.5\ \Omega/\phi$. The a-phase internal voltage of the generator is specified as the reference phasor.

a) Construct the a-phase equivalent circuit of the system.

b) Calculate the three line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

c) Calculate the three phase voltages at the load, $\mathbf{V}_{AN}$, $\mathbf{V}_{BN}$, and $\mathbf{V}_{CN}$.

d) Calculate the line voltages $\mathbf{V}_{AB}$, $\mathbf{V}_{BC}$, and $\mathbf{V}_{CA}$ at the terminals of the load.

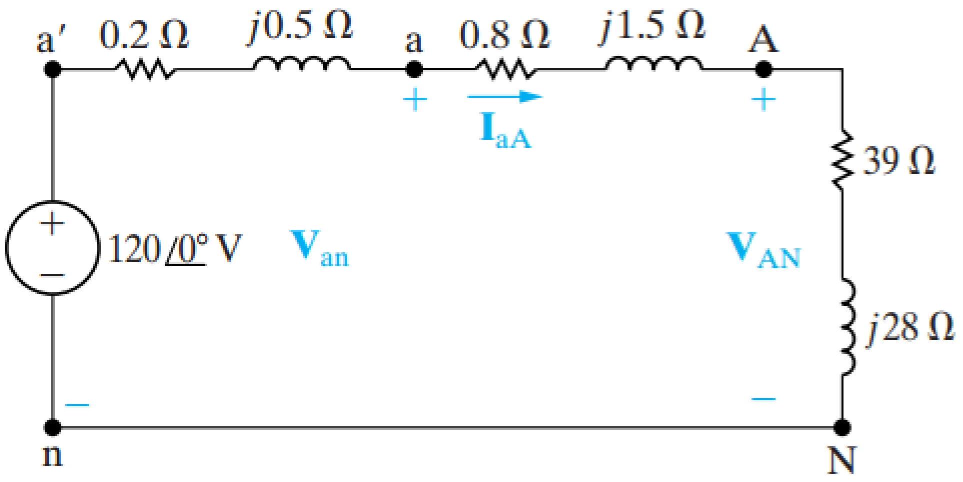
e) Calculate the phase voltages at the terminals of the generator, $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$, and $\mathbf{V}_{cn}$.

f) Calculate the line voltages $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$ at the terminals of the generator.

g) Repeat (a)–(f) for a negative phase sequence.

Example

a)



b)

$$\mathbf{I}_{aA} = \frac{120\angle 0^\circ}{(0.2 + 0.8 + 39) + j(0.5 + 1.5 + 28)}$$

$$= \frac{120\angle 0^\circ}{40 + j30}$$

$$= 2.4\angle -36.87^\circ \text{ A.}$$

$$\mathbf{I}_{bB} = 2.4\angle -156.87^\circ \text{ A,}$$

$$\mathbf{I}_{cC} = 2.4\angle 83.13^\circ \text{ A.}$$

c) The phase voltage at the A terminal of the load is

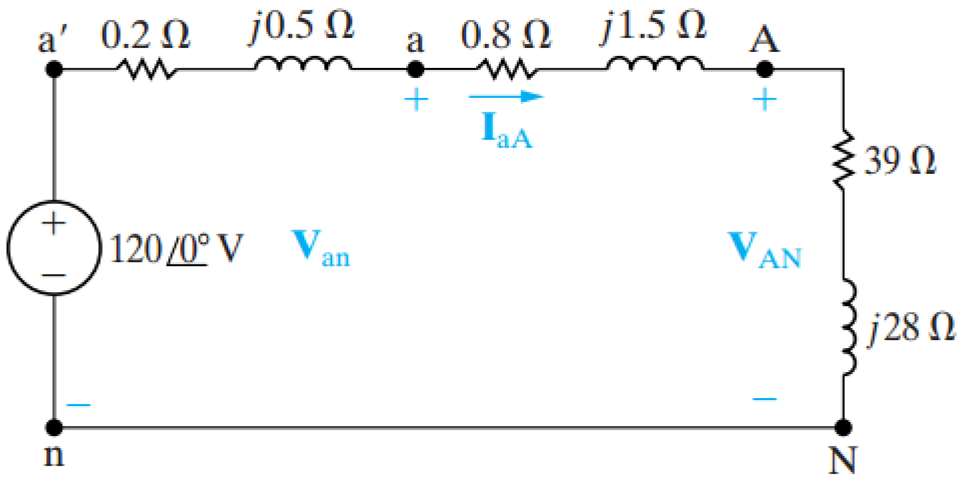
$$\mathbf{V}_{AN} = (39 + j28)(2.4\angle -36.87^\circ)$$

$$= 115.22\angle -1.19^\circ \text{ V.}$$

$$\mathbf{V}_{BN} = 115.22\angle -121.19^\circ \text{ V,}$$

$$\mathbf{V}_{CN} = 115.22\angle 118.81^\circ \text{ V.}$$

Example



d)

$$\mathbf{V}_{AB} = (\sqrt{3}\angle 30^\circ)\mathbf{V}_{AN}$$
$$= 199.58\angle 28.81^\circ \text{ V},$$
$$\mathbf{V}_{BC} = 199.58\angle -91.19^\circ \text{ V},$$
$$\mathbf{V}_{CA} = 199.58\angle 148.81^\circ \text{ V}.$$

e)

$$\mathbf{V}_{an} = 120 - (0.2 + j0.5)(2.4\angle -36.87^\circ)$$
$$= 120 - 1.29\angle 31.33^\circ$$
$$= 118.90 - j0.67$$
$$= 118.90\angle -0.32^\circ \text{ V}.$$

$$\mathbf{V}_{bn} = 118.90\angle -120.32^\circ \text{ V},$$
$$\mathbf{V}_{cn} = 118.90\angle 119.68^\circ \text{ V}.$$

f)

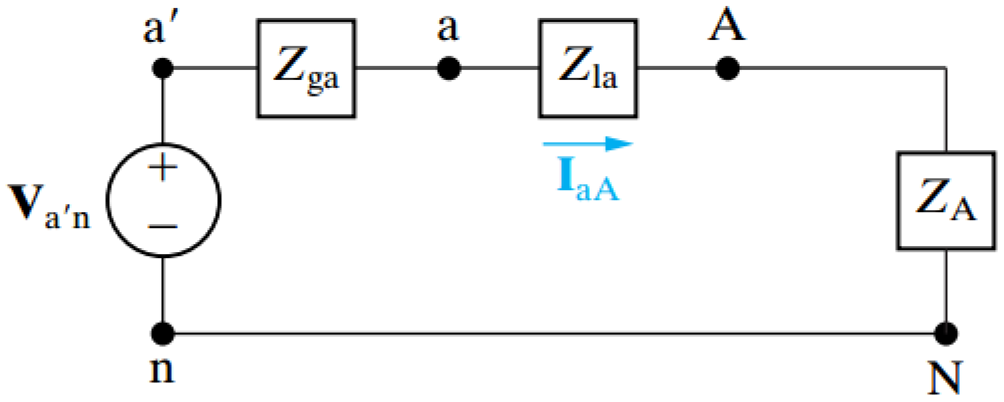
$$\mathbf{V}_{ab} = (\sqrt{3}\angle 30^\circ)\mathbf{V}_{an}$$
$$= 205.94\angle 29.68^\circ \text{ V},$$
$$\mathbf{V}_{bc} = 205.94\angle -90.32^\circ \text{ V},$$
$$\mathbf{V}_{ca} = 205.94\angle 149.68^\circ \text{ V}.$$

Analysis of the Wye-Delta Circuit

- If the load in a three-phase circuit is connected in a delta, it can be transformed into a wye by using the delta-to-wye transformation.

$$Z_Y = \frac{Z_\Delta}{3},$$

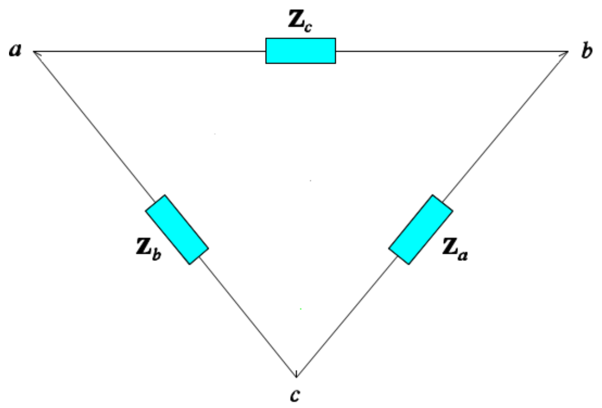
- After the Δ load has been replaced by its Y equivalent, the a-phase can be modeled by the single phase equivalent circuit.



- We use this circuit to calculate the line currents, and we then use the line currents to find the currents in each leg of the original Δ load.

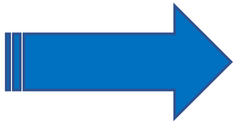
Delta-Wye conversion

- Delta-Wye conversion is very useful when analyzing three-phase systems
- The advantage of wye connection is that it has a **neutral point** which greatly simplifies **per-phase analysis** of three-phase systems
- Delta-Wye conversion is valid for any impedance type as well as resistances



Impedances

Z_a, Z_b, and Z_c are Delta-connected

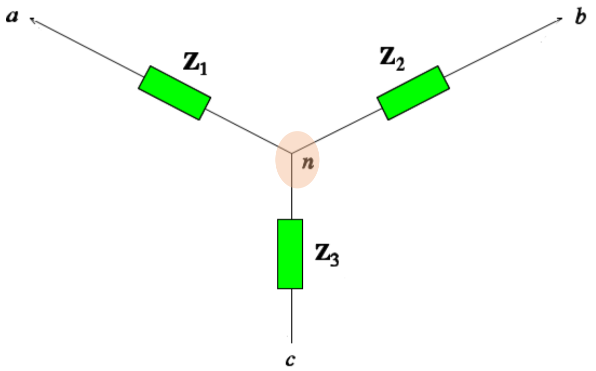


$\Delta - Y$ Conversion

$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c}$$

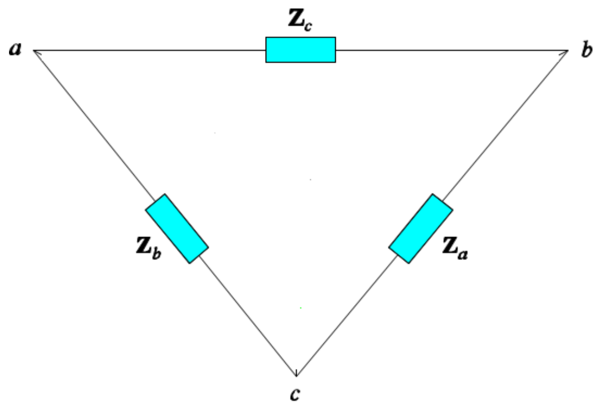
$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$



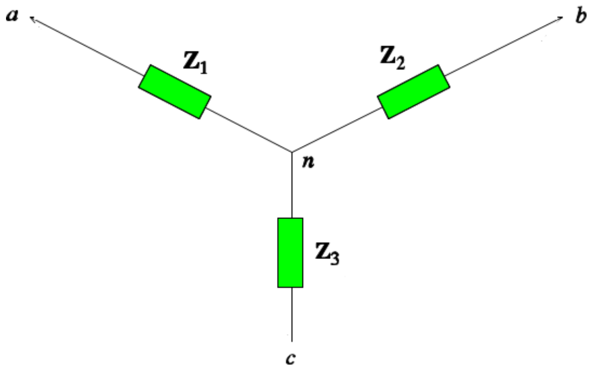
Impedances

Z_1, Z_2, and Z_3 are Wye-connected

Delta-Wye conversion for equal impedances



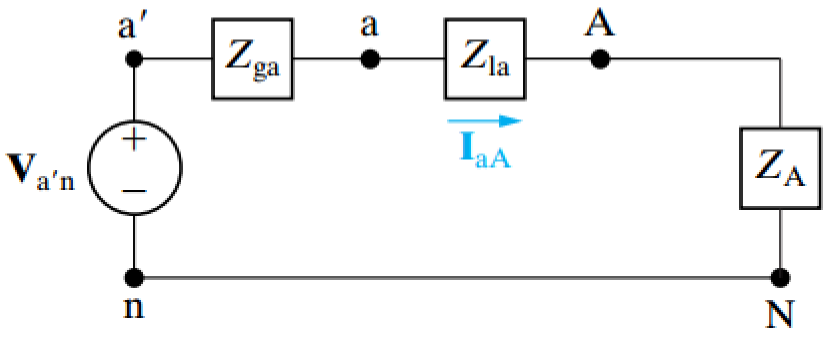
$$Z_Y = \frac{1}{3} Z_D$$

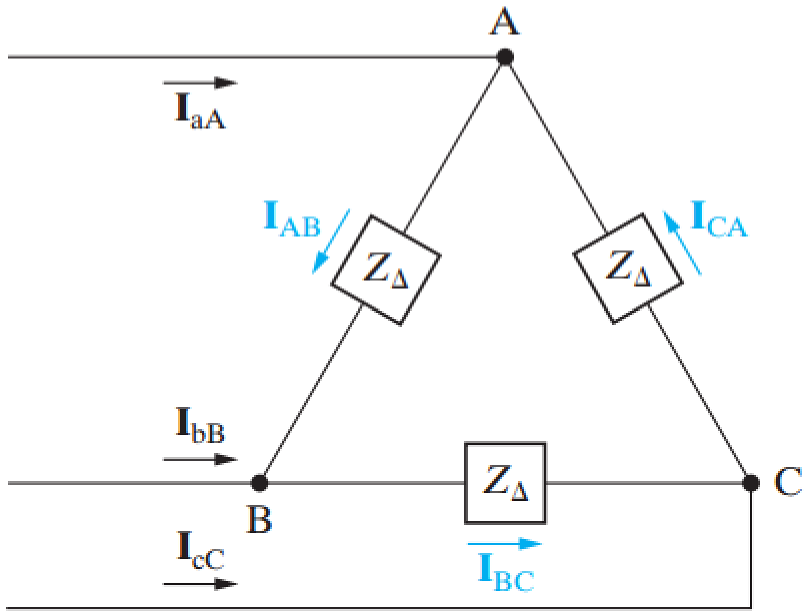


$Z_a = Z_b = Z_c = Z_D$

$Z_1 = Z_2 = Z_3 = Z_Y$

Analysis of the Wye-Delta Circuit



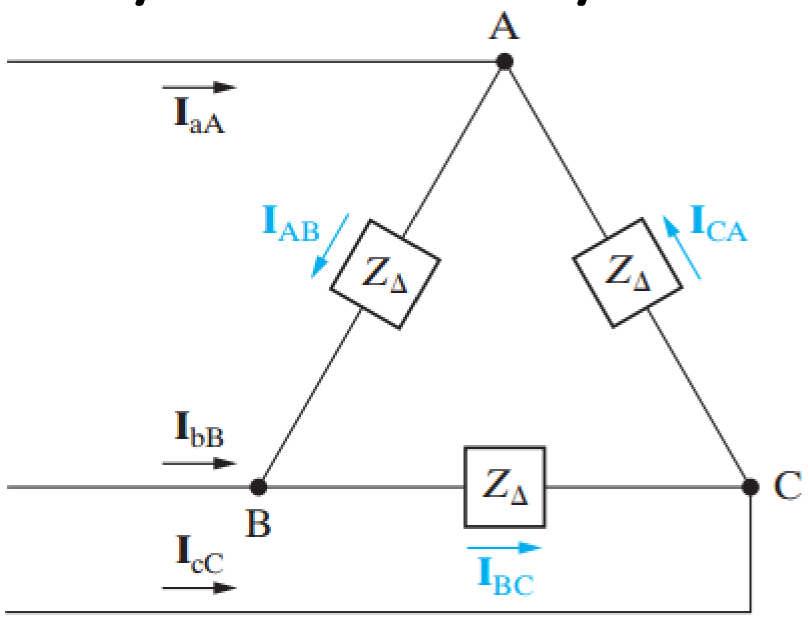


$$\mathbf{I}_{AB} = I_\phi \angle 0^\circ,$$

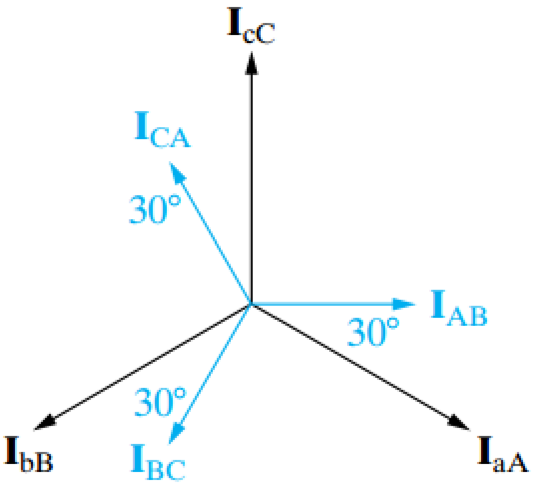
$$\mathbf{I}_{BC} = I_\phi \angle -120^\circ,$$

$$\mathbf{I}_{CA} = I_\phi \angle 120^\circ.$$

Analysis of the Wye-Delta Circuit



$$\mathbf{I}_{AB} = I_\phi \angle 0^\circ,$$
$$\mathbf{I}_{BC} = I_\phi \angle -120^\circ,$$
$$\mathbf{I}_{CA} = I_\phi \angle 120^\circ.$$



$$\begin{aligned}\mathbf{I}_{aA} &= \mathbf{I}_{AB} - \mathbf{I}_{CA} \\ &= I_\phi \angle 0^\circ - I_\phi \angle 120^\circ \\ &= \sqrt{3} I_\phi \angle -30^\circ,\end{aligned}$$

$$\begin{aligned}\mathbf{I}_{bB} &= \mathbf{I}_{BC} - \mathbf{I}_{AB} \\ &= I_\phi \angle -120^\circ - I_\phi \angle 0^\circ \\ &= \sqrt{3} I_\phi \angle -150^\circ,\end{aligned}$$

$$\begin{aligned}\mathbf{I}_{cC} &= \mathbf{I}_{CA} - \mathbf{I}_{BC} \\ &= I_\phi \angle 120^\circ - I_\phi \angle -120^\circ \\ &= \sqrt{3} I_\phi \angle 90^\circ.\end{aligned}$$

Example

The Y-connected source in Example 11.1 feeds a Δ-connected load through a distribution line having an impedance of $0.3 + j0.9\ \Omega/\phi$. The load impedance is $118.5 + j85.8\ \Omega/\phi$. Use the a-phase internal voltage of the generator as the reference.

a) Construct a single-phase equivalent circuit of the three-phase system.

b) Calculate the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

c) Calculate the phase voltages at the load terminals.

d) Calculate the phase currents of the load.

e) Calculate the line voltages at the source terminals.

Example

The Y-connected source in Example 11.1 feeds a Δ-connected load through a distribution line having an impedance of $0.3 + j0.9\ \Omega/\phi$. The load impedance is $118.5 + j85.8\ \Omega/\phi$. Use the a-phase internal voltage of the generator as the reference.

a) Construct a single-phase equivalent circuit of the three-phase system.

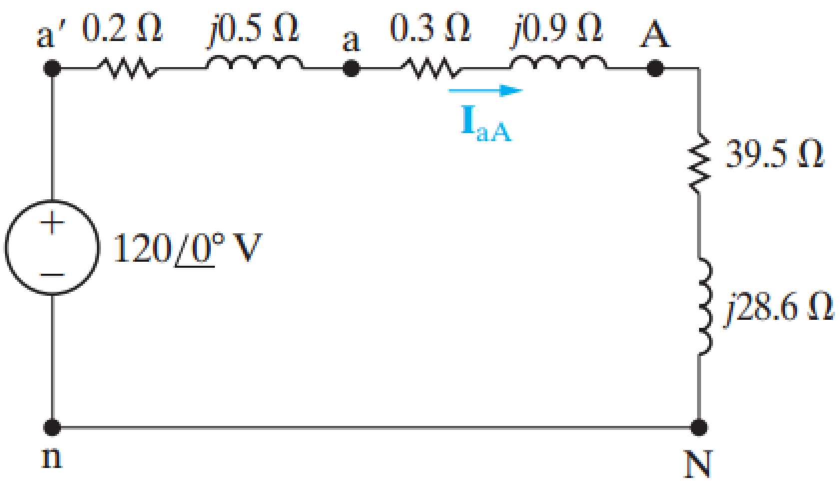
b) Calculate the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

c) Calculate the phase voltages at the load terminals.

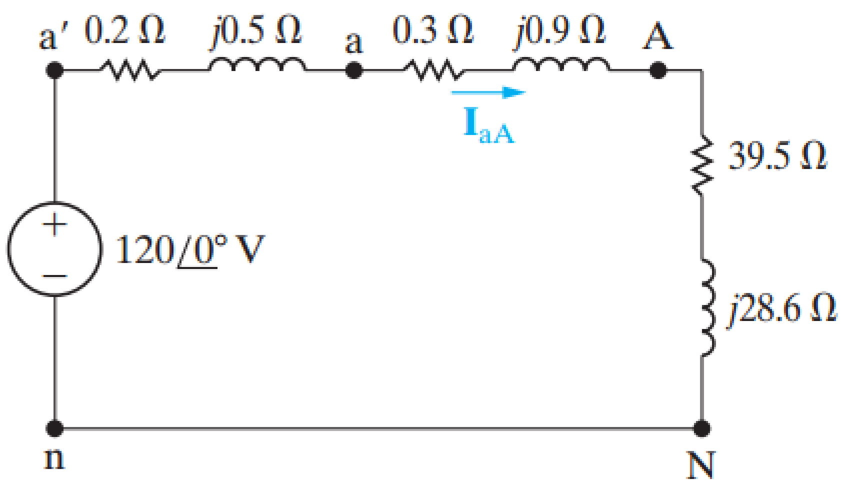
d) Calculate the phase currents of the load.

e) Calculate the line voltages at the source terminals.

a) $$\frac{118.5 + j85.8}{3} = 39.5 + j28.6\ \Omega/\phi.$$



Example



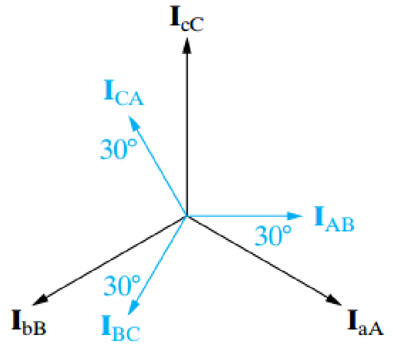
b)

$$\mathbf{I}_{aA} = \frac{120\angle 0^\circ}{(0.2 + 0.3 + 39.5) + j(0.5 + 0.9 + 28.6)}$$

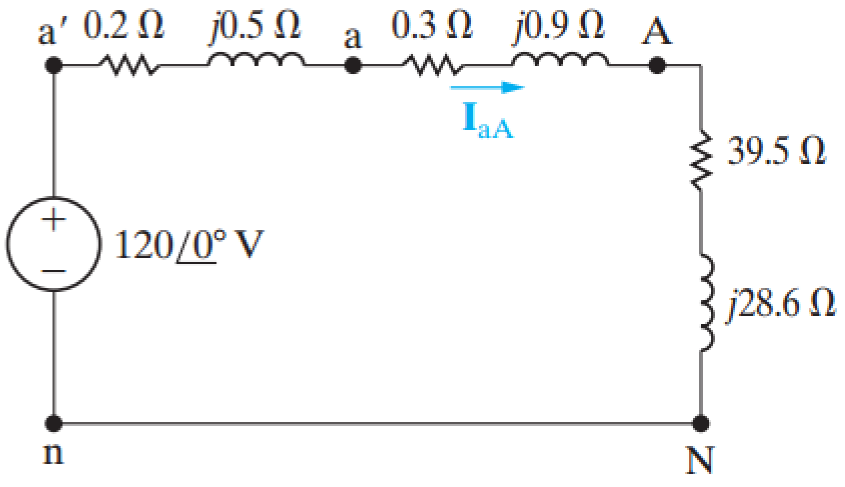
$$= \frac{120\angle 0^\circ}{40 + j30} = 2.4\angle -36.87^\circ \text{ A}.$$

$$\mathbf{I}_{bB} = 2.4\angle -156.87^\circ \text{ A},$$

$$\mathbf{I}_{cC} = 2.4\angle 83.13^\circ \text{ A}.$$



Example



c) $\mathbf{V}_{AN} = (39.5 + j28.6)(2.4 \angle -36.87^\circ)$

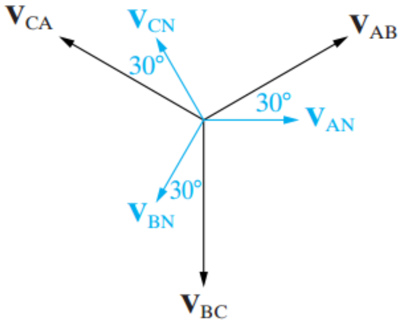
$= 117.04 \angle -0.96^\circ$ V.

$\mathbf{V}_{AB} = (\sqrt{3} \angle 30^\circ)\, \mathbf{V}_{AN}$

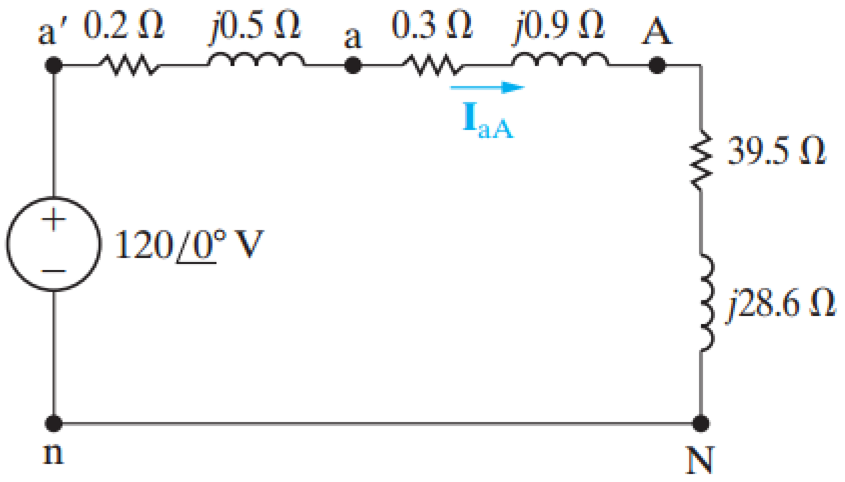
$= 202.72 \angle 29.04^\circ$ V.

$\mathbf{V}_{BC} = 202.72 \angle -90.96^\circ$ V,

$\mathbf{V}_{CA} = 202.72 \angle 149.04^\circ$ V.



Example



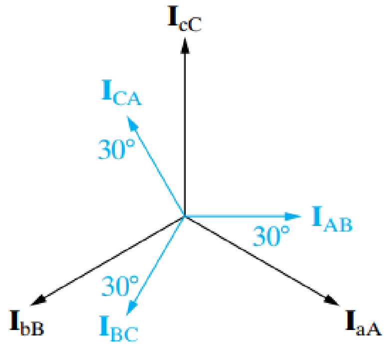
d)

$$I_{AB} = \left(\frac{1}{\sqrt{3}} \angle 30^\circ\right) I_{aA}$$

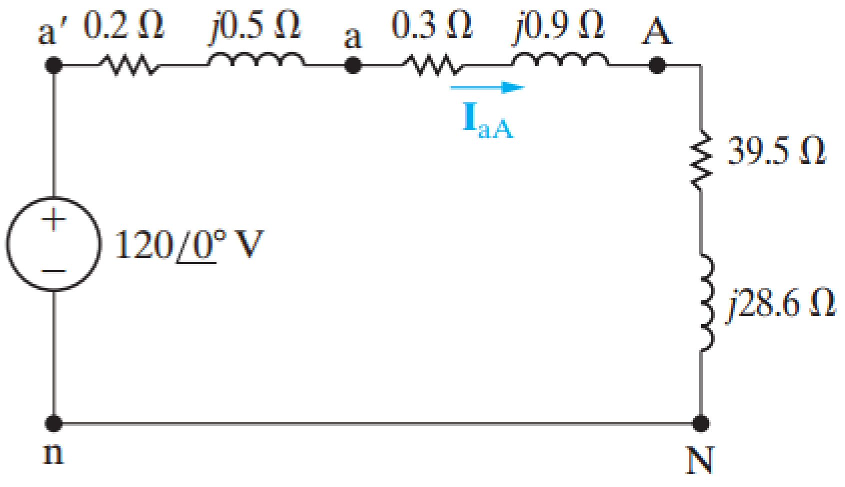
$$= 1.39 \angle -6.87^\circ \text{ A.}$$

$$I_{BC} = 1.39 \angle -126.87^\circ \text{ A,}$$

$$I_{CA} = 1.39 \angle 113.13^\circ \text{ A.}$$



Example



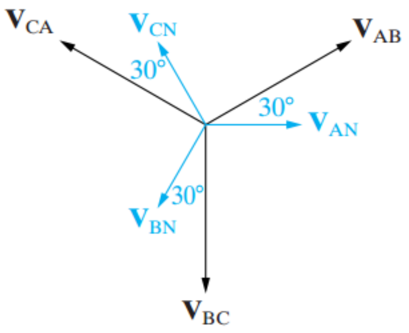
e) $$\mathbf{V}_{an} = (39.8 + j29.5)(2.4 \angle -36.87^\circ) = 118.90 \angle -0.32^\circ \text{ V}.$$

$$\mathbf{V}_{ab} = (\sqrt{3} \angle 30^\circ)\mathbf{V}_{an},$$

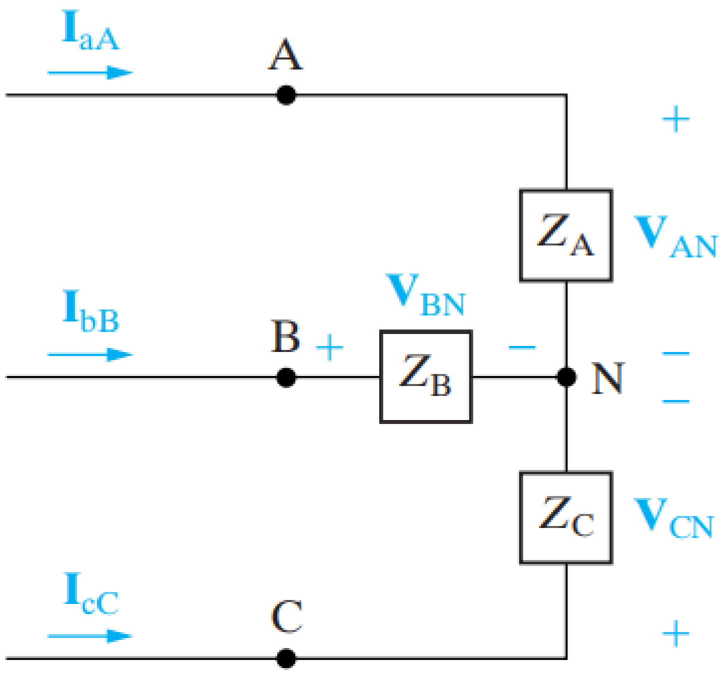
$$\mathbf{V}_{ab} = 205.94 \angle 29.68^\circ \text{ V}.$$

$$\mathbf{V}_{bc} = 205.94 \angle -90.32^\circ \text{ V},$$

$$\mathbf{V}_{ca} = 205.94 \angle 149.68^\circ \text{ V}.$$



Power Calculations - Average Power in a Balanced Wye Load



$$P_A = |\mathbf{V}_{AN}||\mathbf{I}_{aA}| \cos(\theta_{vA} - \theta_{iA}),$$

$$P_B = |\mathbf{V}_{BN}||\mathbf{I}_{bB}| \cos(\theta_{vB} - \theta_{iB}),$$

$$P_C = |\mathbf{V}_{CN}||\mathbf{I}_{cC}| \cos(\theta_{vC} - \theta_{iC}).$$

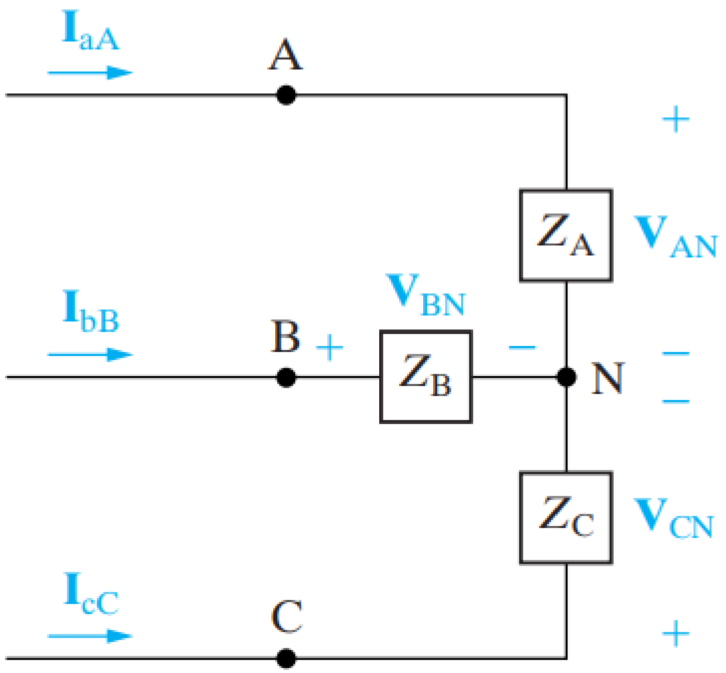
$$V_\phi = |\mathbf{V}_{AN}| = |\mathbf{V}_{BN}| = |\mathbf{V}_{CN}|,$$

$$I_\phi = |\mathbf{I}_{aA}| = |\mathbf{I}_{bB}| = |\mathbf{I}_{cC}|,$$

$$\theta_\phi = \theta_{vA} - \theta_{iA} = \theta_{vB} - \theta_{iB} = \theta_{vC} - \theta_{iC}.$$

$$P_A = P_B = P_C = P_\phi = V_\phi I_\phi \cos\theta_\phi,$$

Power Calculations - Average Power in a Balanced Wye Load



$$\theta_\phi = \theta_{vA} - \theta_{iA} = \theta_{vB} - \theta_{iB} = \theta_{vC} - \theta_{iC}.$$

$$P_A = P_B = P_C = P_\phi = V_\phi I_\phi \cos\theta_\phi,$$

$$P_T = 3P_\phi = 3V_\phi I_\phi \cos\theta_\phi.$$

If we let V_L and I_L and represent the rms magnitudes of the line voltage and current, respectively

$$P_T = 3\left(\frac{V_L}{\sqrt{3}}\right) I_L \cos\theta_\phi$$

$$= \sqrt{3} V_L I_L \cos\theta_\phi.$$

Total real power in a balanced three-phase load

Power Calculations - Complex Power in a Balanced Wye Load

$$Q_\phi = V_\phi I_\phi \sin\theta_\phi,$$

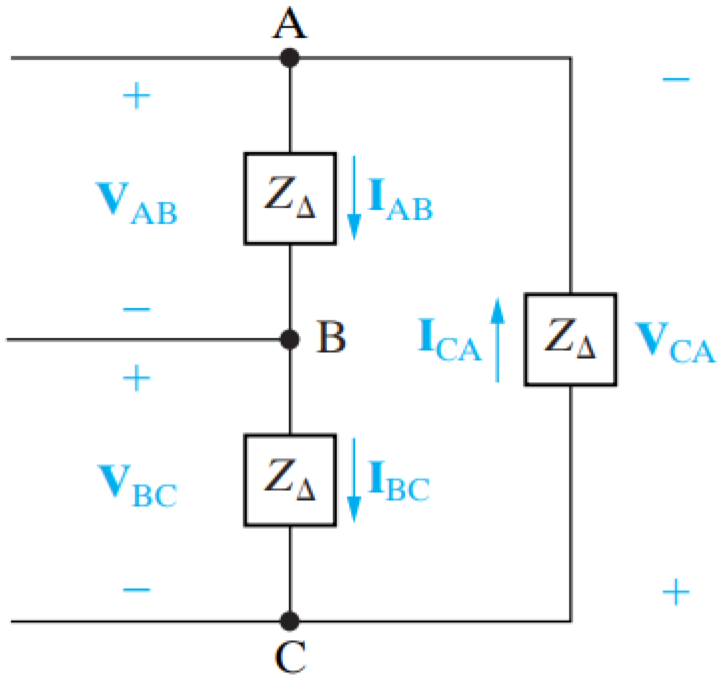
$$Q_T = 3Q_\phi = \sqrt{3}V_L I_L \sin\theta_\phi.$$

$$S_\phi = \mathbf{V}_{AN}\mathbf{I}_{aA}^* = \mathbf{V}_{BN}\mathbf{I}_{bB}^* = \mathbf{V}_{CN}\mathbf{I}_{cC}^* = \mathbf{V}_\phi\mathbf{I}_\phi^*,$$

$$S_\phi = P_\phi + jQ_\phi = \mathbf{V}_\phi\mathbf{I}_\phi^*,$$

$$S_T = 3S_\phi = \sqrt{3}V_L I_L \angle\theta_\phi^\circ.$$

Power Calculations - Average Power in a Balanced Delta Load



$$P_A = |\mathbf{V}_{AB}||\mathbf{I}_{AB}| \cos(\theta_{vAB} - \theta_{iAB}),$$

$$P_B = |\mathbf{V}_{BC}||\mathbf{I}_{BC}| \cos(\theta_{vBC} - \theta_{iBC}),$$

$$P_C = |\mathbf{V}_{CA}||\mathbf{I}_{CA}| \cos(\theta_{vCA} - \theta_{iCA}).$$

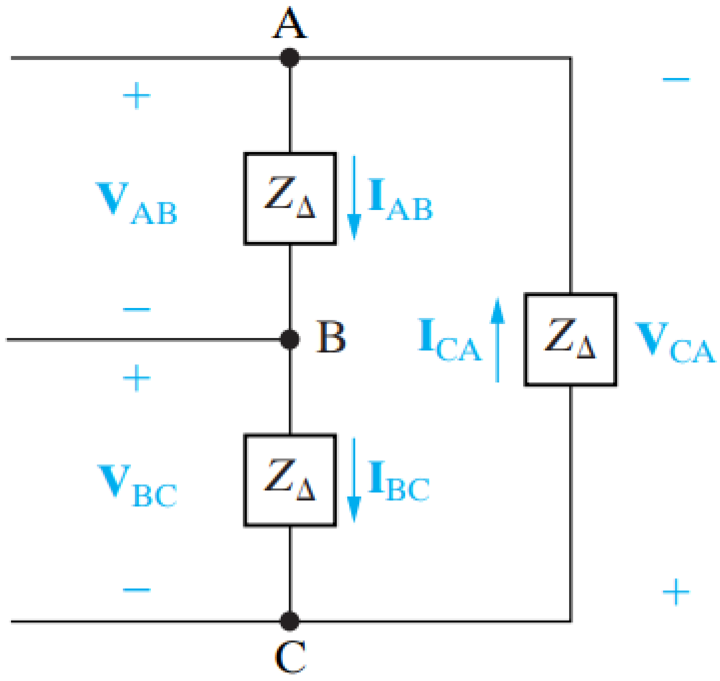
$$|\mathbf{V}_{AB}| = |\mathbf{V}_{BC}| = |\mathbf{V}_{CA}| = V_\phi,$$

$$|\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}| = I_\phi,$$

$$\theta_{vAB} - \theta_{iAB} = \theta_{vBC} - \theta_{iBC} = \theta_{vCA} - \theta_{iCA} = \theta_\phi,$$

$$P_A = P_B = P_C = P_\phi = V_\phi I_\phi \cos\theta_\phi.$$

Power Calculations - Average Power in a Balanced Delta Load



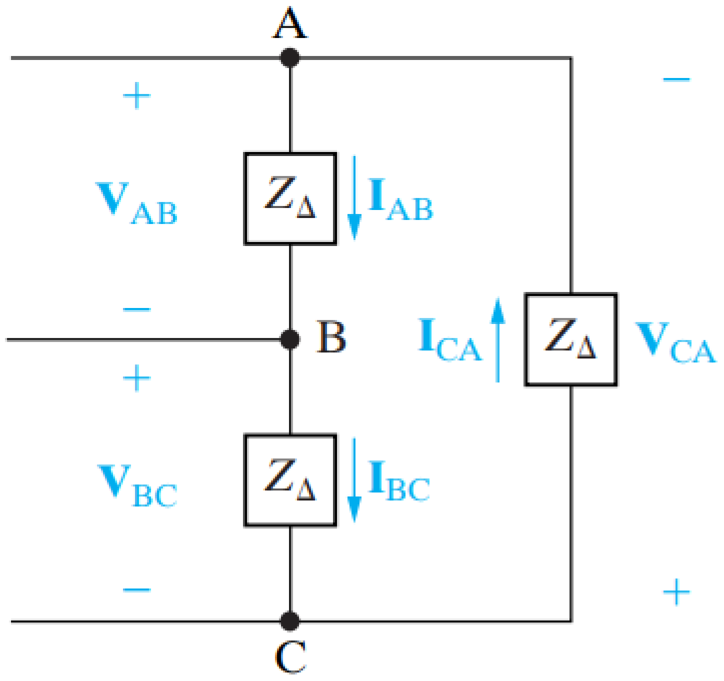
$$P_A = P_B = P_C = P_\phi = V_\phi I_\phi \cos\theta_\phi .$$

$$P_T = 3P_\phi = 3V_\phi I_\phi \cos\theta_\phi$$

$$= 3V_L\left(\frac{I_L}{\sqrt{3}}\right)\cos\theta_\phi$$

$$= \sqrt{3}V_L I_L \cos\theta_\phi .$$

Power Calculations - Complex Power in a Balanced Delta Load



$$Q_\phi = V_\phi I_\phi \sin\theta_\phi;$$

$$Q_T = 3Q_\phi = 3V_\phi I_\phi \sin\theta_\phi;$$

$$S_\phi = P_\phi + jQ_\phi = \mathbf{V}_\phi \mathbf{I}_\phi^*;$$

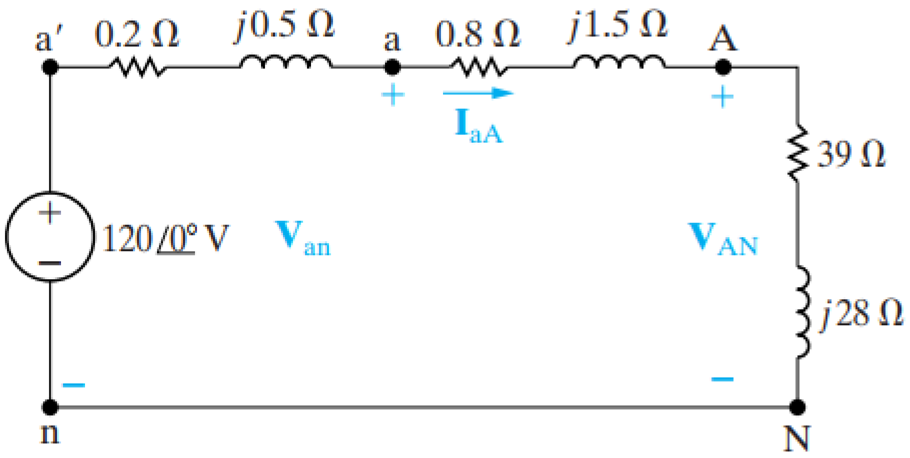
$$S_T = 3S_\phi = \sqrt{3}V_L I_L \angle\theta_\phi.$$

Example

Example 11.1 **Analyzing a Wye-Wye Circuit**

A balanced three-phase Y-connected generator with positive sequence has an impedance of $0.2 + j0.5\ \Omega/\phi$ and an internal voltage of $120\ V/\phi$. The generator feeds a balanced three-phase Y-connected load having an impedance of $39 + j28\ \Omega/\phi$. The impedance of the line connecting the generator to the load is $0.8 + j1.5\ \Omega/\phi$. The a-phase internal voltage of the generator is specified as the reference phasor.

a) Calculate the average power per phase delivered to the Y-connected load of Example 11.1.
b) Calculate the total average power delivered to the load.
c) Calculate the total average power lost in the line.
d) Calculate the total average power lost in the generator.
e) Calculate the total number of magnetizing vars absorbed by the load.
f) Calculate the total complex power delivered by the source.



$$\mathbf{I}_{aA} = 2.4\angle{-36.87^\circ}\ \text{A}.$$

$$\mathbf{V}_{AN} = 115.22\angle{-1.19^\circ}\ \text{V}.$$

Example

a) $V_\phi = 115.22 \text{ V}$

$I_\phi = 2.4 \text{ A},$

$\theta_\phi = -1.19 - (-36.87) = 35.68°.$

$$P_\phi = (115.22)(2.4)\cos 35.68° = 224.64 \text{ W}.$$

b) $P_T = 3P_\phi = 673.92 \text{ W}.$

$$P_T = \sqrt{3}(199.58)(2.4)\cos 35.68° = 673.92 \text{ W}.$$

c) $P_{\text{line}} = 3(2.4)^2(0.8) = 13.824 \text{ W}.$

d) $P_{\text{gen}} = 3(2.4)^2(0.2) = 3.456 \text{ W}.$

e) $$Q_T = \sqrt{3}(199.58)(2.4)\sin 35.68° = 483.84 \text{ VAR}.$$

f) $$S_T = 3S_\phi = -3(120)(2.4)\angle 36.87° = -691.20 - j518.40 \text{ VA}.$$

Example

a) Calculate the total complex power delivered to the Δ-connected load of Example 11.2.

b) What percentage of the average power at the sending end of the line is delivered to the load?

$$\mathbf{I}_{AB} = \left(\frac{1}{\sqrt{3}}\angle 30^\circ\right)\mathbf{I}_{aA}$$

$$= 1.39 \angle -6.87^\circ \text{ A.}$$

$$\mathbf{I}_{BC} = 1.39 \angle -126.87^\circ \text{ A,}$$

$$\mathbf{I}_{CA} = 1.39 \angle 113.13^\circ \text{ A.}$$

a) $\mathbf{V}_\phi = \mathbf{V}_{AB} = 202.72 \angle 29.04^\circ$ V,

$\mathbf{I}_\phi = \mathbf{I}_{AB} = 1.39 \angle -6.87^\circ$ A.

$$S_T = 3(202.72 \angle 29.04^\circ)(1.39 \angle 6.87^\circ)$$

$$= 682.56 + j494.21 \text{ VA.}$$

b) $P_{\text{input}} = 682.56 + 3(2.4)^2(0.3)$

$$= 687.74 \text{ W.}$$

$$682.56/687.74 = 99.25\%$$

Example

A balanced three-phase load requires 480 kW at a lagging power factor of 0.8. The load is fed from a line having an impedance of $0.005 + j0.025\ \Omega/\phi$. The line voltage at the terminals of the load is 600 V.

a) Construct a single-phase equivalent circuit of the system.
b) Calculate the magnitude of the line current.
c) Calculate the magnitude of the line voltage at the sending end of the line.
d) Calculate the power factor at the sending end of the line.

Example

A balanced three-phase load requires 480 kW at a lagging power factor of 0.8. The load is fed from a line having an impedance of $0.005 + j0.025\ \Omega/\phi$. The line voltage at the terminals of the load is 600 V.

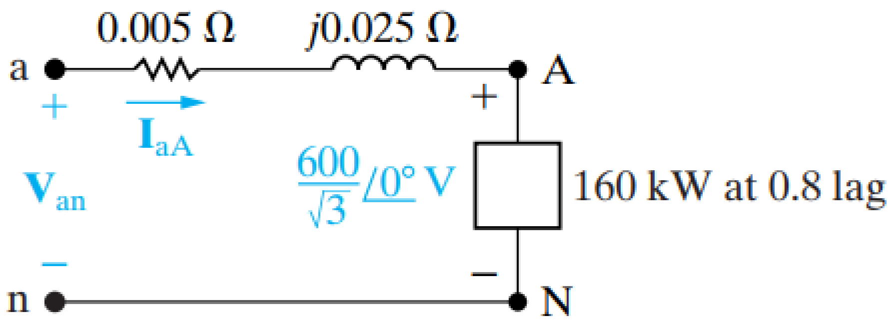
a) Construct a single-phase equivalent circuit of the system.

b) Calculate the magnitude of the line current.

c) Calculate the magnitude of the line voltage at the sending end of the line.

d) Calculate the power factor at the sending end of the line.

a)



b) $\left(\frac{600}{\sqrt{3}}\right)\mathbf{I}_{aA}^{*} = (160 + j120)10^3,$

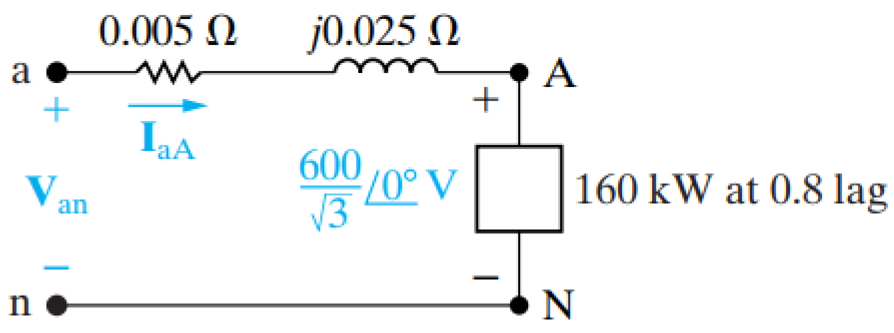
$$\mathbf{I}_{aA}^{*} = 577.35\ \angle 36.87^\circ\ \text{A}.$$

$$\mathbf{I}_{aA} = 577.35\ \angle -36.87^\circ\ \text{A}.$$

$$P_T = \sqrt{3}V_L I_L \cos\theta_p$$
$$= \sqrt{3}(600)I_L(0.8)$$
$$= 480{,}000\ \text{W};$$

$$I_L = \frac{480{,}000}{\sqrt{3}(600)(0.8)}$$
$$= \frac{1000}{\sqrt{3}}$$
$$= 577.35\ \text{A}.$$

Example



d) $$\text{pf} = \cos[1.57^\circ - (-36.87^\circ)]$$
$$= \cos 38.44^\circ$$
$$= 0.783 \text{ lagging.}$$

c) $$\mathbf{V}_{\text{an}} = \mathbf{V}_{\text{AN}} + Z_\ell \mathbf{I}_{\text{aA}}$$
$$= \frac{600}{\sqrt{3}} + (0.005 + j0.025)(577.35 \angle -36.87^\circ)$$
$$= 357.51 \angle 1.57^\circ \text{ V.}$$

$$V_{\text{L}} = \sqrt{3}|\mathbf{V}_{\text{an}}|$$
$$= 619.23 \text{ V.}$$

END OF THE LECTURE

Any questions ?