EEE270
Introduction to Electrical Energy Systems

Lecture 3 - SINUSOIDAL STEADY-STATE POWER CALCULATIONS

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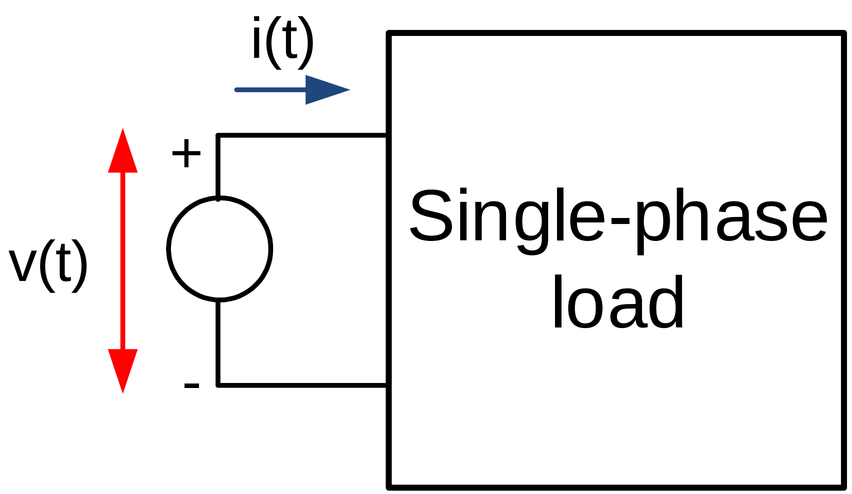
The Sinusoidal Steady-State Power Calculations

- Understanding and the calculation of the following power related definitions are very important:
  - Instantaneous power;
  - Average (real) power;
  - Reactive power;
  - Complex power;
  - Power factor.

Instantaneous power



v(t) = Vmcos(wt+θv) volts
i(t) = Imcos(wt+θi) amps

p(t) = v(t).i(t) ➔ **instantaneous power delivered to the load**

p(t) = Vm.Im.cos(wt+Qv).cos(wt+Qi)

This equation has many terms and not practical to use

Power Calculations

Instantaneous Power

$$p = vi$$

$$v = V_m \cos(\omega t + \theta_v)$$

$$i = I_m \cos(\omega t + \theta_i)$$

$$v = V_m \cos(\omega t + \theta_v - \theta_i)$$

$$i = I_m \cos \omega t$$

$$p = V_m I_m \cos(\omega t + \theta_v - \theta_i) \cos \omega t$$

Power Calculations

Instantaneous Power

$$p = V_m I_m \cos(\omega t + \theta_v - \theta_i) \cos \omega t$$ ⟵ $$\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha - \beta) + \frac{1}{2}\cos(\alpha + \beta)$$

$$\alpha = \omega t + \theta_v - \theta_i$$

$$\beta = \omega t$$

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(2\omega t + \theta_v - \theta_i)$$ ⟵ $$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\alpha = \theta_v - \theta_i$$

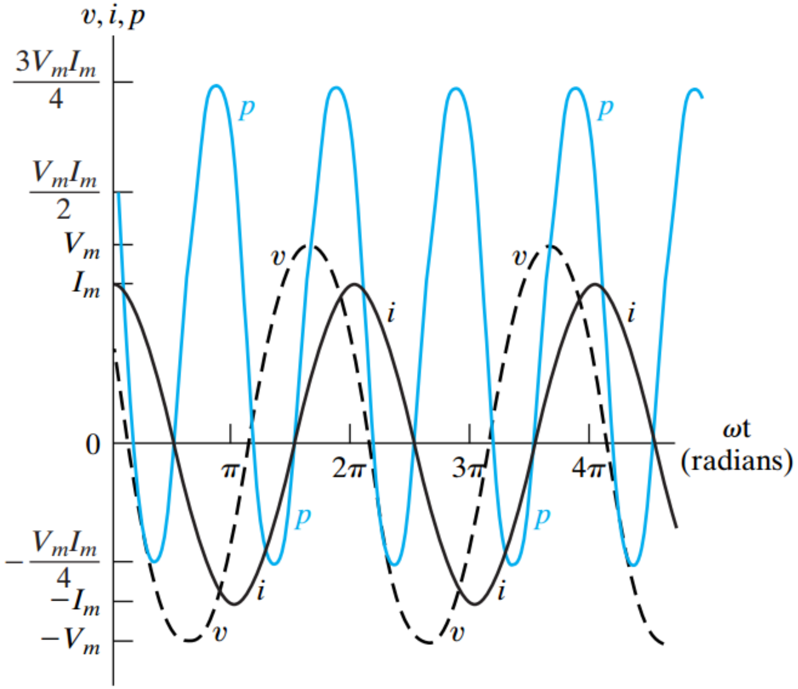
$$\beta = 2\omega t$$

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t$$

Power Calculations

Instantaneous Power

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t$$



Instantaneous power, voltage, and current versus ωt

! The instantaneous power goes through two complete cycles for every cycle of either the voltage or the current.

! The instantaneous power may be negative for a portion of each cycle, even if the network between the terminals is passive.

In a completely passive network, negative power implies that energy stored in the inductors or capacitors is now being extracted

Average and Reactive Power

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t$$

Instantaneous power

Assign $P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$ and $Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$

Average Power
Real Power
Active Power

Reactive Power

$$p = P + P\cos 2\omega t - Q\sin 2\omega t$$

Instantaneous power

The average power associated with sinusoidal signals is the average of the instantaneous power over one period

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p\,dt$$

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$$

Average Power

Power for Purely Resistive Circuits

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t$$

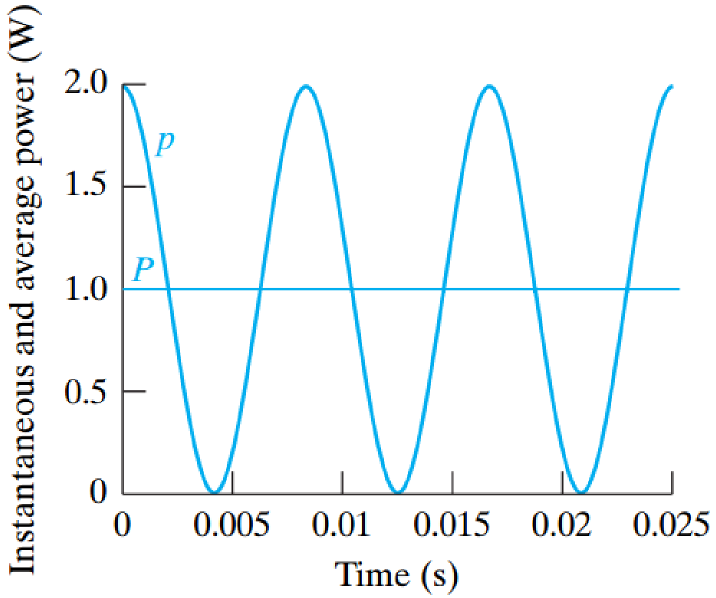
Instantaneous power

$$\theta_v = \theta_i$$

Voltage and current are in phase

$$p = P + P\cos 2\omega t$$

Instantaneous power for purely resistive circuits



The instantaneous real power can never be negative for purely resistive circuits

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) = 0$$

$$\theta_v = \theta_i$$

Reactive power of resistive elements is ZERO

Power for Purely Inductive Circuits

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t$$

← Instantaneous power

If the circuit between the terminals is purely inductive, the voltage and current are out of phase by 90 degrees

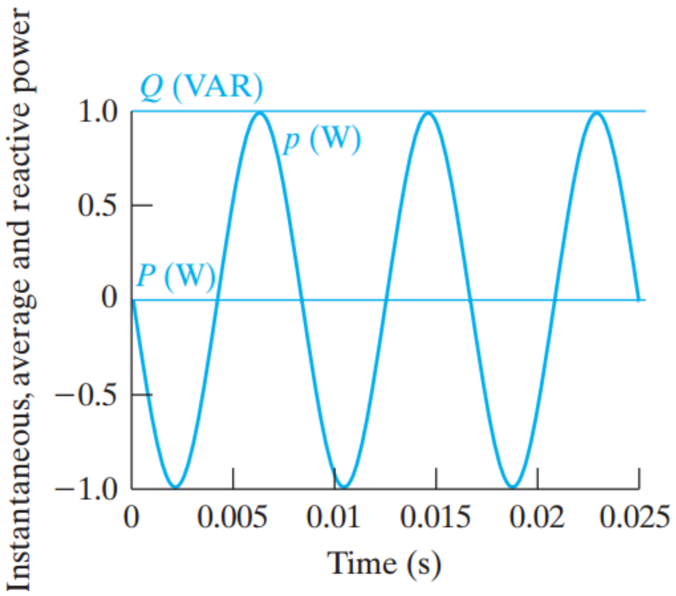
$\theta_i = \theta_v - 90^\circ$ → $\theta_v - \theta_i = +90^\circ$ → **The current lags the voltage by 90 degrees**

$$p = -Q\sin 2\omega t$$

$$p = -\frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t$$

Power for Purely Inductive Circuits

$$p = -Q \sin 2\omega t \qquad p = -\frac{V_m I_m}{2} \sin(\theta_v - \theta_i) \sin 2\omega t$$



In a purely inductive circuit, the average power is zero. Therefore no transformation of energy from electric to nonelectric form takes place.

The instantaneous power at the terminals in a purely inductive circuit is continually exchanged between the circuit and the source driving the circuit, at a frequency of 2ω

In other words, when p is positive, energy is being stored in the magnetic fields associated with the inductive elements, and when p is negative, energy is being extracted from the magnetic fields.

Power for Purely Inductive Circuits

A measure of the power associated with purely inductive circuits is the reactive power *Q*. The name reactive power comes from the characterization of an inductor as a reactive element; its impedance is purely reactive. Note that average power *P* and reactive power *Q* carry the same dimension.

watt (W) for **average power**

var (volt-amp reactive, or VAR) for **reactive power**.

Power for Purely Capacitive Circuits

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t$$

Instantaneous power

If the circuit between the terminals is **purely capacitive**, the voltage and current are out of phase by 90 degrees

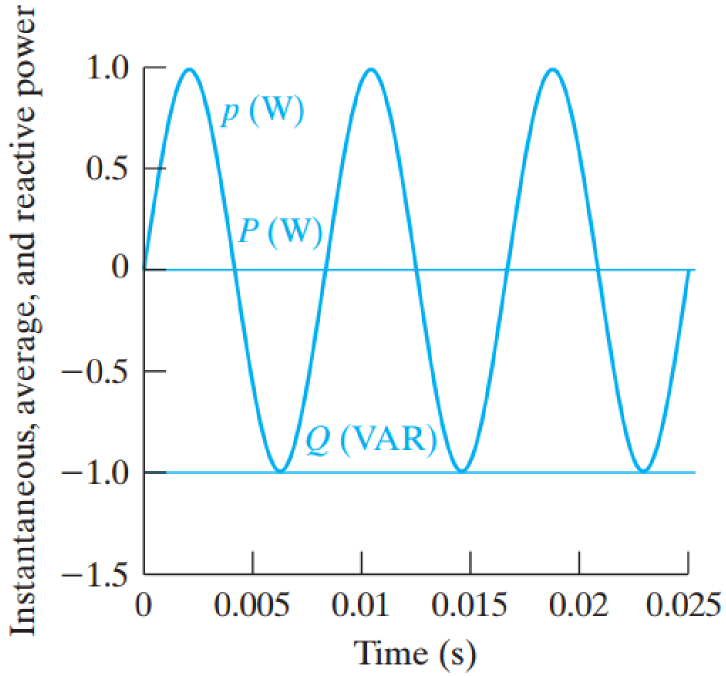
$$\theta_i = \theta_v + 90^\circ \quad \rightarrow \quad \theta_v - \theta_i = -90^\circ$$

The current leads the voltage by 90 degrees

$$p = -Q\sin 2\omega t \qquad Q = -\frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t$$

The average power is zero, there is no transformation of energy from electric to nonelectric form.
In a purely capacitive circuit, the power is continually exchanged between the source driving the circuit and the electric field associated with the capacitive elements.

Power for Purely Capacitive Circuits



The average power is zero, there is no transformation of energy from electric to nonelectric form.
In a purely capacitive circuit, the power is continually exchanged between the source driving the circuit and the electric field associated with the capacitive elements.

Power Factor

$\theta_v - \theta_i$ → **Power factor angle**

$\cos(\theta_v - \theta_i)$ → **Power factor**

$\sin(\theta_v - \theta_i)$ → **Reactive power factor**

The value of the power factor does not tell you the value of the power factor angle because $\cos(\theta_v - \theta_i) = \cos(\theta_i - \theta_v)$

To completely describe this angle, we use the descriptive phrases **lagging power factor** and **leading power factor**.

Lagging power factor implies that current lags voltage, hence an **inductive load**.

Leading power factor implies that current leads voltage, hence a **capacitive load**.

Both the power factor and the reactive factor are convenient quantities to use in describing electrical loads.

Example

Calculating Average and Reactive Power

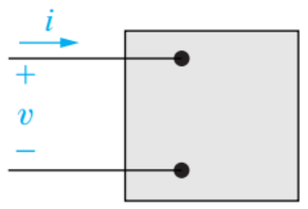
a) Calculate the average power and the reactive power at the terminals of the network shown in Fig. 10.6 if

$$v = 100 \cos(\omega t + 15^\circ) \text{ V},$$

$$i = 4 \sin(\omega t - 15^\circ) \text{ A}.$$

b) State whether the network inside the box is absorbing or delivering average power.

c) State whether the network inside the box is absorbing or supplying magnetizing vars.



a) $$i = 4 \cos(\omega t - 105^\circ) \text{ A}.$$

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i),$$

$$Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i).$$

$$P = \frac{1}{2}(100)(4) \cos[15 - (-105)] = -100 \text{ W},$$

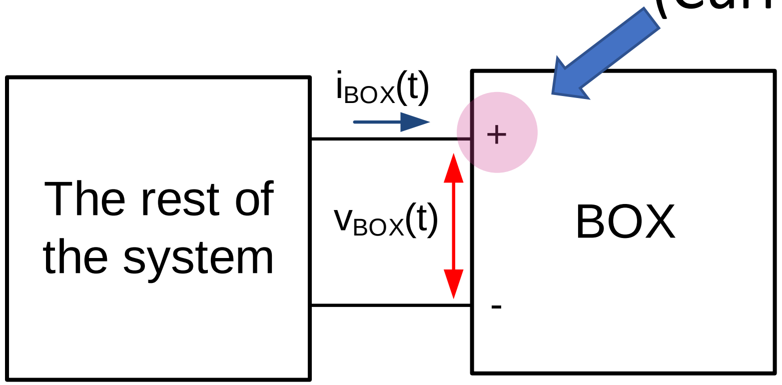
$$Q = \frac{1}{2}100(4) \sin[15 - (-105)] = 173.21 \text{ VAR}.$$

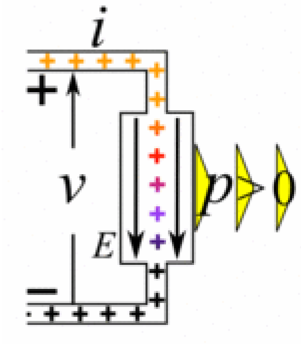
b) The negative value of means that the network inside the
box is delivering average power to the terminals.

c) The passive sign convention means that, because *Q* is positive, the network inside the box is absorbing magnetizing vars at its terminals.

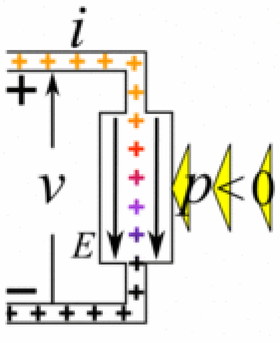
Passive sign convention

(Current is entering into BOX at positive terminal)





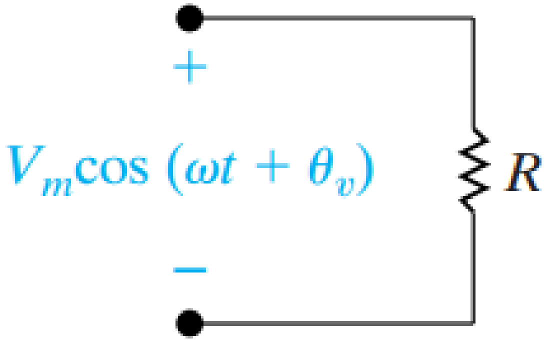
P is consumed



P is generated

P>0	P<0	Q>0	Q<0
P is consumed (absorbed) by the BOX	P is generated (delivered) by the BOX	Q is consumed (absorbed) by the BOX	Q is generated (delivered) by the BOX

The rms Value and Power Calculations



$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p\,dt,$$

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} \frac{V_m^2 \cos^2(\omega t + \phi_v)}{R} dt$$

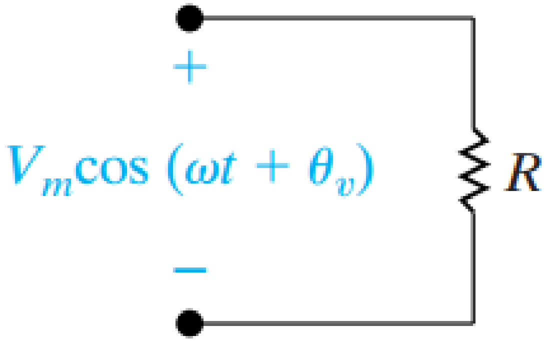
$$= \frac{1}{R}\left[\frac{1}{T}\int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \phi_v) dt\right].$$

$$P = \frac{V_{\text{rms}}^2}{R}.$$

$$P = I_{\text{rms}}^2 R.$$

The rms value is also referred to as the **effective value**

The rms Value and Power Calculations



$$P = \frac{V_{\text{rms}}^2}{R}. \qquad P = I_{\text{rms}}^2 R.$$

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i)$$

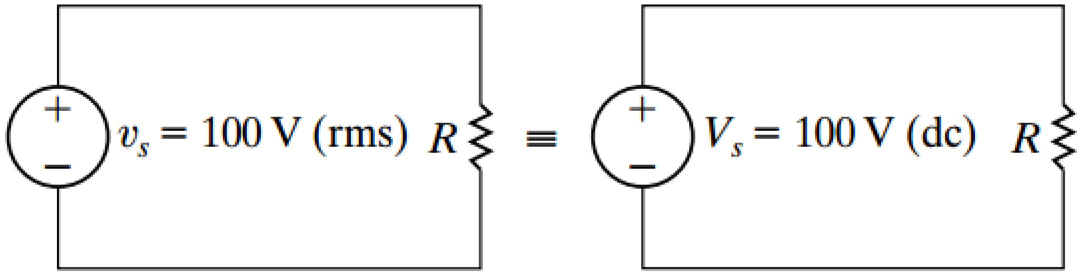
$$= V_{\text{eff}} I_{\text{eff}} \cos(\theta_v - \theta_i);$$

→ **Average Power** in terms of effective values of voltage and current

$$Q = V_{\text{eff}} I_{\text{eff}} \sin(\theta_v - \theta_i).$$

→ **Reactive Power** in terms of effective values of voltage and current

The rms Value and Power Calculations



The rms Value and Power Calculations

The effective value of the sinusoidal signal in power calculations is so widely used that voltage and current ratings of circuits and equipment involved in power utilization are given in terms of rms values.

For example, the voltage rating of residential electric wiring is often **220 V** service.

These voltage levels are the rms values of the sinusoidal voltages supplied by the utility company.

Low-voltage appliances (such as televisions, computers, toasters)

For example: A light bulb, 220 V, 100 W lamp has a resistance of

$$P = \frac{V_{\text{rms}}^2}{R}. \quad \Rightarrow \quad R = V_{rms}^2/P \qquad R = \frac{220^2}{100} = 484\Omega \quad \Rightarrow \quad I_{rms} = \frac{220}{484} = 0.4545A$$

$$I_m = \sqrt{2}.\,4545 = 0.6428$$

Example

a) A sinusoidal voltage having a maximum amplitude of 625 V is applied to the terminals of a 50 Ω resistor. Find the average power delivered to the resistor.

b) Repeat (a) by first finding the current in the resistor.

Example

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b) Repeat (a) by first finding the current in the resistor.

a) The rms value of the sinusoidal voltage is $625/\sqrt{2}$, → 441.94 V.

$$P = \frac{(441.94)^2}{50} = 3906.25 \text{ W}.$$

b) The maximum amplitude of the current in the resistor is 625/50, or 12.5 A. The rms value of the current is $12.5/\sqrt{2}$, → 8.84 A.

$$P = (8.84)^2 50 = 3906.25 \text{ W}.$$

Complex Power

Complex power is the complex **sum** of **real power** and **reactive power**

$$S = P + jQ.$$

Complex power is the same as average or reactive power.

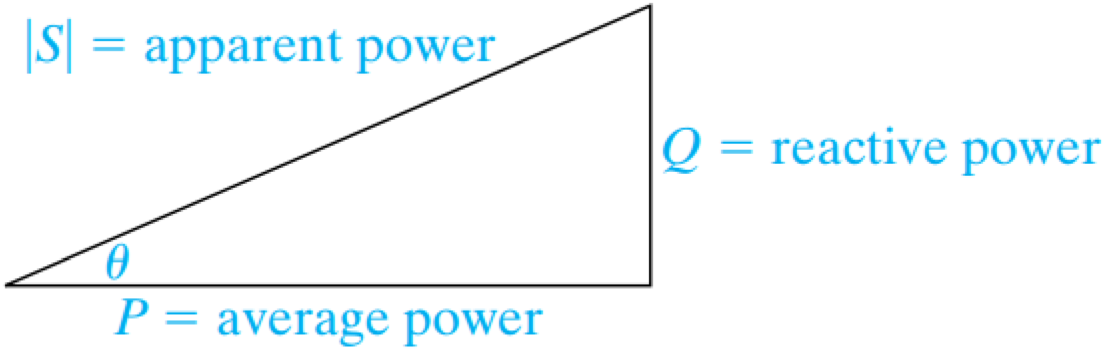
However, to distinguish complex power from either average or reactive power, we use the units **volt-amps (VA).**

We use volt-amps for complex power, watts for average power, and vars for reactive power,

Quantity	Units
Complex power	volt-amps
Average power	watts
Reactive power	var

Complex Power

$$S = P + jQ.$$



Power triangle

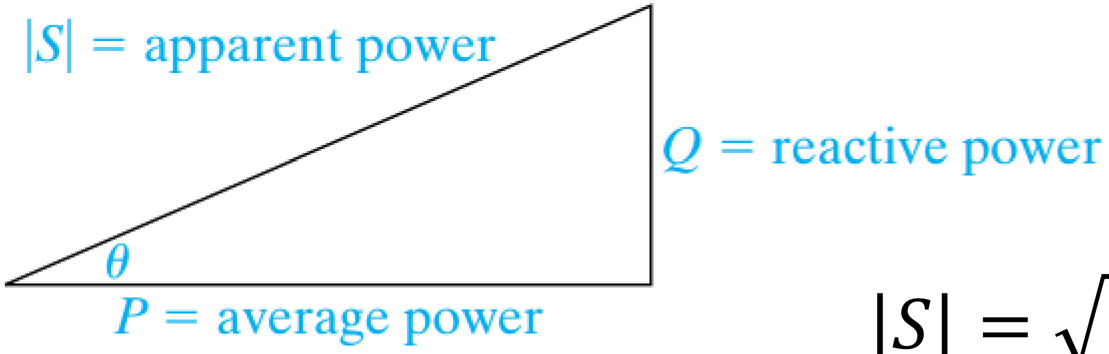
$$\tan\theta = \frac{Q}{P}.$$

$$\frac{Q}{P} = \frac{(V_m I_m/2)\sin(\theta_v - \theta_i)}{(V_m I_m/2)\cos(\theta_v - \theta_i)}$$

$$= \tan(\theta_v - \theta_i).$$

Complex Power

- The magnitude of complex power is referred to as **apparent power**



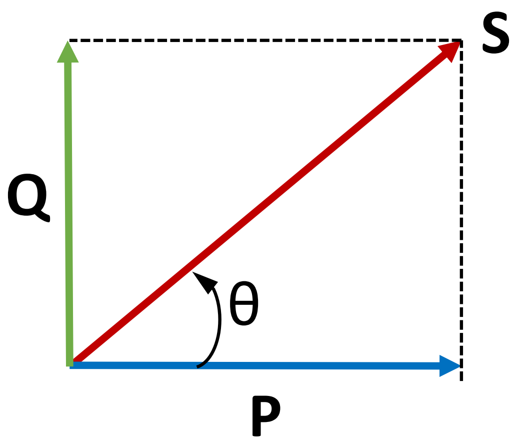
→ Power triangle

$$|S| = \sqrt{P^2 + Q^2}$$ → Apparent Power

$$P = Scos(\theta)$$ → Average Power

$$Q = Ssin(\theta)$$ → Reactive Power

Complex power and power triangle

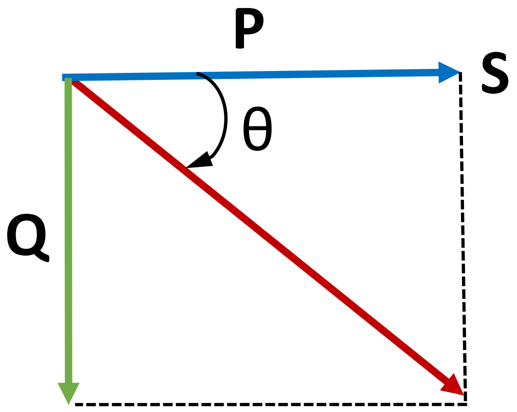


Power triangle of an inductive load

Q>0 and **θ>0**

PF = cosθ = P/|S|

PF = real power/apparent power



Power triangle of a capacitive load

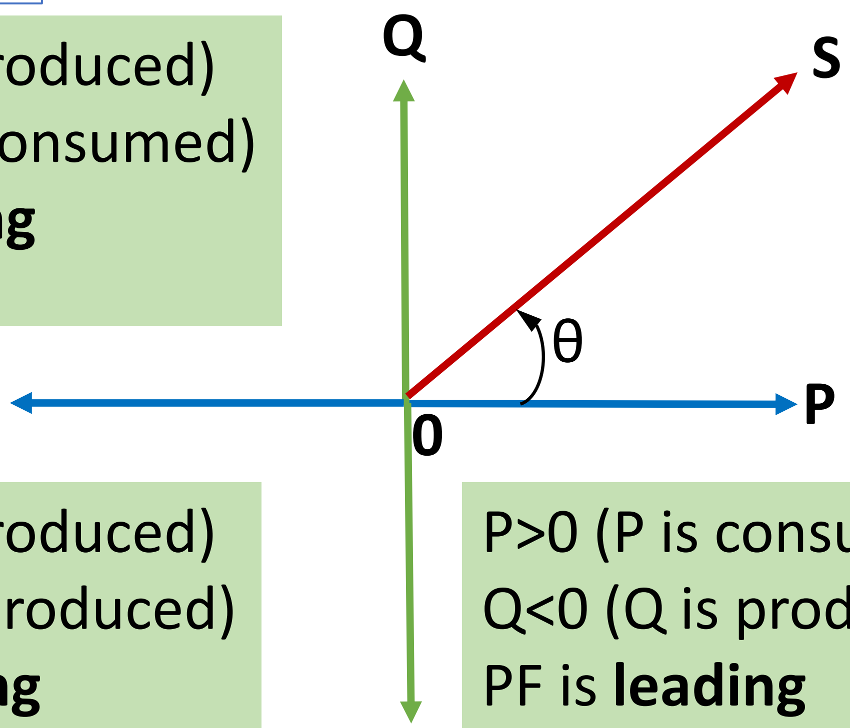
Q<0 and **θ<0**

90°<θ≤180°

P<0 (P is produced)
Q>0 (Q is consumed)
PF is **lagging**

0°≤θ≤90°

P>0 (P is consumed)
Q>0 (Q is consumed)
PF is **lagging**



P<0 (P is produced)
Q<0 (Q is produced)
PF is **leading**

180°<θ≤270°

P>0 (P is consumed)
Q<0 (Q is produced)
PF is **leading**

270°≤θ<360°

Complex Power

- Example

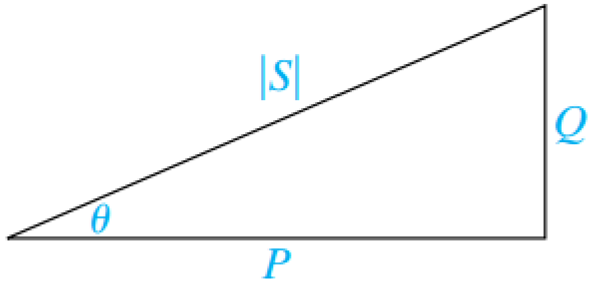
An electrical load operates at 240 V rms. The load absorbs an average power of 8 kW at a lagging power factor of 0.8.

a) Calculate the complex power of the load.

b) Calculate the impedance of the load.

$$P = |S| \cos \theta,$$

$$Q = |S| \sin \theta.$$



$$|S| = \frac{P}{\cos \theta} = \frac{8\,\text{kW}}{0.8} = 10\,\text{kVA},$$

$$Q = 10 \sin \theta = 6\,\text{kVAR},$$

$$S = 8 + j6\,\text{kVA}.$$

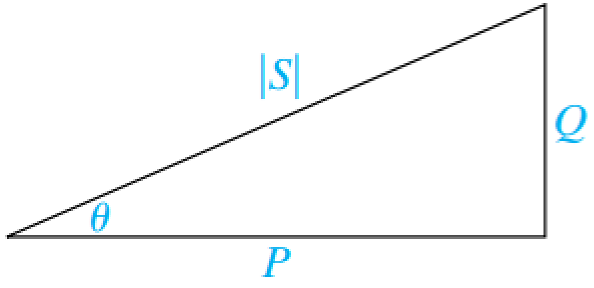
Complex Power

- Example

An electrical load operates at 240 V rms. The load absorbs an average power of 8 kW at a lagging power factor of 0.8.

a) Calculate the complex power of the load.

b) Calculate the impedance of the load.



$$\theta = \cos^{-1}(0.8) = 36.87^\circ.$$

$$\begin{aligned} P &= V_{\text{eff}} I_{\text{eff}} \cos(\theta_v - \theta_i) \\ &= (240) I_{\text{eff}} (0.8) \\ &= 8000 \text{ W}. \end{aligned}$$

$$I_{\text{eff}} = 41.67 \text{ A}.$$

$$|Z| = \frac{|V_{\text{eff}}|}{|I_{\text{eff}}|} = \frac{240}{41.67} = 5.76.$$

$$Z = 5.76 \angle 36.87^\circ\ \Omega = 4.608 + j3.456\ \Omega.$$

Power Calculations

$$S = P + jQ.$$

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i),$$

$$Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i).$$

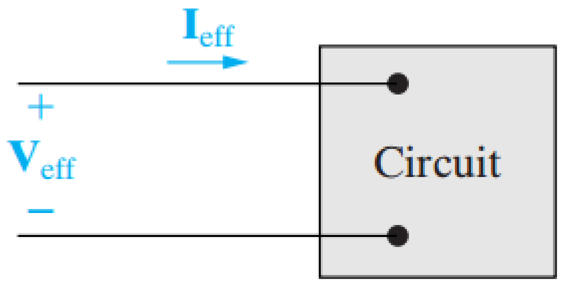
$$S = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + j\frac{V_m I_m}{2} \sin(\theta_v - \theta_i)$$

$$= \frac{V_m I_m}{2} [\cos(\theta_v - \theta_i) + j \sin(\theta_v - \theta_i)]$$

$$= \frac{V_m I_m}{2} e^{j(\theta_v - \theta_i)} = \frac{1}{2} V_m I_m \underline{/(\theta_v - \theta_i)}.$$

$$S = V_{\text{eff}} I_{\text{eff}} \underline{/(\theta_v - \theta_i)}.$$

Complex Power



$$S = V_{\text{eff}} I_{\text{eff}} \angle (\theta_v - \theta_i)$$

$$= V_{\text{eff}} I_{\text{eff}} e^{j(\theta_v - \theta_i)}$$

$$= V_{\text{eff}} e^{j\theta_v} I_{\text{eff}} e^{-j\theta_i}$$

$$= \mathbf{V}_{\text{eff}} \mathbf{I}^*_{\text{eff}}.$$

→ Complex Power

$$I_{\text{eff}} e^{-j\theta_i} = I_{\text{eff}} \cos(-\theta_i) + j I_{\text{eff}} \sin(-\theta_i)$$

$$= I_{\text{eff}} \cos(\theta_i) - j I_{\text{eff}} \sin(\theta_i)$$

$$= \mathbf{I}^*_{\text{eff}}.$$

Complex Power

$$S = V_{\text{eff}} I_{\text{eff}} \angle(\theta_v - \theta_i)$$

$$= V_{\text{eff}} I_{\text{eff}} e^{j(\theta_v - \theta_i)}$$

$$= V_{\text{eff}} e^{j\theta_v} I_{\text{eff}} e^{-j\theta_i}$$

$$= \mathbf{V}_{\text{eff}} \mathbf{I}^*_{\text{eff}}.$$

$$S = \frac{1}{2} \mathbf{V} \mathbf{I}^*.$$

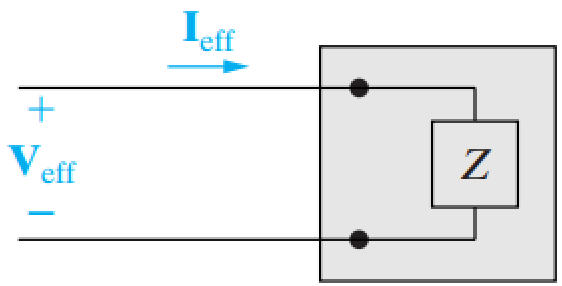
$$\mathbf{V} = 100 \angle 15^\circ \text{ V},$$

$$\mathbf{I} = 4 \angle -105^\circ \text{ A}.$$

$$S = \frac{1}{2}(100 \angle 15^\circ)(4 \angle +105^\circ) = 200 \angle 120^\circ$$

$$= -100 + j173.21 \text{ VA}.$$

Alternate Forms for Complex Power



$$\mathbf{V}_{\text{eff}} = Z\mathbf{I}_{\text{eff}}.$$

$$S = Z\mathbf{I}_{\text{eff}}\mathbf{I}^*_{\text{eff}}$$

$$= |\mathbf{I}_{\text{eff}}|^2 Z$$

$$= |\mathbf{I}_{\text{eff}}|^2 (R + jX)$$

$$= |\mathbf{I}_{\text{eff}}|^2 R + j|\mathbf{I}_{\text{eff}}|^2 X = P + jQ,$$

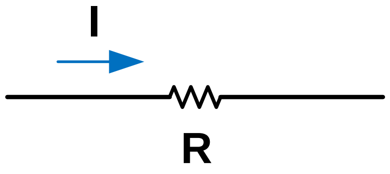
$$P = |\mathbf{I}_{\text{eff}}|^2 R = \frac{1}{2} I_m^2 R,$$

$$Q = |\mathbf{I}_{\text{eff}}|^2 X = \frac{1}{2} I_m^2 X.$$

Loss and Efficiency

Electrical Losses in single-phase power systems can be of two types:

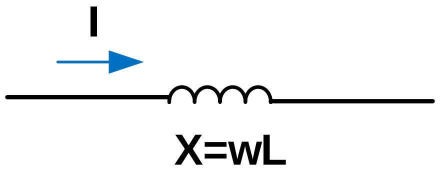
a) **Real power loss** due to heating which results from current flow



$R.I^2$

I is either
- ✓ RMS AC current
- ✓ DC current

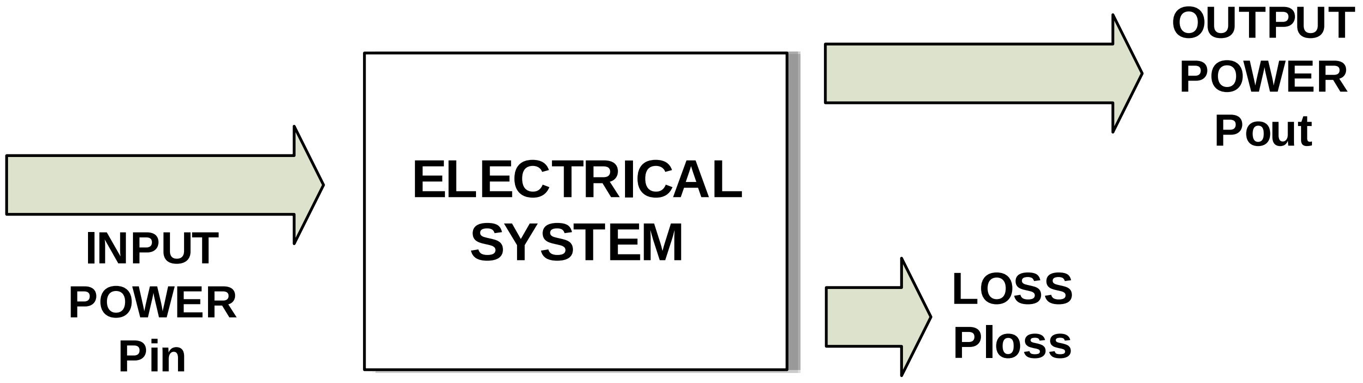
b) **Reactive power loss** due to
- ✓ Reactance of cables and lines
- ✓ Magnetizing of transformers, electrical motors, and generators



$X.I^2$

I is either
- ✓ RMS AC current
- ✓ DC current

Loss and Efficiency



Pin = Pout + Ploss

Efficiency = η = Pout/Pin x 100%

- ✓ Efficiency is related with the active (real) power
- ✓ Efficiency is generally represented by letter "**η**" (eta)
- ✓ 0≤**η** ≤1
- ✓ 0 % ≤**η** ≤100 %
- ✓ If Ploss is ignored (Ploss=0) ➔ **η = 100 % (ideal case)**

Calculating Average and Reactive Power

In the circuit shown in Fig. 10.13, a load having an impedance of $39 + j26\ \Omega$ is fed from a voltage source through a line having an impedance of $1 + j4\ \Omega$. The effective, or rms, value of the source voltage is 250 V.

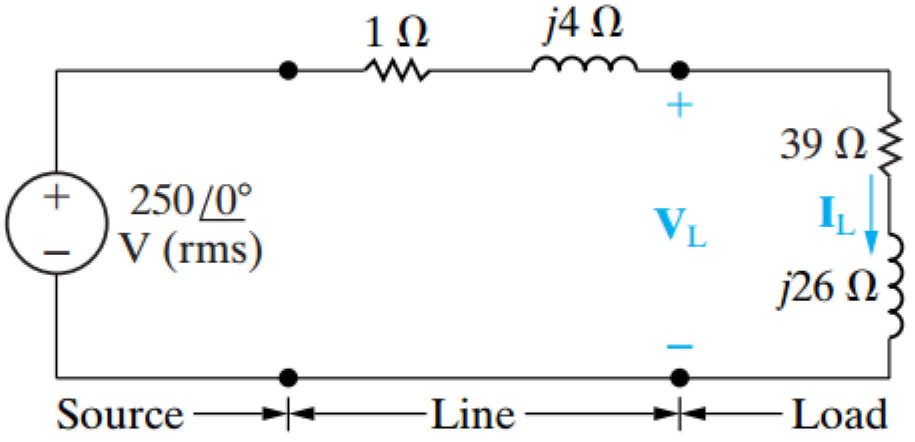
a) Calculate the load current $\mathbf{I}_L$ and voltage $\mathbf{V}_L$.

b) Calculate the average and reactive power delivered to the load.

c) Calculate the average and reactive power delivered to the line.

d) Calculate the average and reactive power supplied by the source.

e) Calculate the losses and the efficiency of this system.



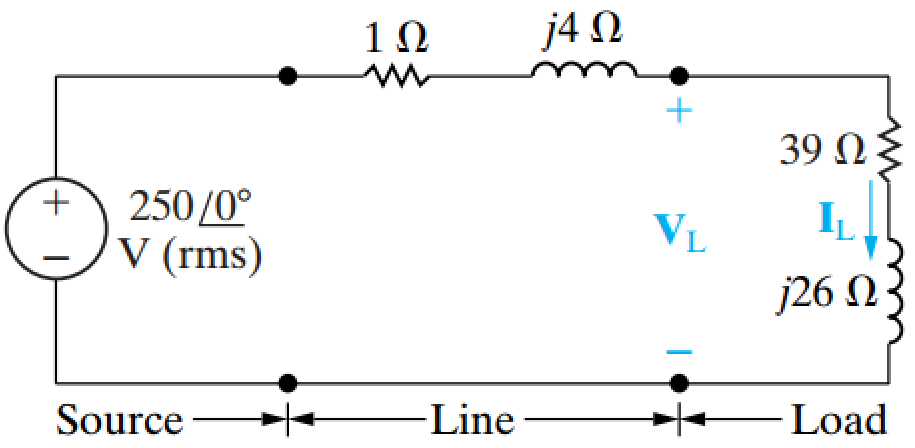
Calculating Average and Reactive Power

a) Calculate the load current $\mathbf{I}_L$ and voltage $\mathbf{V}_L$.

$$\mathbf{I}_L = \frac{250\,\angle 0^\circ}{40 + j30} = 4 - j3 = 5\,\angle -36.87^\circ \text{ A (rms)}.$$

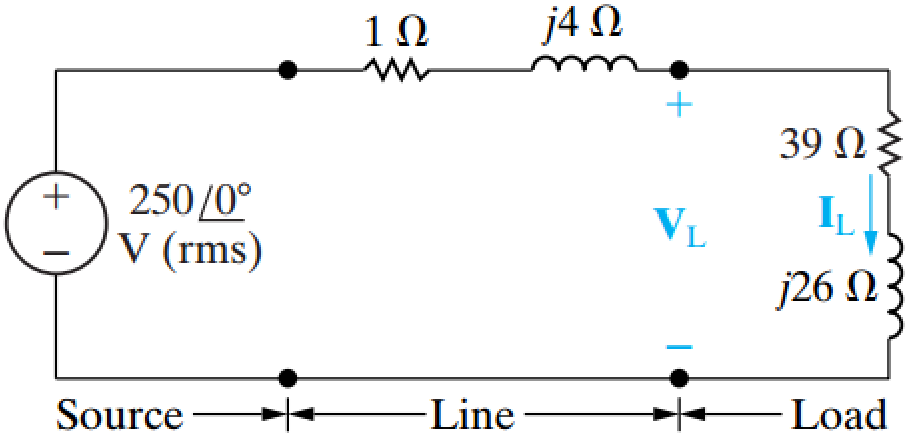
$$\mathbf{V}_L = (39 + j26)\mathbf{I}_L = 234 - j13$$

$$= 234.36\,\angle -3.18^\circ \text{ V (rms)}.$$



Calculating Average and Reactive Power

b) Calculate the average and reactive power delivered to the load.



$$\mathbf{I}_L = \frac{250\angle 0^\circ}{40 + j30} = 4 - j3 = 5\angle -36.87^\circ \text{ A (rms)}.$$

$$\mathbf{V}_L = (39 + j26)\mathbf{I}_L = 234 - j13$$

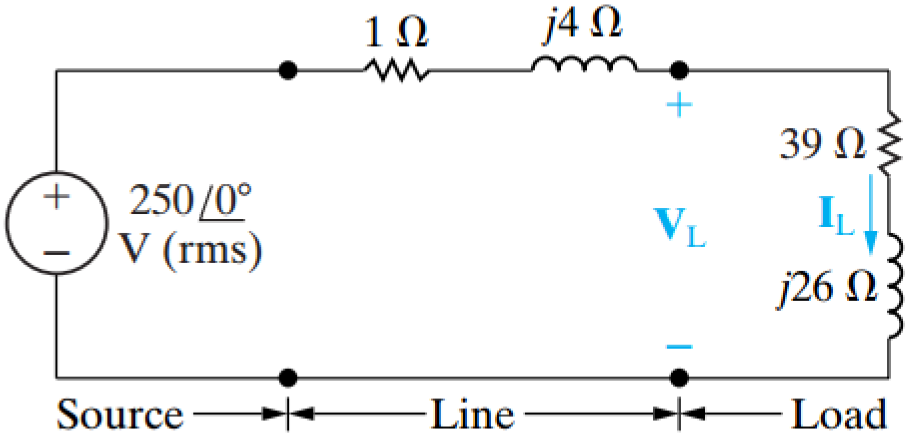
$$= 234.36\angle -3.18^\circ \text{ V (rms)}.$$

$$S = \mathbf{V}_L\mathbf{I}_L^* = (234 - j13)(4 + j3)$$

$$= 975 + j650 \text{ VA}.$$

Calculating Average and Reactive Power

c) Calculate the average and reactive power delivered to the line.



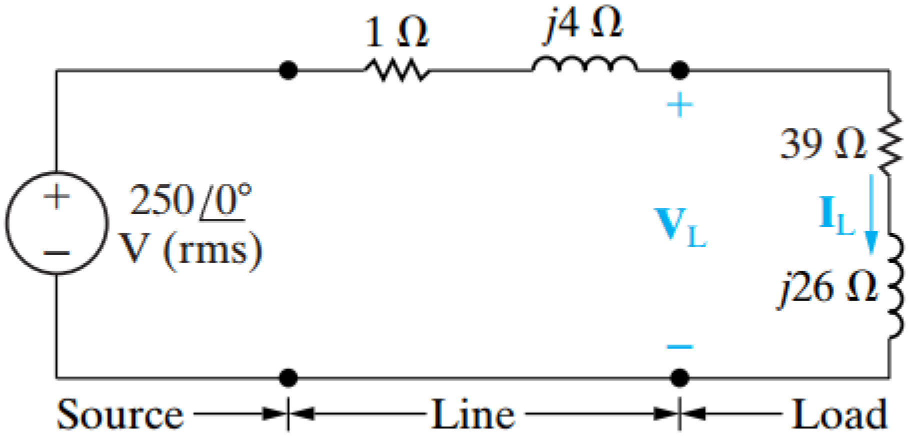
$$\mathbf{I}_L = \frac{250\angle 0^\circ}{40 + j30} = 4 - j3 = 5\angle -36.87^\circ \text{ A (rms)}.$$

$$P = (5)^2(1) = 25 \text{ W},$$

$$Q = (5)^2(4) = 100 \text{ VAR}.$$

Calculating Average and Reactive Power

d) Calculate the average and reactive power supplied by the source.



$$S = 25 + j100 + 975 + j650$$

$$= 1000 + j750 \text{ VA}.$$

$$S_s = -250\mathbf{I}_L^*.$$

$$S_s = -250(4 + j3) = -(1000 + j750) \text{ VA}.$$

END OF THE LECTURE

Any questions ?