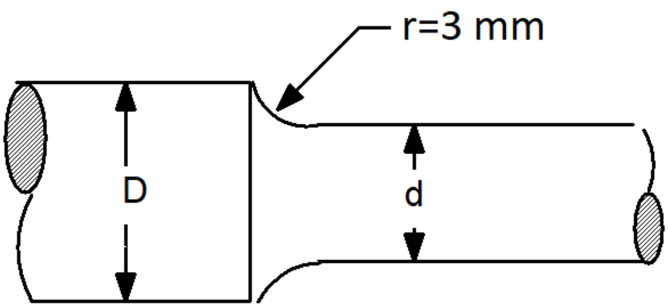
TUTORIAL – FATIGUE DESIGN -1

Problem 1: The figure shows a section of an AISI 1050 CD steel shaft with dimensions D=30mm and d=25mm. The shaft section at the shoulder is subjected to a completely reversed bending moment of 60 Nm and a torsion fluctuating between -30 and 50 Nm. Determine the factor of safety for infinite life based on the modified Goodman diagram. (Operating temperature is 100 °C and reliability is %90).



Answer 1:

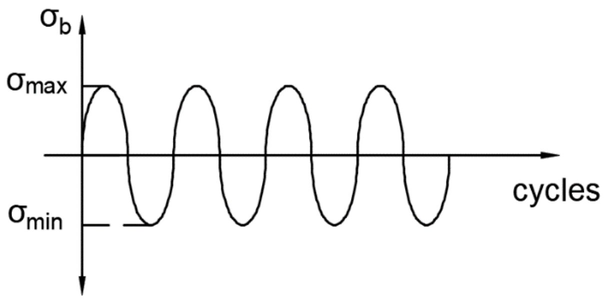
AISI 1050 CD → Sut=690 MPa , Sy=580 MPa (Shigley A-20)

Table A–20

Deterministic ASTM Minimum Tensile and Yield Strengths for Some Hot-Rolled (HR) and Cold-Drawn (CD) Steels [The strengths listed are estimated ASTM minimum values in the size range 18 to 32 mm ($\frac{3}{4}$ to $1\frac{1}{4}$ in). These strengths are suitable for use with the design factor defined in Sec. 1–10, provided the materials conform to ASTM A6 or A568 requirements or are required in the purchase specifications. Remember that a numbering system is not a specification.] *Source:* 1986 SAE Handbook, p. 2.15.

1 UNS No.	2 SAE and/or AISI No.	3 Processing	4 Tensile Strength, MPa (kpsi)	5 Yield Strength, MPa (kpsi)	6 Elongation in 2 in, %	7 Reduction in Area, %	8 Brinell Hardness
G10060	1006	HR	300 (43)	170 (24)	30	55	86
		CD	330 (48)	280 (41)	20	45	95
G10100	1010	HR	320 (47)	180 (26)	28	50	95
		CD	370 (53)	300 (44)	20	40	105
G10150	1015	HR	340 (50)	190 (27.5)	28	50	101
		CD	390 (56)	320 (47)	18	40	111
G10180	1018	HR	400 (58)	220 (32)	25	50	116
		CD	440 (64)	370 (54)	15	40	126
G10200	1020	HR	380 (55)	210 (30)	25	50	111
		CD	470 (68)	390 (57)	15	40	131
G10300	1030	HR	470 (68)	260 (37.5)	20	42	137
		CD	520 (76)	440 (64)	12	35	149
G10350	1035	HR	500 (72)	270 (39.5)	18	40	143
		CD	550 (80)	460 (67)	12	35	163
G10400	1040	HR	520 (76)	290 (42)	18	40	149
		CD	590 (85)	490 (71)	12	35	170
G10450	1045	HR	570 (82)	310 (45)	16	40	163
		CD	630 (91)	530 (77)	12	35	179
G10500	1050	HR	620 (90)	340 (49.5)	15	35	179
		CD	690 (100)	580 (84)	10	30	197
G10600	1060	HR	680 (98)	370 (54)	12	30	201
G10800	1080	HR	770 (112)	420 (61.5)	10	25	229
G10950	1095	HR	830 (120)	460 (66)	10	25	248

Bending Stress (Reversed)



Maximum and minimum stresses;

$$\sigma_{max} = \frac{Mc}{I} = \frac{32M_{max}}{\pi d^3} = \frac{32.60000}{\pi.25^3} = 39.1\ MPa$$

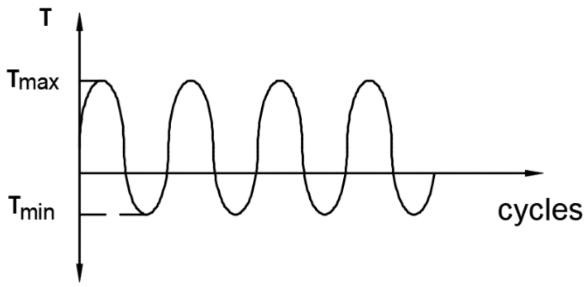
$$\sigma_{min} = -39.1\ MPa$$

Alternating and mean stress components;

$$\sigma_{b_a} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{39.1 - (-39.1)}{2} = 39.1\ MPa$$

$$\sigma_{b_m} = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{39.1 + (-39.1)}{2} = 0\ MPa$$

Torsional Stress (Fluctuating)



Maximum and minimum stresses;

$$\tau_{max} = \frac{Tr}{J} = \frac{16T_{max}}{\pi d^3} = \frac{16.50000}{\pi.25^3} = 16.3\ MPa$$

$$\tau_{min} = \frac{Tr}{J} = \frac{16T_{min}}{\pi d^3} = \frac{16.-30000}{\pi.25^3} = -9.7\ MPa$$

Alternating and mean stress components;

$$\tau_a = \frac{\tau_{max} - \tau_{min}}{2} = \frac{16.3 - (-9.7)}{2} = 13\ MPa$$

$$\tau_m = \frac{\tau_{max} + \tau_{min}}{2} = \frac{16.3 + (-9.7)}{2} = 3.3\ MPa$$

Endurance Limit (S_e);

$$S_e = k_a k_b k_c k_d k_e S_e'$$

$$S_e' = 0.5 S_{ut} = 0.5.690 = 345\ MPa \quad (S_{ut} < 1400\ MPa)$$

- Surface factor (k_a)

$k_a = aS_{ut}{}^b$ a=4.51, b=-0.265 for machined surface

$$k_a = 4.51(690)^{-0.265} = 0.79$$

Surface Finish	a	b
Ground	1.58	-0.065
Machined or Cold Drawn	4.51	-0.265
Hot Rolled	57.7	-0.718
As Forged	272	-0.995

- Size factor (k_b)

$$k_b = 1.189 d^{-0.097} \quad (8 < d < 250\ mm, beam\ is\ rotating)$$

$$k_b = 1.189(25)^{-0.097} = 0.87$$

- Reliability factor (k_c)

$$k_c = 0.897 \quad (for\ 90\%\ reliability, from\ table\ A3-19)$$

Table A3-19 Reliability factors kc corresponding to an 8 percent standard deviation of the endurance limit

Reliability R	Standardized variable zr	Reliability factor kc
0,50	0,000	1,000
0,90	1,288	0,897
0,95	1,645	0,868
0,99	2,326	0,814
0,999	3,090	0,753
0,999 9	3,719	0,702
0,999 99	4,265	0,659
0,999 999	4,753	0,620
0,999 999 9	5,199	0,584

- Temperature factor (k_d)

$$k_d = 1.02 \;\; (T = 100\,°C < 350\,°C)$$

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Table 6–4

Effect of Operating Temperature on the Tensile Strength of Steel.* (S_T = tensile strength at operating temperature; S_{RT} = tensile strength at room temperature; $0.099 \le \hat{\sigma} \le 0.110$)

Temperature, °C	S_T/S_{RT}	Temperature, °F	S_T/S_{RT}
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

- Stress concentration factor (k_e)

$$k_e = 1$$

(Stress concentration factor must be taken as 1, because there are two alternating stress components. Therefore, fatigue strength reduction factor values (K_f) for each types of stress will be calculated and multiplied with the corresponding alternating stress components).

- Fatigue strength reduction factors (K_f)

<u>For bending</u>

$q = 0.82$

From table A3-17 based on r&S_{ut}

Table A3-17 Notch-sensitivities for steels and 2024 Wrought Aluminum alloys subjected to reversed bending and reversed axial loads *

		Steels			
Notch rad,(mm)	Aluminum alloys	Sut 0,4GPa	Sut 0,7GPa	Sut 1,0GPa	Sut 1,4GPa
0,000	-	-	-	-	-
0,100	0,200	0,360	0,540	0,670	0,810
0,150	0,250	0,440	0,590	0,710	0,840
0,250	0,300	0,480	0,620	0,740	0,850
0,350	0,380	0,530	0,640	0,760	0,860
0,500	0,410	0,550	0,670	0,790	0,870
0,625	0,450	0,600	0,700	0,810	0,900
0,750	0,490	0,620	0,730	0,830	0,910
0,875	0,520	0,640	0,740	0,840	0,920
1,000	0,540	0,650	0,750	0,850	0,930
1,250	0,590	0,660	0,760	0,860	0,930
1,500	0,630	0,670	0,780	0,870	0,940
2,000	0,680	0,710	0,810	0,890	0,950
2,500	0,730	0,730	0,830	0,900	0,960
4,000	0,830	0,780	0,860	0,930	0,970

<u>For torsion</u>

$q = 0.995$

From table A3-18 for Quenched and drawn steel, r=3mm

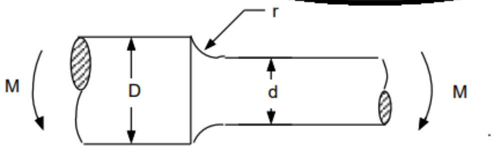
Table A3-18 Notch-sensitivities for materials in reversed torsion.

For larger notch radii use the values of q corresponding to r= 4mm.*

Notch radius mm	Quenched and drawn steel q	Annealed steel q	Aluminum alloys q
0,050	0,600	0,400	0,100
0,100	0,800	0,480	0,220
0,250	0,860	0,600	0,370
0,300	0,890	0,670	0,460
0,500	0,915	0,760	0,570
0,750	0,950	0,820	0,670
1,000	0,960	0,860	0,715
1,250	0,970	0,880	0,760
1,500	0,980	0,900	0,790
2,000	0,985	0,930	0,840
2,500	0,990	0,950	0,860
3,000	0,995	0,960	0,890
4,000	0,995	0,990	0,910

K_t=1.5	K_t=1.5
From table A3-10 based on r/d & D/d	(from table A3-9 based on r/d & D/d)

Table A3-10 Stress Concentration factors for round shaft with shoulder fillet in bending



σ_o= Mc/I, where c=d/2 and I= $\pi d^4/64$

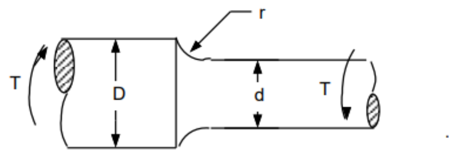
r/d=0.12 D/d=1.2

	D/d =1,02	D/d =1,05	D/d =1,1	D/d =1,5	D/d =3
r/d	Kt	Kt	Kt	Kt	Kt
0,012	2,290	2,553	2,700	-	-
0,017	2,120	2,378	2,500	3,000	-
0,021	2,000	2,240	2,366	2,774	3,000
0,025	1,926	2,134	2,260	2,600	2,862
0,036	1,760	1,936	2,046	2,310	2,600
0,050	1,644	1,782	1,865	2,060	2,310
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2,140
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1,986
0,087	1,472	1,563	1,630	1,728	1,880
0,100	1,440	1,534	1,580	1,660	1,804
0,125	1,380	1,468	1,500	1,584	1,684
0,150	1,330	1,412	1,450	1,510	1,584

$K_{f_b} = 1 + q(K_t - 1)$

$K_{f_b} = 1 + 0.82(1.5 - 1) = 1.41$

Table A3-9 Stress concentration factors for round shaft with shoulder fillet in torsion



τ_o= Tc/J, where c=d/2 and J=$\pi d^4/32$

r/d=0.12 D/d=1.2

	D/d =1,09	D/d =1,20	D/d =1,33	D/d =2,0
r/d	Kt	Kt	Kt	Kt
0,009	-	-	-	-
0,012	1,800	2,300	-	2,600
0,030	1,566	2,040	2,144	2,288
0,025	1,472	1,894	2,020	2,122
0,033	1,384	1,761	1,878	1,966
0,042	1,322	1,644	1,755	1,828
0,050	1,283	1,576	1,677	1,750
0,062	1,244	1,500	1,600	1,644
0,075	1,206	1,434	1,516	1,572
0,087	1,184	1,378	1,458	1,510
0,100	1,166	1,342	1,412	1,466
0,125	1,144	1,275	1,344	1,400

$K_{f_b} = 1 + q(K_t - 1)$

$K_{f_b} = 1 + 0.995(1.3 - 1) = 1.3$

According to Modified Goodman Theory; (for infinite life, N>10^6 cycles)

$$n = \frac{1}{\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}} \qquad \sigma'_a = \sqrt{{\sigma_a}^2 + 3{\tau_a}^2} \qquad \sigma'_m = \sqrt{{\sigma_m}^2 + 3{\tau_m}^2}$$

Now we need to calculate equivalent stresses (Von Misses Stresses) for both alternating and mean components;

Alternating	Mean
$\sigma_a = \sigma_{b_a}.K_{f_b} = 39.1(1.41) = 55.1\ MPa$	$\sigma_m = \sigma_{b_m} = 0$
$\tau_a = \tau_a.K_{f_t} = 13(1.3) = 16.9\ MPa$	$\tau_m = \tau_m.K_{f_t} = 3.3(1.3) = 4.29\ MPa$
$\sigma'_a = \sqrt{{\sigma_a}^2 + 3{\tau_a}^2} = \sqrt{55.1^2 + 3(16.9)^2}$	$\sigma'_m = \sqrt{{\sigma_m}^2 + 3{\tau_m}^2} = \sqrt{0^2 + 3(4.29)^2}$
$\sigma'_a = 62.4\ MPa$	$\sigma'_a = 7.43\ MPa$

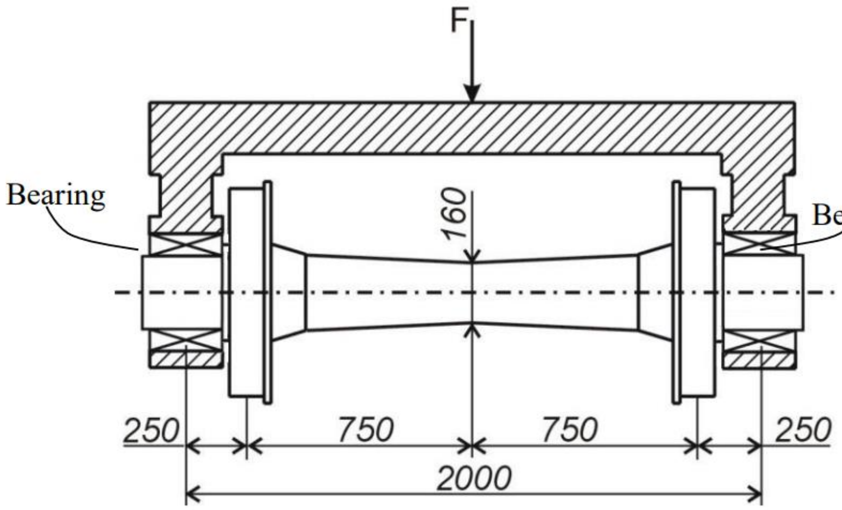
Note: Stress concentration effects both alternating and mean components, because the material is not annealed material, it is quenched material!!!

$S_e = k_a k_b k_c k_d k_e S_e{}'$ =(0.79) (0.87) (0.897) (1.02) (1) (345) =212.7 MPa

$$n = \frac{1}{\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}} = \frac{1}{\frac{62.4}{212.7} + \frac{7.43}{690}} = 3.28 > 1 \qquad \textit{safe for fatigue design !!!}$$

Problem 2: A car axle carries a load of 235 kN. The material is medium carbon forged steel. Its Brinell Hardness number is 179, the ultimate strength is 620 MPa and the yield strength is 500MPa. All dimensions are in mm.

a) Find the value of factor of safety for infinite life at the center of the axle as the car is proceeding along a smooth straight-level track.
b) Estimate the expected life under a completely reversed stress of 200MPa and 400MPa separately.

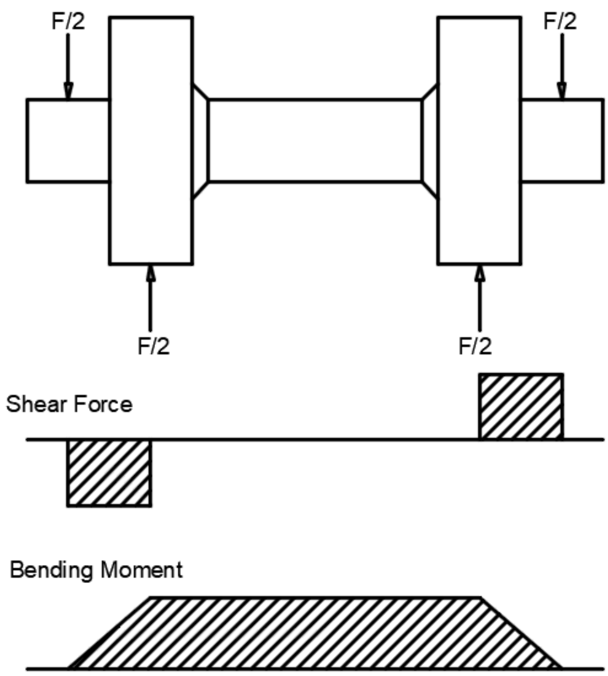


Answer 2:

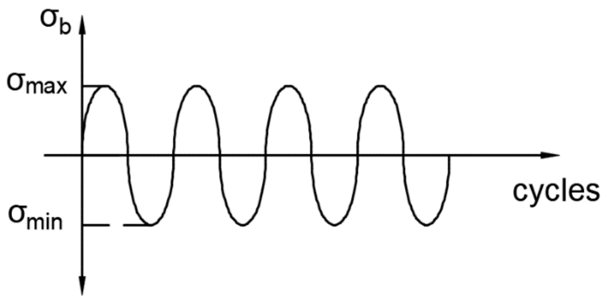
Material= Medium carbon forged steel

HB=179

Sut=620 MPa, Sy=500 MPa



Reversed type of bending stress occurs;



Alternating and mean stresses;

$$\sigma_a = \sigma_{max} = \frac{32M}{\pi d^3} = \frac{32\left(\frac{235}{2}250\right)1000}{\pi 160^3} = 73.1\ MPa$$

$$\sigma_m = 0$$

Von Misses stresses;

$$\sigma_a{}' = \sigma_a = 73.1\ MPa$$

$$\sigma_m{}' = 0$$

Endurance Limit (S_e);

$$S_e = k_a k_b k_c k_d k_e S_e{}'$$

$$S_e{}' = 0.5S_{ut} = 0.5.620 = 310\ MPa \quad (S_{ut} < 1400\ MPa)$$

- Surface factor (k_a)

  $k_a = aS_{ut}{}^b$ a=272, b=-0.995 for forged steel

  $k_a = 272(620)^{-0.995} = 0.453$

Surface Finish	a	b
Ground	1.58	-0.065
Machined or Cold Drawn	4.51	-0.265
Hot Rolled	57.7	-0.718
As Forged	272	-0.995

- Size factor (k_b)
  $k_b = 1.189d^{-0.097} \quad (8 < d < 250mm, beam\ is\ rotating)$
  $k_b = 1.189(160)^{-0.097} = 0.72$
- Reliability factor (k_c)
  $k_c = 1 \quad (nothing\ is\ mentioned)$

- Temperature factor (k_d)
  $k_d = 1 \quad (nothing\ is\ mentioned)$

- Stress concentration factor (k_e)
  $k_e = 1$
  There is no stress concentration at the middle point, such as;
    - Reduction in diameter,
    - Keyway
    - Hole, etc.

$S_e = k_a k_b k_c k_d k_e S_e{}'$

$S_e = (0.453)(0.72)(1)(1)(1)(310) = 101.1\ MPa$

From Modified Goodman Theory;

$$n = \frac{1}{\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}} = \frac{1}{\frac{73.1}{101.1} + 0} = 1.38 > 1 \qquad safe\ for\ fatigue\ design\ !!!$$