LESSON 1

Equilibrium & Free Body Diagrams

Shear Force & Bending Moment Diagrams

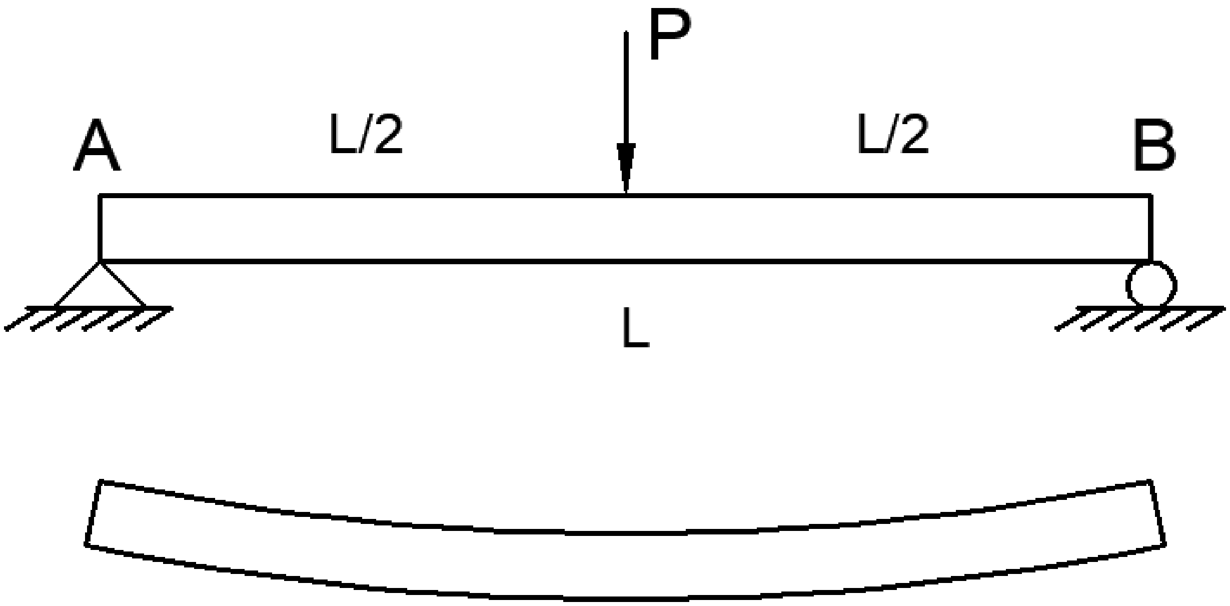
Equilibrium & Free Body Diagrams

- If we assume that the system to be studied is motionless or, has constant velocity, then the system has zero acceleration.
- Under this condition, the system is said to be in equilibrium
- For equilibrium, the forces and moments acting on the system is in balance such that;

  - $\Sigma F=0$
  - $\Sigma M=0$

Equilibrium & Free Body Diagrams

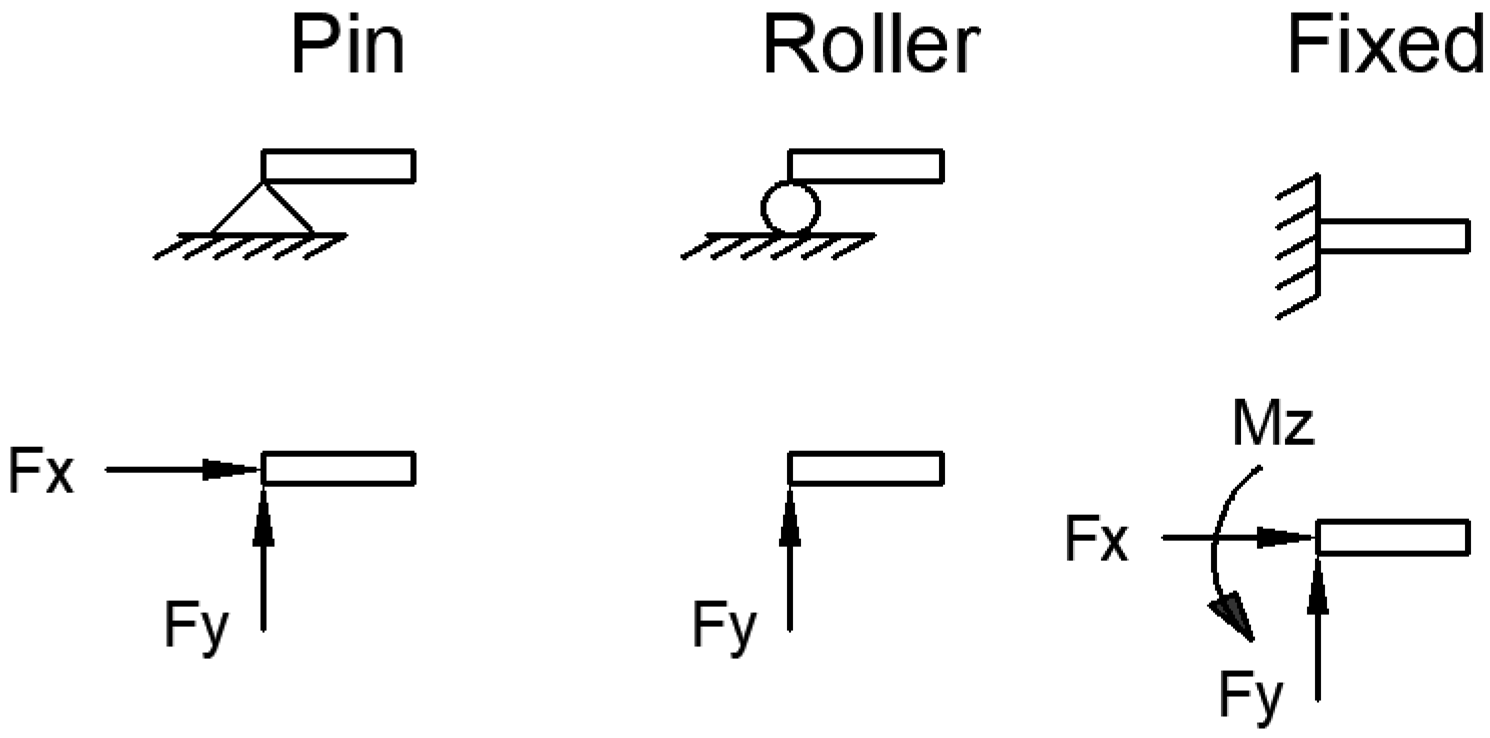
- What is a statically determinated beam and how to analyze it?



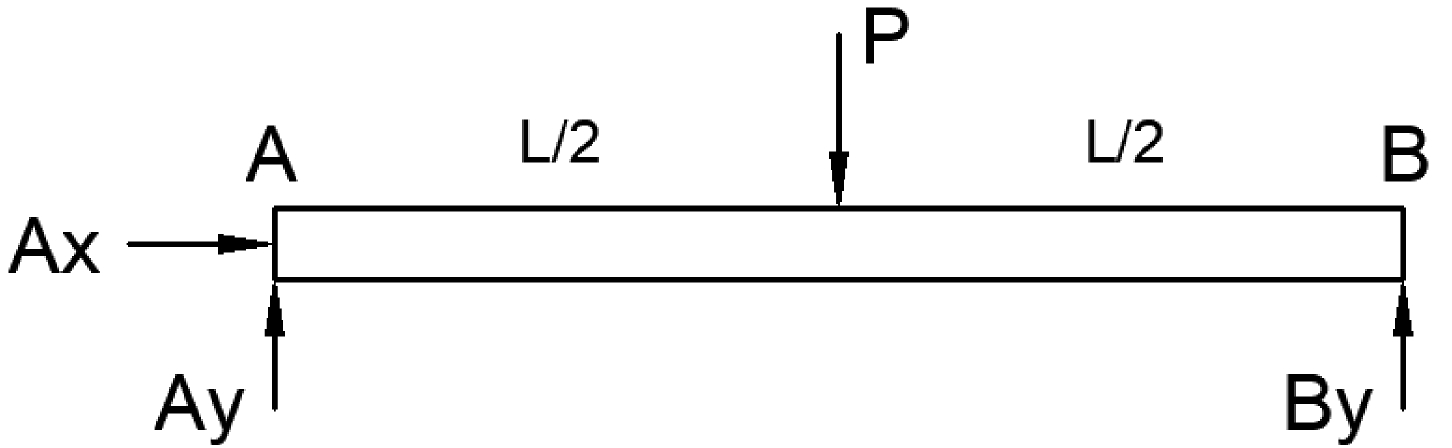
Simply supported beam

Equilibrium & Free Body Diagrams

- The most common types of supports:



Equilibrium & Free Body Diagrams



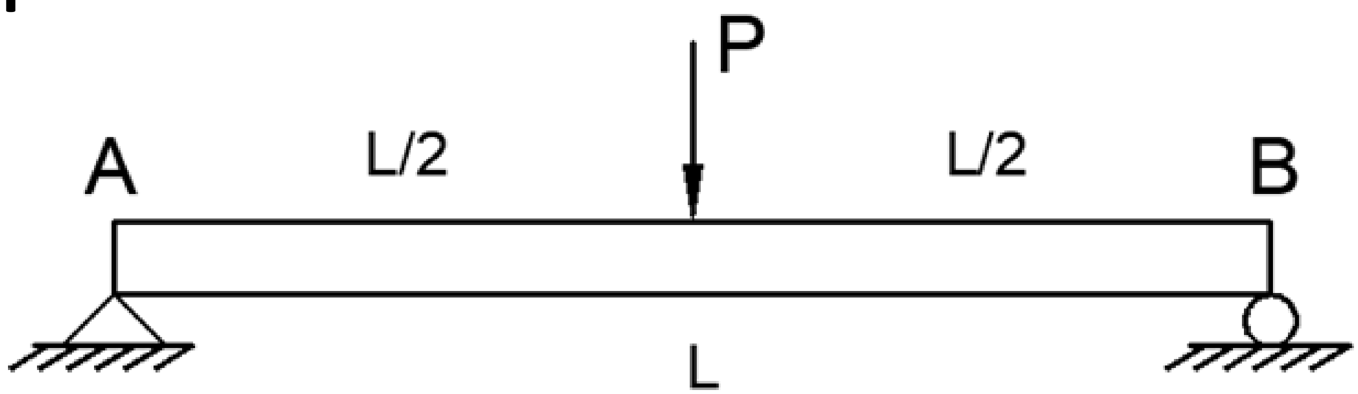
- Unknowns;
  - A_x
  - A_y
  - B_y
- Equations;
  - $\Sigma F_x=0$
  - $\Sigma F_y=0$
  - $\Sigma M_A=0$

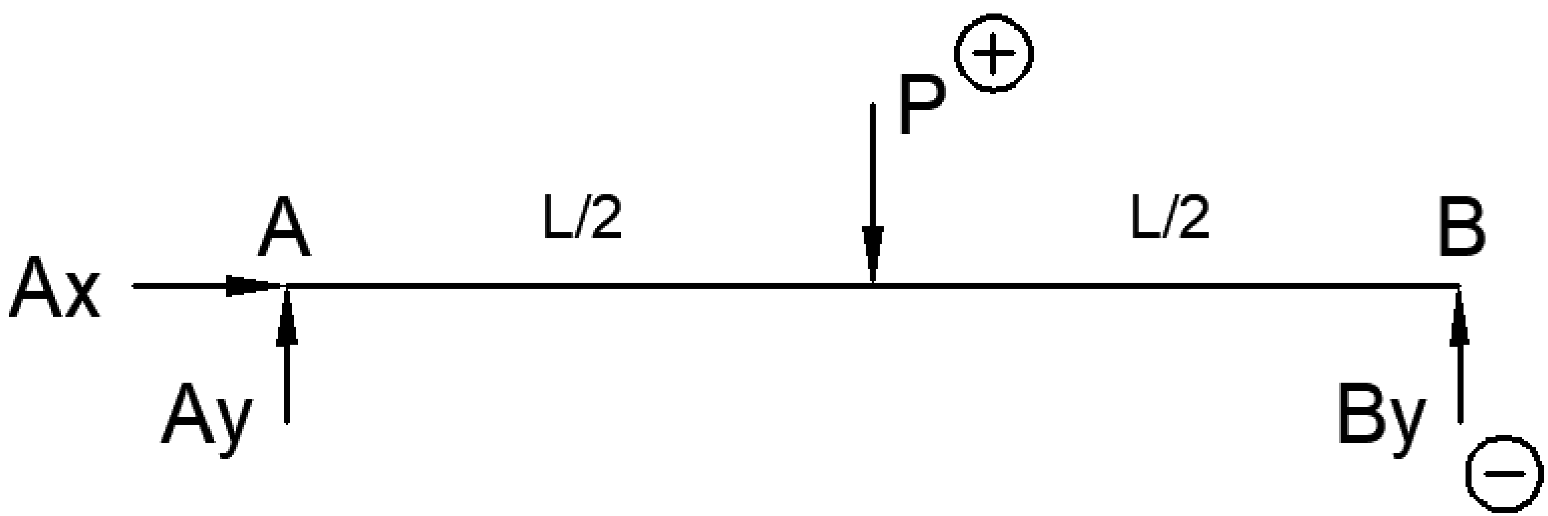
If the unknowns can be calculated, then the beam is statically determinated beam!!!

Equilibrium & Free Body Diagrams

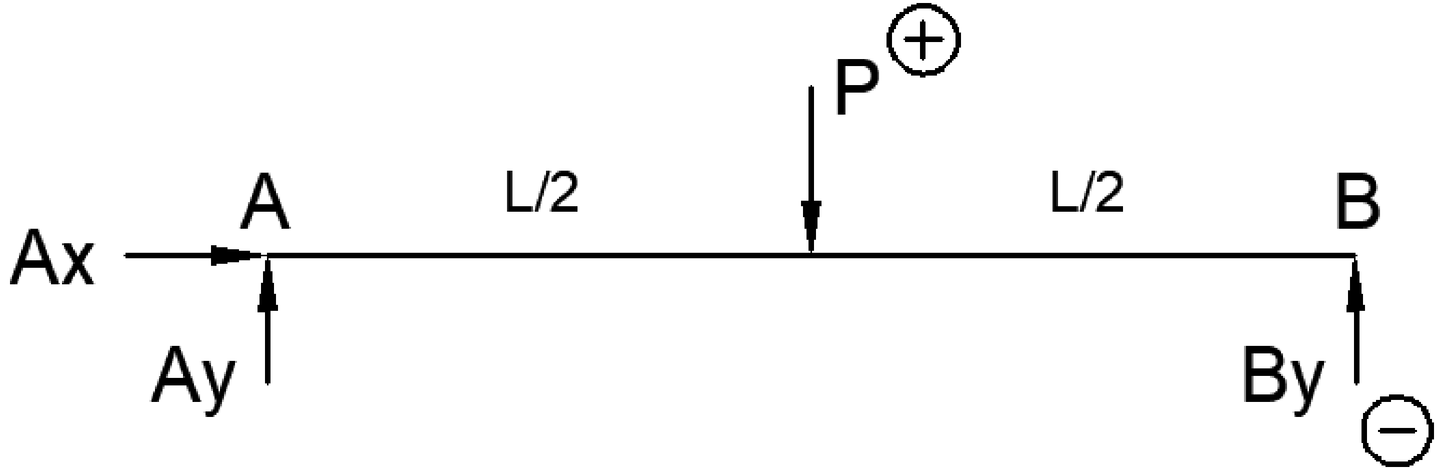
- Free Body Diagrams (FBD)
  - FBD is used to simplfy the analysis of a very complex structure or machine by isolating each element.
  - In order to analyze a beam, it is customery to draw a free body diagram.

Example 1





FBD of the beam



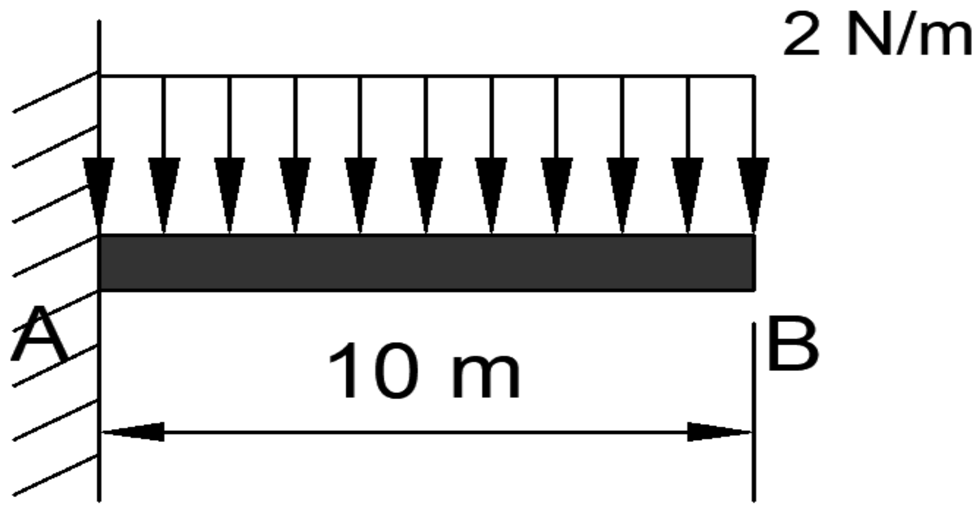
3 unknowns; need 3 equations:

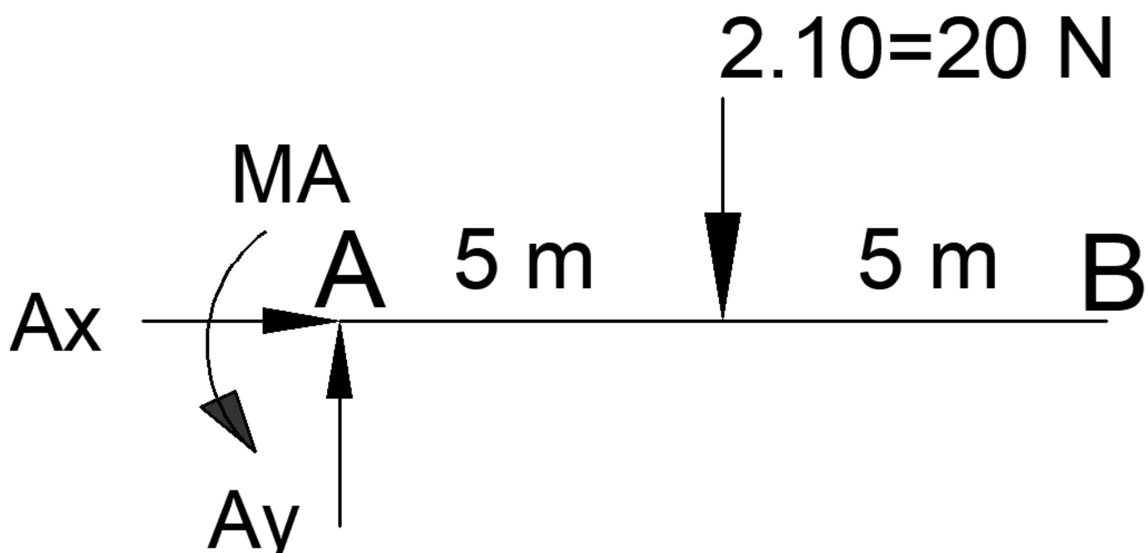
$\Sigma Fx=0 \quad \Rightarrow A_x=0$

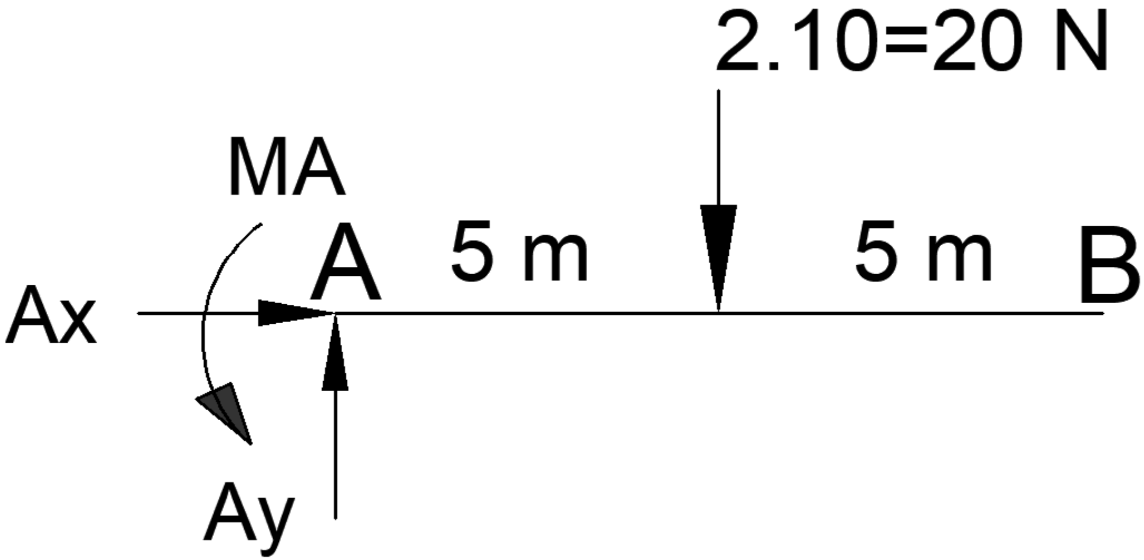
$\Sigma Fy=0 \quad \Rightarrow A_y+B_y-P=0 \quad => A_y=P/2$

$\Sigma MA=0 \quad \Rightarrow P.(L/2)-B_y.L=0 \quad => B_y=P/2$

Example 2





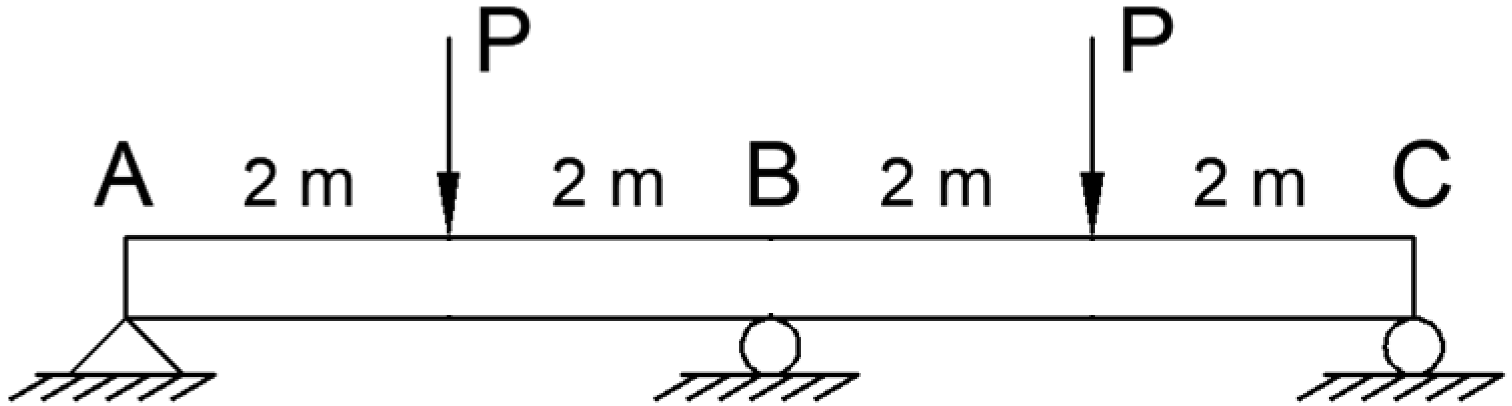


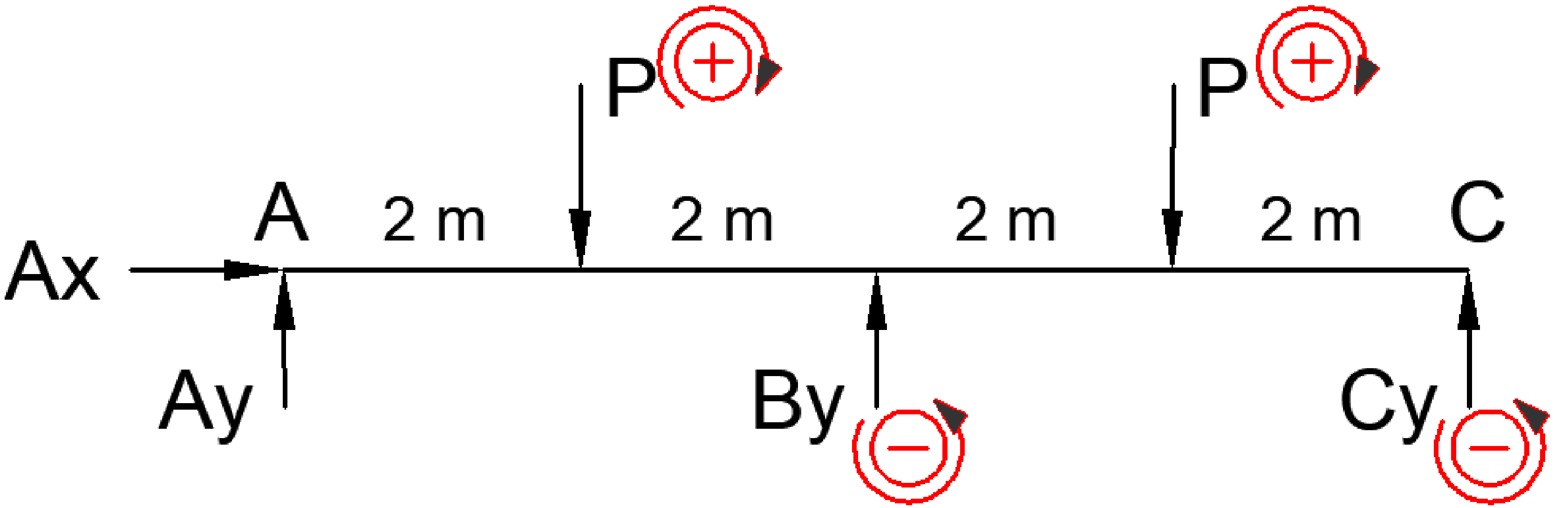
$\Sigma Fx=0 \quad \Rightarrow A_x=0$

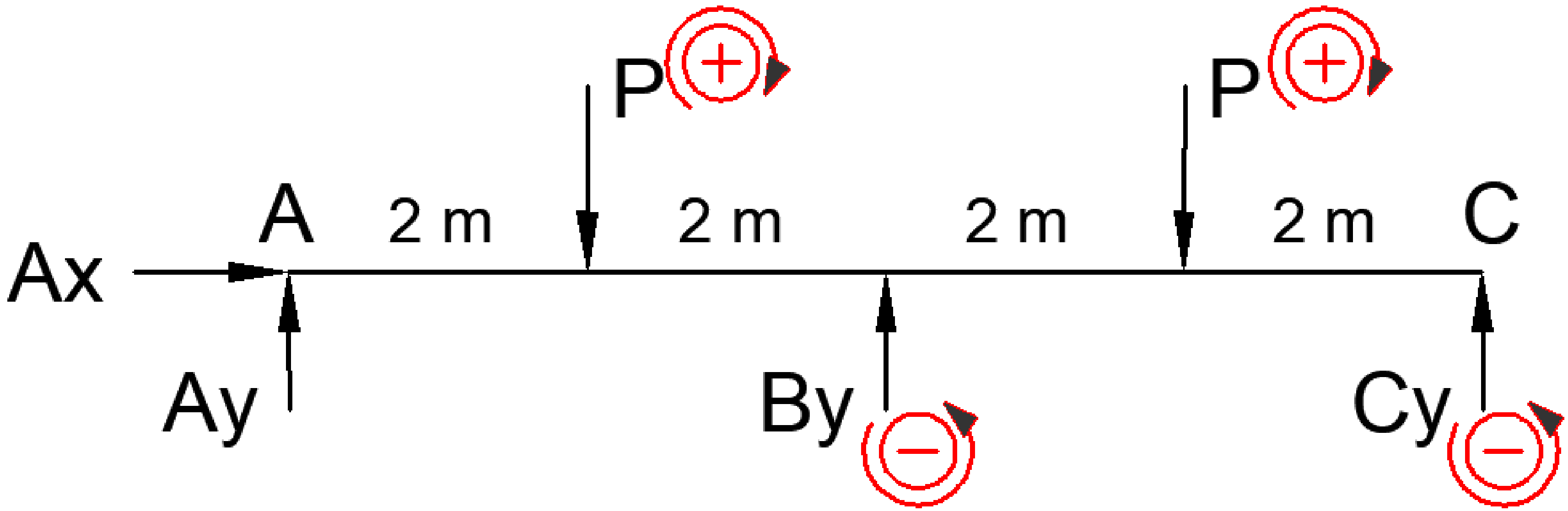
$\Sigma Fy=0 \quad \Rightarrow A_y-20=0 \quad => A_y=20\ N$

$\Sigma MA=0 \quad \Rightarrow 20.5-M_A=0 \quad => M_A=100\ Nm$

- Discussion;







$\Sigma Fx=0 \quad \Rightarrow A_x=0$

$\Sigma Fy=0 \quad \Rightarrow A_y+B_y+C_y-P-P=0$

$\Sigma MA=0 \quad \Rightarrow P.2+P.6-B_y.4-C_y.8=0$

3 equations,
4 unknown;

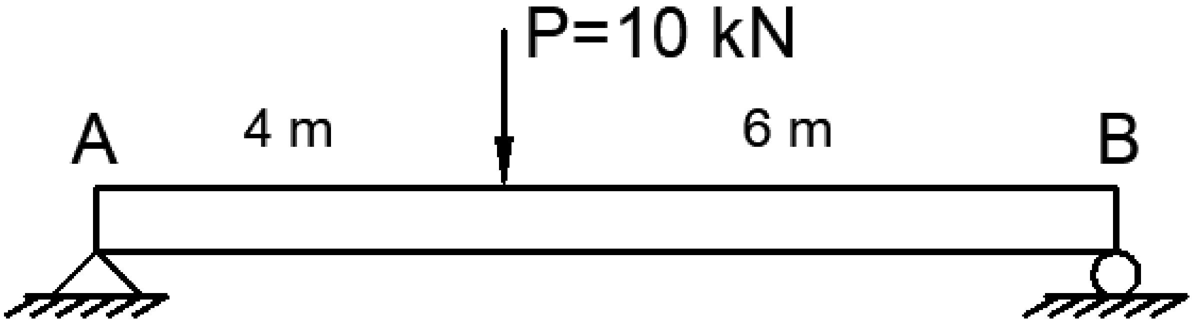
Statically Indeterminate Beam!!!

Should be used some techniques, such as hinged beams

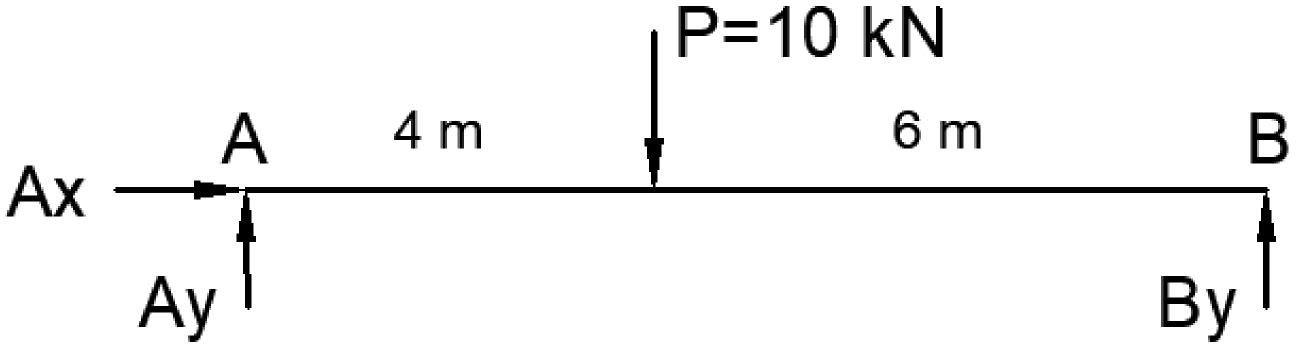
Shear Force & Bending Moment Diagrams

- Shear force & bending moment diagrams are used to determinate the maximum shear and bending moment point on the beam.
- These points give us **an idea** about the critical section on the beam.
- There are some methods to draw the diagrams, such as;
    1. Cutting method
    2. Area method
    3. Singularity functions

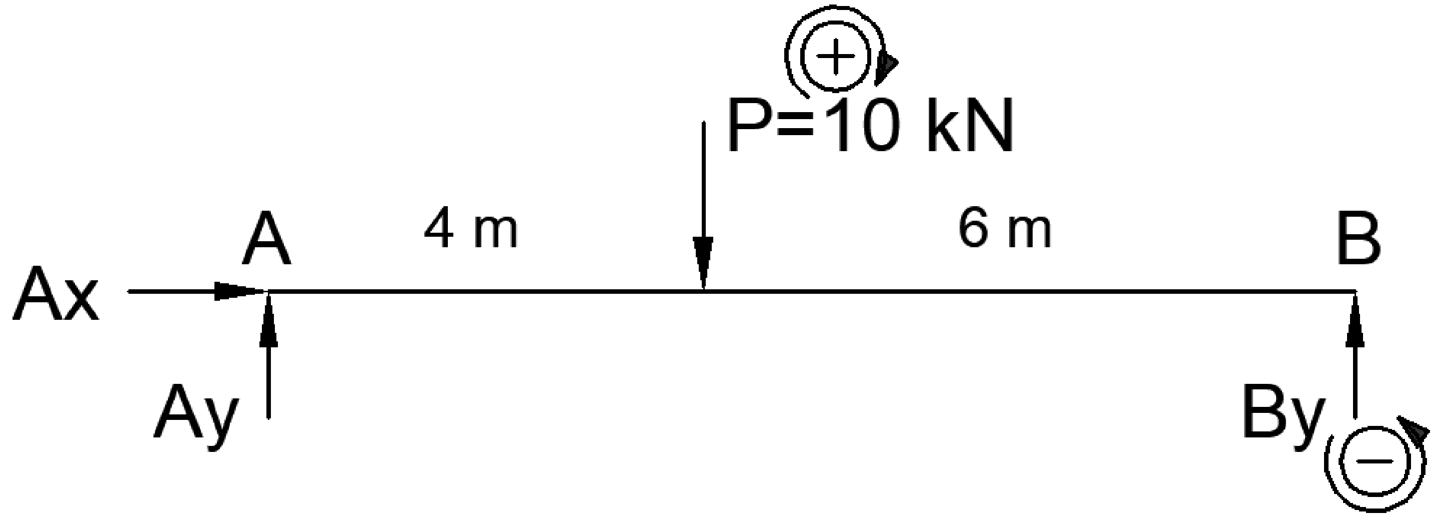
1. Cutting Method

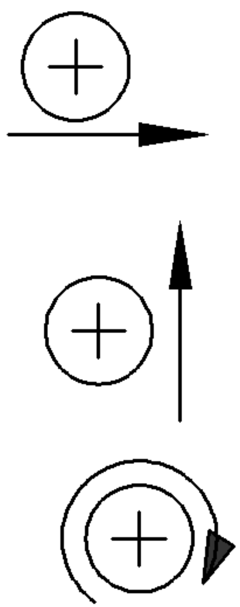


1- FBD of the beam is drawn



2- Reaction forces are calculated





$\Sigma Fx=0 \quad \Rightarrow A_x=0$

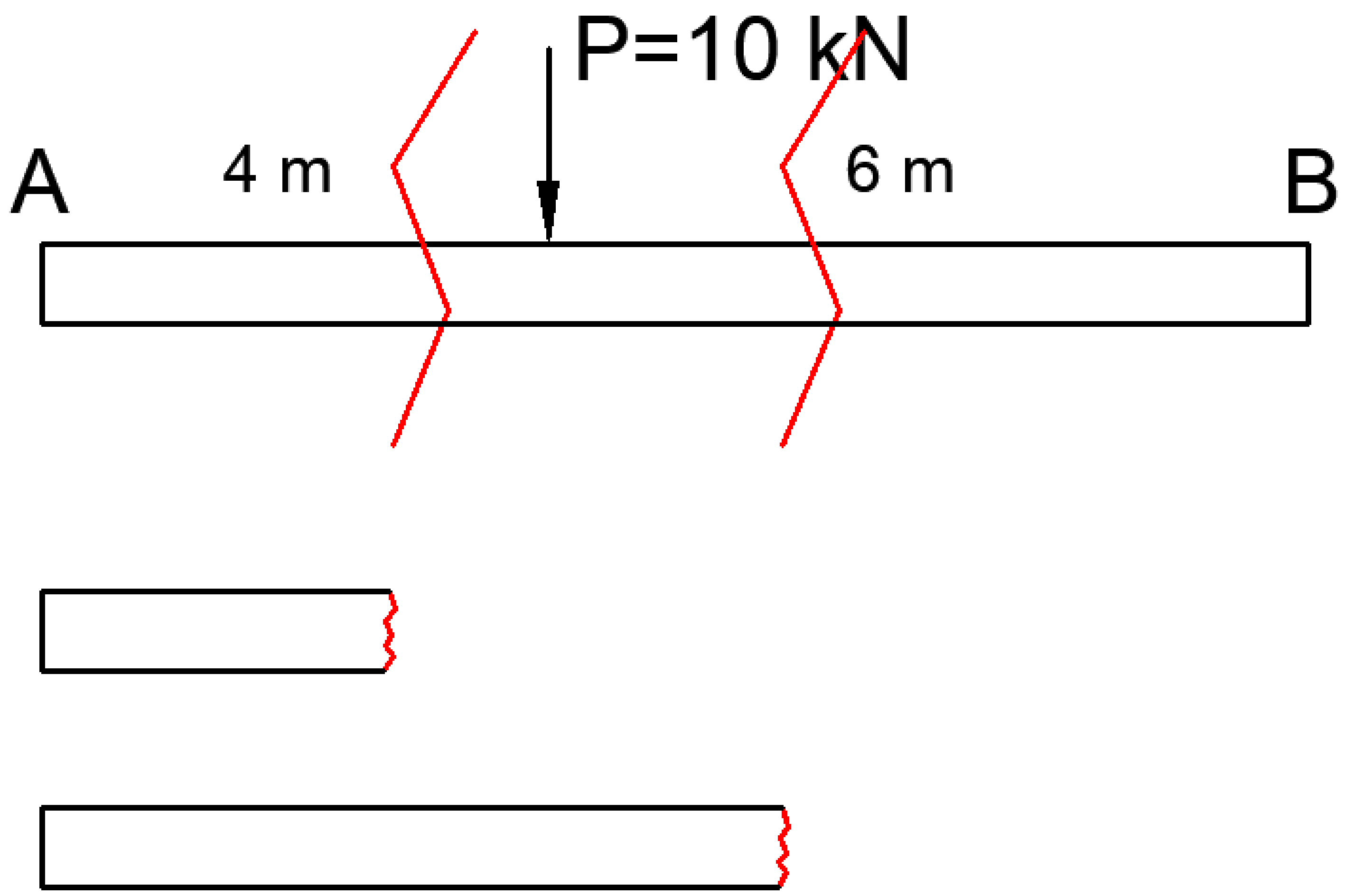
$\Sigma Fy=0 \quad \Rightarrow A_y+B_y-10=0 \quad => A_y=6$ kN

$\Sigma MA=0 \quad \Rightarrow 10.4-B_y.10=0 \quad => B_y=4$ kN

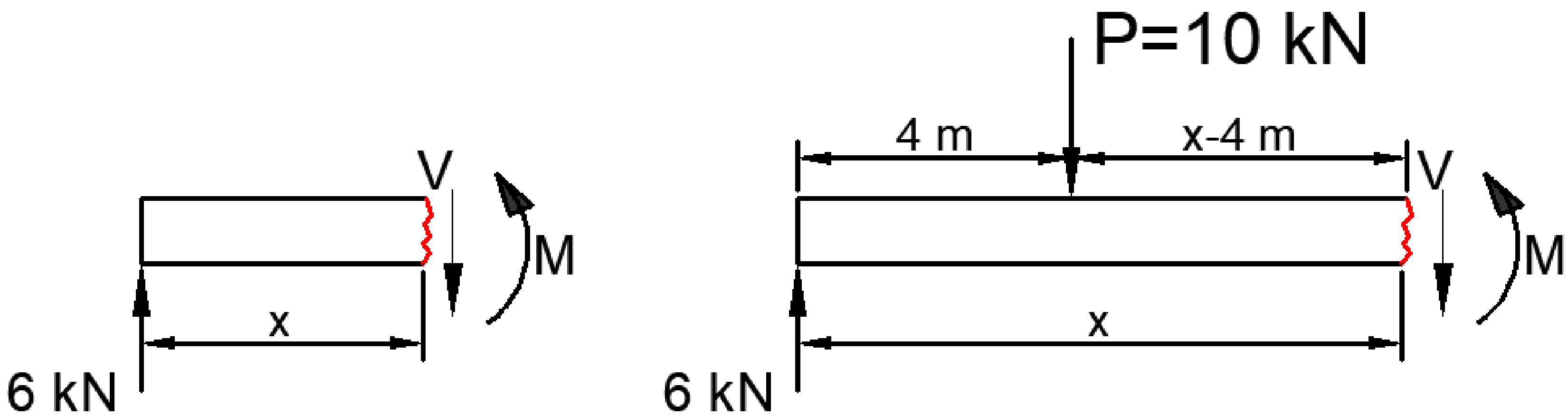
3- The beam is cut at the point of interest.

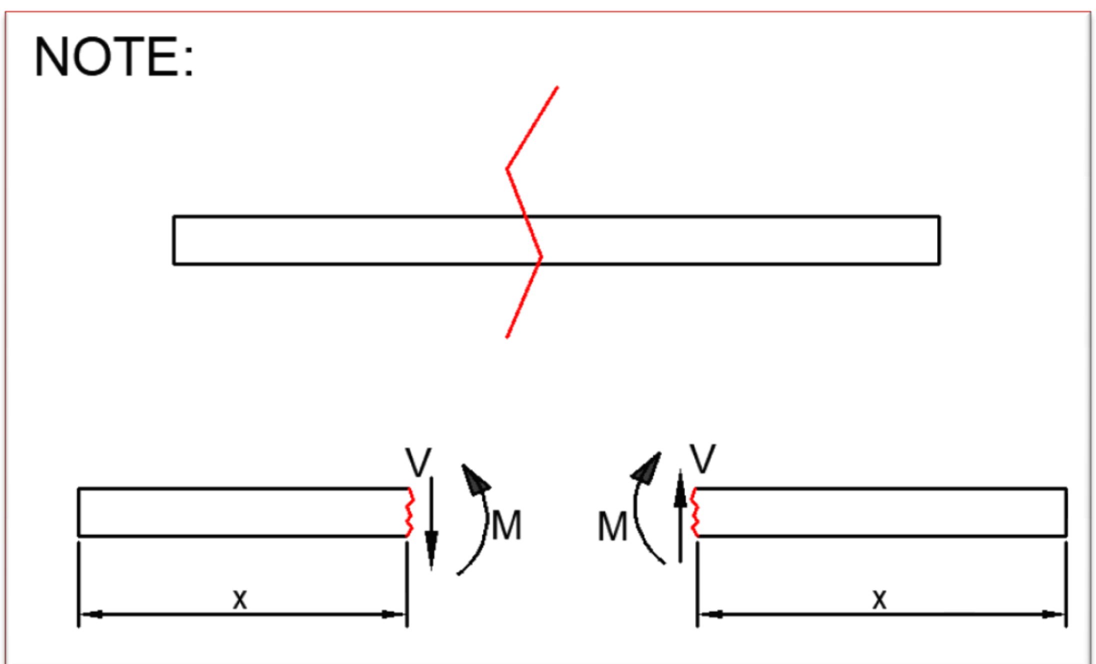
Cut the beam before/after each discountunity such as:

- Concentrated loads
- Distributed loads
- Reactions
- Moments


P=10 kN
A
4 m
6 m
B

3- Show the internal shear force and bending moment,





4- Formulate the equilibrium equations

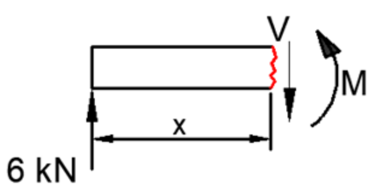
- <u>First Part: $(0 \le x \le 4)$</u>

$\Sigma F_x = 0 \Rightarrow A_x = 0$

$\Sigma F_y = 0 \Rightarrow 6 - V = 0, \; V = 6 \text{ kN}$

$\Sigma M_O = 0 \Rightarrow$ At the cut point

$\Rightarrow 6x - M = 0, \; M = 6x$



- <u>Second part: $(4 \le x \le 10)$</u>

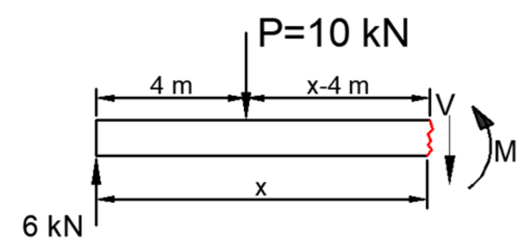
$\Sigma F_x = 0 \Rightarrow A_x = 0$

$\Sigma F_y = 0 \Rightarrow 6 - 10 - V = 0, \; V = -4 \text{ kN}$

$\Sigma M_O = 0 \Rightarrow$ At the cut point

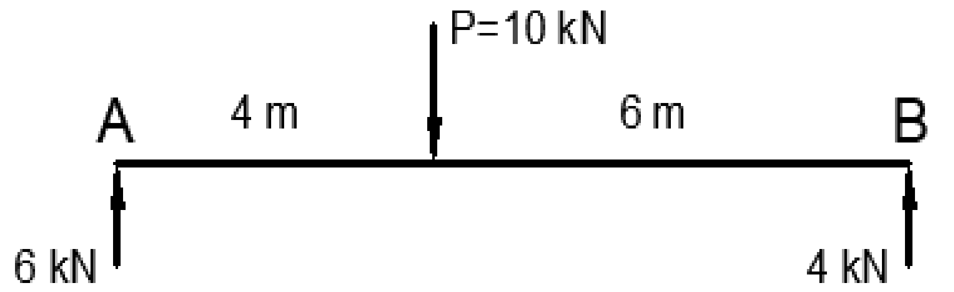
$\Rightarrow 6x - 10(x-4) - M = 0$

$\Rightarrow M = 40 - 4x$



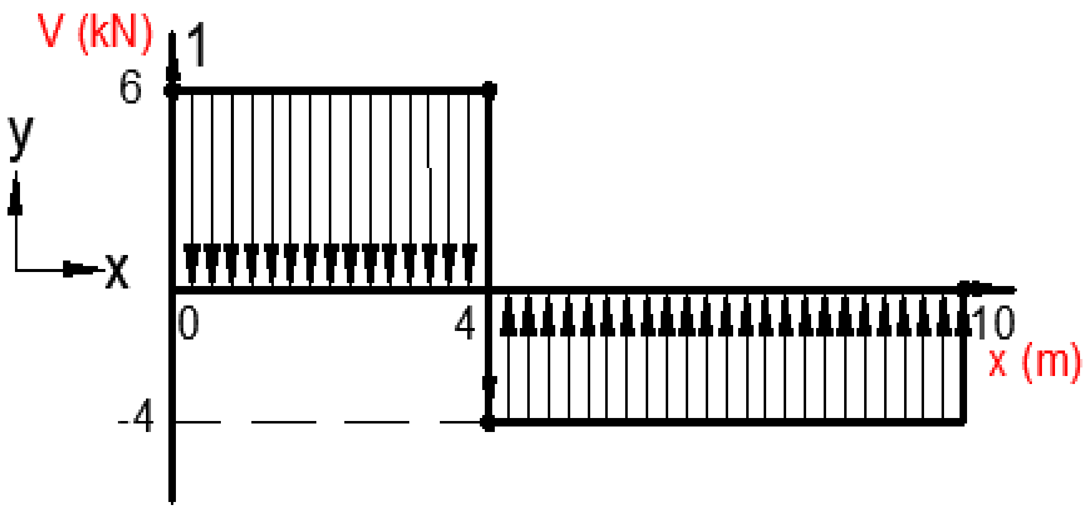
$$V = \begin{cases} 6 & 0 < x < 4 \\ -4 & 4 < x < 10 \\ 0 & x = 10 \end{cases}$$

$$M = \begin{cases} 6x & 0 < x < 4 \\ 40 - 4x & 4 < x < 10 \\ 0 & x = 10 \end{cases}$$



$$V = \begin{cases} 6 & 0<x<4 \\ -4 & 4<x<10 \\ 0 & x=10 \end{cases}$$

$$M = \begin{cases} 6x & 0<x<4 \\ 40-4x & 4<x<10 \\ 0 & x=10 \end{cases}$$

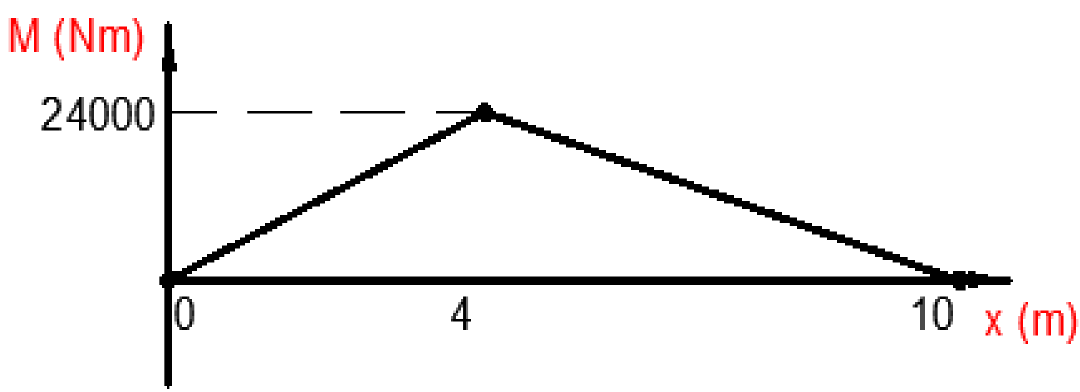


For V:

X=0 => V=6 kN

X=4 => V=6 kN , V=-4 kN

X=10 =>V=-4 kN , V=0



For M:

X=0 => M=0 Nm

X=4 => M=6.4=24 kNm

M=40-(4.4)=24 kNm

X=10 => M=40-(4.10)=0

NOTE:

– Cutting method is usable if there is a small number of discountunities, but if there is a problem includes so many discountunities (Figure A), then it is better to use another mothod.

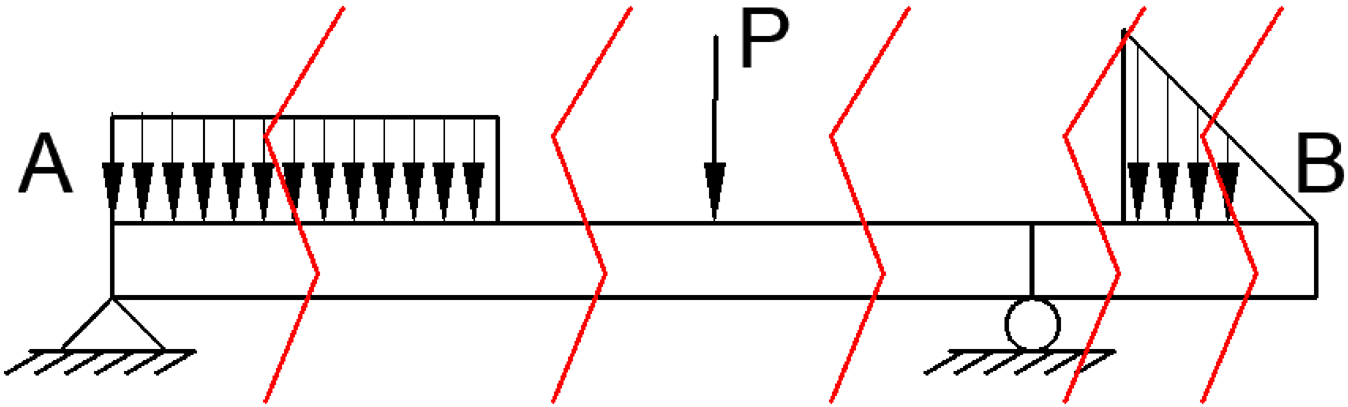
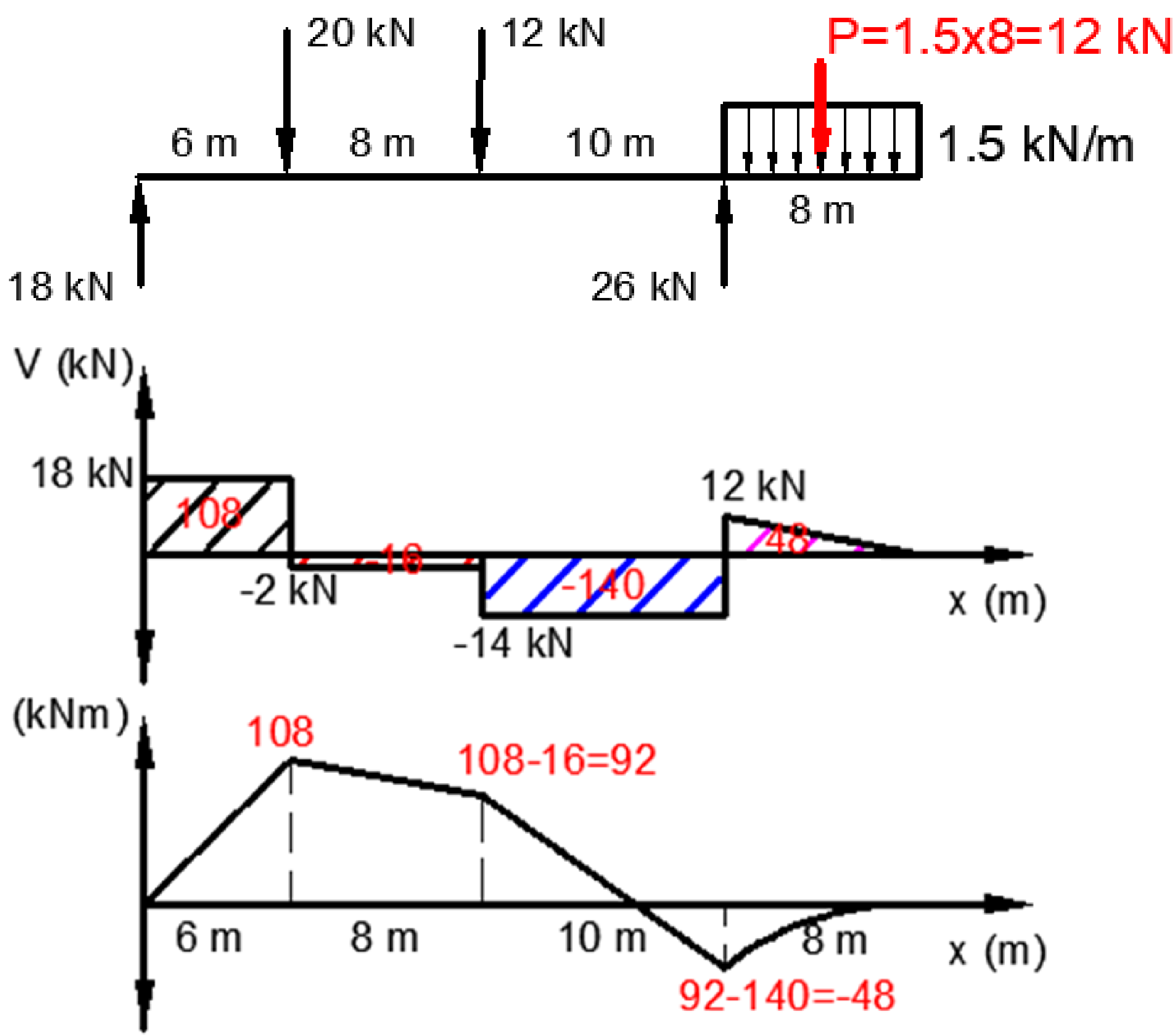


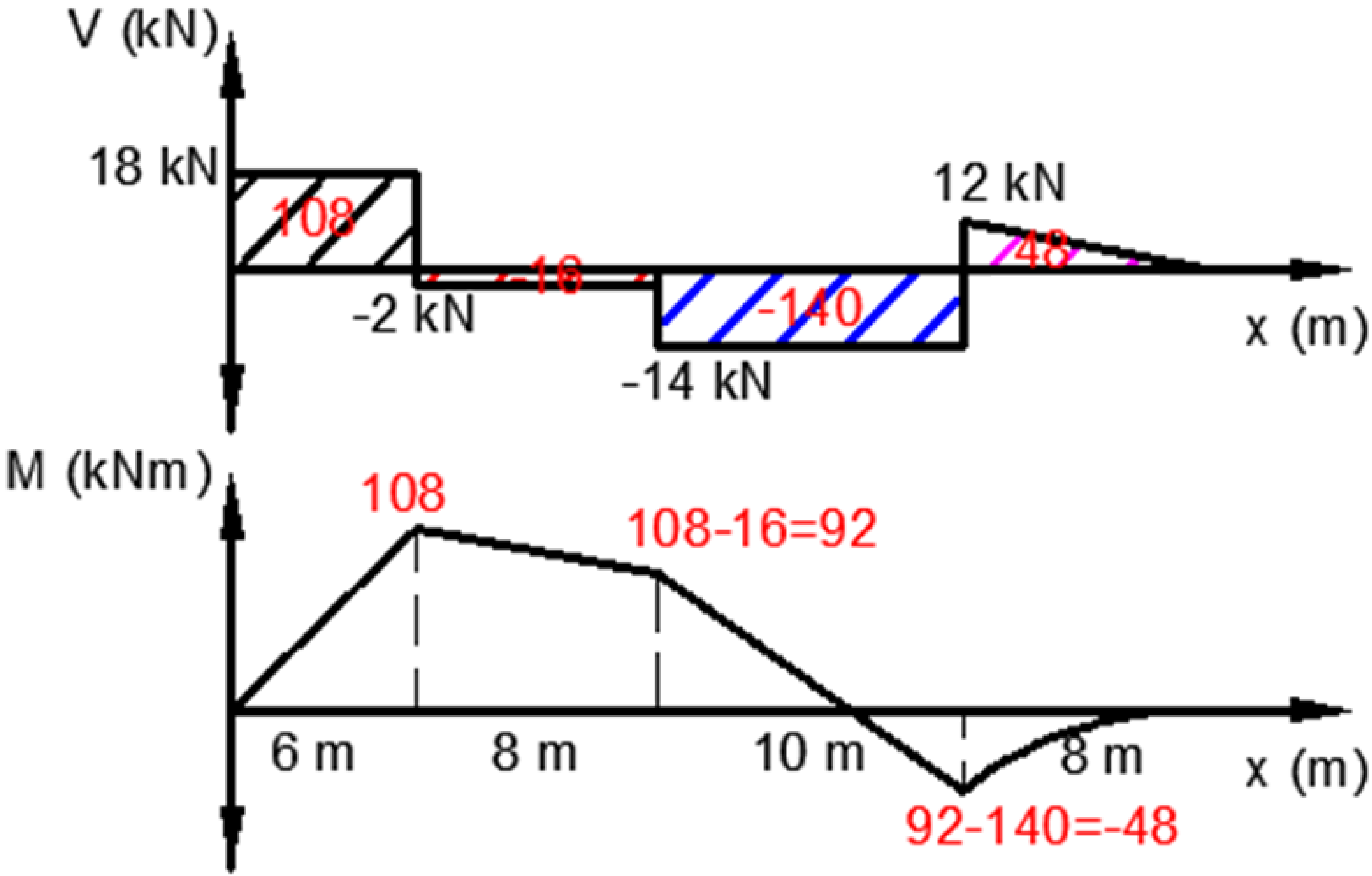
Figure A

There will be so many equations!!!

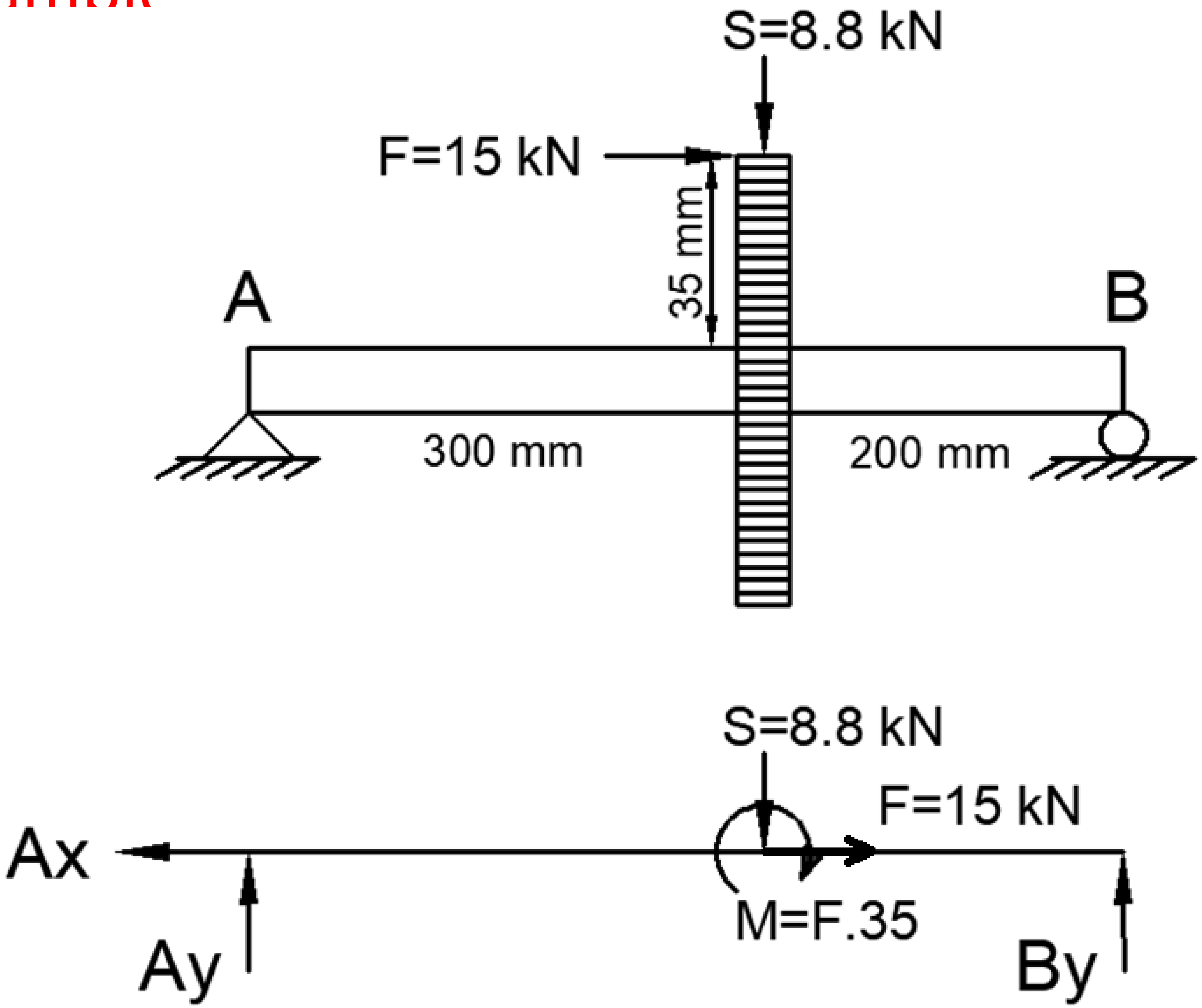
1. Area Method

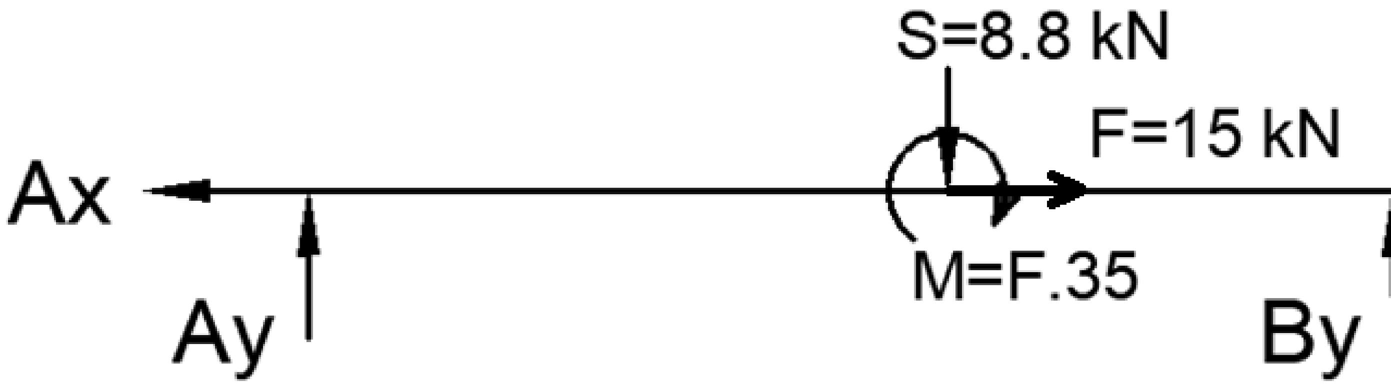
- As an example; FBD of a beam is given;




V (kN)
18 kN
108
-2 kN
-16
-140
-14 kN
12 kN
48
x (m)
M (kNm)
108
108-16=92
92-140=-48
6 m
8 m
10 m
8 m
x (m)

- Example





$\Sigma Fx=0 \Rightarrow 15-A_x=0 \quad => A_x= 15$ kN

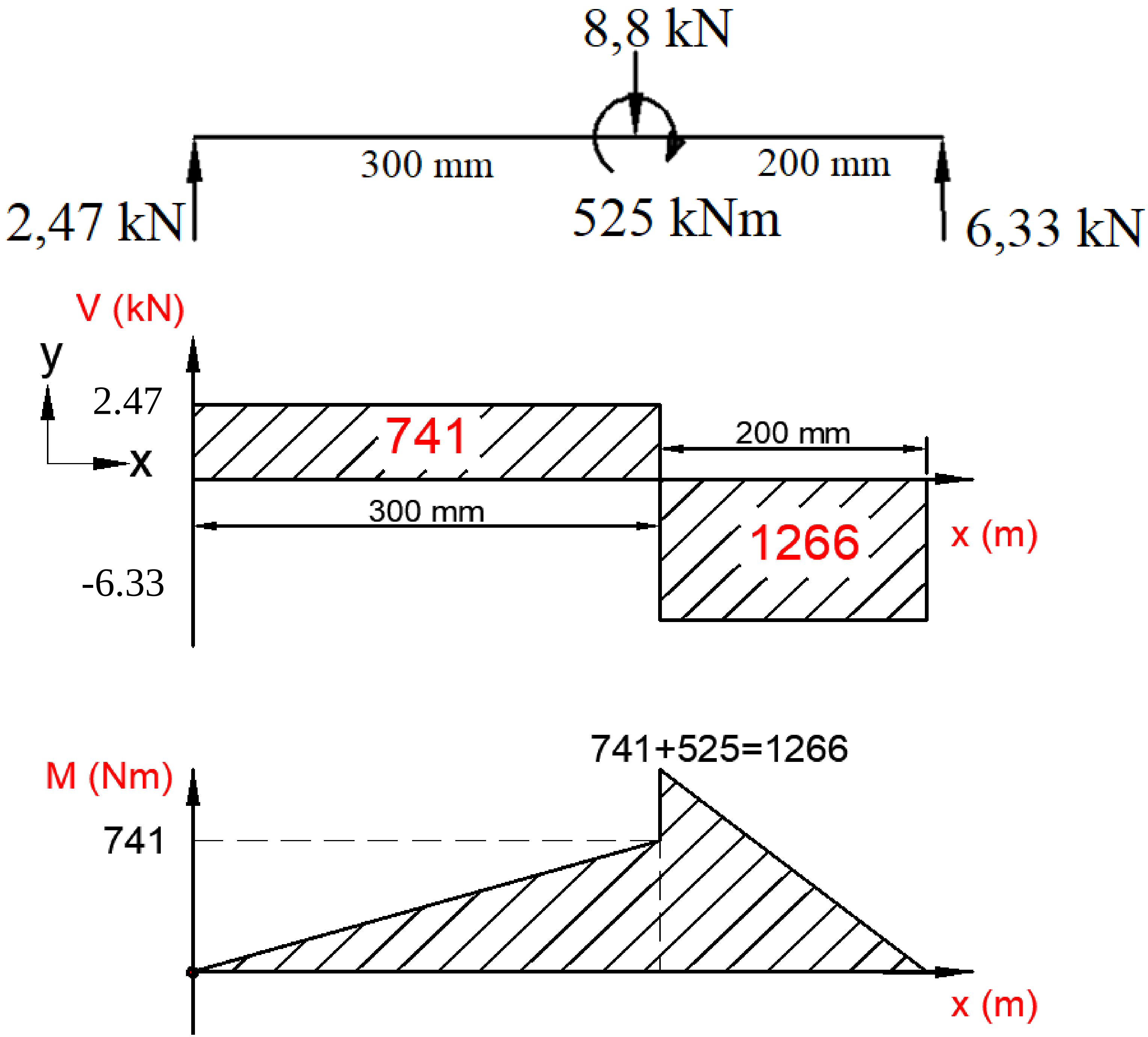
$\Sigma Fy=0 \Rightarrow A_y+B_y-S = 0 \quad => \quad A_y+B_y = 8,8$ kN

$\Sigma MA=0 \Rightarrow 15.35+8,8.300-By.500 = 0$

$\Rightarrow 525+2640-500.By = 0$

$\Rightarrow$ **By=6,33 kN**

$\Rightarrow$ **Ay=2,47 kN**



8,8 kN
300 mm
200 mm
2,47 kN
525 kNm
6,33 kN
V (kN)
y
x
2.47
741
200 mm
300 mm
1266
x (m)
-6.33
741+525=1266
M (Nm)
741
x (m)