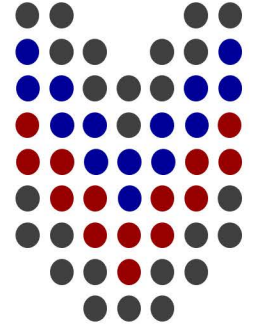


[ ME 472 ]

Engineering Metrology & Quality Control



[ CHAPTER 8 ]

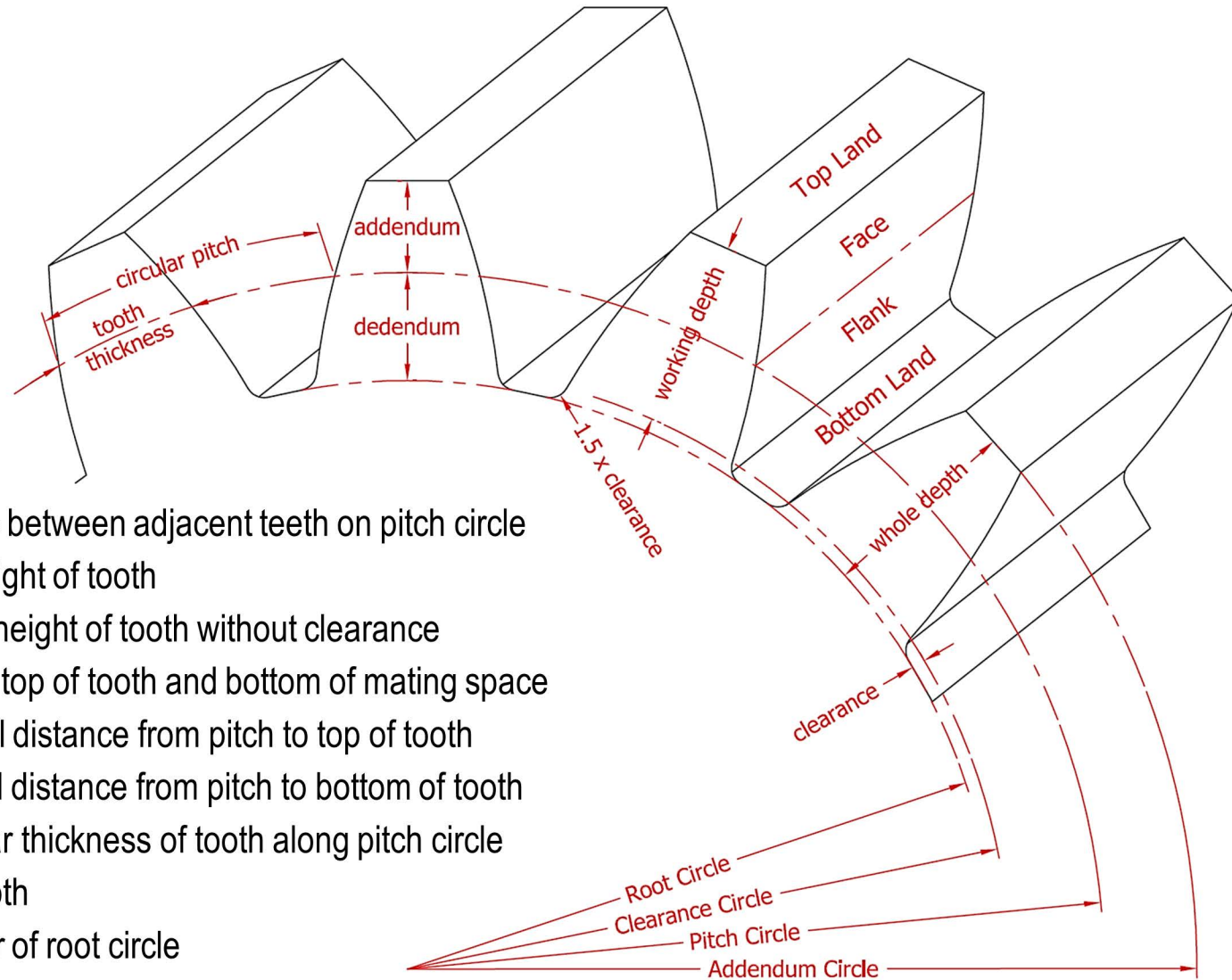
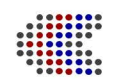
Measurement of Gears



**Assoc. Prof. Dr. A. Tolga BOZDANA**  
**Mechanical Engineering Department**

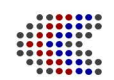
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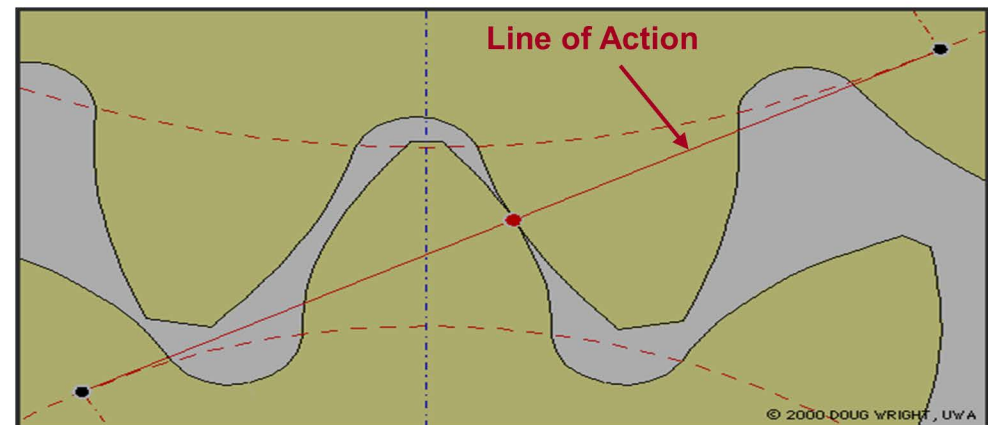
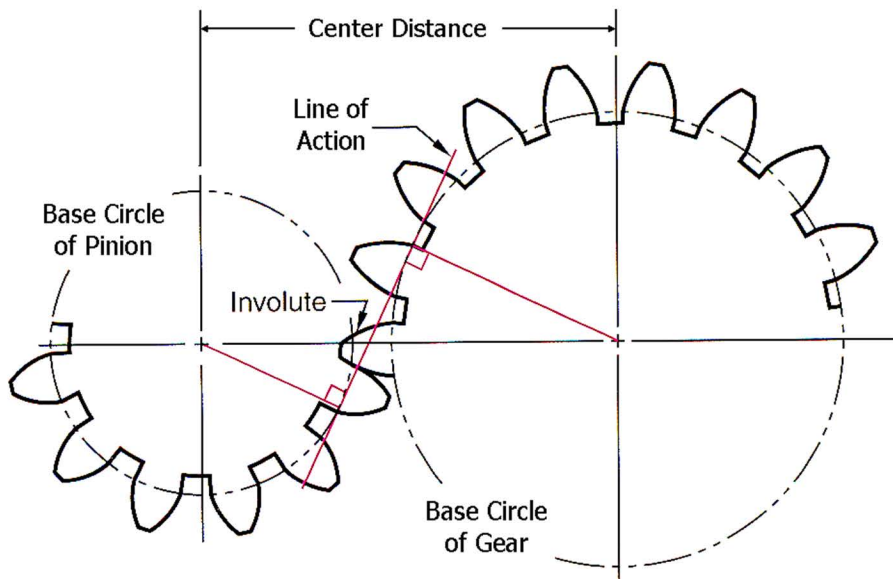
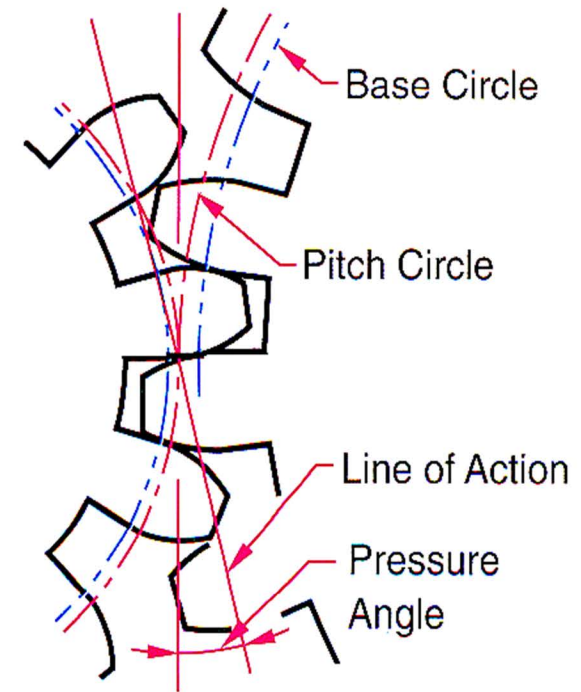
## Terminology (Spur Gear)

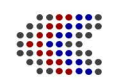
- **Circular Pitch (adım):** circular distance between adjacent teeth on pitch circle
- **Whole Depth (diş yüksekliği):** total height of tooth
- **Working Depth (çalışma yüksekliği):** height of tooth without clearance
- **Clearance (diş açıklığı):** distance from top of tooth and bottom of mating space
- **Addendum (diş üstü yükseklik):** radial distance from pitch to top of tooth
- **Dedendum (diş altı yükseklik):** radial distance from pitch to bottom of tooth
- **Tooth Thickness (diş kalınlığı):** circular thickness of tooth along pitch circle
- **Face Width (diş genişliği):** width of tooth
- **Root Diameter (diş dibi çap):** diameter of root circle
- **Outside Diameter (diş üstü çap):** diameter of addendum circle
- **Pitch Diameter (bölüm çapı):** diameter of imaginary pitch circle



## Meshing of Gears

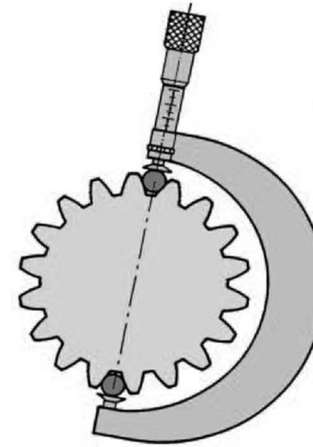
- The standard gear size is defined by **module** & **number of teeth**.
- **Only the gears having identical module** can be meshed.
- Meshing of two spur gears with center distance is shown below (smaller one is called "**pinion**" whereas "**gear**" refers to the larger)
- **Line of action (pressure line)** is drawn tangent to the base circle of pinion and gear.
- The kinematic principle of gearing applies when gear teeth are in contact: **angular velocity ratio of the meshing gears is constant along the line of action**
- **Pressure angle** is the angle between tangent to the pitch circles and the line of action.



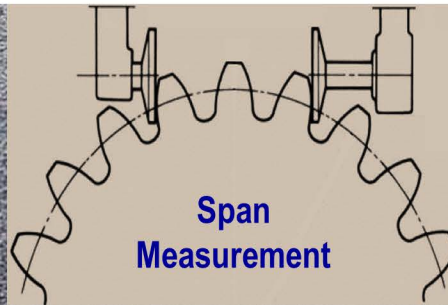
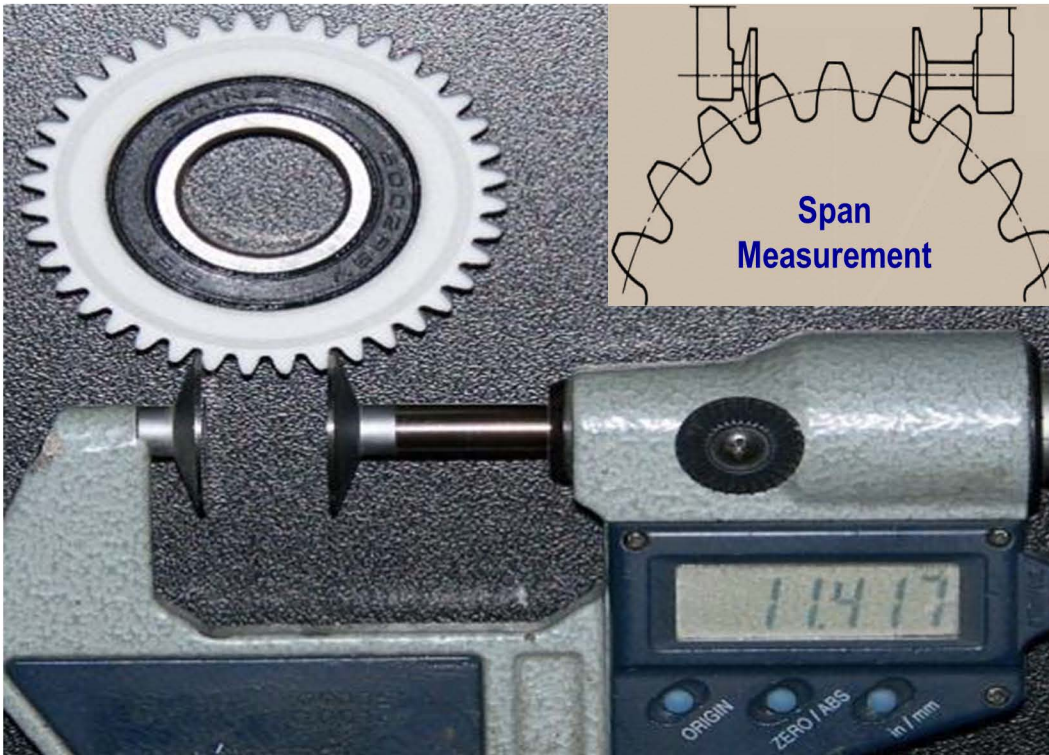


## Measurement of Gears

- Gear measurement according to **ANSI/AGMA 2002-C16:**  
(standard by **American Gear Manufacturers Association**)
  - **Measurement over Pins:** distance over pins placed within teeth
  - **Gear Tooth Vernier Caliper:** chordal tooth thickness & height
  - **Span Measurement:** distance between a number of teeth



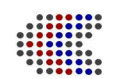
Measurement over Pins



Span Measurement



Gear Tooth Vernier Caliper



### Measurement over Pins (Spur Gears)

➤ **Tooth Thickness (t)** is calculated by:

$$t = D_p \left[ \text{inv}(e) - \text{inv}(\Psi) - \frac{D_g}{D_p * \cos(\Psi)} + \frac{\pi}{z} \right]$$

➤ **Pin Diameter (D<sub>g</sub>)** is defined as:

➤ External gears: D<sub>g</sub> = 13.97 m

➤ Internal gears: D<sub>g</sub> = 13.58 m

➤ **Distance over Pins (H)** equals to:

➤ Even no. of teeth: H<sub>e</sub> = 2h + D<sub>g</sub>

➤ Odd no. of teeth: H<sub>o</sub> = 2h \* cos(90°/z) + D<sub>g</sub>

➤ **Involute of an angle** is found as:

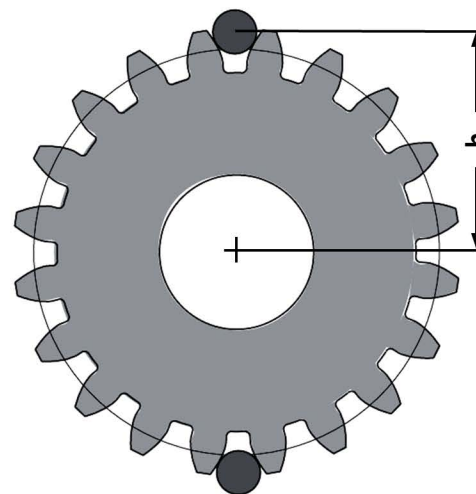
$$\text{inv}(x) = \tan(x) - [x * (\pi/180^\circ)]$$

➤ **Angle "e"** is specified by:

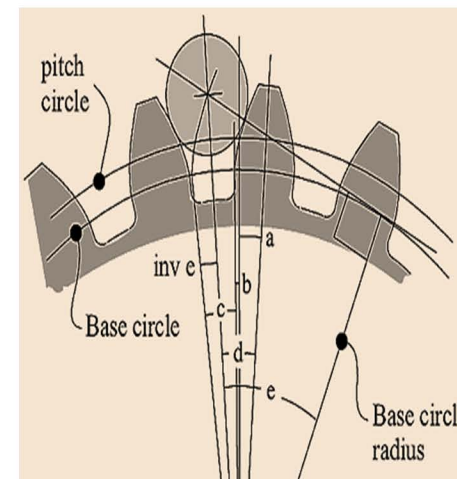
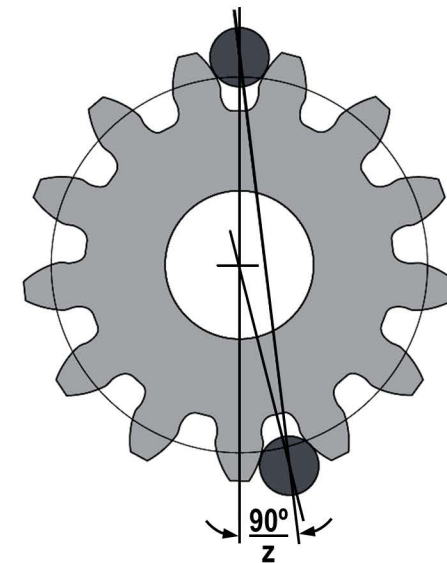
$$\text{➤ Even no. of teeth: } e = \arccos \left[ \frac{z * \cos(\Psi)}{D_p * (H_e - D_g)} \right]$$

$$\text{➤ Odd no. of teeth: } e = \arccos \left[ \frac{z * \cos(\Psi)}{D_p * (H_o - D_g)} * \cos \left( \frac{90^\circ}{z} \right) \right]$$

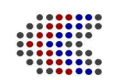
Even number of teeth



Odd number of teeth



- t: tooth thickness
- D<sub>p</sub>: pitch diameter
- D<sub>g</sub>: pin diameter
- H: distance over pins
- z: number of teeth
- m: module
- Ψ: pressure angle
- e: angle of pin location



### Gear Tooth Vernier Caliper (Spur Gears)

- Usual practice is to measure **Chord Thickness & Height**:

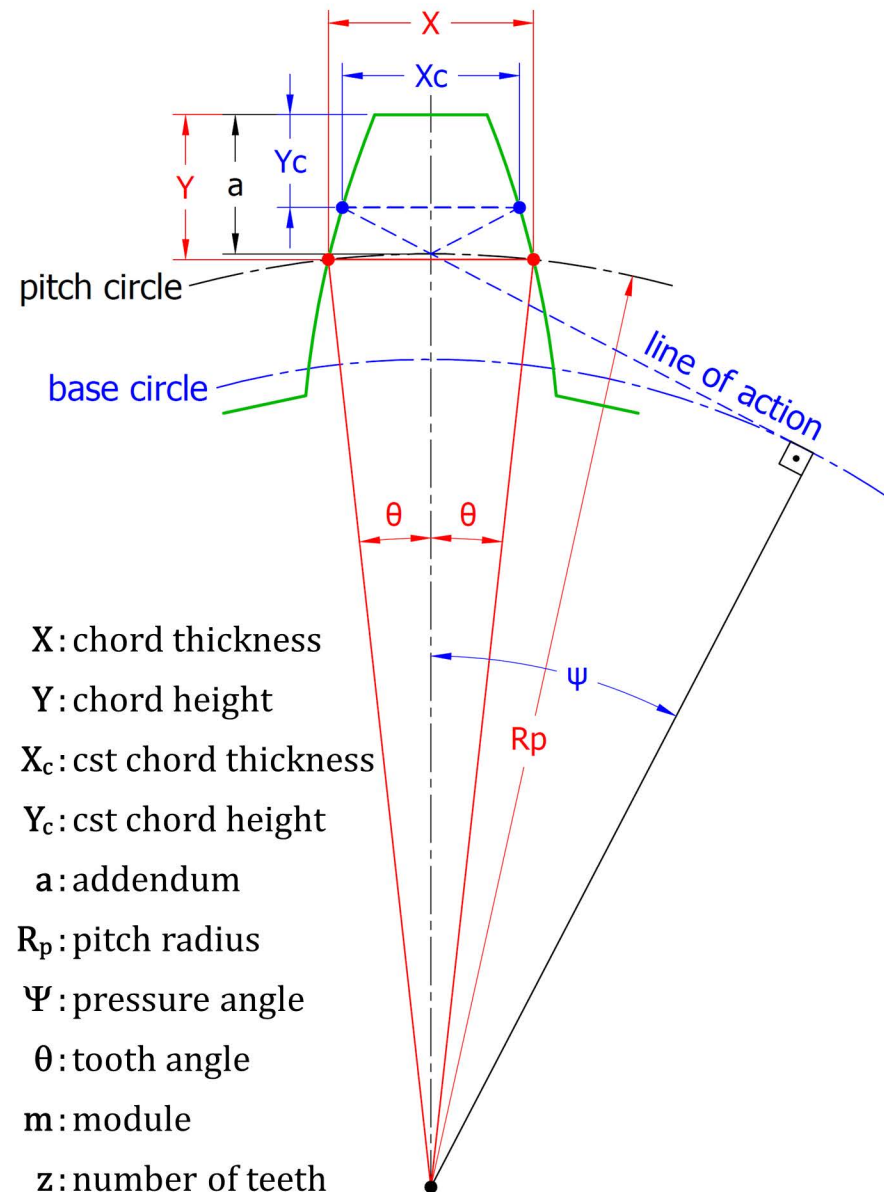
$$X = 2R_p * \sin(\theta) \quad \& \quad Y = a + R_p [1 - \cos(\theta)]$$

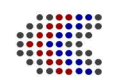
$$\text{where } a = m = \frac{2R_p}{z} \quad \& \quad \theta = \frac{\pi/2}{z} = \frac{90^\circ}{z}$$

- When many gears (with different number of teeth) to be measured, the calculations would become laborious. Thus, **Constant (Fixed) Chord Thickness & Height** are found as independent from number of teeth:

$$X_c = \frac{\pi}{4} * m * \cos^2(\Psi) \quad \& \quad Y_c = m - \frac{\pi}{8} * m * \sin(2\Psi)$$

- **Constant (Fixed) Chord** is the length joining the points on opposite faces of tooth, making contact with the mating teeth at the line of action.





## Span Measurement (Spur Gears)

- **Span Length (M)** refers to the **base tangent length**, expressed in terms of **base tooth thickness ( $t_b$ )** and **base pitch ( $p_b$ )** depending on **teeth in span length ( $k$ )**:

$$M = t_b + p_b * (k - 1)$$

- **Base Tooth Thickness ( $t_b$ )** is expressed as:

$$t_b = m * z * \cos(\Psi) * \left[ \frac{\pi/2}{z} + \text{inv}(\Psi) \right]$$

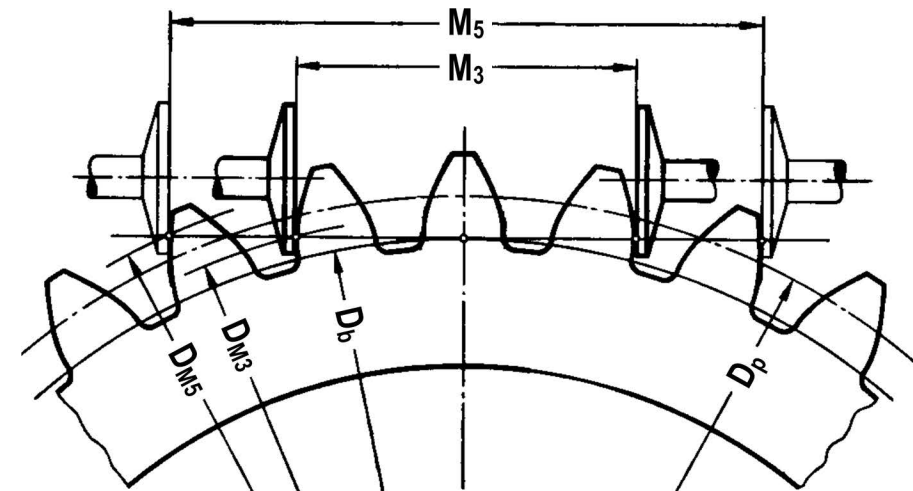
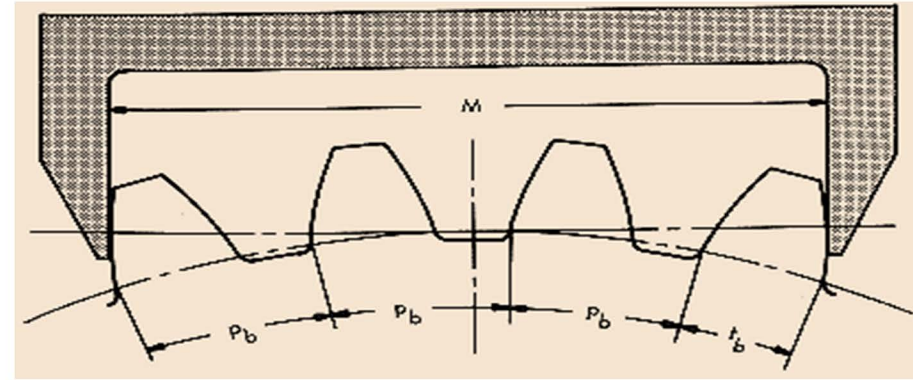
- **Base Pitch ( $p_b$ )** is expressed as:

$$p_b = m * \pi * \cos(\Psi)$$

- Rearranging the terms gives the span length:

$$M = D_p * \cos(\Psi) * \left[ \frac{\pi/2}{z} + \text{inv}(\Psi) + \frac{\pi}{z} * (k - 1) \right]$$

- As seen from figure, **span length defines the points at which measuring device grabs the teeth during measurement.**

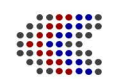


$M_3$ : span length ( $k = 3$ )

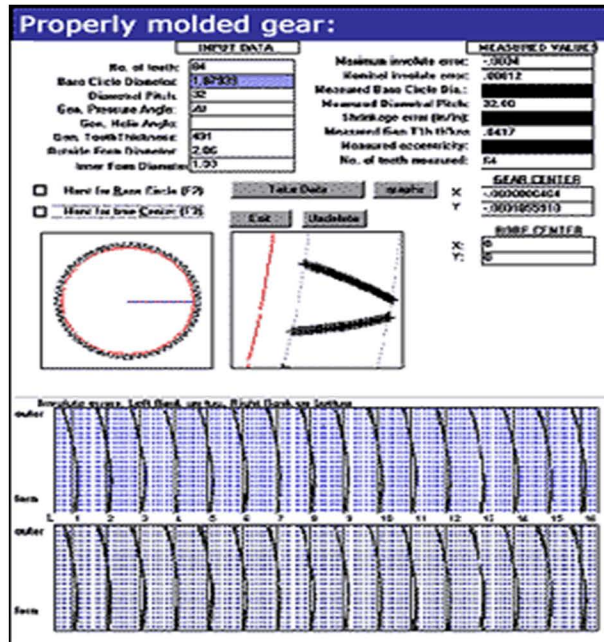
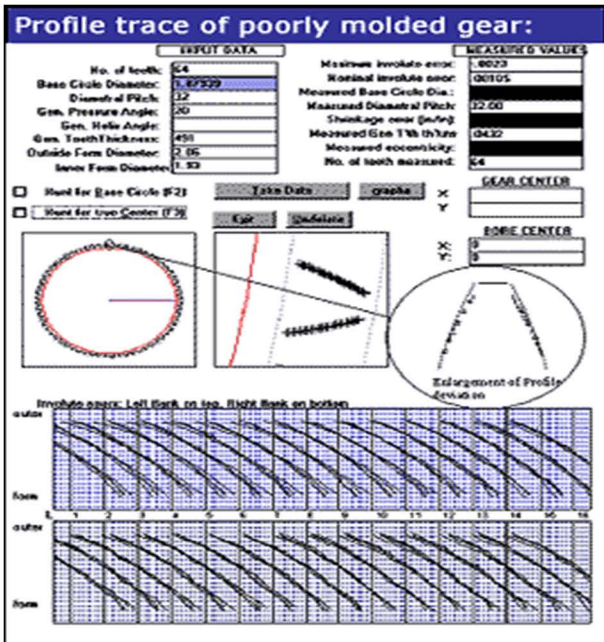
$M_5$ : span length ( $k = 5$ )

$D_b$ : base diameter

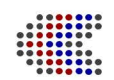
$D_p$ : pitch diameter



## Gear Measurement by CMM







METHOD	ADVANTAGES	DISADVANTAGES
<b>Measurement over Pins</b>	☺ <b>Measurement not affected</b> by form errors and/or runout along the outside diameter	☹ <b>The most appropriate pins must be selected</b> (a bottleneck for gears having non-standard features)
<b>Gear Tooth Vernier Caliper</b>	☺ Relatively <b>cheaper &amp; easier</b> to use	☹ <b>Precision of caliper</b> directly affects results ☹ Measurement depend on <b>two vernier readings</b> , each of which is a function of the other. ☹ Measuring with <b>“edge”</b> of caliper jaw ( <b>not “face”</b> ), which does not lend itself to accurate measurement
<b>Span Measurement</b>	☺ <b>Measurement not affected</b> by form errors and/or runout along the outside diameter	☹ Not usable for <b>high helix angle &amp; narrow face width</b> (spanning a sufficient number of teeth is the problem) ☹ Affected by <b>errors in base pitch &amp; tooth profile</b> (due to modified profile from true involute shape)
<b>Measurement by CMM</b>	☺ <b>Very accurate</b> in case of undamaged teeth ☺ Measurement of <b>dimensions &amp; form of gear</b> ☺ Can be used for <b>almost all gear profiles</b>	☹ <b>Time consuming</b> for set-up & process control ☹ <b>Appropriate probe sizes</b> must be employed (i.e. small size of gears cannot be measured)