ME 308
MACHINE ELEMENTS II

CHAPTER 2

SPRING DESIGN
PART_1

OBJECTIVES OF THE CHAPTER

- Identify, describe, and understand principles of several types of springs including
  - helical compression springs,
  - helical extension springs and
  - torsion springs.
- Design and analyze helical compression springs, including compatibility with allowable stresses.
- Develop necessary analytical tools for spring design.
- Review principles of design for other types of springs, such as extension springs and torsion springs.
- Select predesigned springs from manufacturers' catalogs and
- Incorporate them in appropriate designs.

SPRINGS

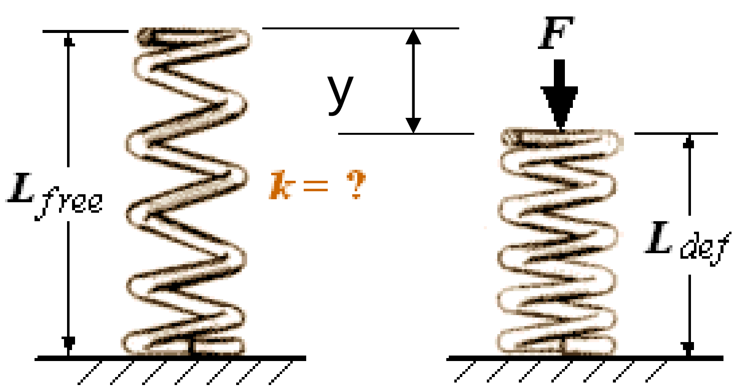
NO RIGID BODY BEHAVIOUR(AS IN STATICS),

ON THE CONTRARY,

A LOT OF DEFLECTION AND DEFORMATION

BUT MAINLY ELASTIC

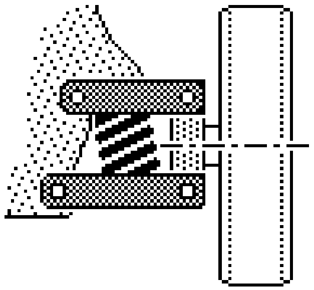
PLASTIC DEFORMATION IN SPRINGS MEANS “FAILURE”

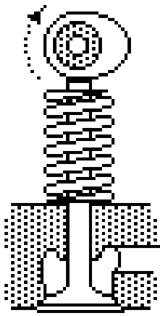


2.1 INTRODUCTION

When a designer wants rigidity, negligible deflection is an acceptable approximation as long as it does not compromise function. Flexibility is sometimes needed and is often provided by metal bodies with cleverly controlled geometry. These bodies can exhibit flexibility to the degree the designer seeks. Such flexibility can be linear or nonlinear in relating deflection to load. These devices allow controlled application of force or torque; the storing and release of energy can be another purpose. Flexibility allows temporary distortion for access and the immediate restoration of function. Because of machinery's value to designers, springs have been intensively studied; moreover, they are mass-produced (and therefore low cost), and ingenious configurations have been found for a variety of desired applications. In this chapter we will discuss the more frequently used types of springs, their necessary parametric relationships, and their design.

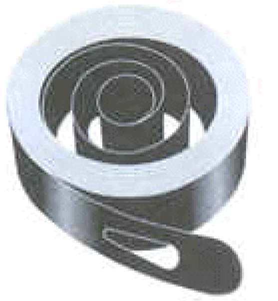
MECHANICAL SPRINGS

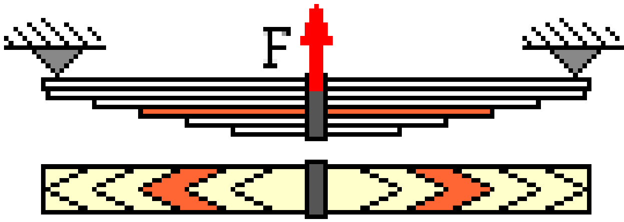




Springs are mechanical elements used in machines for

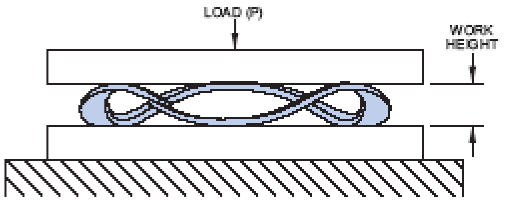
- exerting force,
- storing and absorbing energy, and
- providing flexibility.





Springs are classified as

- Wire springs,
- Flat springs, or
- special-shaped springs



SPRING RATE

- Springs are the mechanical elements which transfer a tensile or compressive force with a certain linear deflection or torque with angular deformation.
- They also store energy and release it when the load or torque is removed from the system.
- They have a characteristic called ‘’spring rate’’ , ‘’spring constant’’ or

‘’scale of spring’’ $k = \frac{F}{y}$ or $k = \frac{dF}{dy}$

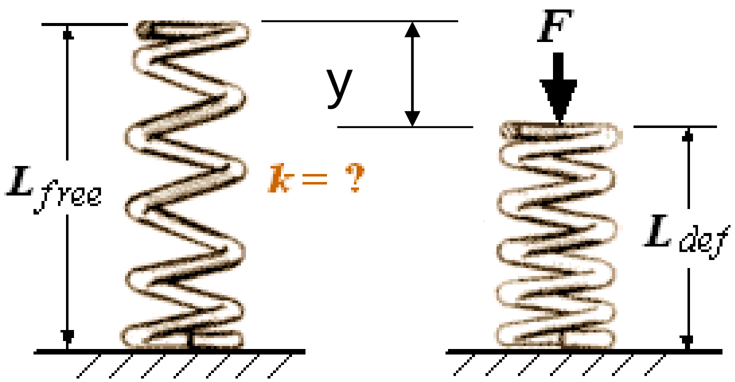


Fig. 2.1 Deflection of compression spring under load

Fig. 2.2 Linear springs
(with constant spring rate)

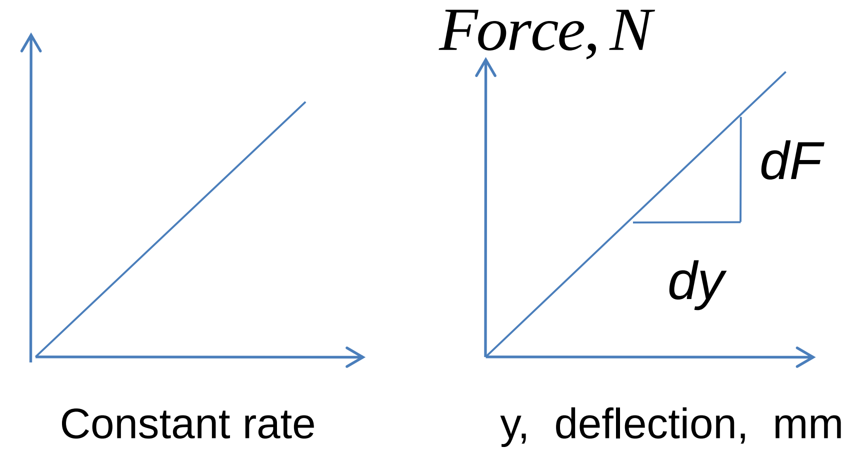
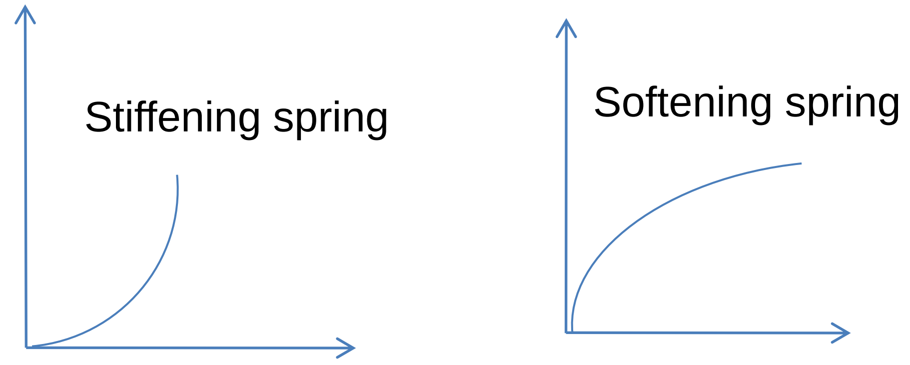


Fig. 2.3 Non-linear springs
(with variable spring rate)



2.2 SPRING TYPES

In general, springs may be classified as:

1. wire springs,
2. flat springs, or
3. special-shaped springs, and

there are variations within these divisions.

Wire springs include helical springs of round or square wire, made to resist and deflect under tensile, compressive, or torsional loads.

Flat springs include cantilever and elliptical types, wound motor- or clock-type power springs, and flat spring washers, usually called Belleville springs.

2.2.1 OTHER SPRING TYPES

A. Wire springs

1)Helical compression springs

- Standard constant rate
- Variable pitch-variable rate
- Barrel
- Hour glass
- Conical

2)Helical extension springs

- Extension springs with hooks
- Draw bar springs (compression springs) used in tensile loading are a kind of extension springs applications.

3)Torsion springs

B. Other springs

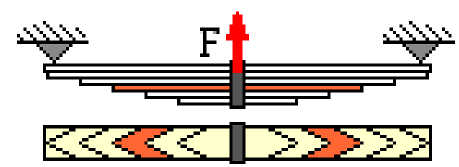
4)Spring washers
(Belleville, wave, slotted finger, curved etc.)

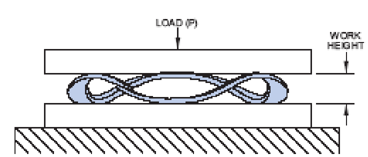
5)Beam springs

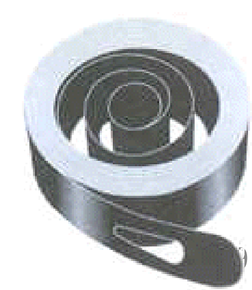
6)Volute springs

7)Constant force

8)Power or motor springs







Here are some examples for wire springs

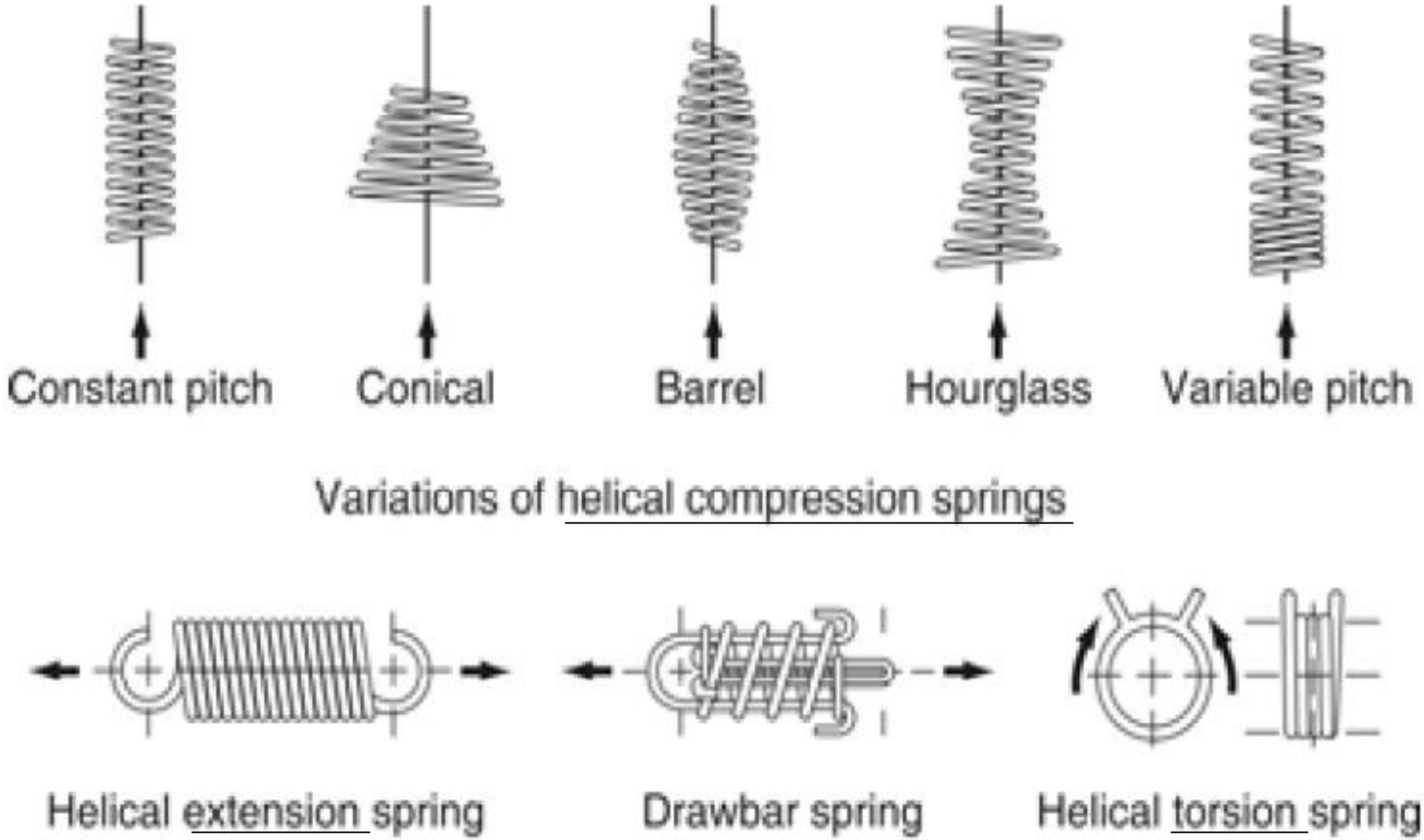


Fig. 2.4 Wire springs of compression, extension and torsion types

These are the other springs (non-wire)

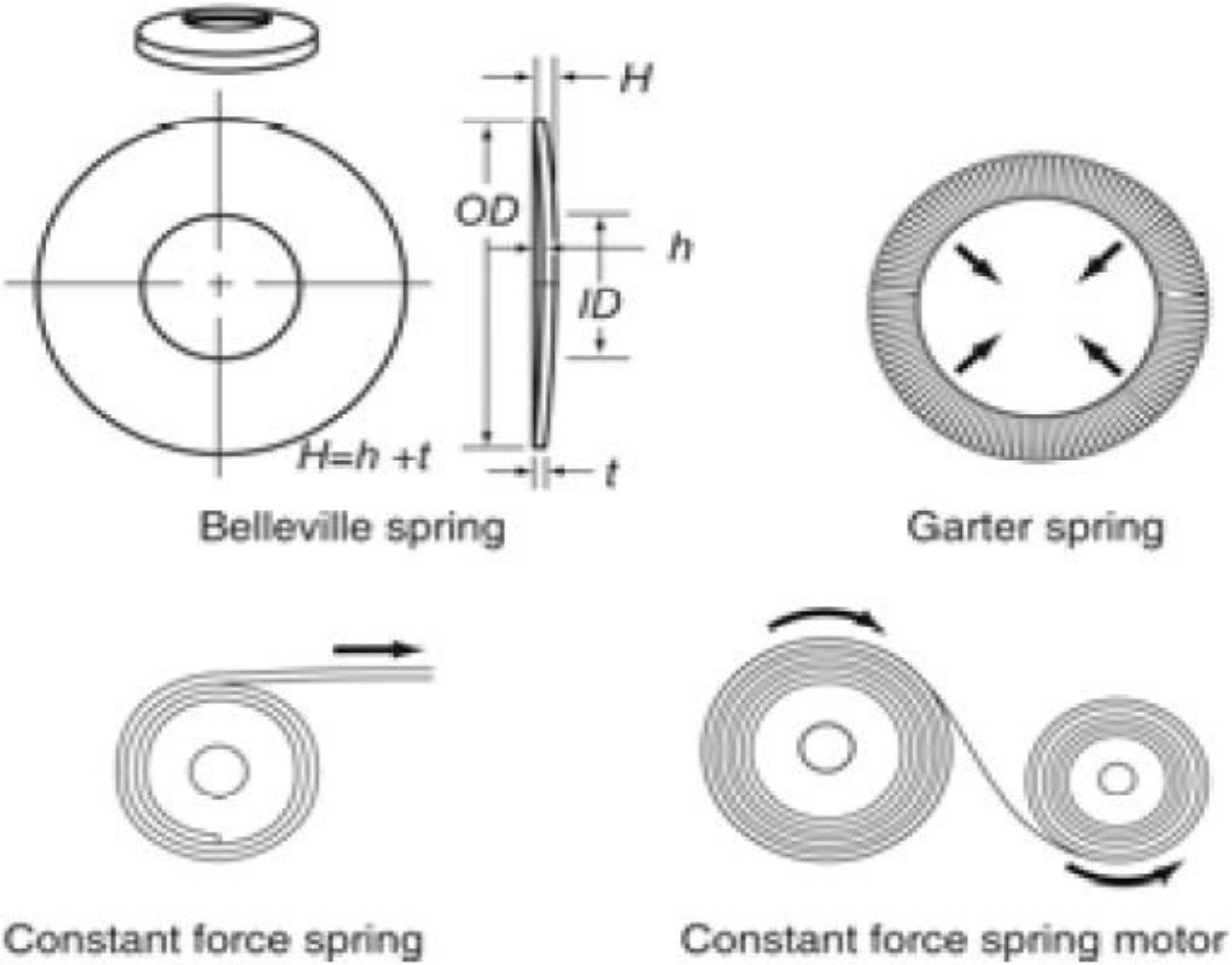


Fig. 2.6 Other type springs

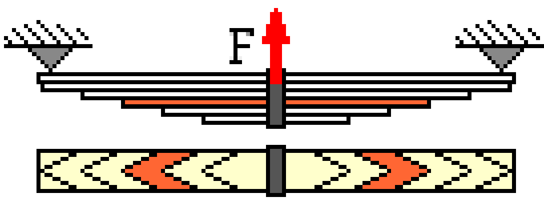
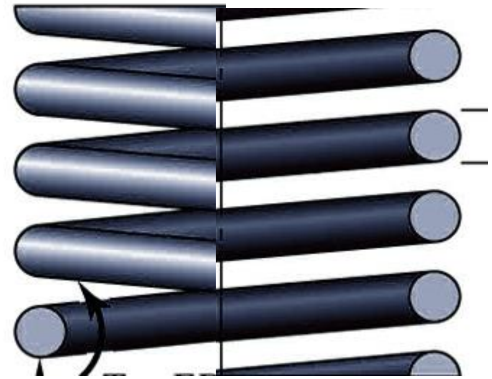


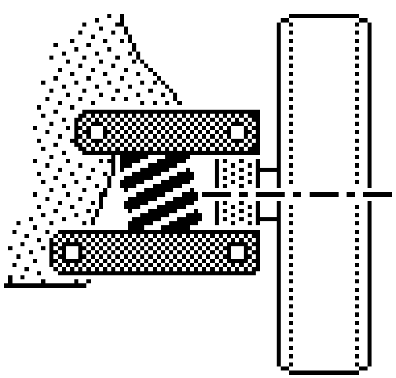
Fig. 2.7 Beam springs

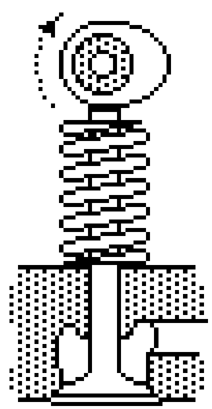
2.2.2 Wire Springs

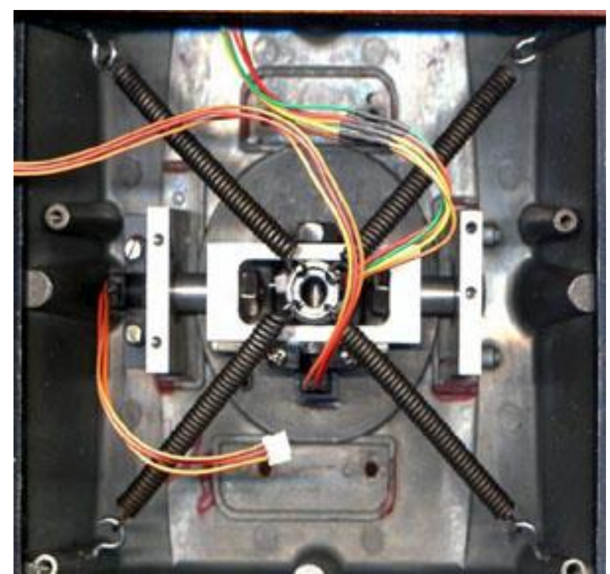
Wire springs are usually manufactured from circular crossection wires in different configurations like

- helical compression springs
- helical extension springs and
- torsion springs

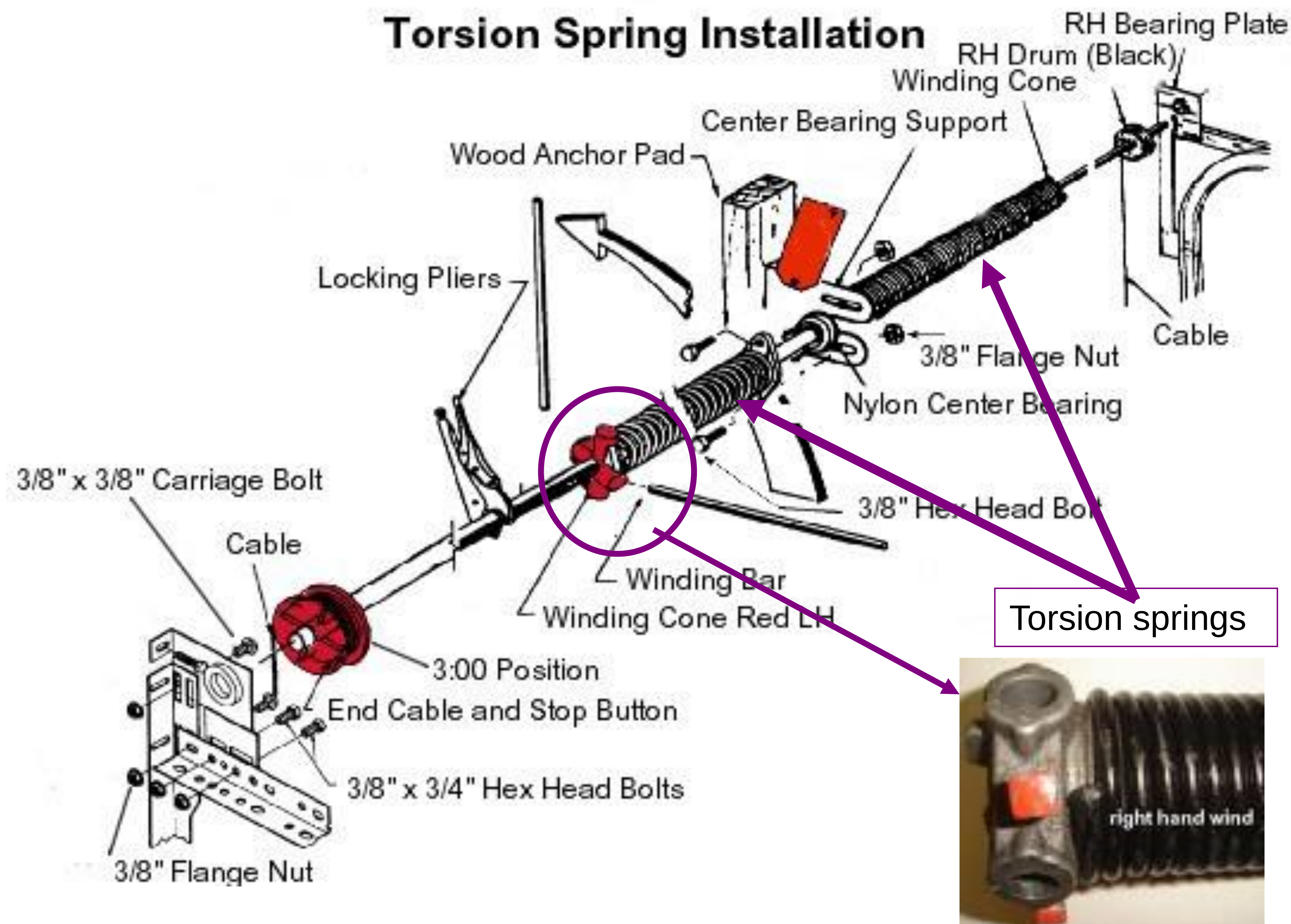




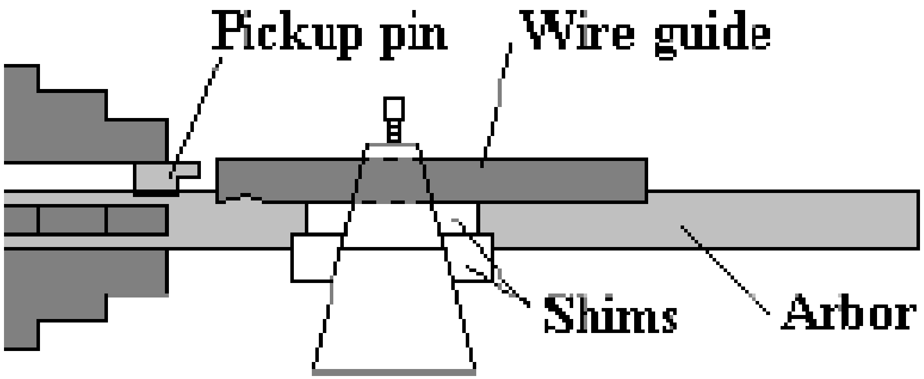


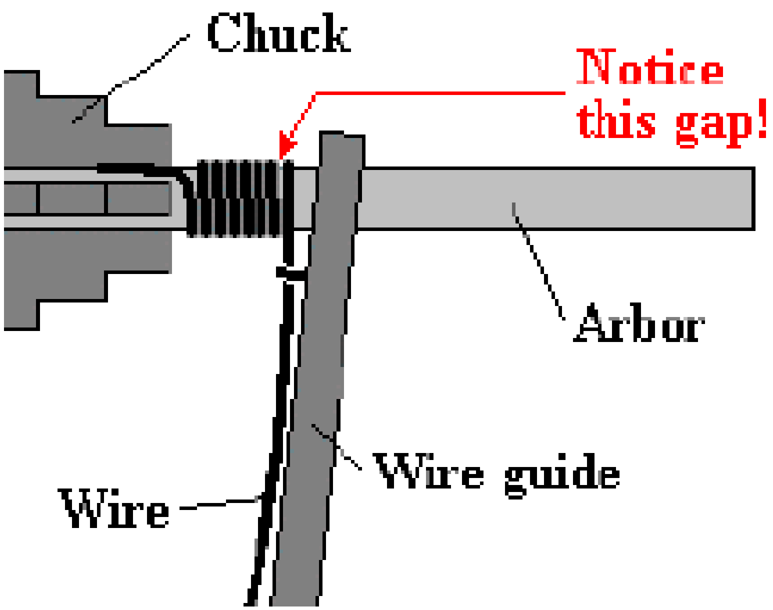


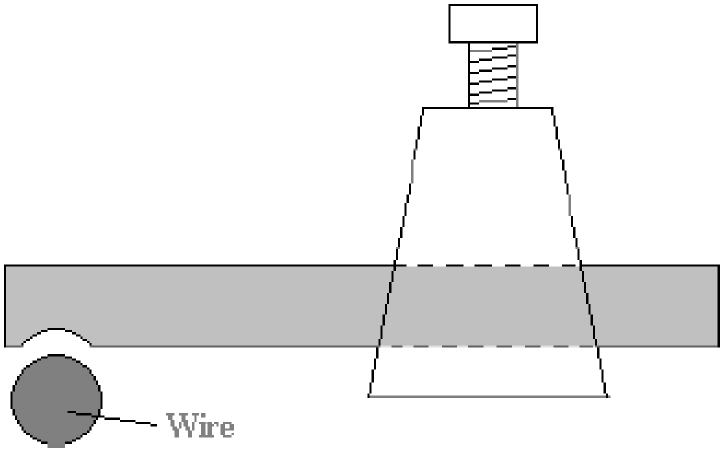
2.2.3 A torsion spring application in a garage door

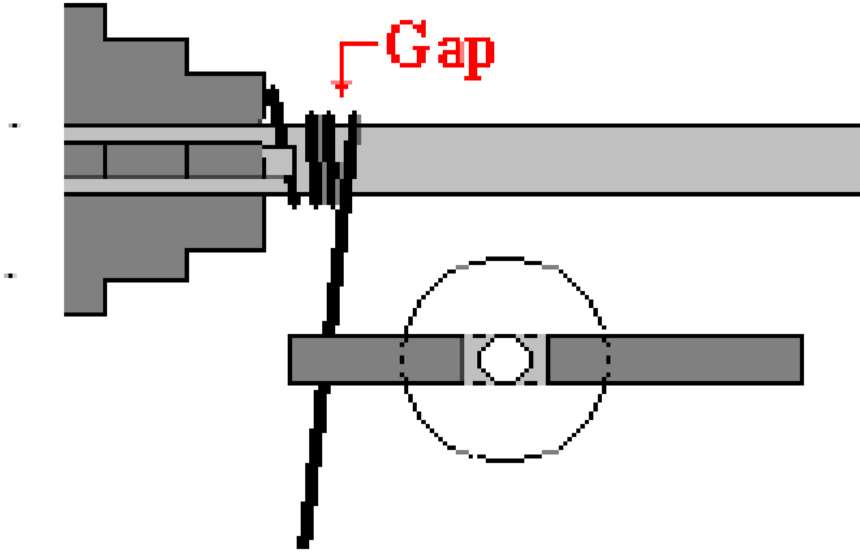


This is how coil springs are manufactured on a lathe





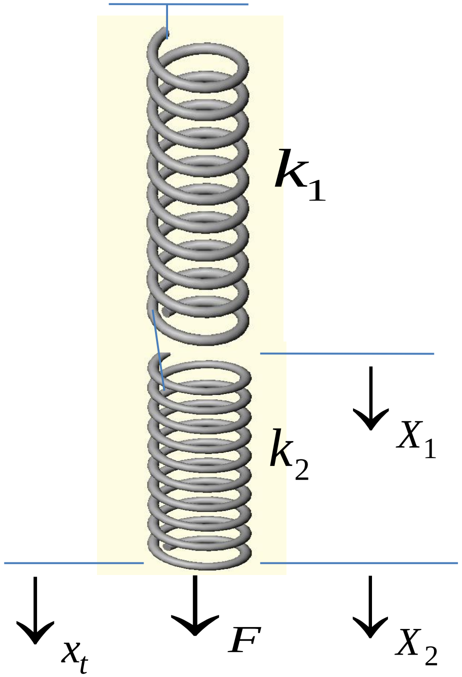




2.3 SPRING CONFIGURATIONS

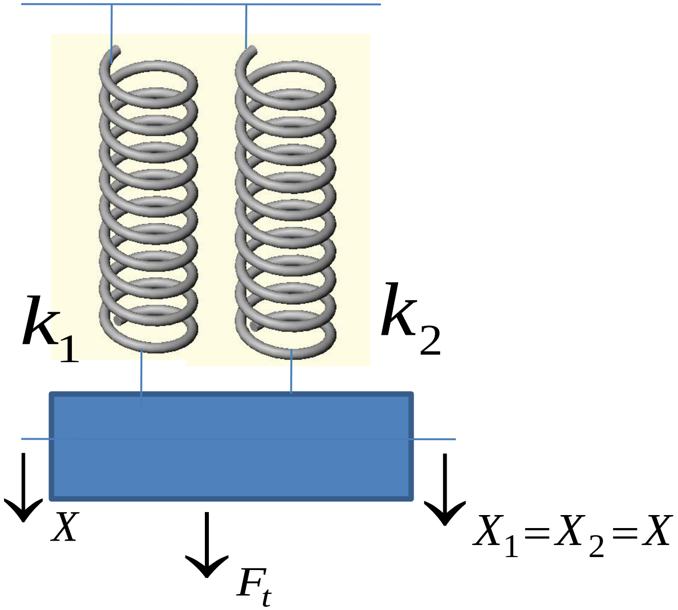
There are two configurations of springs

- a) In series



$X_t = X_1 + X_2$

b) in parallel



a) In series

$$x_t = x_1 + x_2 \boxed{1}$$

$$F_1 = F_2 = F_t \boxed{2}$$

$$F_1 = k_1.x_1 \rightarrow x_1 = \frac{F_1}{k_1}$$

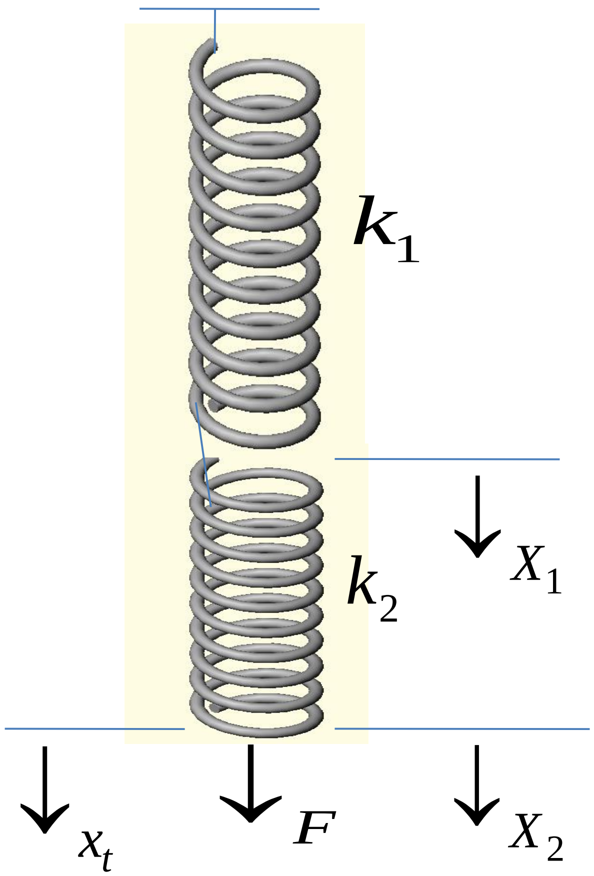
$$F_2 = k_2.x_2 \rightarrow x_2 = \frac{F_2}{k_2}$$

$$F_t = k_t.x_t \rightarrow x_t = \frac{F_t}{k_t}$$

into eqn 1

$$\frac{F_t}{k_t} = \frac{F_1}{k_1} + \frac{F_2}{k_2}, F_t = F_1 = F_2$$

$$\frac{1}{k_t} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots \quad (2.1)$$



k_t is the total system stiffness in series applications.

b) In parallel

$$x_1 = x_2 \, (1)$$

$$F_t = F_1 + F_2 \, (2)$$

$$F_1 = k_1.x_1 = k_1.x$$

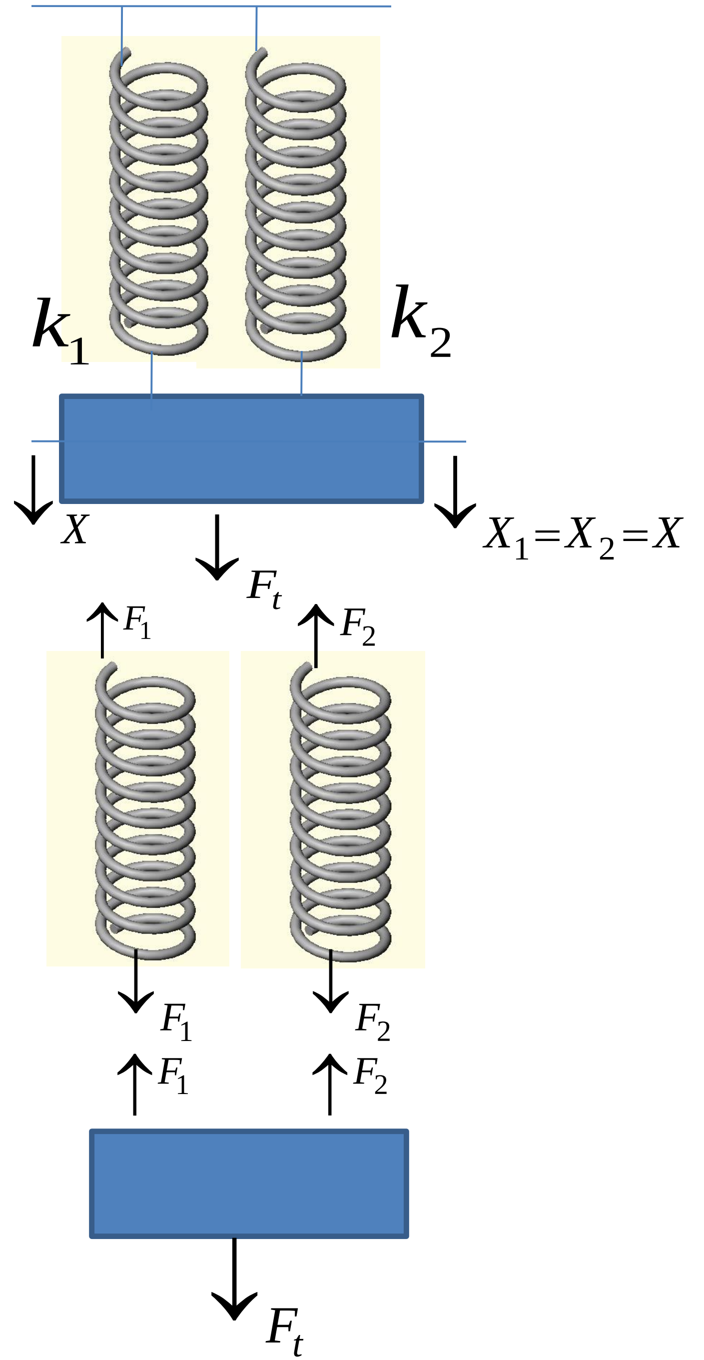
$$F_2 = k_2.x_2 = k_2.x \quad \text{into}(2)$$

$$F_t = k_t.x$$

$$k_t.x = k_1.x + k_2.x$$

$$k_t = k_1 + k_2 + \ldots \qquad (2.2)$$

k_t total system stiffness in parallel applications



2.4 SPRING MATERIALS

- Springs store potential energy while deflecting a noticable amount under reasonably high loads.
- By doing so, they provide maximum elastic energy storage while not failing due to high stresses in material.
- Elastic energy storage capacity or Modulus of resilience was defined as the area under the *σ-ε* curve within elastic range

$$R = \frac{1}{2}\sigma_y \varepsilon \quad = \frac{1}{2}\sigma_y\left(\frac{\sigma_y}{E}\right) \quad = \frac{1}{2}\frac{S_y^2}{E} \qquad (2.3)$$

Therefore springs are required to have:

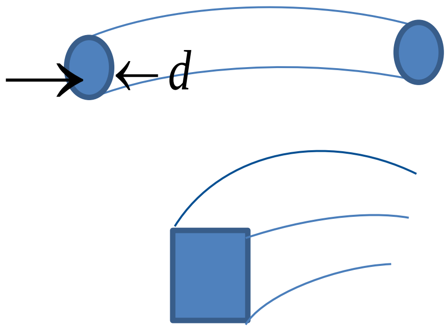
- High yield strength (and hence high ultimate strength) and
- Low modulus of elasticity

There is, however, a limited number of materials and alloys suitable for such applications of springs. Examples are:

- Medium to high carbon steels
- Alloys steels
- Few of stainless steel alloys

2.4.1 SPRING WIRE

Spring wires are usually "round" cross-section despite the fact that some rectangular cross-sections are available.



Wire diameters vary from 0.1 mm to 16 mm.

Common spring wire materials designated in different standards are:

SAE	ASTM No	Materials
1066	A227	cold drawn (hard drawn) wire
1085	A228	music wire
1065	A229	oil tempered wire (general purpose use)
1070	A230	oil tempered wire (valve spring quality-fatigue loading)
6150	A232	chrome vanadium
30302	A313	stainless steel
5254	A401	chrome silicon
	etc.	etc.

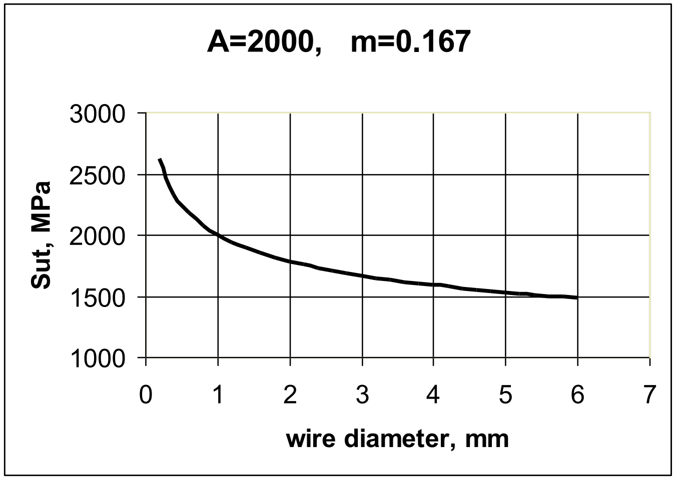
2.4.2 Tensile Strength of Spring Materials

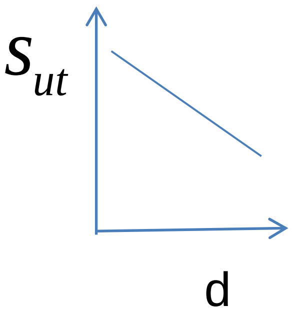
As known from ME 215, Engineering Material I;

•Larger the material or specimen size higher the risk of having non-homogenous material hence lower the material strength (S_y or S_{ut})

•On the contrary, smaller the material or specimen size lower the risk of having non-homogenous material or higher the chance of having a more homogenous and cleaner material hence higher the material strength.

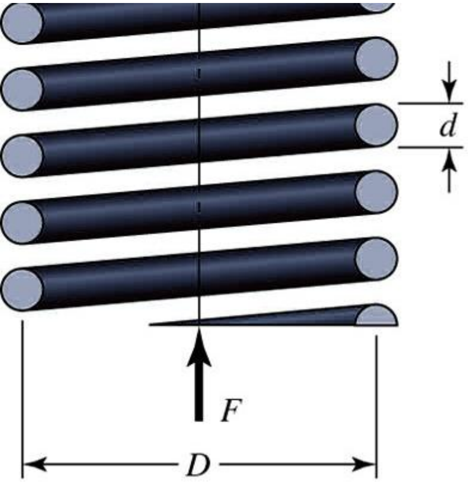
If d $\uparrow$; $S_{ut}\downarrow$ or If d $\downarrow$; $S_{ut}\uparrow$





For spring materials S_{ut} is defined as

$$S_{ut} = \frac{A}{d^m} \qquad (2.4)$$



where d is wire diameter in mm and constants A and m are given in tables for different spring materials
(A= 1750 - 2150 MPa, m= 0.112 - 0.192)

The yield strength in tensile loading (stress) is given as:

$$S_y = 0.75 S_{ut} \qquad (2.5)$$

When applying distortion-energy theory the **yield strength in shear** loading (stress) is given as:

$$S_{sy} = 0.577 S_y \quad or \quad \text{nearly} \quad S_{sy} = 0.60 S_y \qquad (2.6)$$

Similarly, the **ultimate shear strength** (S_{us}) could be taken as

$$S_{us} = 0.60 S_{ut} \qquad (2.7)$$

2.4.3 Materials for Helical Springs

- Springs are most commonly manufactured by hot- or cold-working processes depending upon the material size, the spring index (C), and the desired properties.
- There are a numerous spring materials available for the designer. These include: plain carbon steels, alloy steels, corrosion resisting steels, phosphor bronze (nonferrous alloy), spring brass, beryllium copper and various nickel alloys.

The most common way for selecting spring materials is by looking at their tensile strength, a property that is only defined once the wire diameter is chosen:

$$S_{ut} = \frac{A}{d^m} \qquad (2.4)$$

Table 2.1 Spring materials and constant for estimating tensile strength

Material	**Size range (mm)**	**Exponent, *m***	**Constant, *A* (*MPa.mm^m^*)**
Music wire	0.10-6.5	0.146	2170
Oil-tempered wire	0.50-12	0.186	1880
Hard-drawn wire	0.70-12	0.192	1750
Chrome-vanadium	0.80-12	0.167	2000
Chrome silicone	1.60-10	0.112	2000

$$S_{ut} = \frac{A}{d^m} \qquad (2.4)$$

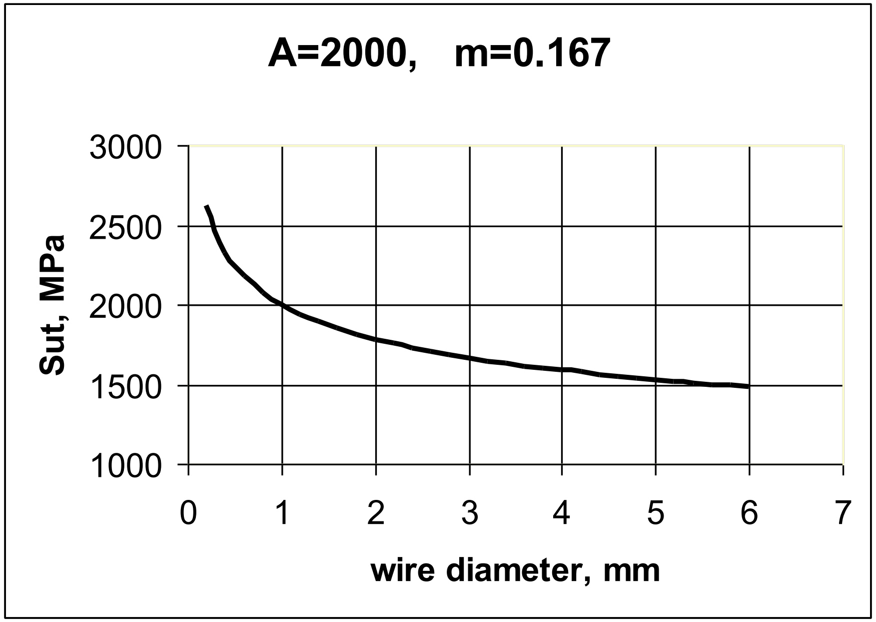
Where

A (MPa.mm^{-m}) is a constant defined through experimentation,

d (mm) is the diameter of the wire and

m is the slope for the Force vs. Displacement graph for the wire.

Strength(S_{ut}) has units of MPa.



S_y usually varies between 60% to 95% of S_{ut}.

Torsional yield strength (*S_{sy}*) of wire can be estimated using distortion energy theory ($S_{sy} = 0.577S_y$).

This then results in the range (steel only):

$$0.35S_{ut} \le S_{sy} \le 0.52S_{ut} \qquad (2.8)$$

Here are the lower limits of S_{sy} for different spring materials:

For Music & Hard-drawn wire:

$$S_{sy} = 0.45 S_{ut} \qquad (2.9)$$

Valve Spring Wire (Cr-Va, Cr-Si), hardened and tempered carbon and low alloy steel wire:

$$S_{sy} \geq 0.50 S_{ut} \qquad (2.10)$$

Nonferrous Materials:

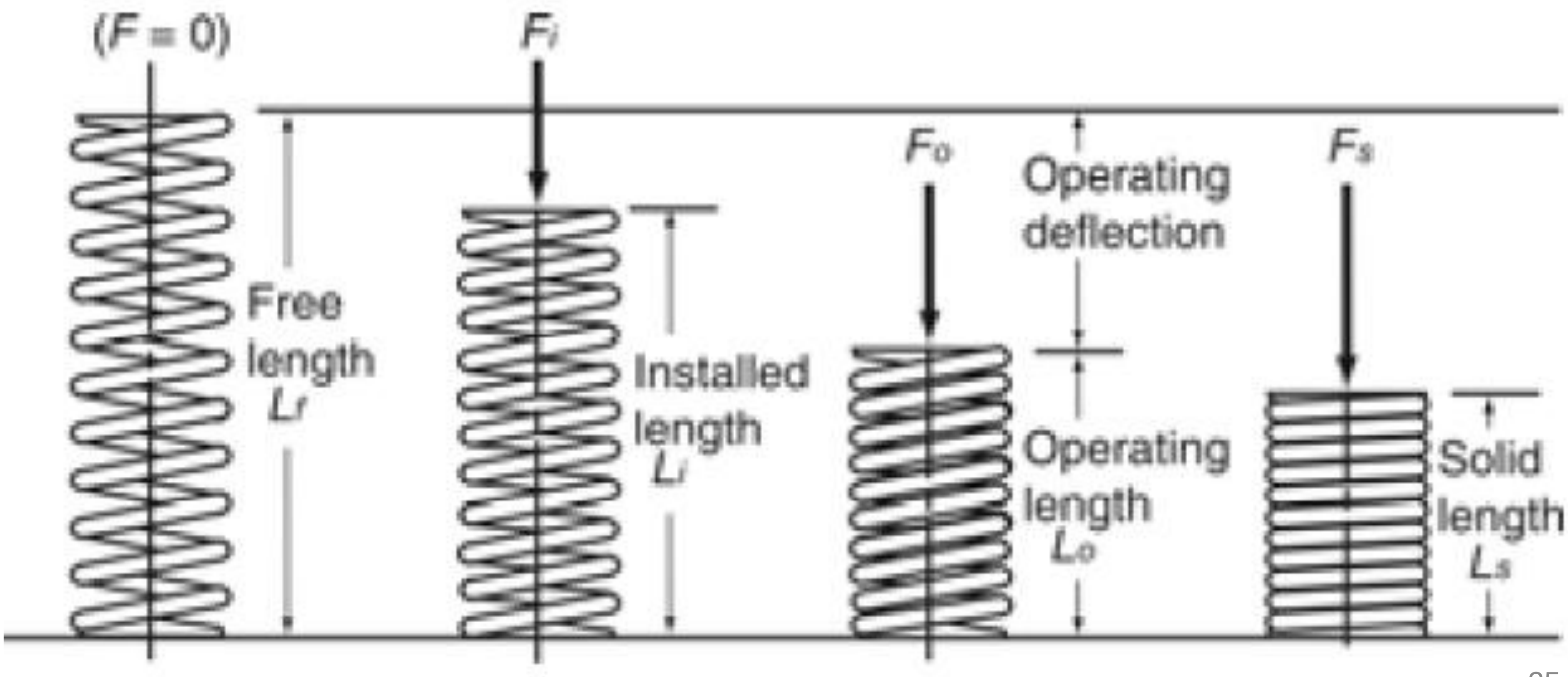
$$S_{sy} \geq 0.35 S_{ut} \qquad (2.11)$$

2.5 Helical Compression Spring Geometry

Compression springs carry only compression loads.

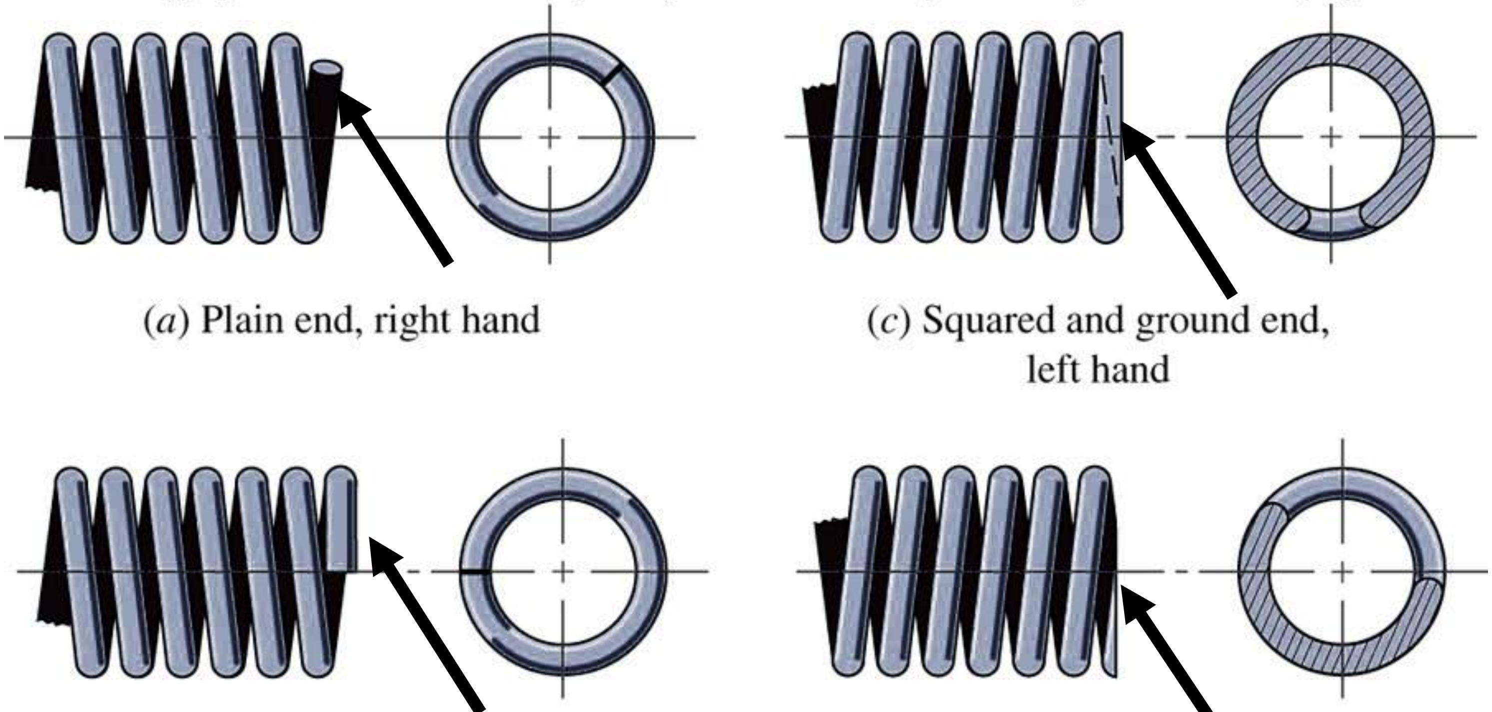
Compression springs are the wire springs wound helically with <u>coils not touching each other under no load and while operating.</u>

The figure below shows compression spring under different loads



There are 4 common types of ends of compressions springs:

- Plain end
- Squared end
- Plain & ground end
- Squared & ground end



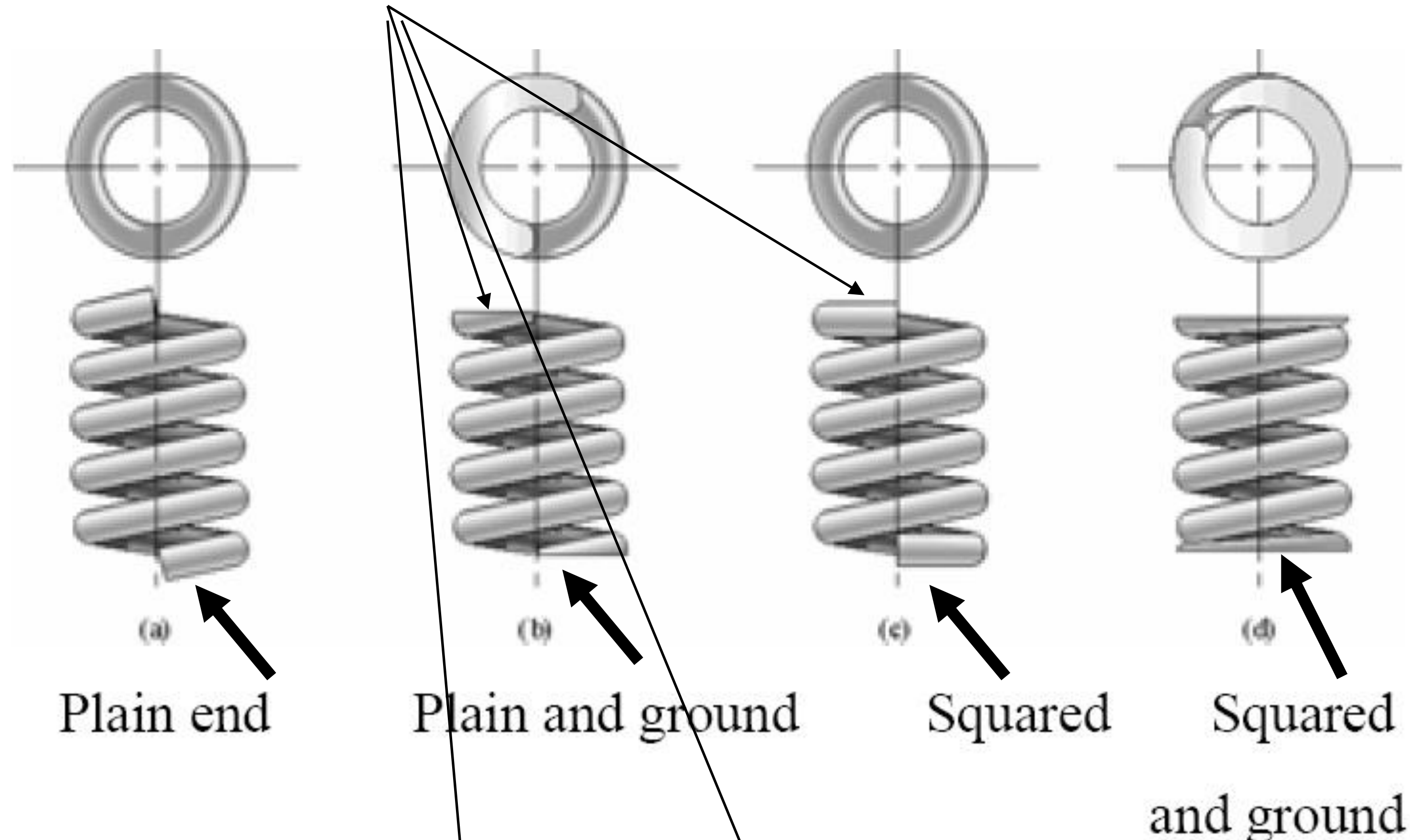
(*a*) Plain end, right hand

(*c*) Squared and ground end, left hand

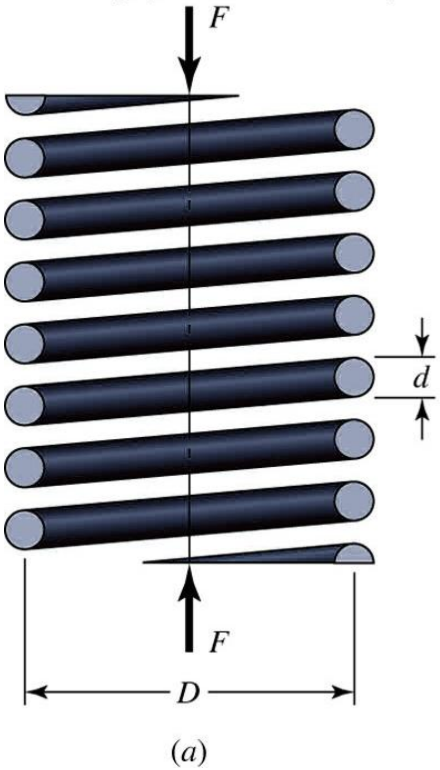
(*b*) Squared or closed end, right hand

(*d*) Plain end, ground, left hand

Here again 4 common types of compressions springs ends and corresponding inactive (dead) coils due to squaring and grinding



Term	Plain	Plain and Ground	Squared or Closed	Squared and Ground
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$



"d" is the wire diameter

"D" is the mean coil diameter

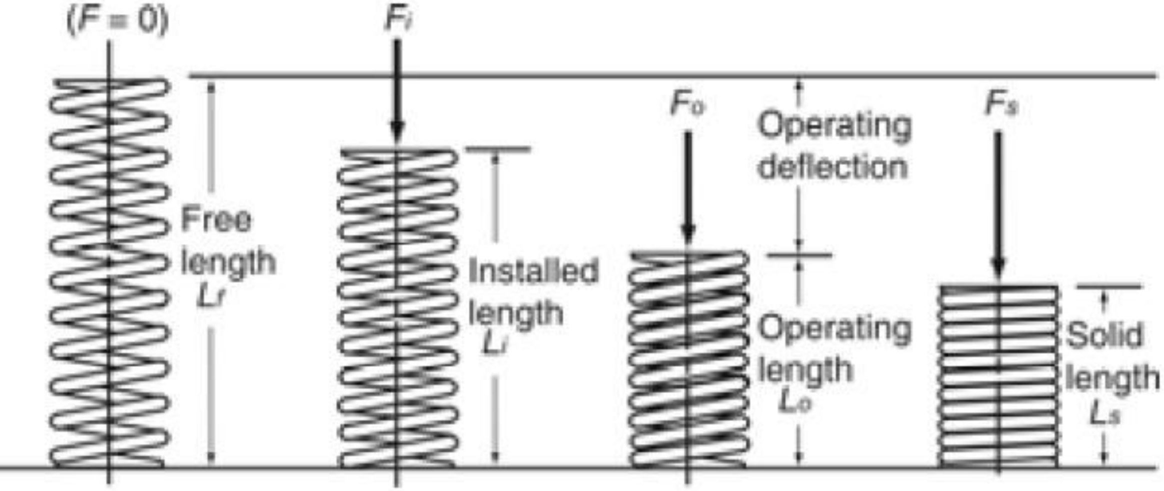
N_t is the number of coils (7.5 coils here)

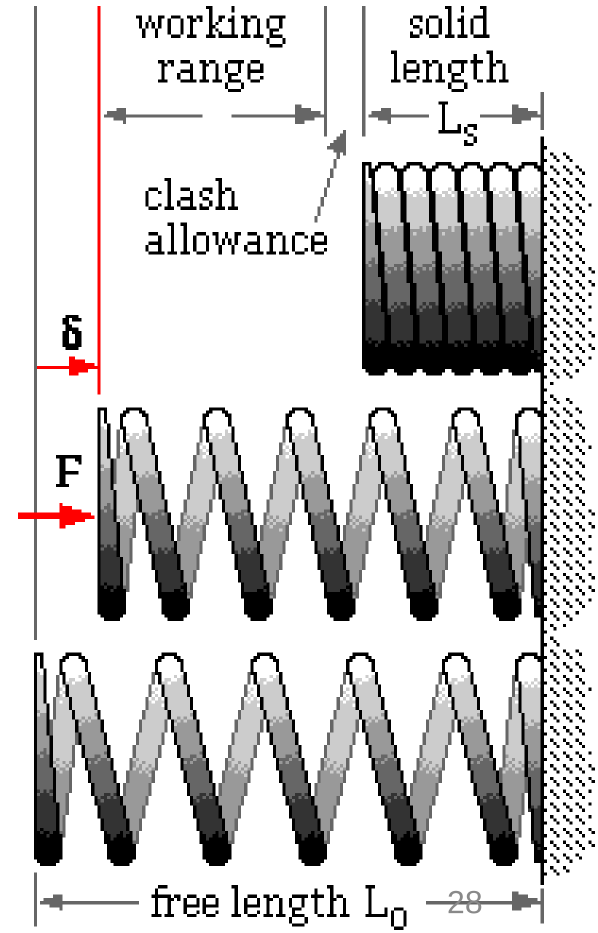
L_f (or L_o) is the free length of the spring

L_s is the solid length of the spring

and spring index, C, is a parameter defined as:

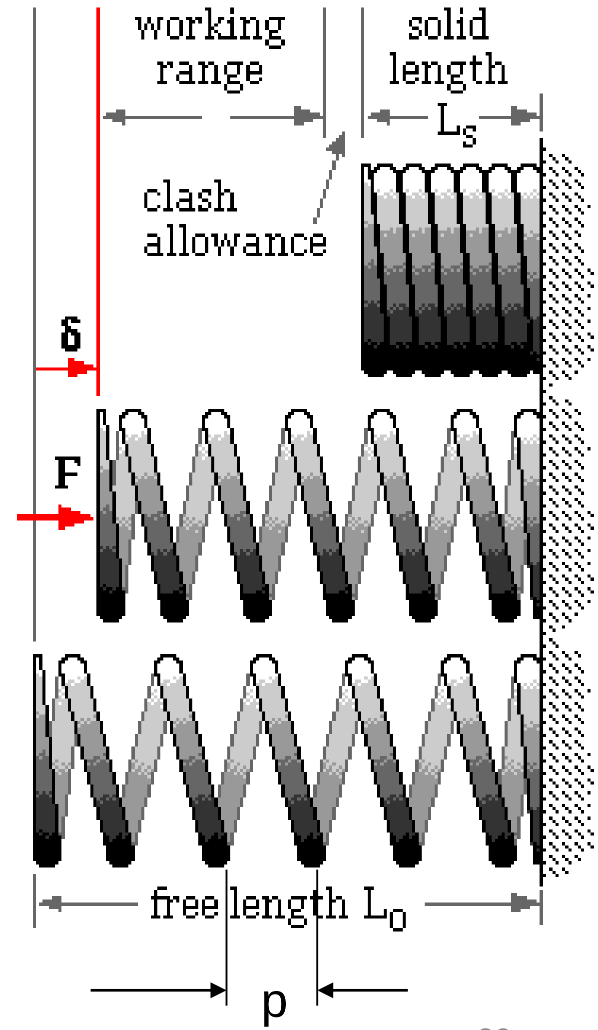
$$C = \frac{D}{d}$$





The solid height/length of a spring is the height of a compression spring when under sufficient load to bring all the coils into contact with adjacent coils.

For squared ground ends $$L_s = (N_t - N_e)d$$



Where N_t = total number of coils,

“N_e” represents the dead coils and can vary depending on the amount ground off (see Table).

“d” is the spring wire diameter.

The free length is represented by L_o.

For a squared and ground end spring this free length (L_o) takes the form:

$$L_o = pN_a + 2d$$

Here are the similar equations for different end conditions:

Term	Type of Spring Ends Plain	Plain and Ground	Squared or Closed	Squared and Ground
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, L_0	$pN_a + d$	$p(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
Pitch, p	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$(L_0 - 3d)/N_a$	$(L_0 - 2d)/N_a$

$$L_s = (N_t - N_e)d \qquad L_o = pN_a + 2d$$

2.5 Helical Compression Springs Under Loads
(Recommended Design Conditions)

Spring Index factor *C* usually takes a value between 4 and 12.

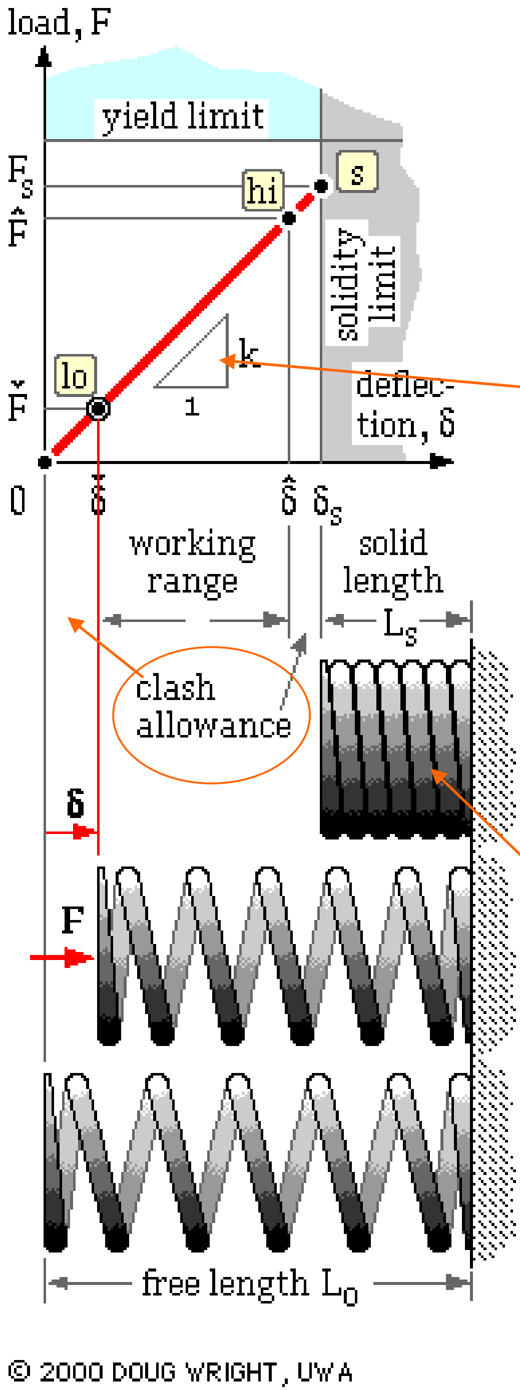
$$C = \frac{D}{d} \qquad 4 \le C \le 12 \qquad (2.12)$$

- For C>>12 springs tend to tangle and thereby require individual packaging (spring are likely to buckle)

- For C<<4 spring wire diameters become too large compared to the diameter of the coil thus increasing the risk of surface cracking when winding the spring (spring are difficult to manufacture)

The number of active coils (when designing springs) is suggested as:

$$3 \le N_a \le 15 \qquad (2.13)$$



Spring rate or stiffness of the spring is the ratio of force applied to corresponding deflection (or the slope of the curve)

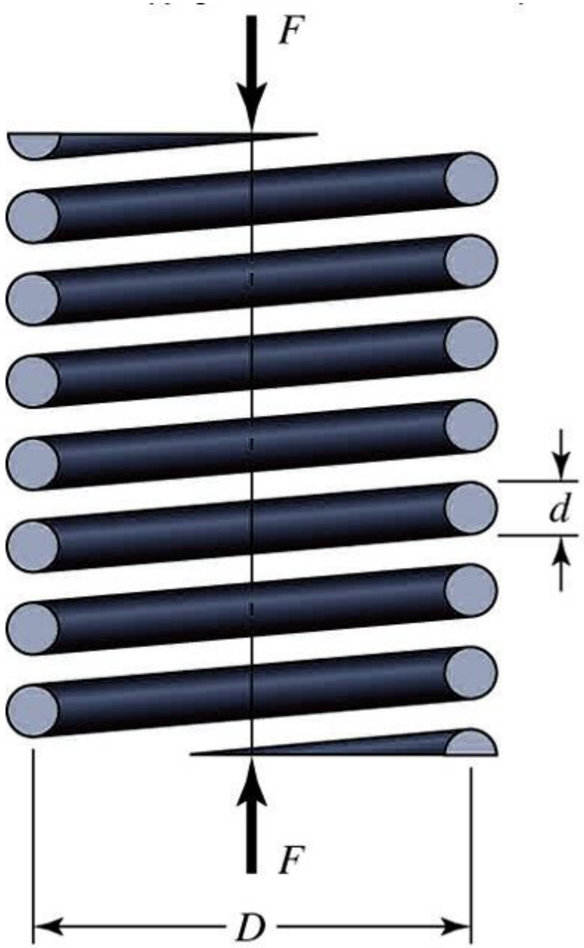
$$k = \frac{F}{y} \qquad (2.14)$$

Springs are not to be used in first and last 15% of the deformation range hence leaving a “**clash allowance**” before the solid condition

$$\xi \geq 0.15 \qquad (2.15)$$

In case the spring is forced to solid condition, design factor of solid height, n_s *is also suggested as*

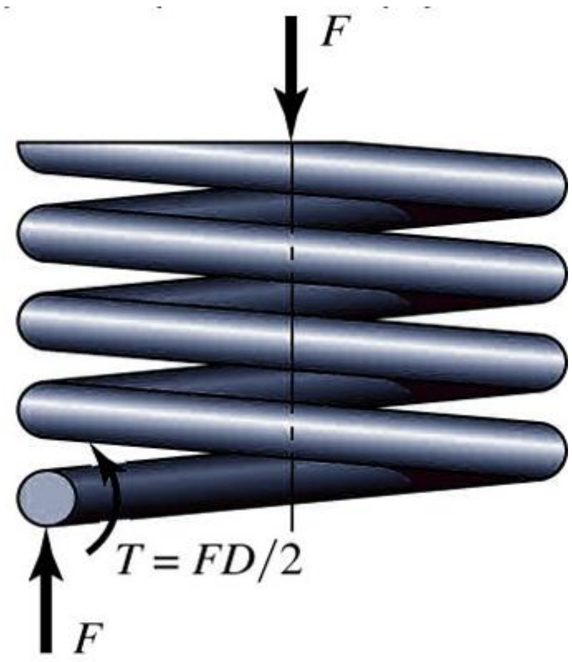
$$n_s \geq 1.2 \qquad (2.16)$$



(a)

Compression spring under
axial load F

Applying section method and taking upper half of the spring



$F_{ree}B_{ody}D_{iagram}$ of the same spring wire being exposed to

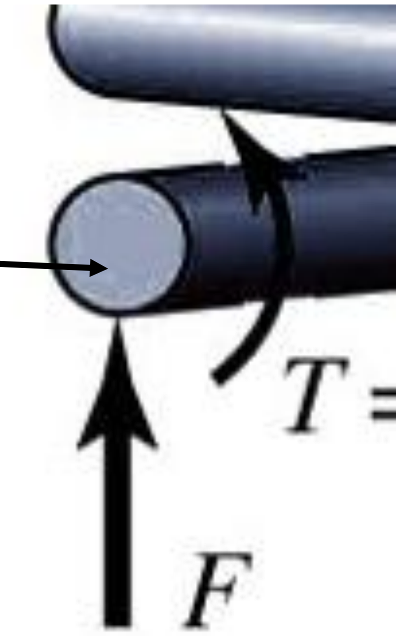
- direct shear (F) and
- torsional shear (T) loads

- Wire cross section resists both direct shear stress and torsional shear stress at the same time

$$\tau_{max} = \frac{F}{A} + \frac{Tr}{J} \qquad (2.17)$$

Direct shear part ($\frac{F}{A}$)

Torsional shear part ($\frac{Tr}{J}$)



Since $T = \frac{FD}{2}$ $\quad r = \frac{d}{2}$ $\quad J = \frac{\pi d^4}{32}$ $\quad A = \frac{\pi d^2}{4}$

The shear stress is then expressed as

$$\tau_{max} = \frac{4F}{\pi d^2} + \frac{8FD}{\pi d^3} \qquad (2.18)$$

Direct shear part ($\frac{4F}{\pi d^2}$)

Torsional shear part ($\frac{8FD}{\pi d^3}$)

Defining the spring index, C=D/d and using it in the stress equation

$$\tau_{\max} = \frac{4F}{\pi d^2} + \frac{8FD}{\pi d^3} \qquad (2.18)$$

leads to the following expression for the maximum stress:

$$\tau_{\max} = \frac{8FD}{\pi d^3}\left(1 + \frac{0.5}{C}\right) \qquad (2.19)$$

Now defining the shear stress correction factor, K_s, as

$$K_s = \left(1 + \frac{0.5}{C}\right) = \left(\frac{2C+1}{2C}\right) \qquad (2.20)$$

the maximum shear stress in the spring element is then given as

$$\tau_{\max} = K_s \frac{8FD}{\pi d^3} \qquad (2.21)$$

or in general terms

$$\tau_{\max} = K \frac{8FD}{\pi d^3} \qquad (2.22)$$

Within the general equation

$$\tau_{\max} = K\frac{8FD}{\pi d^3} \qquad (2.22)$$

$$K_W = \frac{4C-1}{4C-4} + \frac{0.615}{C} \qquad (2.23)$$

K_W is called "Wahl correction factor" and includes two types of effects:

- Shear stress concentration (direct shear) effect (K_s) and
- curvature effect (K_c) due to circular coil shape

K_s was derived for a straight wire and does not include curvature effect.

Thus the actual multiplication factor for coil springs is K (including both direct shear and curvature effects) and can be defined as:

$$K_W = K_S K_C \qquad (2.24)$$

K_C is the curvature effect and can be found from

$$K_C = \frac{K_W}{K_S} \qquad (2.25)$$

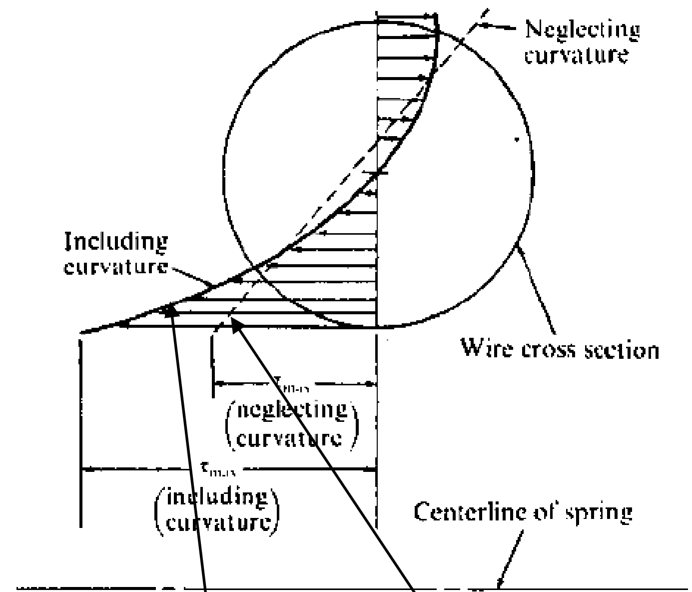
2.5.2 The Effect of Curvature on Stress

$$\tau_{\max} = \frac{4F}{\pi d^2} + \frac{8FD}{\pi d^3} \quad (2.18) \qquad \tau_{\max} = \frac{8FD}{\pi d^3}\left(1 + \frac{0.5}{C}\right) \quad (2.19) \qquad \tau_{\max} = K_s \frac{8FD}{\pi d^3} \quad (2.21)$$

Original equations above are based upon the assumption of wire being straight.

The curvature of the wire actually <u>increases the stress</u> on the inside of the wire and decreases the stress on the outside of the wire.

Therefore K_s in equations is replaced by K which corrects for both the curvature and the direct shear effects.



Following two equations could be used for factor K since the results are so close to each other.

$$\tau_{\max} = K_s \frac{8FD}{\pi d^3} \quad (2.21)$$

$$K_W = \frac{4C-1}{4C-4} + \frac{0.615}{C} \quad (2.23) \qquad K_B = \frac{4C+2}{4C-3} \quad (2.26) \qquad \tau_{\max} = K \frac{8FD}{\pi d^3} \quad (2.22)$$

K_W is called "Wahl factor", K_B is called "Bergstrasser factor"

- In static type loadings (constant load), the curvature factor K_c **will be neglected** and **only factor** K_s will be used within equation

$$\tau = K_s \frac{8FD}{\pi d^3}$$

- Whereas in fatigue type loadings (varying load), the curvature factor K_c will be used, but not as a stress raiser, on the contrary as a strength reduction factor in S_{se}. Factor K_s will still be used in stress equation

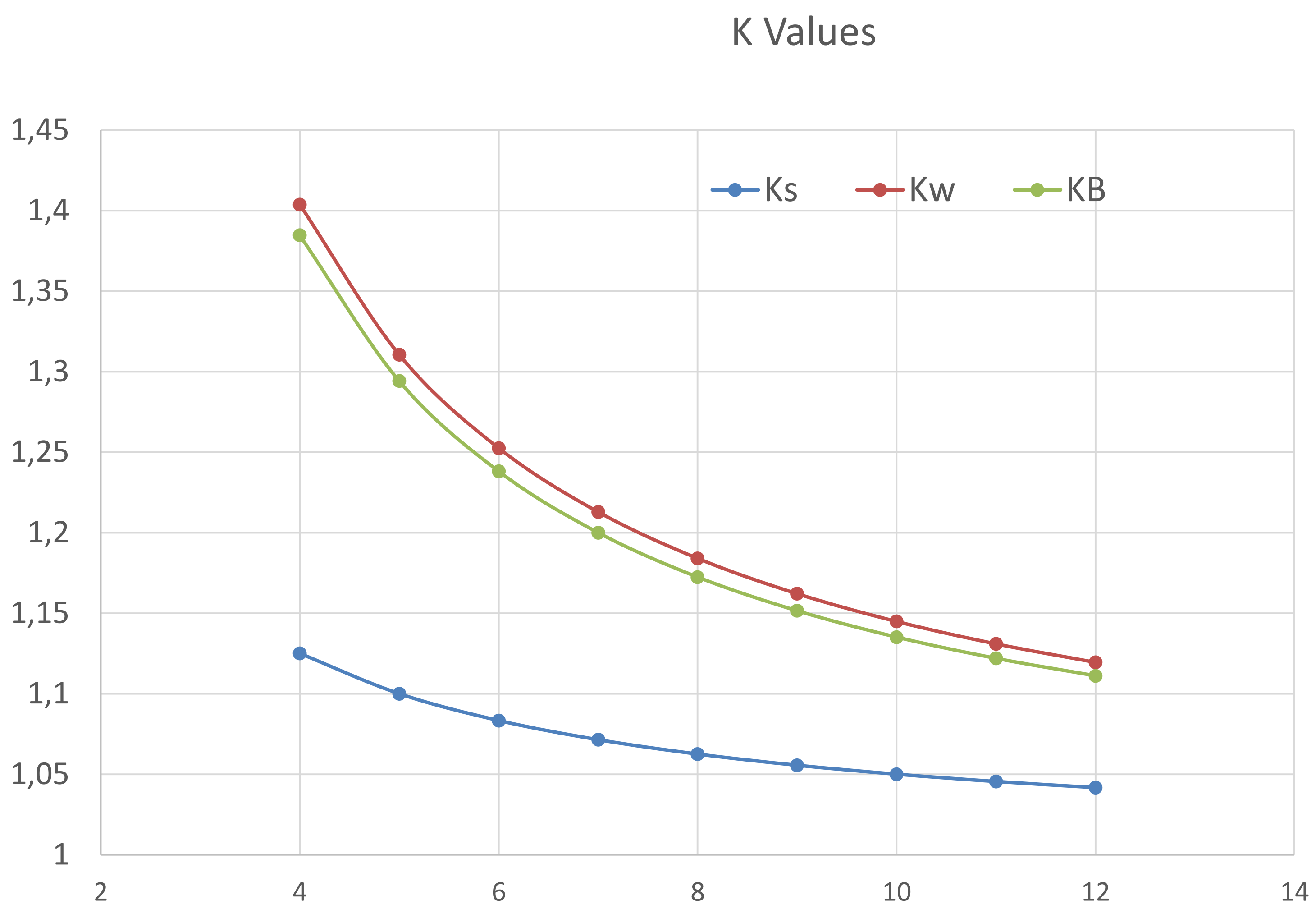
$$K_C = \frac{K_B}{K_S} > 1 \longrightarrow k_e = \frac{1}{K_C} \qquad \tau = K_s \frac{8FD}{\pi d^3}$$

$$S_{se} = k_a * k_b * k_c * k_d * k_e * k_f * S_{se}' \qquad (2.27)$$

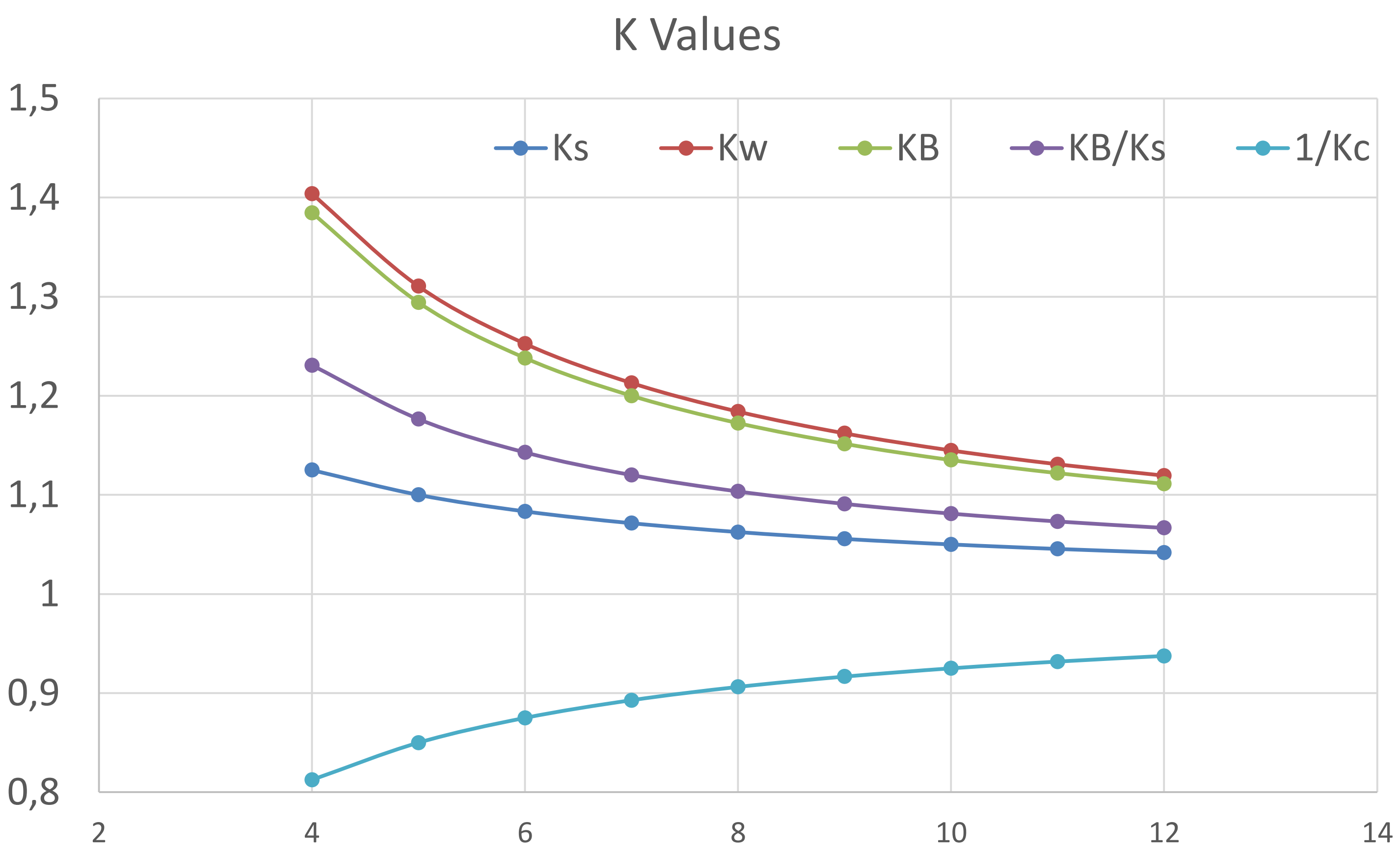
If we calculate K values for different C values:

				Kc	ke
C	**Ks**	**Kw**	**KB**	**KB/Ks**	**1/Kc**
4	1,125	1,40375	1,38462	1,23077	0,81250
5	1,1	1,3105	1,29412	1,17647	0,85000
6	1,08333	1,2525	1,2381	1,14286	0,87500
7	1,07143	1,21286	1,2	1,12	0,89286
8	1,0625	1,18402	1,17241	1,10345	0,90625
9	1,05556	1,16208	1,15152	1,09091	0,91667
10	1,05	1,14483	1,13514	1,08108	0,92500
11	1,04545	1,13091	1,12195	1,07317	0,93182
12	1,04167	1,11943	1,11111	1,06667	0,93750

Graph of K values for different C values:



Graph of K values for different C values:



Remembering from failure theories (static loading case) that:

- For a safe spring under maximum load, the maximum stress created within the spring material should be less than the strength of the spring material,
- Or the ratio of spring material strength to the maximum stress created in the spring should be more than unity.

$$\tau_{\max} \leq S_{sy} \quad or \quad \tau_{\max} = K \frac{8F_{\max} D}{\pi d^3} < S_{sy}$$

$$n_s = \frac{S_{sy}}{\tau_{\max}} \geq 1.0$$

Strain Energy

The external work done on an elastic member in deforming it is transformed into *strain,* or *potential, energy*. If the member is deformed a distance *y*, and if the force-deflection relationship is linear, this energy is equal to the product of the average force and the deflection, or

$$k = \frac{F}{y} \qquad (2.14) \quad \text{or} \quad y = \frac{F}{k}$$

$$U = \frac{F}{2} y = \frac{F^2}{2k}$$

This equation is general in the sense that the force *F* can also mean torque, or moment, provided, of course, that consistent units are used for *k*. By substituting appropriate expressions for *k*, strain-energy formulas for various simple loadings may be obtained. For tension and compression and for torsion, for example, we employ Eqs. (2–29) and (2–30)

$$\delta = \frac{Fl}{AE} \qquad\qquad \theta = \frac{Tl}{GJ}$$

$$k = \frac{AE}{l} \qquad (2.29)$$

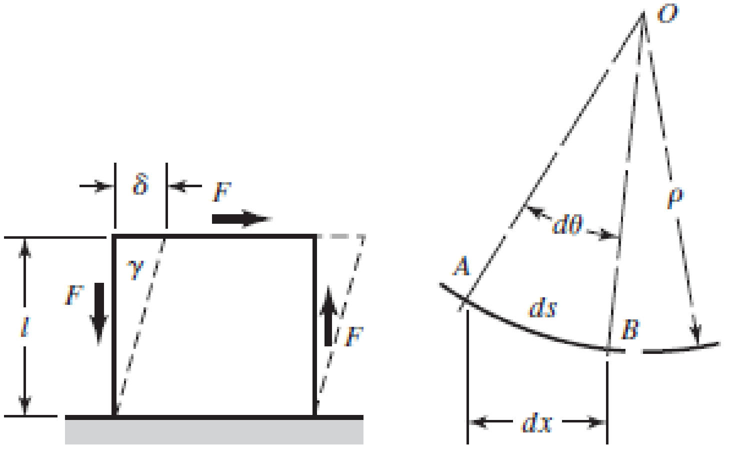
$$k = \frac{T}{\theta} = \frac{GJ}{l} \qquad (2.30)$$

and obtain

$$U = \frac{F^2 l}{2AE} \qquad \text{tension and compression} \qquad (2\text{-}31)$$

$$U = \frac{T^2 l}{2GJ} \qquad \text{torsion} \qquad (2\text{-}32)$$

To obtain an expression for the strain energy due to direct shear, consider the element with one side fixed in Fig. *a*. The force *F* places the element in pure shear, and the work done is $U = F\delta/2$. Since the shear strain is $\gamma = \delta/l = \tau/G = F/AG$, we have



(*a*) Pure shear element (*b*) Beam bending element

$$U = \frac{F^2 l}{2AG} \qquad \text{direct shear} \qquad (2\text{-}33)$$

Castigliano's Theorem

A most unusual, powerful, and often surprisingly simple approach to deflection analysis by an energy method called *Castigliano's theorem*. It is a unique way of analyzing deflections and is even useful for finding the reactions of indeterminate structures. **Castigliano's theorem states that *when forces act on elastic systems subject to small displacements, the displacement corresponding to any force, in the direction of the force, is equal to the partial derivative of the total strain energy with respect to that force.*** The terms *force* and *displacement* in this statement are broadly interpreted to apply equally to moments and angular displacements. Mathematically, the theorem of Castigliano is

$$\delta_i = \frac{\delta U}{\delta F_i} \qquad (2-34)$$

where δ*i* is the displacement of the point of application of the force *Fi* in the direction of *Fi* . For rotational displacement Eq. (2–34) can be written as

$$\theta_i = \frac{\delta U}{\delta M_i} \qquad (2-35)$$

where θ*i* is the rotational displacement, in radians, of the beam where the moment *Mi* exists and in the direction of *Mi* .

As an example, apply Castigliano's theorem using Eqs. (2–31) and (2–32) to get the axial and torsional deflections. The results are

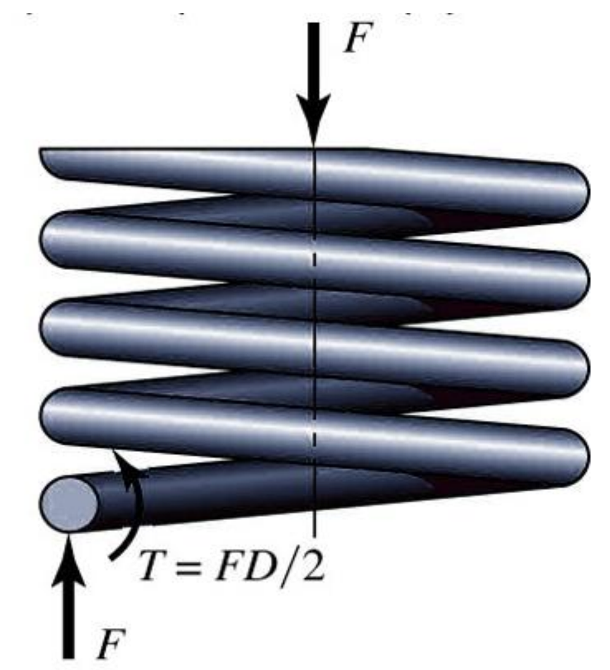
$$\delta = \frac{\delta}{\delta F}\left(\frac{F^2 l}{2AE}\right) = \frac{Fl}{AE}$$

$$\theta = \frac{\delta}{\delta T}\left(\frac{T^2 l}{2GJ}\right) = \frac{Tl}{GJ}$$

$$U = \frac{F^2 l}{2AG} \qquad \text{direct shear} \qquad (2\text{-}33)$$

2.5.3 Deflection and Stiffness of Helical Springs

By using Castiglione's theorem, the total strain energy for a helical spring is composed of a torsional component and a shear component.



Now if the spring is deformed a distance y and if the Force– displacement relationship is elastic (linear), the strain energy is equal to the product of the average force and the deflection.

$$U = \frac{F}{2} y = \frac{F^2}{2k} \qquad U = \frac{T^2 l}{2GJ} + \frac{F^2 l}{2AG}$$

Substituting

$$T = \frac{FD}{2}, l = \pi D N_a, J = \frac{\pi d^4}{32}, A = \frac{\pi d^2}{4}, U = \frac{4F^2 D^3 N_a}{d^4 G} + \frac{2F^2 D N_a}{d^2 G}$$

$N_a = Active\ Coil\ Number$

Therefore to find total deflection the total strain energy is partially derivated wrt the force *F*

$$U = \frac{4F^2D^3N_a}{d^4G} + \frac{2F^2DN_a}{d^2G}$$

$$y = \frac{\partial U}{\partial F} = \frac{8FD^3N_a}{d^4G} + \frac{4FDN_a}{d^2G}$$

Since $C = D/d$

$$y = \frac{8FD^3N_a}{d^4G}\left(1 + \frac{1}{2C^2}\right)$$

$$y \cong \frac{8FD^3N_a}{d^4G} \qquad (2.28)$$

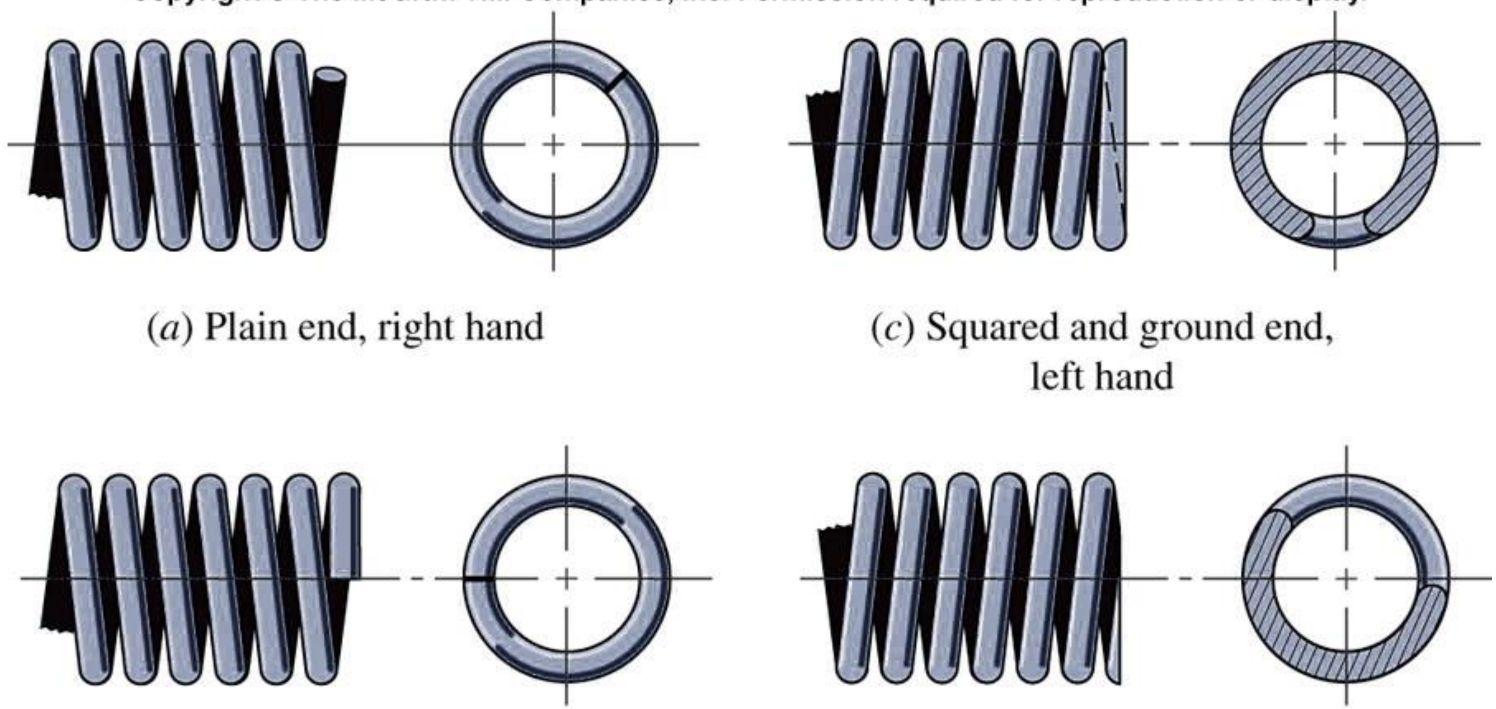
The spring rate (known also as the *scale* of the spring) is : $k = \frac{F}{y}$

$$k \cong \frac{d^4G}{8D^3N_a} \qquad (2.29)$$

k is spring constant

N_a is active coil number

- Remembering that there are 4 common types of compression spring ends with some coils made inactive (dead) due to squaring and grinding:



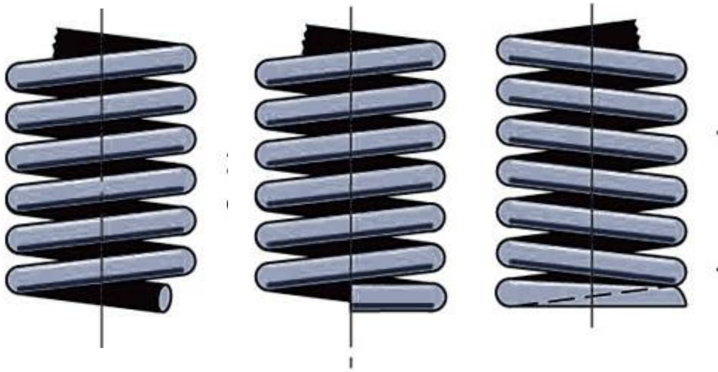
(*a*) Plain end, right hand

(*c*) Squared and ground end, left hand

(*b*) Squared or closed end, right hand

(*d*) Plain end, ground, left hand

- For important or critical applications springs should be both squared and ground for better load transfer <u>and stability.</u>
- Stability means a spring <u>will not buckle</u> under load

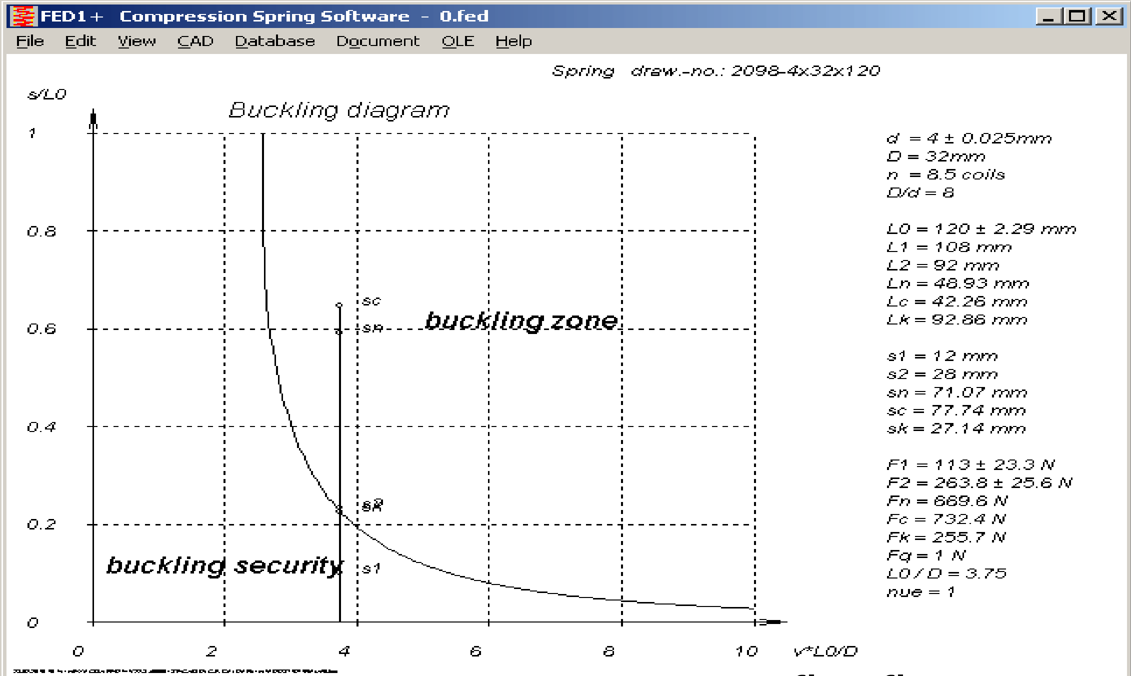


A long/tall spring with small mean diameter will easily buckle (similar to long columns) under load and this will prevent the functioning of the spring.

This condition of buckling (also called spring surge), therefore, <u>**is a failure of the spring.**</u>

To prevent buckling of springs the geometrical ratios between
- free length and mean diameter and L_f /D
- deflection under maximum load and free length, y / L_f

have to be kept in certain limits.



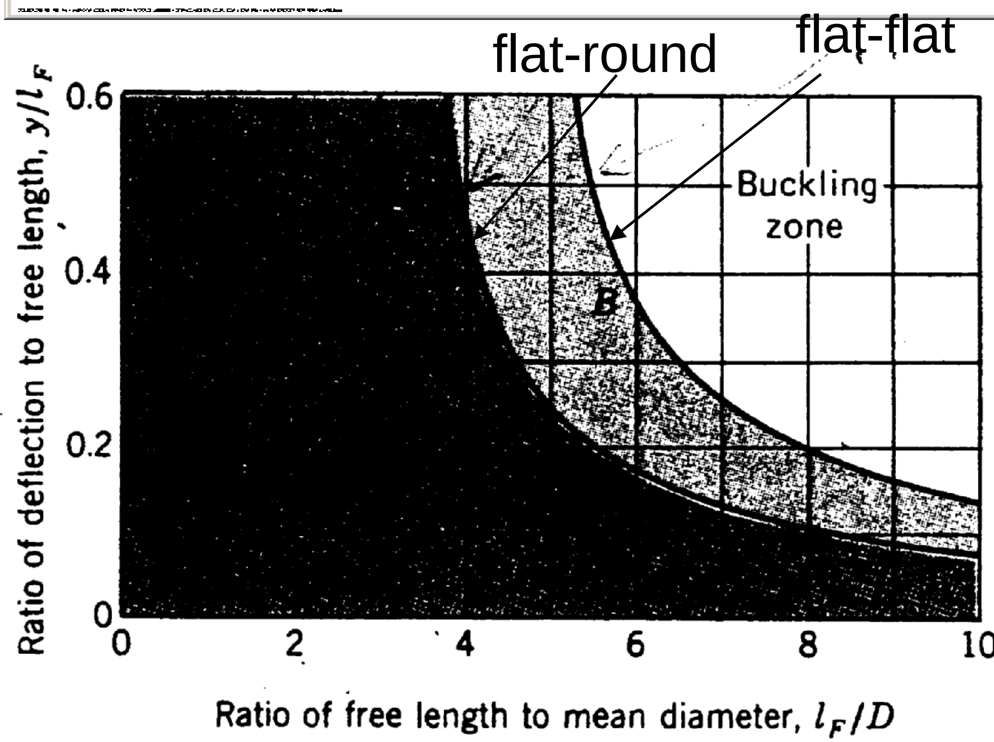


FIGURE 10-4 Curves show when buckling of compression coil springs may occur. Both curves are for springs having squared and ground ends. For curve *A* one end of the spring is compressed against a flat surface, the other against a rounded surface. For curve *B* both ends of the spring are compressed against flat and parallel surfaces.

Since springs are flexible in nature they may buckle depending on the end conditions and L_f/D and y/L_f ratios when they are loaded in compression.

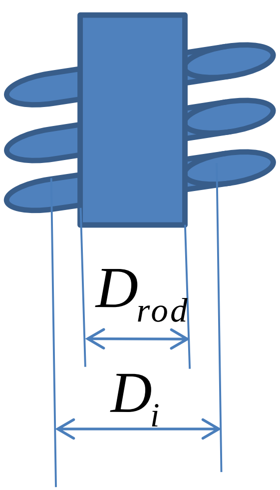
<u>To prevent buckling</u>:

- 1) Either the ratios of y/L_f and L_f/D should be kept in certain limits given in figure in text book.

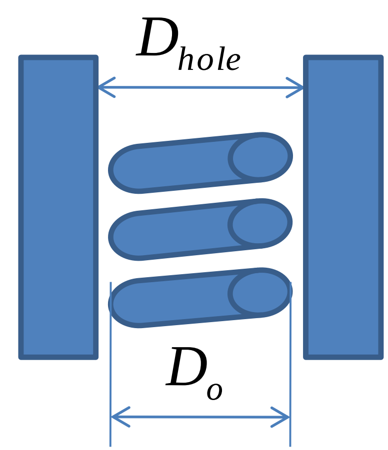
- 2) or a rod should be inserted through the spring to hold it straight $D_{rod} < D_{inside spring}$

- 3) or it should be inserted into a hole

$$D_{hole} > D_{outside spring}$$



$$(D_i - D_{rod}) \geq 0.1d$$



$$(D_{hole} - D_o) \geq 0.1d$$

2.6 Buckling of the Springs in Compression

Springs with

$L_f/D > 3.8$ are likely to fail by buckling

$L_f/D < 3.8$ are likely not to buckle (SAFE).

MECHANICAL SPRINGS

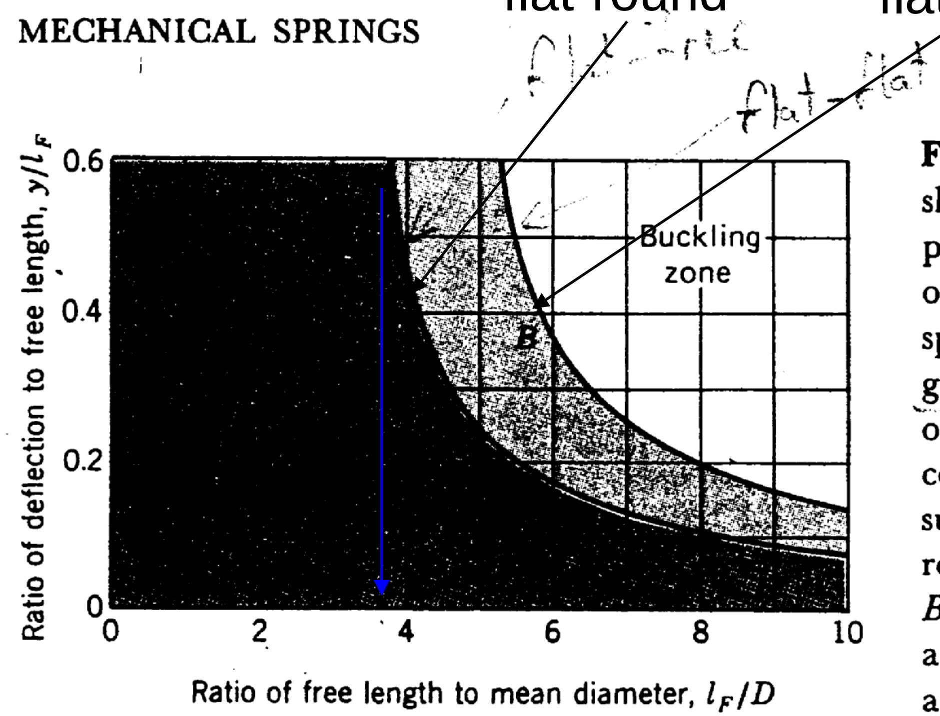
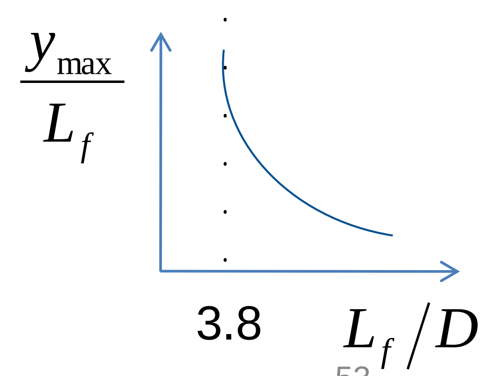


FIGURE 10-4 Curves show when buckling of compression coil springs may occur. Both curves are for springs having squared and ground ends. For curve *A* one end of the spring is compressed against a flat surface, the other against a rounded surface. For curve *B* both ends of the spring are compressed against flat and parallel surfaces.



EXAMPLE 1

A Chromium-Vanadium wire spring has a mean diameter of 10.0 mm, a spring index, C = 5.56, and there are 100 active coils. The pre-load is 20 N and the modulus of elasticity is 207.5 GPa.

Determine:

i. The tensile and torsional yield strengths of the wire
ii. The initial torsional stress in the wire
iii. The spring rate; and
iv. The force required to cause the spring to be stressed to the yield strength

SOLUTION:

Table 2.1 Spring materials and constant for estimating tensile strength			
Material	**Size range (mm)**	**Exponent, *m***	**Constant, *A* (*MPa.mm*^{m})**
Music wire	0.10-6.5	0.146	2170
Oil-tempered wire	0.50-12	0.186	1880
Hard-drawn wire	0.70-12	0.192	1750
Chrome-vanadium	0.80-12	0.167	2000
Chrome silicone	1.60-10	0.112	2000

(This is not a design problem, therefore we do not use 3<Na<15 criterion. This is an analysis problem since spring specifications like material, diameter, coil number, etc. are known)

i. *C* = 5.56 = *D/d* = 10/d

therefore, d = 10/5.56 = 1.8 mm

Using Table 2.1,

$$S_{ut} = \frac{2000}{(1.8)^{0.167}} = 1813.00\ \ MPa$$

Then calculating the tensile Yield Strength:

$$S_y = 0.75 S_{ut} = \left(0.75 \times 1813\right) = 1360\ MPa$$

We can then get the Torsional Yield Strength
(based on distortion energy theory):

$$S_{sy} = 0.577 S_y = \left(0.577 \times 0.75\right) S_{ut} = 0.433\ S_{ut}$$

$$= \left(0.433 \times 1813\right) = 785\ MPa \quad Ans$$

ii. In order to calculate τ we first need to get the <u>shear-stress correction factor *K*</u> (K_s or K)

$$\tau = K\frac{8FD}{\pi d^3}$$

Using K_s given below:

$$K_s = \left(1+\frac{0.5}{C}\right) = \left(1+\frac{0.5}{5.56}\right) = 1.089$$

Initial stress τ_i can be calculated as follows:

$$\tau_i = K_s\frac{8F_iD}{\pi d^3} = 1.089\times\frac{8\times 20\times 10}{\pi\times(1.8)^3} = 95.1\,MPa$$

iii. The spring stiffness can now be calculated (with G= 77.2 GPa):

$$k = \frac{Gd^4}{8D^3 N_a} = \frac{(77.2 \times 10^3) \times (1.8^4)}{8 \times (10^3) \times 100}$$

$$= 1.013\ N/mm \quad Ans$$

iv. Finally the force required to yield the material can now be calculated:

$$F_{sy} = \frac{\pi d^3 S_{sy}}{8 K_s D} = \frac{\pi \times (1.8^3) \times 785}{8 \times 1.089 \times 10}$$

$$\tau = K_s \frac{8FD}{\pi d^3}$$

$$= 165\ N \quad Ans$$

EXAMPLE 2

A helical spring of wire diameter 6 mm and spring index, *C, 6* is acted by an initial load of *800 N*.

After compressing it further by 10 mm the stress in the wire is *500 MPa*.

Find the number of active coils.

G=84 000 MPa.

SOLUTION:

$$C = \frac{D}{d}$$

D= spring index (C) x d= 6x6= 36 mm

$$\tau_{\max} = (K_w)\frac{8FD}{\pi d^3}$$

$$K_s = \left(1 + \frac{0.5}{C}\right)$$

$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C} = 1.2525$$

Note that in the case of static load one can also use K_s instead of K_w

$$K_W = \frac{4C-1}{4C-4} + \frac{0.615}{C} \qquad K_B = \frac{4C+2}{4C-3}$$

C	Ks	Kw	KB
6	1,08333	1,2525	1,2381

$$\tau_{\max} = (K_w)\frac{8FD}{\pi d^3}$$

$$500 = 1.2525 \times \frac{8F \times 36}{\pi 6^3}$$

$$F = \frac{500 \times \pi \times 6^3}{1.2525 \times 8 \times 36}$$

$$\therefore F = 940.6\ N$$

$$k = \frac{F}{y} = \frac{940.6 - 800}{10} = 14\ N/mm$$

$$k = \frac{Gd^4}{8D^3 N_a} \quad or,$$

$$N_a = \frac{Gd^4}{8D^3 k} = \frac{84000 \times 6^4}{8 \times 36^3 \times 14} \cong 21$$

$$N_a \cong 21\ turns$$

SOLUTION:

$$C = \frac{D}{d}$$

D= spring index (C) x d= 6x6= 36 mm

$$\tau_{\max} = (K_w)\frac{8FD}{\pi d^3}$$

$$K_s = \left(1+\frac{0.5}{C}\right) = 1+\frac{0.5}{6} = 1.083$$

Note that in the case of static load one can also use K_s instead of K_w

$$K_W = \frac{4C-1}{4C-4} + \frac{0.615}{C} \quad K_\mathbf{B} = \frac{4C+2}{4C-3}$$

C	Ks	Kw	KB
6	1,08333	1,2525	1,2381

$$\tau_{\max} = (K_w)\frac{8FD}{\pi d^3}$$

$$500 = 1.083 \times \frac{8F \times 36}{\pi 6^3}$$

$$F = \frac{500 \times \pi \times 6^3}{1.083 \times 8 \times 36}$$

$$\therefore F = 1087.8\ N$$

$$k = \frac{F}{y} = \frac{1087.8 - 800}{10} = 28.8\ N/mm$$

$$k = \frac{Gd^4}{8D^3 N_a} \quad or,$$

$$N_a = \frac{Gd^4}{8D^3 k} = \frac{84000 \times 6^4}{8 \times 36^3 \times 28.8} \cong 10.21$$

$$N_a \cong 10.25\ turns$$

2.7 CRITICAL FREQUENCY OF HELICAL SPRINGS

Since the springs are flexible they can vibrate at certain frequencies under the effect of loadings.

When the loading frequency 'f 'of the spring under the dynamic load $F = F^{*}\sin wt$ reaches one of its natural frequencies (f_n) the spring coils will vibrate at large amplitudes until the coils impact each other and create high impact loads and hence fail.

To prevent this resonant condition ($f = f_n$) the forcing frequency f should be much smaller than f_n. $f << f_n$;
the suggested limit is $f_n \geq 15 f$.

The fundamental critical frequency should be greater than 15 to 20 times the frequency of the force or motion of the spring in order to avoid resonance with the harmonics.

If the frequency is not high enough, the spring should be redesigned to increase *k or decrease spring weight W.*

Natural frequency

$$f_n = \frac{1}{2}\sqrt{\frac{kg}{W_a}} \quad Hz\,(cycle/\sec) \qquad (2.30)$$

$$\left(f_n = \frac{1}{2}\sqrt{\frac{k}{m}} \right)$$

where

k = spring rate

g = acceleration due to gravity

W_a= mass of spring in N

$$W_a = A \times L \times \rho = \frac{\pi d^2}{4} \times \pi D N_t \times \rho = \frac{\pi^2 d^2 D N_t \rho}{4}$$

ρ is the material weight density (*N/m*³)

or

$$f_n = \frac{2}{\pi N_t}\frac{d}{D^2}\sqrt{\frac{Gg}{32\rho}} \quad rad/\sec \qquad (2.31)$$

$$k \cong \frac{d^4 G}{8 D^3 N_a} \qquad (2.29)$$

For flat-flat surfaces $$f_n = \frac{1}{2}\sqrt{\frac{kg}{W_a}} \quad Hz\,(cycle/\sec) \qquad (2.32)$$

For flat-free end $$f_n = \frac{1}{4}\sqrt{\frac{kg}{W_a}} \quad Hz\,(cycle/\sec) \qquad (2.33)$$

Example 3: for compression springs

Design a compression spring for a static load known deflection such that the spring must give a minimum force of 500 N and a maximum force of 750 N over an adjustment range of 20 mm deflection.

Solution: Use the least expensive, unpeened, cold drawn (hard drawn) spring wire (ASTM A 227) since the load is static (Table 2.1)

From Table 2.1; for hard drawn wires diameter range is between 0.70 mm-12 mm

The coefficients are: m= 0.192, A= 1750 MPa to be used in eqn. $S_{ut} = \frac{A}{d^m}$

Design criteria: 1) $\tau_{\max} \leq S_{sy} \quad or \quad n_s = \frac{S_{sy}}{\tau_{\max}} \geq 1.0$

2) $4 \leq C \leq 12$

3) check for buckling $\frac{L_f}{D} = ? \qquad \frac{y_{\max}}{L_f} = ? \rightarrow$ (related figure)

4) not a must , but check if $\tau_{solid} \leq S_{sy}$

5) critical frequency

For a spring to be designed, the parameters:

$d=?$ $D=?$ $N_t = ?$ $L_f = ?$ should be determined.

1) Since none of the design parameters are known, we have to start by assuming a wire diameter d between 0.7-12 mm

Let $d= 4\ mm$ and $C= 8$ ⟶ $D= Cxd = 32\ mm$

Thus :

$$\tau_{max} = K_s \frac{8FD}{\pi d^3}$$

$$K_s = 1 + \frac{0.5}{C} = 1.0625$$

$$\tau_{max} = 1.0625 \frac{(8)(750)(32)}{\pi 4^3}$$

$$F = 750\ N$$

$$D = 32\ mm$$

$$\tau_{max} = 1014.6\ MPa$$

$$d = 4\ mm$$

$$S_{ut} = \frac{1750}{(4)^{0.192}} = 1341\ MPa$$

$$S_{sy} = 0.577(0.75 * 1341) = 580\ MPa$$

$$n_s = \frac{S_{sy}}{\tau_{max}} = \frac{580}{1014.6} = 0.57 < 1.0 \quad \underline{FAILURE\ !}$$

Re-size the wire diameter to reduce τ_{max}

Let *d= 5 mm and C= 8* $\longrightarrow$ *D= 40 mm*

$$K_s = 1 + \frac{0.5}{8} = 1.0625 \qquad \rightarrow \quad \tau_{\max} = 1.0625 \frac{8(750)(40)}{\pi 5^3} = 649.4 \; MPa$$

$$S_{ut} = \frac{1750}{5^{0.192}} = 1284.8 \; MPa \rightarrow S_{sy} = (0.577)(0.75)(1285) = 556 \; MPa$$

$$n_s = \frac{S_{sy}}{\tau_{\max}} = \frac{556}{649.4} = 0.85 < 1.0 \quad \underline{FAILURE\,!}$$

Re-size to *d*= 5 mm *and C*= 6 → *D*= 30mm

K_s=1.0833, τ_{max} =496.5MPa, S_{sy}=556MPa,

n_s=556/496.5= 1.12 >1.0 **SAFE**

Thus a spring material of ASTM A 227 with
d= 5 mm *and D*= 30 mm satisfies the criteria of 1 *and* 2 (C=6)

To find out other parameters N_t *and* L_f we proceed as:

Given a deflection of 20 mm over a force range of 500 N - 750 N

$$k = \frac{\Delta F}{\Delta y} = \frac{750 - 500}{20} = \frac{250\, N}{20\, mm} = 12.5\ N/mm \quad or\ k = 12500\ N/m$$

Also

$$k = \frac{d^4 G}{8 D^3 N_a} = 12.5\ N/mm$$

where

$$d = 5\, mm$$

$$G = 79.3 \times 10^9\ N/m^2\ or\ G = 79.3 \times 10^3\ N/mm^2$$

$$D = 30\, mm$$

$$N_a = ?$$

So $N_a = \frac{d^4 G}{8D^3 k} = \frac{(5)^4 (79.3) \times 10^3}{8(30)^3 \times 12.5}$ rounded to nearest quarter

$$N_a = \frac{49562500}{2700000} = 18.356\, coils \qquad N_a = 18.5\, coils$$

3) Check for buckling $\frac{L_f}{D}$ *and* $\frac{y_{\max}}{L_f} = \frac{y_i + y_w}{L_f}$

Assuming squared and ground ends $N_e = 2 \rightarrow N_t = N_a + N_e = 20.5\, coils$

$$L_s = N_t \times d = 20.5 \times 5 = 102.5\, mm$$

$$y_w = 20\, mm\, (given\ in\ problem)$$

$$y_{clash} = 0.15 \times y_w = 0.15 \times 20 = 3\, mm$$

$$y_{initial} = \frac{F_{\min}}{k} = \frac{500}{12.5} = 40\, mm$$

$$L_f = L_s + y_{clash} + y_{work} + y_{initial}$$

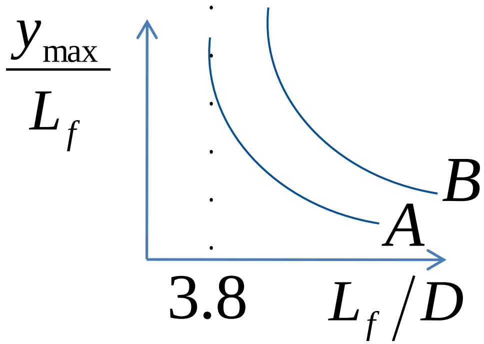
$$L_f = 102.5 + 3 + 20 + 40 = \underline{\underline{165.5\, mm}}$$

$$L_f = \underline{\underline{165.5\, mm}}$$

3) Check for buckling

$$\frac{L_f}{D} = \frac{165.5}{30} = 5.52 > 4 \rightarrow \textit{likely to buckle}$$

$$\frac{y_{max}}{L_f} = \frac{y_i + y_w}{L_f} = \frac{40+20}{165.5} = 0.363$$



MECHANICAL SPRINGS

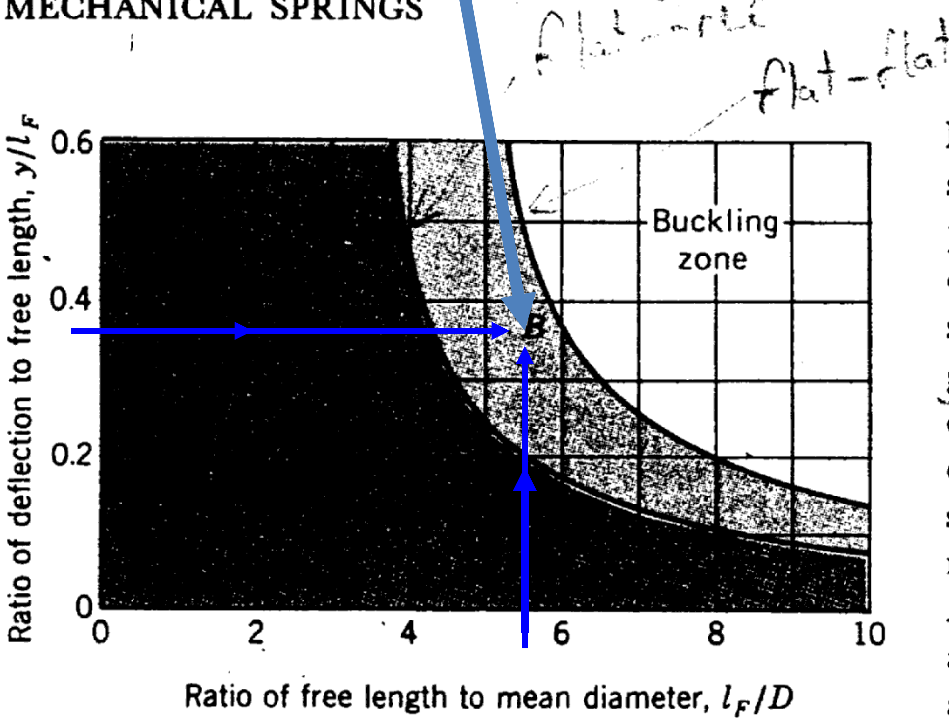


FIGURE 10-4 Curves show when buckling of compression coil springs may occur. Both curves are for springs having squared and ground ends. For curve *A* one end of the spring is compressed against a flat surface, the other against a rounded surface. For curve *B* both ends of the spring are compressed against flat and parallel surfaces.

- If one end of spring is rounded (curve A) it buckles,
- but if both ends are compressed between flat and parallel surfaces (curve B) it does not buckle although near the curve B.

So these springs should be compressed between flat and parallel surfaces.

If not compressed between parallel-flat surfaces then some kind of rod or hole mechanism should be used to fix the springs.

If you use rod inside the spring:

$$d_{rod_{\max}} \le D_i - 0.1 \times d$$

$$d_{rod_{\max}} \le (30 - 5) - 0.1 \times 5 \le 24.5\, mm$$

If you fit the spring in a hole:

$$D_{hol_{\min}} \ge D_o + 0.1 \times d$$

$$D_{hol_{\min}} \ge (30 + 5) + 0.1 \times 5 \ge 35.5\, mm$$

4)

$$\tau_{solid} \overset{?}{\le} S_{sy}$$

S_{sy}=556 MPa

$$\tau_{solid} = K_s \frac{8FD}{\pi d^3}$$

$$\tau_{solid} \textit{ requires the force}$$

$$F_{solid} = k \times \left(L_f - L_s\right)$$

$$= 12.5(165.5 - 102.5) = 787.5\ N$$

$$\tau_{solid} = 1.0833 \times \frac{8 \times 787.5 \times 30}{\pi \times (5)^3} = 521.38\ MPa$$

$$n_{solid} = \frac{S_{sy}}{\tau_{solid}}$$

$$= \frac{556}{521.38} = 1.07 > 1.0 \ \ \underline{OK!} \quad \textit{no failure}$$

5) critical frequency

$$f_n = \frac{1}{2}\sqrt{\frac{kg}{W_a}} \quad Hz\left(cycle/\sec\right)$$

$$f_n = \frac{1}{2}\sqrt{\frac{k}{m_s}} \quad Hz\left(cycle/\sec\right)$$

$$k = \frac{d^4 G}{8D^3 N_a} = 12.5\ N/mm$$

$$m_s = A \times L \times \rho = \frac{\pi(5)^2}{4} \times (\pi \times 30 \times 20.5) \times \left(10^{-9}\ \frac{m^3}{mm^3}\right) \times 7800\,\frac{kg}{m^3}$$

$$m_s = 0.296\ kg$$

$$f_n = \frac{1}{2}\sqrt{\frac{12500\ N/m}{0.296\ kg}} = 102 \quad Hz\left(cycle/\sec\right)$$

$$f_{force} \le \frac{f_n}{15} = \frac{102}{15} \le 7\ Hz\left(cycle/\sec\right)$$

if dynamic force is applied

SOLUTION:

ASTM A 227 hard-drawn wire with

$d = 5\ mm$

$D = 30\ mm$

$C = 6$

$D_i = 25\ mm \quad N_t = 20.5 \quad \text{coils squared and ground ends.}$

$D_o = 35\ mm \quad L_f = 165.5\ mm \quad \text{flat and parallel ends.}$

TO BE CONTINUED