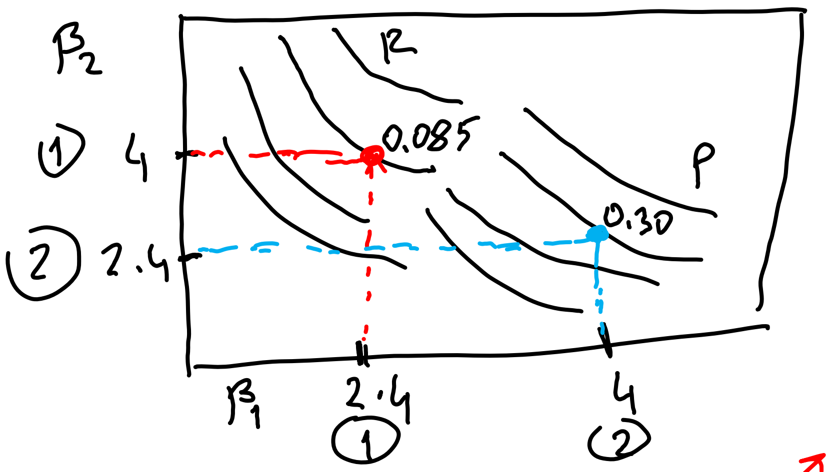
PROBLEMS

1) Lean beef block (a slab) with dimensions of 1mx0.25mx0.6m is being frozen. The following data are available: hc = 30W/m^2.K, $T_{initial}$ = 5ºC, T∞=-30ºC, density = 1050 kg/m^3, Lv = 333.22 kJ/kg, moisture content = 74.5%, k_f = 1.108 W/m.K, T_F = - 1.75ºC. Find the time required to freeze to - 10°C using Plank's equation.

<u>Solution:</u>

$$t_F = \frac{\rho \times L_v}{(T_F - T_\infty)} \times \left[\frac{P \cdot a}{h_c} + \frac{R \times a^2}{k_1} \right]$$

① $\beta_1 = \frac{0.6}{0.25} = 2.4$, $\beta_2 = \frac{1}{0.25} = 4$ and ② $\beta_1 = \frac{1}{0.25} = 4$, $\beta_2 = \frac{0.6}{0.25} = 2.4$



R = 0,085

P = 0,30

a = 0,25 (thickness for slab)

$$L_v = 333,22 \frac{kJ}{\cancel{kg H_2O}} \times 0,745 \frac{\cancel{kg H_2O}}{kg\ beef} = 248,25 \frac{kJ}{kg\ beef}$$

$$t_F = \frac{1050 \frac{kg}{m^3} \times 248,25 \times 10^3 \frac{J}{kg}}{[-1.75 - (-30)]°C} \left[\frac{0,3 \times (0,25m)}{30 \frac{J}{m^2.s.K}} + \frac{0,085 \times (0,25m)^2}{1.108 \frac{J}{m.s.K}} \right] \times \frac{1hr}{3600s}$$

$$t_F = 16.8\ hr.$$

$\boxed{\therefore L_v: \text{Latent heat of fusion per unit mass of food.}}$

2) **Lean beef block with dimensions of 1mx0.25mx0.6m is being frozen. The following data are available: hc = 30W/m^2.K, $T_{initial}$ = 5ºC, T_∞= - 30ºC, ρ_u = 1050 kg/m^3, ρ_f = 955 kg/m^3, Cpf = 2.5 kJ/kg beef.K, Cpu = 3.52 kJ/kg beef.K, Lv = 333.22 kJ/kgH_2O, moisture content = 74.5%, k_f = 1.108 W/m.K, T_F = - 1.75ºC. Find the time required to freeze to - 15°C using Pham's equation.**

Solution:

$$N_{Bi} = \frac{h_c \times d_c}{k_f} = \frac{30 \times \overbrace{\frac{1}{2} \times 0{,}25}^{d_c:\ \text{half thickness}}}{1.108} = 3{,}3845$$

$$T_{fm} = 1.8 + 0{,}263 \times T_c + 0{,}105 \times \underbrace{T_a}_{T_\infty}$$

$$T_{fm} = 1.8 + 0.263 \times (-15) + 0{,}105 \times (-30) = -5.295\,°C$$

$$\Delta H_1 = C_{pu} \times \rho_u (T_i - T_{fm})$$

$$= 3520 \frac{J}{kg.°C} \times 1050 \frac{kg}{m^3} \times [5 - (-5.295)]\,°C$$

$$= 38050320\ J/m^3$$

$$\Delta H_2 = \rho_f [L_f + C_{pf} (T_{fm} - T_c)]:$$

$$L_f = 333.22 \times 0{,}745 = 248{,}3 \frac{kJ}{kg\ beef} \equiv 248300 \frac{J}{kg\ beef}$$

$$\Delta H_2 = 955 \frac{kg}{m^3} \left[248300 \frac{J}{kg} + 2500 \frac{J}{kg.°C} \times [-5.295 - (-15)]\,°C\right]$$

$$= 260297187\ J/m^3$$

$$\Delta T_1 = \frac{T_i + T_{fm}}{2} - T_a = \frac{5 + (-5.295)}{2} - (-30) = 29.8\,°c$$

$$\Delta T_2 = T_{fm} - T_a = -5.295 - (-30) = 24.705\,°c$$

$$t_F = \frac{d_c}{\varepsilon_f . h_c}\left[\frac{\Delta H_1}{\Delta T_1} + \frac{\Delta H_2}{\Delta T_2}\right]\left(1 + \frac{N_{Bi}}{2}\right)$$

d_c: for slab: half thickness

ε_f: " " : 1

$$t_F = \frac{\frac{1}{2} \times (0,25)}{1 \times 30}\left[\frac{38050320}{29.8} + \frac{260297187}{24.7}\right] \cdot \left[1 + \frac{3.3845}{2}\right]$$

$$t_F = 132503\ s \equiv 36.7\ hr$$

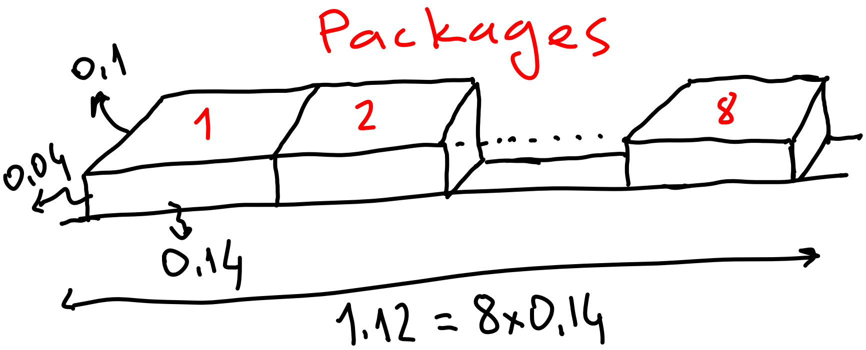
3) A continuous plate freezing system is being designed to freeze 0.5 kg fish packages at a rate of 500 kg fish/h. Package dimensions are 0.04x0.1x0.14 m. Each package enters the freezer at 4.4ºC initially. Width of each plate is 1.12 m and can hold 8 packages. Plate temperature is – 23 ºC, surface heat transfer coefficient 28 W/m^2.K, thickness of packaging material is 8x10^{-4} m, thermal conductivity of packaging material is 0.05 W/m.K, thermal conductivity of frozen fish is 1.125 W/m.K, enthalpy change between initial and plate temperatures is 306 kJ/kg fish, ρ_u = 880 kg/m^3.

Compute the number of freezing stations (plates) required and the refrigeration requirement.

Solution: Using modified Plank's equation. It accounts for time required to remove sensible heat in pre-cooling and post-freezing period.

$$t_F = \frac{\rho \cdot \Delta H'}{(T_i - T_\infty)}\left[\frac{P \times a}{h_c} + \frac{R \times a^2}{k_1}\right], \quad L_v \rightarrow \Delta H'$$

$$\beta_1 = \frac{0{,}1}{0{,}04} = 2.5, \quad \beta_2 = \frac{8 \times 0{,}14}{0{,}04} = 28 \Rightarrow$$ β_1 and β_2 are not present in the chart $\Rightarrow$ It is assumed the plate chain to be an infinite slab.



$P = 1/2$, $R = 1/8$

$T_i = 4.4\,°C$

$T_\infty = -23\,°C$

$a = 0{,}04\ m$

$k = 1.125\ W/m.K$

In one plate $= \dfrac{1{,}12\ \cancel{m}}{0{,}14\ \cancel{m}/\text{package}} = 8$ packages

$$\frac{1}{h_c} = \frac{1}{h_{surface}} + \frac{\Delta x}{k}$$ for packaging material

$$\frac{1}{h_c} = \frac{1}{28} + \frac{8 \times 10^{-4}}{0{,}05} \Rightarrow h_c = 19.34\ W/m^2.K$$

kJ → J

$$t_F = \frac{880 \times 306 \times 1000}{[4.4 - (-23)] \times 3600} \times \left[\frac{1/2 \times (0{,}04)}{19.34} + \frac{1/8 \times (0{,}04)^2}{1.125}\right]$$

s → h

thermal conduct. of frozen fish

$t_F = 3.263\,hr.$

$500\,\frac{kg\,fish}{\cancel{hr}} \times 3.263\,\cancel{hr} = 1631\,kg\ fish\ capacity$

$$\frac{1631\,kg\ \cancel{fish}}{8\,\frac{\cancel{package}}{plate} \times 0.5\,\frac{kg\,\cancel{fish}}{\cancel{package}}} = 408\ stations\ (plates)$$

Refrigeration requirement:

$$500\,\frac{\cancel{kg\,fish}}{hr} \times 306\,\frac{kJ}{\cancel{kg\,fish}} = 153000\,\frac{kJ}{\cancel{hr}} \times \frac{1\,\cancel{hr}}{3600\,s} = 42.5\,kW.$$

4) A food product at 25°C is being frozen with liquid nitrogen (product final temperature = - 35°C). Food product is in brick shape (0.05x0.1x0.2 m). If the freezing time is 3 minutes, calculate the rate of N_2 utilization for this product (kg N_2/kg product).
Cp of N_2 = 1 kJ/kg N_2, λ for N_2 = 198 kJ/kg N_2, $T_{liq.Nitrogen}$ = -196°C density of product = 800 kg/m^3, h_c= 200 W/m^2.K, k = 1.2 W/m.K.

<u>Solution:</u>

$T_i = 25°c,\ T_f = -35°c,\ a = 0{,}05\,m,\ t_F = 3\,min \equiv 0{,}05\,hr.$

$$t_F = \frac{\rho . \Delta H'}{(T_i - T_\infty)} \times \left[\frac{P.a}{h_c} + \frac{R \times a^2}{k}\right]$$

$\beta_1 = \frac{0{,}2}{0{,}05} = 4$

$\beta_1 = \frac{0.1}{0{,}05} = 2,\quad \beta_2 = \frac{0.2}{0{,}05} = 4 \Rightarrow$

$\beta_2 = 0{,}1/0{,}05 = 2$

from chart; $P = 0{,}282,\ R = 0{,}083$

$$0{,}05 = \frac{800 \times \Delta H' \times 1000}{[25-(-196)]} \times \left\{\frac{0{,}282 \times 0{,}05}{200} + \frac{0{,}083 \times (0{,}05)^2}{1{,}2}\right\} \times \frac{1\,hr}{3600\,s}$$

$$\Rightarrow \Delta H' = 204.09 \ kJ/kg \ product$$

$$\Delta H_{N_2} = \lambda_{N_2} + c_p (T_f - T_i)_{N_2}$$

$$\Delta H_{N_2} = 198 \ kJ/kg \ N_2 + 1 \frac{kJ}{kg N_2 \cdot \cancel{°C}} \times [\underbrace{-35}_{T_{final} \ N_2} - (-196)] \cancel{°C}$$

$$= 359 \ kJ/kg \ N_2$$

$$\frac{\text{Rate of } N_2 \text{ utilized}}{\text{kg product}} = \frac{204.09 \ \cancel{kJ}/kg \ product}{359 \ \cancel{kJ}/kg \ N_2}$$

$$= 0.568 \ kg \ N_2/kg \ product$$

5) Determine the refrigeration requirement for freezing of 50 kg lean beef with 74.5 % moisture content from an initial temperature of 5°C to a final temperature of - 15°C. Assume enthalpies of beef at these temperatures are 317 and 58 kJ/kg beef, respectively.

Solution:

$$\Delta H' = (317 - 58) \frac{kJ}{kg} = 259 \ kJ/kg \ beef$$

$$\text{Refrigeration requirement} = 259 \frac{kJ}{\cancel{kg \ beef}} \times 50 \ \cancel{kg \ beef}$$

$$= 12950 \ kJ$$

6) 100 kg of tomato was frozen by using liquid nitrogen. Initial and final temperatures of tomato were 25°C and -15°C, respectively. Tomato are assumed to be spherical with a diameter 5 cm and density 0.8 g/cm^3. Tomato solids makes up 35 % of the total weight of tomato and tomato juice solids makes up 5 % of

total weight of tomato juice. Liquid nitrogen is sprayed on to tomato at a rate of 0.02 kg N_2/tomato.min. Calculate the thermal efficiency of this process. Following data is also given:

h_c = 100 W/m^2.K, k = 2.5 W/m.K, Cp of N_2 = 1 kJ/kgN_2.K, enthalpy of evaporation of N_2 = 200 kJ/kg N_2, $T_{liq.Nitrogen}$ = -196°C.

Solution: For fruits and vegetables ⇒

liquid (juice)

$$\Delta H' = \left[1 - \frac{X}{100}\right]\Delta H_j + 1.21\left(\frac{X}{100}\right) \times \Delta T$$

Solid

X : solids content of fruit.

(Tomato juice) : $T_i = 25°C \Rightarrow H_i = 525$ kJ/kg tomato juice

$T_f = -15°C \Rightarrow H_f = 80$ kJ/kg tomato juice

ΔH_j

$$\Delta H_{\text{tomato juice}} = 525 - 80 = 445 \text{ kJ/kg tomato juice}$$

$$\Delta H' = \left[1 - \frac{35}{100}\right] \times 445 + \underbrace{1.21}_{\text{Cp solid}} \cdot \left(\frac{35}{100}\right) \times [25 - (-15)]$$

$\Rightarrow \Delta H' = 306.19 \dfrac{\text{kJ}}{\text{kg tomato}}$ heat utilized by the product.

$P = 1/6$ $R = 1/24$ for sphere

$a = 0.05$ m, $T_i = 25°C$, $T_\infty = -196°C$

$$t_F = \frac{800 \times 306.19 \times 10^3}{[\underset{\searrow T_i}{25} - (-196)]} \times \left[\frac{1/6 \times 0{,}05}{100} + \frac{1/24 \times (0{,}05)^2}{2.5}\right] \times \frac{1\,hr}{3600\,s}$$

$$t_F = 0{,}0385\ hr \equiv 2.309\ min$$

Enthalpy of liq. $N_2 = \lambda_{N_2}$ + sensible heat

$$= 200 + 1 \times [\underset{\searrow T_{final\ N_2}}{-15} - (-196)] = 381\ kJ/kg$$

$$V \text{ of 1 tomato} = \frac{4}{3}\pi \times r^3 = \frac{4}{3}\pi \times \left(\frac{0{,}05}{2}\right)^3 = 6{,}5 \times 10^{-5}\ m^3$$

$$wt \text{ of 1 tomato} = 6{,}5 \times 10^{-3}\ m^3 \times 800\ \frac{kg}{m^3} = 0{,}052\ kg$$

$$\frac{0{,}02\ \cancel{kg N_2}}{1\ \cancel{tomato} \cdot \cancel{min}} \times 2.309\ \cancel{min} \times \frac{1\ \cancel{tomato}}{0{,}052\ kg} \times 381\ \frac{kJ}{\cancel{kg N_2}}$$

$= 338{,}3\ kJ/kg$ tomato heat input totally.

$$\text{Thermal efficiency} = \frac{\text{Heat utilized}}{\text{Heat input}} = \eta$$

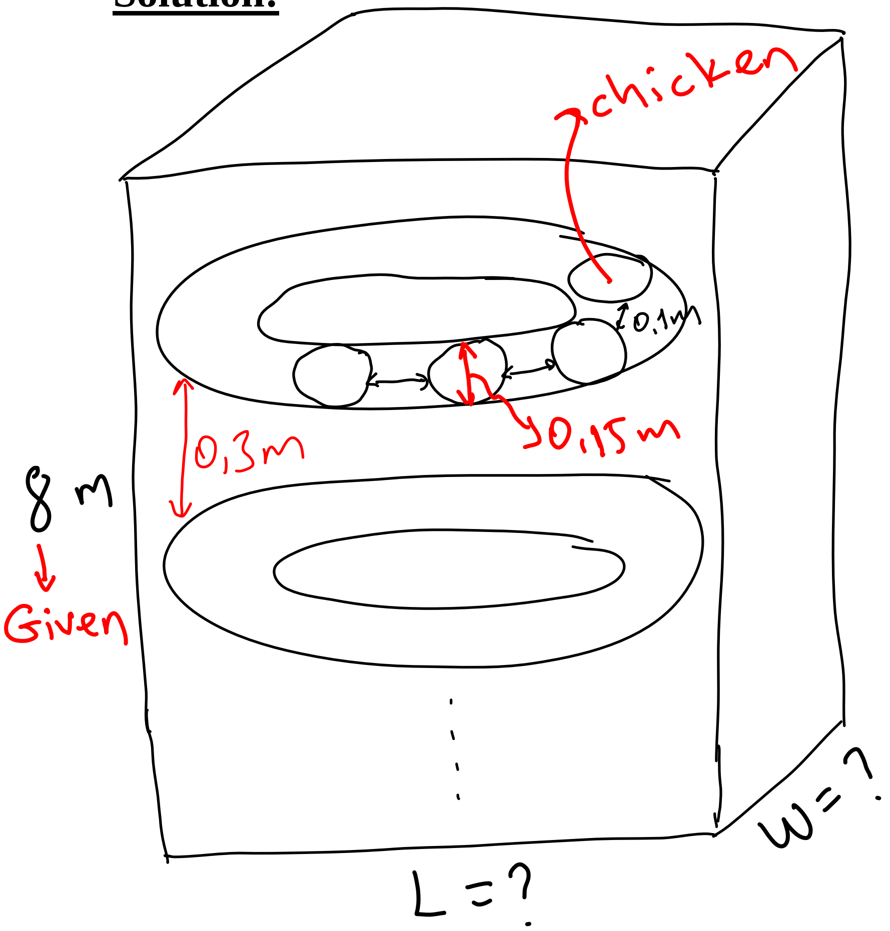
$$\eta = \frac{306.19}{338.19} \times 100 = 90.5\%$$

7) A continuous freezing system for whole chickens in a plastic film is designed using a spiral conveyor and high velocity cold air. The initial product temperature is 5°C, final temperature is - 2°C and medium (air) temperature is - 30°C. The conveyor that carries the product to freezing chamber is designed to operate at a

rate of 3 m/min. Assume the chicken has a spherical shape. Determine the approximate dimensions of freezing room, the capacity of the refrigeration system and total number of chickens on 15 circular sections. The following data were given:

h_c = 22 W/m^2.K, height of the freezing chamber = 8 m, clerance between sections of spiral conveyor = 0.3 m, k = 1.298 W/m.K, density = 855 kg/m^3, assume the diameter of a chicken is 0.15 m and the distance between each chicken is 0.1 m, in the system there are 15 circular sections, ΔH^{l} = 278.6 kJ/kg chicken.

Solution:



For a sphere $\Rightarrow$

$$P = \frac{1}{6}, \quad R = \frac{1}{24}$$

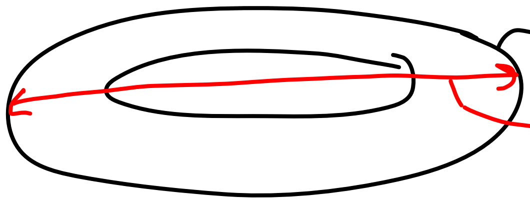
$$t_F = \frac{\Delta H' \times \rho}{T_i - T_\infty}\left[\frac{P \cdot a}{h_c} + \frac{R \times a^2}{k}\right]$$

$$= \frac{278.6\times10^3 \times 855}{[5-(-30)]} \times \left[\frac{1/6 \times 0.15}{22} + \frac{1/24 \times (0.15)^2}{1.298}\right] \times \frac{1\,hr}{3600\,s}$$

$t_F = 3.5 \text{ hr}$

$$\text{Total conveyor length} = \frac{3\,m}{min} \times 3.5\,hr \times \frac{60\,min}{1\,hr} = 630\,m$$

$$\text{Length of each circle} = \frac{630}{15 \text{ circle}} = 42\,m/\text{circle}$$



length = 42 m

D = 13.36 m

$$\text{Circumference} = 2\pi r = \pi . D = 42 \Rightarrow D = 13.36\,m$$

Dimensions: L = 14 m, width = 14 m at least

$$\text{Approximate dimensions} \cong \underset{L}{14} \times \underset{W}{14} \times \underset{H\text{ (given)}}{8}\; m^3$$

$\Rightarrow 0.15 + 0.1 = 0.25\,m$

$$\frac{42\,\cancel{m}/\text{circle}}{0.25\,\cancel{m}/\text{chicken}} = 168 \text{ chickens on one circle.}$$

Total # of chicken on 15 circles ⇒

$$\frac{168 \text{ chicken}}{1 \text{ circle}} \times 15 \text{ circle} = 2520 \text{ chickens}$$

$$M: \text{Mass of a chicken} = V \times \rho = \frac{4}{3}\pi r^3 \times \rho$$

$$M = \frac{4}{3} \times \pi \times \left(\frac{0.15}{2}\right)^3 \times 855 = 1.51 \text{ kg/chicken}$$

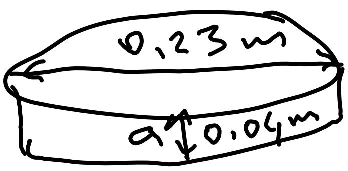
$$\frac{3 \text{ m/min}}{0.25 \text{ m/chicken}} = 12 \ \frac{\text{chicken frozen}}{\text{min}}$$

Refrigerant requirements (RR) = ?

$$RR = 278.6 \frac{\text{kJ}}{\text{kg}\cancel{\text{chic}}} \times \frac{1.51 \ \cancel{\text{kg}}\text{chic}}{\cancel{\text{chic}}} \ \frac{12 \ \cancel{\text{chic}}}{\text{min}} = 5048 \ \frac{\text{kJ}}{\text{min}}$$

8) A pecon coffee cake (each cake = 0.372 kg) is being frozen in liquid nitorgen in 1.7 minutes. Initial temperature of cake is 22°C, the final temperature is - 18°C. The temperature of freezing medim (liquid nitrogen) is - 196°C. If 0.665 kg N_2 is required per kg product, estimate the surface heat transfer coefficient of the system. R = 1/8, P = 1/2, k = 1.731 W/m.K, latent heat of nitrogen = 197.98 kJ/kg N_2, Cp of N_2 is 1.044 kJ/kg N_2.

<u>Solution:</u>



$\Rightarrow$ Assume infinite slab $\Rightarrow$

$R = 1/8, \ P = 1/2$

$V = \pi . r^2 \times a$

$$\rho = \frac{m}{V} = \frac{0{,}372\ kg}{\pi\left(\frac{0{,}23}{2}\right)^2 \times 0{,}04\ m^3} = 224\ kg/m^3$$

$$\lambda_{N_2} = 197.98\ kJ/kg\,N_2$$

$0{,}665 \dfrac{kg\,N_2}{kg\ product}$ is evaporated after it is used.

$$\text{Sensible heat}_{N_2} = C_p \Delta T = 1.044 \times [\overset{T_{final\,N_2}}{-18} - (\overset{T_{\infty}}{-196})]$$

$$= 185.83\ kJ/kg\,N_2$$

$$\Delta H_{N_2} = \lambda_{N_2} + \text{Sensible heat of } N_2$$

$$= 197{,}98 + 185{,}83 = 338{,}81\ kJ/kg\,N_2$$

$$\Delta H'_{product} = 0{,}665 \frac{kg\,\cancel{N_2}}{kg\ product} \times 338{,}81 \frac{kJ}{kg\,\cancel{N_2}}$$

$$= 255.23\ kJ/kg\ product$$

$$t_F = 1{,}7\ min \equiv 0{,}028\ hr\ (\text{given})$$

Using Plank's modified equation =>

$$0{,}028\ hr = \frac{\overset{\rho}{224} \times \overset{\Delta H'}{253{,}23 \times 10^3}}{[22-(-196)]} \times \left[\frac{\frac{1}{2}(0{,}04)}{h_c} + \frac{\frac{1}{8}(0{,}04)^2}{1{,}731}\right] \times \frac{1\ hr}{3600\ s}$$

$$\Rightarrow h_c = 73{,}2\ W/m^2.K$$

9) Calculate the total refrigeration requirement (in Btu) when 2000 lb of beef slices are frozen from an initial temperature of 40°F to freezing temperature of 23°F and then stored at - 4°F (Cp of beef above freezing temperature is 0.82 Btu/(lb.°F), Cp of frozen beef is 0.46 Btu/(lb.°F), the latent heat of fusion of water to ice is 144 Btu/lb water, moisture content of beef is 75 %).

Solution:

1) To bring the T of beef down to freezing T =>

$H_1 = M \cdot Cp_u \times W \times (T_{initial} - T_{freezing})$; W is water fraction

$$H_1 = 2000\ \cancel{lb} \times 0{,}82 \frac{Btu}{\cancel{lb}.\cancel{°F}} \times 0{,}75 \times (40-23)\cancel{°F} = 9840\ Btu$$

2) To freeze the beef => Heat of fusion:

$$H_2 = L_v \times W \times m = 144 \frac{Btu}{\cancel{lb}} \times 0{,}75 \times 2000\ \cancel{lb} = 216000\ Btu$$

$-4°C$

3) To lower T of frozen beef to storage $T \Rightarrow$

$$H_3 = m \cdot Cp_f \times W \times (T_{freezing} - T_{storage}) \rightsquigarrow T_{final}$$

$$H_3 = 2000 \times 0.46 \times 0{,}75 \times [23 - (-4)] = 18630 \text{ Btu}$$

$$\text{Total load} = H_T = H_1 + H_2 + H_3 \Rightarrow$$

$$H_T = 9840 + 216000 + 18630 \Rightarrow$$

$$H_T = 244470 \text{ Btu}$$

The equations that we used in this problem set:

1. Plank's equation

2. Pham's equation

3. Modified Plank's equation