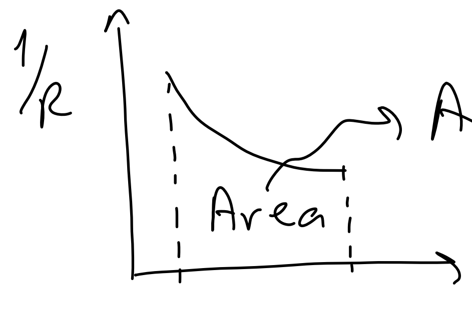
Calculation Methods for Falling Rate Drying Period:

* Method using graphical integration:

$$t_f = \frac{M_s}{A} \cdot \int_{X_2}^{X_1} \frac{dx}{R} = \frac{M_s}{A} \int_{X_2}^{X_1} \frac{1}{R} \cdot dx$$

From drying data find several R values $\Rightarrow$

i.e., R vs X and calculate 1/R



$$\text{Area} = \int_{X_2}^{X_1} \frac{dX}{R} \Rightarrow t_f = \underbrace{\frac{M_s}{A}}_{\text{assume constant } A} \times \text{Area}$$

We assume negligible shrinkage $\Rightarrow$

A = constant.

If $A \neq$ constant $\Rightarrow A = f(x) = A(x)$

$\Rightarrow$ integrate it.

$$t_f = M_s \times \int_{X_2}^{X_1} \frac{dx}{A(x) \cdot R} \rightsquigarrow \text{if } A \neq \text{constant}$$

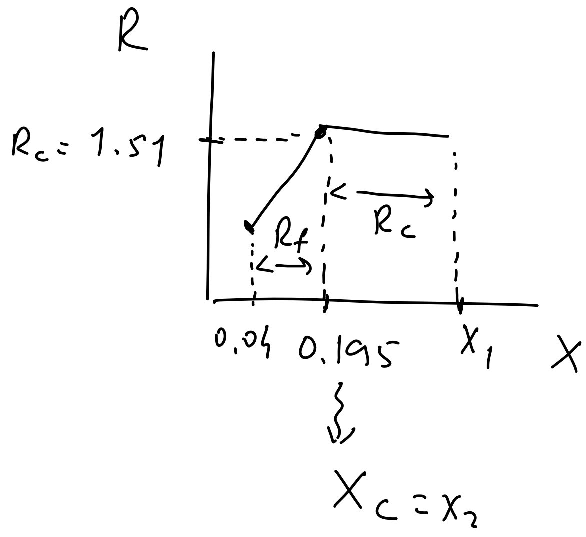
Example: A batch of wet solid whose data are given below is to be dried from a moisture content of $X_1 = 0.38$ to $X_f = 0.04$ kg H_2O/kg DS. The weight of the dry solids is $M_s = 399$ kg and $A = 18.58$ m^2 of top drying surface.

Calculate the time for drying. (Given that $R_c = 1.51$ kg $H_2O/h.m^2$).

For falling rate period:

X	R	$1/R$
X_c 0.195 →	1.51	✓
0.150 →	1.21	✓
0.100 →	0.90	✓
0.065 →	0.71	✓
0.50 →	0.37	✓
0.04 →	0.27	✓

$\Rightarrow$



Solution: Find $1/R$ values $\Rightarrow$

$$\frac{M_s}{A} = \frac{399}{18.58} = 21.5 \ kg/m^2 .$$

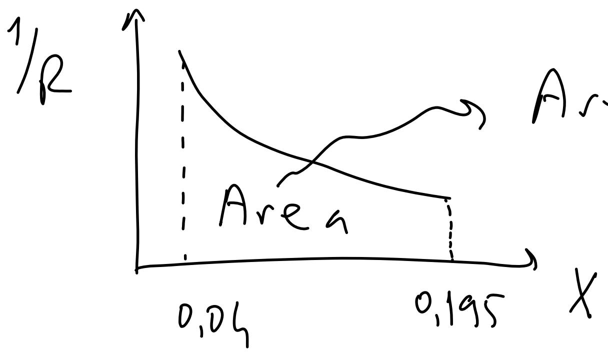
$$t_c = \frac{M_s}{A \times R_c} \times (X_1 - X_2) \ , \quad X_1 = 0.38, \ X_2 = X_c = 0.195$$

$$t_c = 21.5 \times \frac{1}{1.51} \times (0.38 - 0.195) = 2.63 \ h$$

In the falling rate period $\Rightarrow t_f = ?$

Find t_f graphically $\Rightarrow$

$$t_f = \frac{M_s}{A} \int_{X_2}^{X_1} \frac{dx}{R} \Rightarrow$$

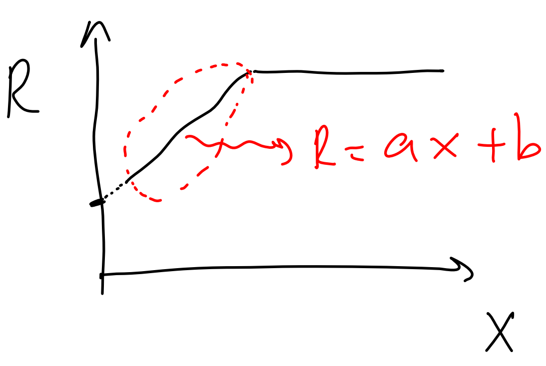


$$\text{Area} = \int \frac{dx}{R} = 0{,}189$$

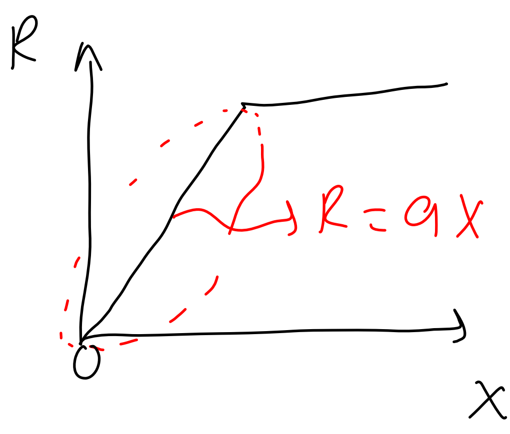
$$t_f = 21.5 \times (0.189) = 4.06 \text{ h}$$

$$\text{total drying time} = t_c + t_f = 2.63 + 4.06$$
$$= 6.69 \text{ h}.$$

⊛ Calculation methods for special cases in falling rate period:



OR



1) Rate is linear function of X.

i.e., $R = aX + b \Rightarrow$ differentiate it $\Rightarrow$

$$t_f = \frac{M_s}{A} \int_{X_2}^{X_1} \frac{dx}{R}$$

$$dR = a \cdot dx \Rightarrow dx = \frac{dR}{a} \Rightarrow$$

$$t_f = \frac{M_s}{A} \times \int_{X_2}^{X_1} \frac{1}{a} \times \frac{dR}{R} = \frac{M_s}{a \cdot A} \cdot \int_{R_2}^{R_1} \frac{dR}{R} \Rightarrow$$

$$t_f = \frac{M_s}{a \cdot A} \times \ln \frac{R_1}{R_2}$$

Since $R_1 = a \cdot X_1 + \cancel{b}$

$$+ \quad \mp R_2 = \mp a \cdot X_2 \mp \cancel{b}$$

$$R_1 - R_2 = a\,(x_1 - X_2) \Rightarrow a = \frac{R_1 - R_2}{X_1 - X_2}$$

$$t_f = \frac{M_s}{A} \times \frac{(x_1 - x_2)}{(R_1 - R_2)} \times \ln \frac{R_1}{R_2}$$

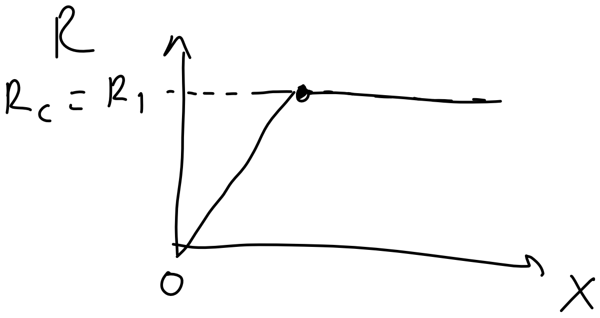
2) Rate is a linear function through origin:

$R = aX \Rightarrow$ differentiate $\Rightarrow dR = a \cdot dx \Rightarrow$

$$dx = \frac{dR}{a}$$

$$t_f = \frac{M_s}{A} \int_{x_2}^{x_1} \frac{dX}{R} \Rightarrow$$

$$t_f = \frac{M_s}{A}\int_{R_2}^{R_1}\frac{1}{a}\times\frac{dR}{R} = \frac{M_s}{a\cdot A}\times\ln\frac{R_1}{R_2}$$



$$R_1 = a X_1$$

$$\mp R_2 = \mp a X_2$$

$$+\quad R_1 - R_2 = a(x_1 - x_2)$$

$$a = \frac{R_1 - R_2}{x_1 - x_2}$$

$$R_f \quad R_1 = R_c \Rightarrow x_1 = X_c$$

$$X_2 = 0 \Rightarrow R_2 = 0 \Rightarrow a = \frac{R_1}{x_1} = \frac{R_c}{X_c}$$

$$t_f = \frac{M_s}{A\times\frac{R_c}{X_c}}\times\ln\frac{R_c}{R_2} \Rightarrow \boxed{t_f = \frac{M_s}{A}\times\frac{X_c}{R_c}\times\ln\frac{X_c}{X_2}}$$

$$\frac{R_c}{R_2} = \frac{\cancel{a}\cdot X_c}{\cancel{a}\cdot X_2} = \frac{X_c}{X_2}$$

$$\boxed{R_f = R_c\times\frac{X}{X_c}}$$

Example: Repeat the previous example, but as an approximation assume a straight line of the rate R vs X through the origin from point Xc to X = 0 for the falling rate.

Solution: $R = a \cdot x$ → straight line passing through origin.

$R_c = 1.51$, $X_c = 0.195$, $X_2 = 0.04$

$$t_f = \frac{M_s}{A} \times \frac{X_c}{R_c} \times \ln \frac{X_c}{X_2} = \left(\frac{399}{18.58}\right) \times \frac{0.195}{1.51} \times \ln \frac{0.195}{0.04}$$

$t_f = 4.39$ h.

Moisture Diffusivities in Foods.

- If a diffusion transport mechanism is assumed ⇒

  the rate of moisture movement is described by an effective diffusivity value, Deff.

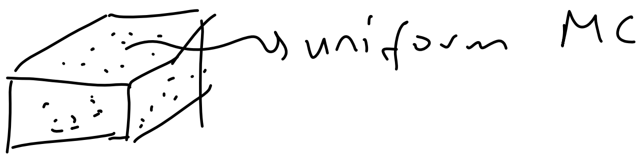
- Deff represents the overall transport property of water in the food. It includes the effect of liquid diffusion, vapor diffusion, etc.

* Fick's law is often used to describe a moisture diffusion process.

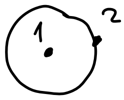
$$\frac{\partial X}{\partial t} = D_{eff} \cdot \frac{\partial^2 X}{\partial L^2}$$

<u>Assumptions:</u>

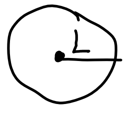
1) Constant diffusivity
2) Uniform initial MC



3) A zero moisture gradient at the center of the sphere.



* Solution of Fick's Law for a Sphere:



$$m^* = \frac{X - X^*}{X_0 - X^*} = \frac{6}{\pi^2} \cdot \sum_{n=1}^{\infty} \exp\left(-n^2 \cdot \frac{D_{eff} \cdot \pi^2}{L^2} \cdot t\right)$$

for long drying periods take $n = 1$

$$M^* = \frac{X - X^*}{X_0 - X^*} = \frac{6}{\pi^2} \cdot \exp\left(-\frac{D_{eff} \cdot \pi^2 \cdot t}{L^2}\right)$$

X = MC at any time

X^* = equilibrium MC

X_0 = initial MC

t = drying time

D_{eff} = effective diffusion coefficient

D_{eff} : m^2/s or cm^2/s

L = radius of sphere

For most of the foods D_{eff} : 10^{-9} to 10^{-12} m^2/s.

If $\underbrace{D_{eff_1} = 10^{-9}}_{\text{drying is faster.}}$ and $D_{eff_2} = 10^{-11}$ m^2/s $\Rightarrow$

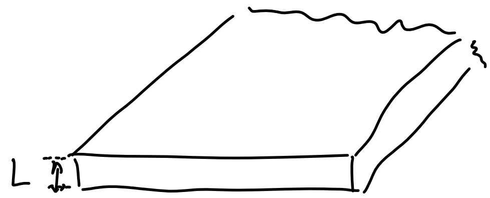
(*) Solution of Fick's Law for a Slab:

$$M^* = \frac{X - X^*}{X_0 - X^*} = \frac{8}{\pi^2} \cdot \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \cdot \exp\left[-\frac{(2n+1)^2 \cdot \pi^2 \cdot D_{eff} \cdot t}{4 \cdot L^2}\right]$$

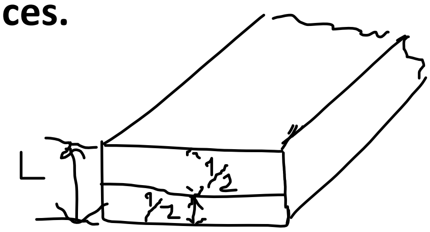
For long drying times $\Rightarrow$ $n = 0$ only $\Rightarrow$

$$m^* = \frac{X - X^*}{X_0 - X^*} = \frac{8}{\pi^2} \cdot \exp\left(-\frac{\pi^2 \cdot D_{eff} \cdot t}{4 \cdot L^2}\right)$$

L: thickness of the slab.



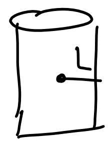
L = 1/2 the thickness of the slab when drying occurs from the top and the bottom parallel faces.



Solution of Fick's Law for a Cylinder:

$$m^* = \frac{X - X^*}{X_0 - X^*} = \frac{4}{L^2} \cdot \sum_{n=1}^{\infty} \frac{1}{\beta_n^2} \exp\left(-D_{eff} \cdot \beta_n^2 \cdot t\right)$$

L: radius of cylinder



β_n: Bessel function

at large drying times $\Rightarrow n = 1 \Rightarrow$

$$m^* = \frac{X - X^*}{X_0 - X^*} = \frac{4}{L^2} \cdot \frac{1}{\beta_n^2} \cdot \exp\left(-D_{eff} \cdot \beta_n^2 \cdot t\right)$$