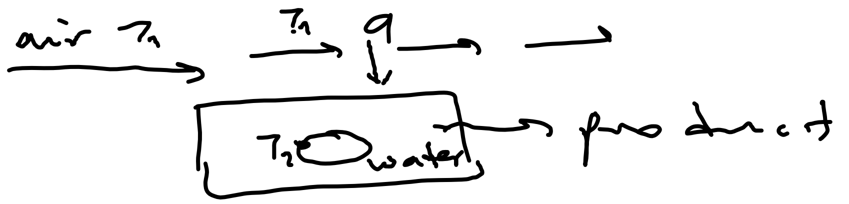
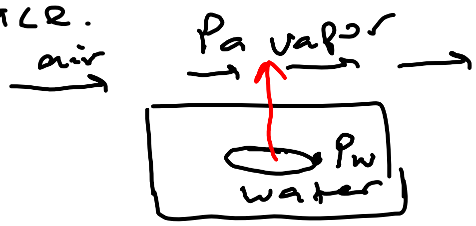
Heat and Mass Transfer

Heat transfer occurs within the product structure and is related to the temperature gradient between product surface and water surface at the same location within the product.



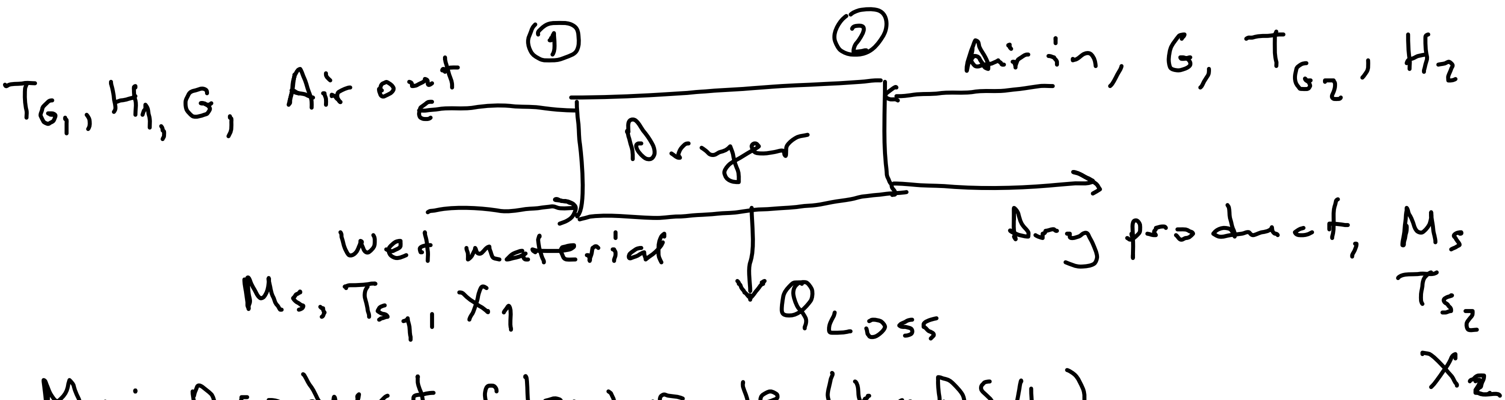
$T_1 > T_2$

⊗ The vapors are transported from water surface within the product to product surface.



$P_w > P_a$

The gradient causing moisture-vapor diffusion is vapor P_w at water surface and at P_a of air at product surface.



M_s: product flow rate (kg DS/h)

X : MC (kg H_2O/kg DS)

T_s : Solids T (K)

G : Air flow rate (kg DA/h)

H : Absolute humidity (kg H_2O/kg DA)

T_G : Air T (K)

H'_G : Enthalpy of air (kJ/kg DA)

H'_s : " " solids (kJ/kg DS)

λ_0 : Latent heat of vaporization of water at Tref (kJ/kg H_2O)

(usually $T_{ref} = 0°C \Rightarrow \lambda_0 = 2501$ kJ/kg H_2O)

⊛ Overall Moisture Balance:

$$G \cdot H_2 + M_s \cdot X_1 = G H_1 + M_s \cdot X_2$$

Enthalpy of air

$$H'_G = C_s (T_G - T_{ref}) + H \cdot \lambda_0$$

C_s: Humid heat $= 1.005 + 1.88 \cdot H \rightarrow$ kJ/kg DA.K

Enthalpy of wet solid

$$H'_s = Cp_s (T_s - T_{ref}) + X \cdot Cp_A (T_s - T_{ref})$$

Cp_s: heat capacity of the dry solid in kJ/kg DS.K

Cp_A: " " " liquid moisture in kJ/kg H_2O.K

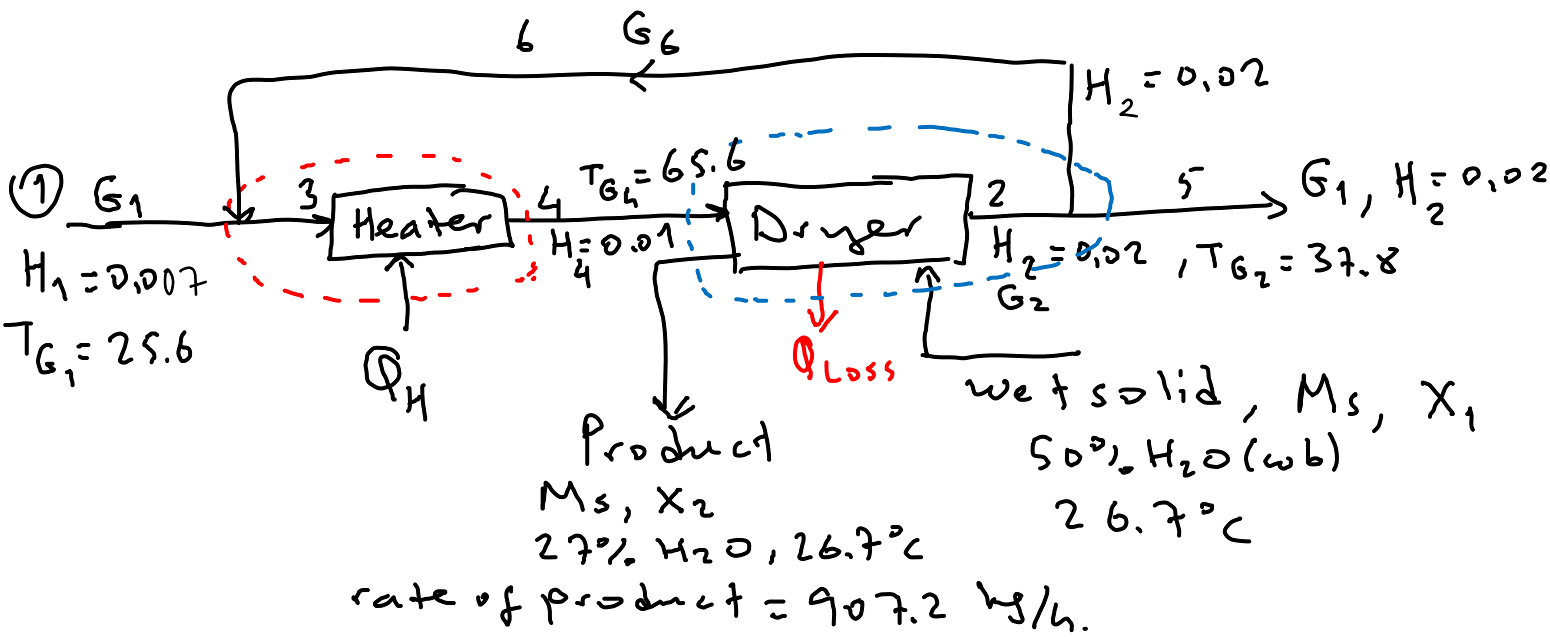
Heat Balance on the Dryer

$$G \times H'_{G_2} + M_s \times H'_{S_1} = G H'_{G_1} + M_s \times H'_{S_2} + Q_{Loss}$$

For an adiabatic process ⇒ $Q_{Loss} \cong 0$.

Example: The wet feed material to a continuous dryer contains 50 wt % water on a wet basis and is dried to 27 wt % by countercurrent air flow. The dried product leaves at the rate of 907.2 kg/h. Fresh air to the system is at 25.6°C and has a humidity of H = 0.007 kg H_2O/kg DA. The moist air leaves the dryer at 37.8°C and H = 0.020 and part of it is recirculated and mixed with the fresh air before entering a heater. The heated mixed air enters the dryer at 65.6°C, and H = 0.01. The solid enters at 26.7°C and leaves at 26.7°C. Calculate the fresh air flow, the percent air leaving the dryer that is recycled, the heat added in the heater and the heat loss from the dryer.

Solution:



$$M_s = 907.2 \times (0{,}73) = 662.25 \text{ kg DS/h}$$

(0,73 ⇒ DS)

$$X_1 = \frac{50}{100-50} = 1 \text{ kg } H_2O/\text{kg DS}$$

$$X_2 = \frac{27}{100-27} = 0{,}37 \text{ kg } H_2O/\text{kg DS}$$

Material balance for H_2O on heater:

$$G_1 \times H_1 + G_6 \times H_2 = (G_1 + G_6) \times H_4$$

$$G_1 \times (0{,}007) + G_6 \times (0{,}02) = (G_1 + G_6) \cdot (0{,}01) \rightsquigarrow (1)$$

Material balance for H_2O on dryer:

$$(G_1 + G_6) \times H_4 + M_s \times X_1 = (G_1 + G_6) \times H_2 + M_s \times X_2$$

$$(2) \rightsquigarrow (G_1 + G_6)(0{,}01) + 662.25 \times 1 = (G_1 + G_6) \times (0{,}02) + 662.25 \times 0.37$$

Solving eqns (1) and (2) =>

$G_1 = 32094$ kg fresh air/h

$G_6 = 9628$ kg air/h.

$$\text{Recycled air} = \frac{9628}{32094 + 9628} \times 100 = \underline{\underline{23\%}}$$

Heat balance on heater, $T_{ref} = 0°C$ =>

$$H'_{G_1} = C_s (T_G - T_{ref}) + H \cdot \lambda_0$$

$$= [(1.005 + 1.88 H)(T_G - T_{ref}) + H(2501)]$$

$$G_1 \times H'_{G_1} + G_6 \times H'_{G_6} + Q_H = (G_1 + G_6) \times H'_{G_4} \Rightarrow$$

$$32094 \times [(1.005 + 1.88 \times 0{,}007)(25.6 - 0) + 0{,}007 \times (2501)] +$$

$$9628 [(1.005 + 1.88 \times 0.02)(37.8 - 0) + 0.02 (2501)] + Q_H$$

$$= (32094 + 9628) [(1.005 + 1.88 \times 0.01)(65.6 - 0) + 0{,}01 (2501)]$$

$$Q_H = 1586138 \frac{kJ}{h} = 1586138 \frac{kJ}{h} \times \frac{1h}{3600\,s} \Rightarrow$$

$$Q_H = 440.6\ kW$$

Heat balance on dryer, $T_{ref} = 0°C \Rightarrow$

$$H'_s = Cp_s (T_s - T_{ref}) + X \cdot Cp_A (T_s - T_{ref})$$

$$(G_1 + G_6) H'_{G_4} + M_s \cdot H'_{s_1} = (G_1 + G_6) \cdot H'_{G_2} + M_s \cdot H'_{s_2} + Q_{Loss}$$

$$(32094 + 9628)\left[(1.005 + 1.88 \times 0.01)(65.6 - 0) + 0.01 \times (2501)\right] +$$
$$662.25\left[Cp_s(26.7 - 0) + 1 \times 4.187\,(26.7 - 0)\right] =$$
$$(32094 + 9628)\left[(1.005 + 1.88 \times 0.02)(37.8 - 0) + 0.02\,(2501)\right] +$$
$$662.25\left[Cp_s(26.7 - 0) + 0.37 \times (4.187)(26.7 - 0)\right] + Q_{Loss}$$

$$Q_{Loss} = 161001.9 \frac{kJ}{h} \equiv 44.7\ kW$$

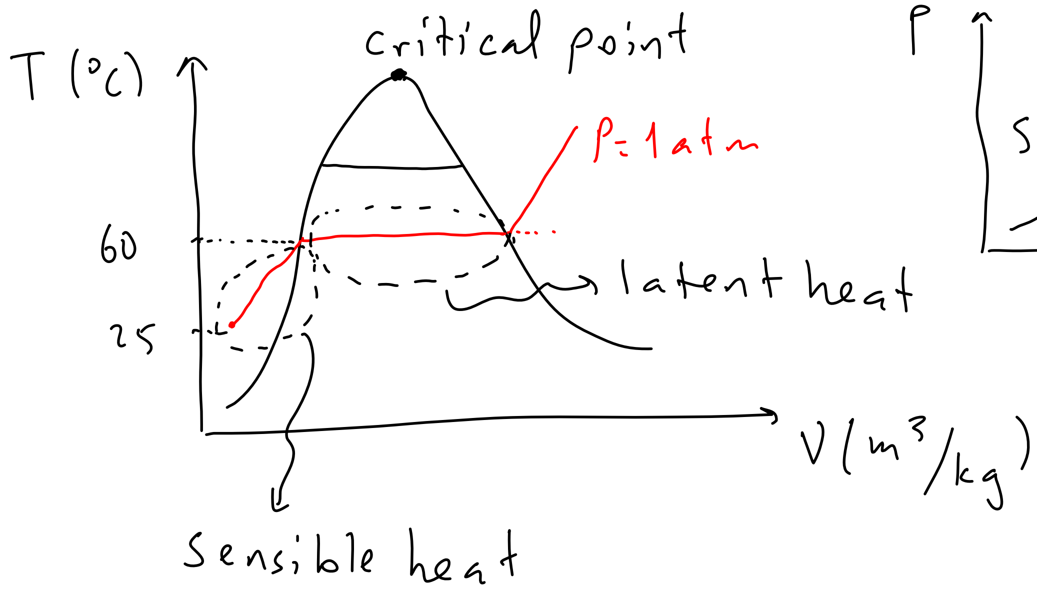
Energy Efficiency in Drying

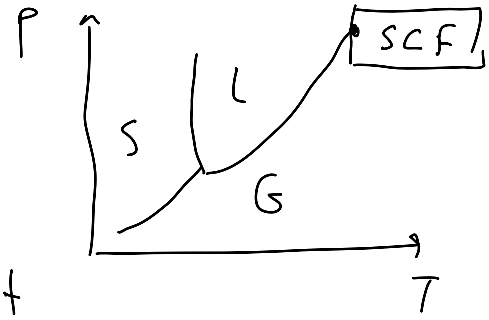
Energy represents the major part of the operating cost of industrial dryers (about 60 %). Large amounts of energy are used for drying food and agricultural materials.

Energy is used mainly for the evaporation of free water, desorption of sorbed water or sublimation of ice in freze drying.

Thermal dehydration requires theoretically 2.3 Mj/kg water and practically 3-6 Mj/kg water evaporated.

Thermodynamically, evaporation of free water requires 2.26 Mj/kg water at 100°C, 2.36 Mj/kg at 60°C and 2.50 Mj/kg at 0°C. The heat of sublimation of ice at 0°C is 2.84 Mj/kg and higher energies are required for desorption of water bound on food biopolymers.





From saturated steam tables, at 60°C, the latent heat of vaporization ≅ 2609.6 − 251.13 ≅ 2360 kJ/kg water

Temp (C)	Vapor Pressure (kPa)	Specific Volume (m3/kg) Liquid	Specific Volume (m3/kg) Sat'd Vapor	Enthalpy (kJ/kg) Liquid	Enthalpy (kJ/kg) Sat'd Vapor
30	4.246	0.0010043	32.894	125.79	2556.3
33	5.034	0.0010053	28.011	138.33	2561.7
36	5.947	0.0010063	23.940	150.86	2567.1
40	7.384	0.0010078	19.523	167.57	2574.3
45	9.593	0.0010099	15.258	188.45	2583.2
50	12.349	0.0010121	12.032	209.33	2592.1
55	15.758	0.0010146	9.568	230.23	2600.9
60	19.940	0.0010172	7.671	251.13	2609.6
65	25.03	0.0010199	6.197	272.06	2618.3

In addition to the latent heat, energy is required for sensible heating of the food material, the dryer, and the exhaust air, and for mechanical movement of the process air (operation of fans).

Thus, the total energy consumption varies in the range of 3.2 – 4.5 Mj/kg water in continuous convective dryers.

It is higher in batch dryers (4 – 6 Mj/kg water) and it may reach 10 Mj/kg water in vacuum and freeze dryers, where energy-consuming vacuum pumps and refrigerated condensers are needed.

Overall Thermal Efficiency of Some Dryers

The cost of drying is an important factor in dryer design. For evaporation of water, 3 Mj/kg water (spray dryers) to 6 Mj/kg water (tray dryers) energy is needed.

The energy efficiency of the dryers (ratio of the heat of evaporation to heat input to the dryer):

- **It is higher in contact (40-80 %) than convective drying (20-40 %).**
- **Rotary dryers are more efficient than tray and spray dryers.**

Dryer Efficiency: That fraction of the total heat supplied during a drying operation which is usefully used in evaporating moisture.

Efficiency $= \frac{2}{5} \times 100 = 40\%$

e.g.,

- **Drum dryers: 35 - 80 %**
- **Spray dryers: 20 - 50 %**
- **Radiant dryers (IR, MW, Radio frequency): 30 – 40 %**

Cost Picture For Some Dryers

If rough costs per kg water evaporated = x,

- **Cost of forced air drying = 0.70x**
- **Cost of drum drying = 0.80x**
- **Cost of spray drying = 1.0x**
- **Cost of vacuum drying = 2.0x**
- **Cost of freeze drying = 4.0x**

Energy Requirements: Another point of view

Heat energy required for 1 kg original material (Q) ⇒

Q = heat energy to raise T to drying temperature (sensible heat)

+

Latent heat to remove H_2O

<u>Example</u>: A food (1 kg) containing 80 % water is to be dried at 100°C down to a moisture content of 10 %. The initial temperature of the food is 21°C, the latent heat of vaporisation of water at 100°C and standard atmospheric pressure is 2257 kJ/kg. The specific heat capacity of the food is 3.8 kJ/kg.°C and of water is 4.186 kJ/kg.°C

> **a) Calculate the quantity of heat energy required per unit weight of the original material for drying under atmospheric pressure.**
>
> **b) Calculate the energy requirement/kg water removed.**

<u>Solution</u>: a)

Initial MC = 1000 g × 0.8 = 800 g } initially.
DS = 1000 − 800 = 200 g

In product ⇒ 10% MC, 90% DS

Moisture in product $= \frac{200\text{ g}}{90\%} \times 10\% = 22.2$ g moisture

H_2O removed = 800 − 22.2 = 777.8 g ≡ 0.778 kg

$Q = m \cdot C_p (T_2 - T_1) + \lambda \cdot MC_{(removed)}$

$= 1\text{ kg} \times 3.8 \frac{\text{kJ}}{\text{kg}\cdot°\text{C}} \times (100 - 21)°\text{C} + 2257 \frac{\text{kJ}}{\text{kg } H_2O} \times 0.778\text{ kg } H_2O$

$Q = 2056$ kJ/kg.

b)

$$\frac{\text{Energy}}{\text{kg } H_2O \text{ removed}} = \frac{2056 \text{ kJ}}{0.778 \text{ kg}} \cong 2643 \text{ kJ/kg } H_2O$$

<u>Example</u>: Using the same data, calculate the heat energy required to remove the moisture per unit weight of raw material, if vacuum drying is to be carried out at 60°C under the corresponding saturation pressure of 20 kPa abs. [The latent heat of vaporisation of water at 60°C (20 kPa) is 2358 kJ/kg].

<u>Solution</u>:

$$Q = m c_p \cdot \Delta T + \lambda \times (\text{moisture evaporated})$$
$$= 1 \times 3.8\,(60-21) + 2358 \times 0.778$$
$$= 1983 \text{ kJ}/1 \text{ kg raw material.}$$

<u>Example</u>: How much energy is required per kg of raw material, if the same foodstuff is to be freeze-dried at 0°C? (Latent heat of sublimation at 0°C is 2838 kJ/kg).

<u>Solution</u>:

$$Q = m c_p \Delta T + \lambda \cdot \text{Moisture removed}$$
$$= m \cdot c_p (0-0) + 2838 \times 0.778$$
$$= 2208 \text{ kJ}$$

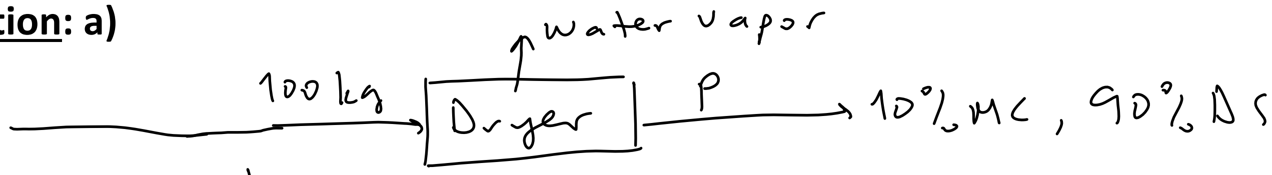
<u>Example</u>: A dryer reduces the moisture content of 100 kg of a potato product from 80 % to 10 % moisture. If 250 kg of steam at 70 kPa gauge is used to heat 49,800 m^3 of air to 80^{o}C, and if the air is cooled to 71^{o}C in passing through the dryer, calculate the efficiency of the dryer in terms of the heat supplied

a) from the air

b) from the steam

<u>Data supplied</u>: Cp of potato = 3.43 kJ/kg.^{o}C, Assume potato enters at 24^{o}C and leaves at the same temperature as the exit air, Latent heat of vaporization at 71^{o}C is 2331 kJ/kg water, Cp of air = 1 kJ/kg.^{o}C, density of air = 1.06 kg/m^3.

Solution: a)



80% MC = 80 kg

20% DS = 20 kg

Dry solid balance ⇒

$100 \times 0.2 = P \times 0.9 \Rightarrow P = 22.2\ kg\ (DS + H_2O)$

Mass of H_2O in product $= 22.2 - 20 = 2.22\ kg\ H_2O$

H_2O removed $= 80 - 2.2 = 77.8\ kg$

⊛ Heat supplied to potato $= m\,C_p\,\Delta T + \lambda \times (kg\ H_2O\ removed)$

$= 100 \times 3.43 \times (71 - 24) + 2331 \times 77.8 = 1.97 \times 10^5\ kJ.$

⊛ Heat given up by air / 100 kg potato $= m\,C_p\,\Delta T$

$= \underbrace{49800\ m^3 \times 1.06 \frac{kg}{m^3}}_{m} \times 1 \frac{kJ}{kg.°C} \times (80 - 71)\ °C = 4.75 \times 10^5\ kJ$

Thermal efficiency of air drying $= \dfrac{1.97 \times 10^5}{4.75 \times 10^5} \times 100 = 41\%$

b) λ of steam at ($70 + 100 = 170$ kPa absolute) is

(100 = atmospheric P)

2216 kJ/kg steam. (From saturated steam table)

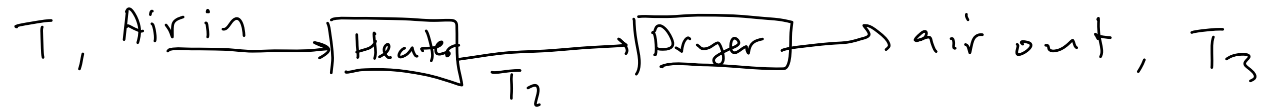
$$\text{heat in steam} = 250\ \text{kg steam} \times \frac{2216\ \text{kJ}}{1\ \text{kg steam}} = 5.54\times10^{5}\ \text{kJ}$$

$$\text{Overall efficiency} = \frac{1.97\times10^{5}}{5.54\times10^{5}} \times 100 = 35.5\%$$

<u>Heat Recovery</u>:

Recirculation of the exhaust air from the dryer can recover part of the heat rejected to the environment. Convective dryers use some of recirculation, recovering only the sensible heat, since it is difficult to recover the latent heat of evaporation of water in the exhaust gases.

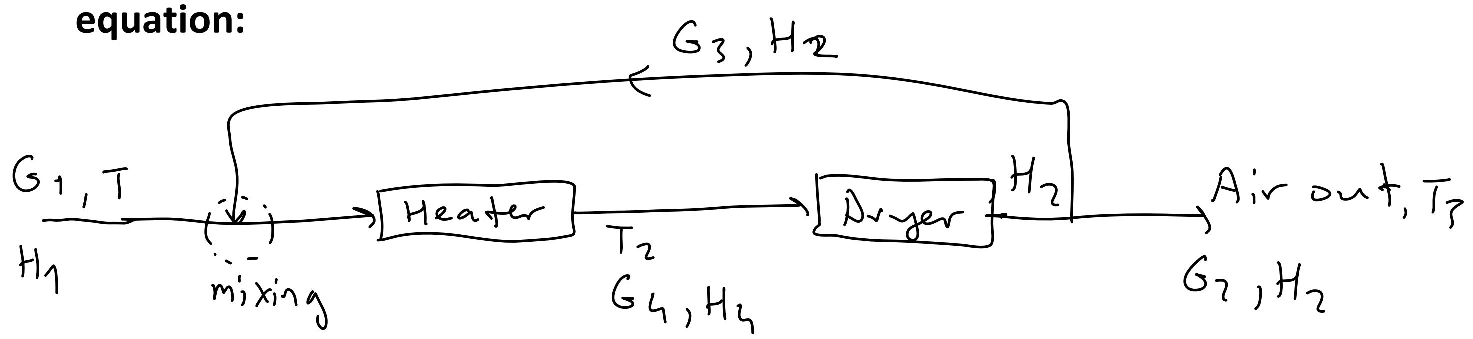
- **The theoretical thermal efficiency n of a convective dryer without recirculation is**



$$n = \frac{(T_2 - T_3)}{(T_2 - T)} \times 100$$

where, T, T_2 and T_3 are the ambient, inlet, and exit air temperatures, respectively.

- **The thermal efficiency of a dryer with recirculation is given by the following equation:**



$$\text{Around mixing} \Rightarrow G_1 \times H_1 + G_3 \times H_2 = \underbrace{(G_1 + G_3)}_{G_4} \times H_4$$

$$\eta = \frac{(T_2 - T_3)}{[(T_2 - T_3) + (1-w)\cdot(T_3 - T)]} \times 100$$

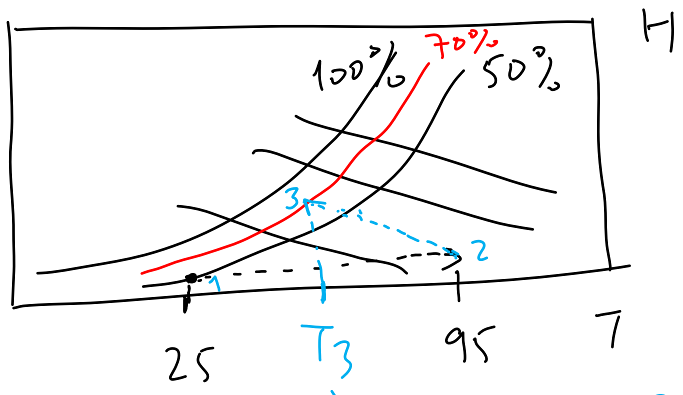
where, w is the ratio of recirculated to total air flow.

Example: A convective air dryer is used to dry a food product. Atmospheric air at 25°C and 50 % RH is heated to 95°C and passed adiabatically through the dryer, leaving at 70 % RH.

a) Calculate thermal efficiency for operation without recirculation.

b) Calculate thermal efficiency for operation with 50 % air recirculation. The recirculated air is heated to 95°C and exits the dryer at 70 % RH.

Solution: a) Find T_3 ?

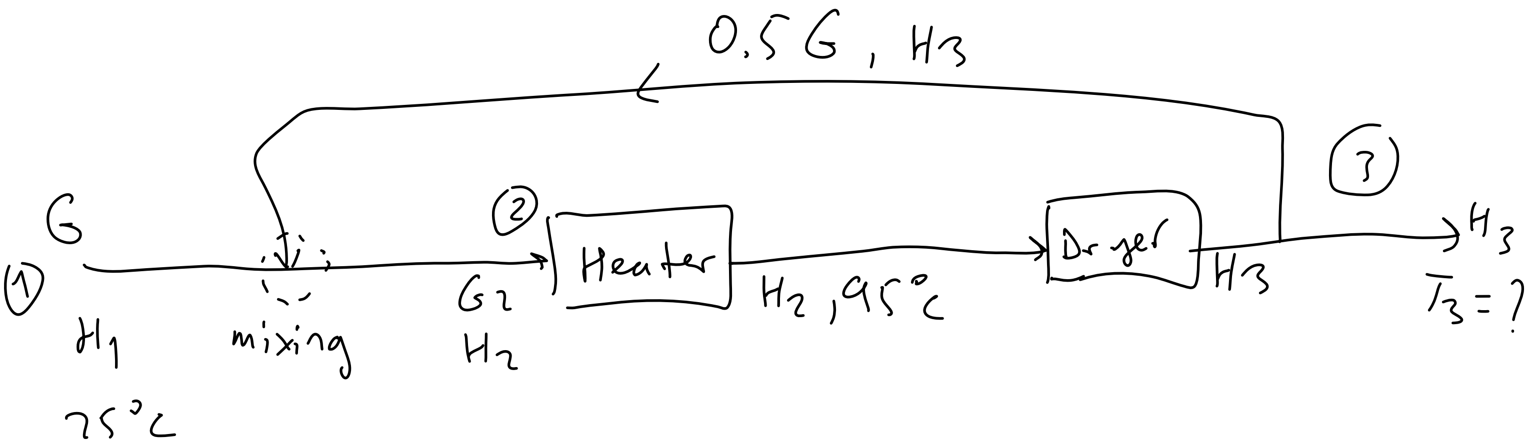


$\rightarrow T_3 = 40°C$ exit Temp.

$$\eta = \frac{(95-40)}{(95-25)} \times 100 = 78.57\%$$

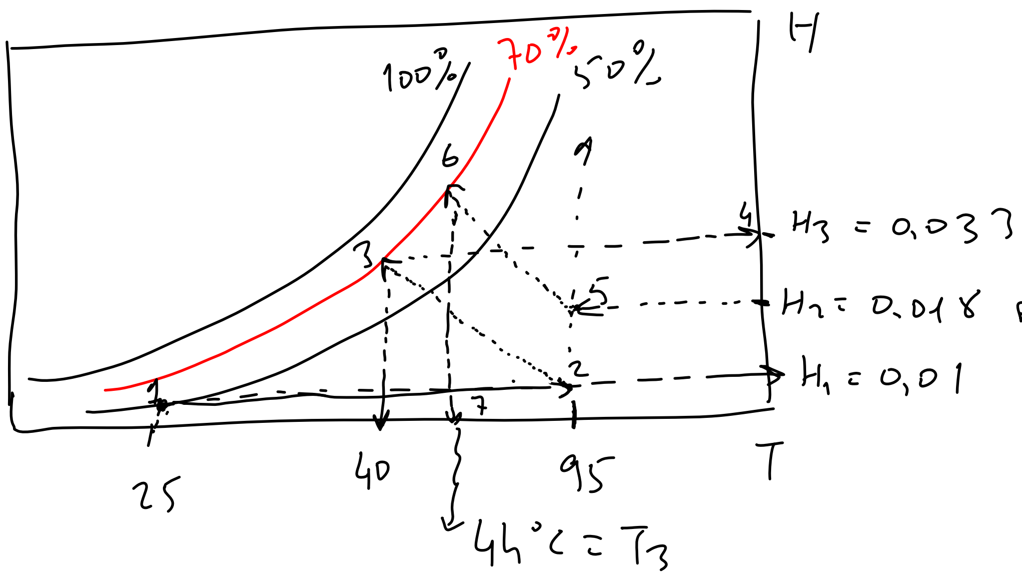
b) With 50% recirculation of air $\Rightarrow T_3$?

$$w = \frac{0.5\,G}{1\,G} = 0.5$$



Around mixing =)

$$G \times H_1 + (0{,}5\,G) \cdot H_3 = (G + 0{,}5\,G) \times H_2$$



$$0{,}01 + 0{,}5 \times 0{,}033 = (1 + 0{,}5) \times H_2 =)$$

$$H_2 = 0{,}018$$

$$\eta = \frac{(95-44)}{\left[(95-44) + (1-0{,}5)(44-25)\right]} \times 100 \cong 84\%$$