Example: Assume $s_1^2 = 90.1$, $V_1 = 8$, $s_2^2 = 118.3$ and $V_2 = 6$, then, test if sample #1 is less variable at $\alpha = 0.05$.

Solution:

$$\left.\begin{array}{l} H_0: S_1^2 = S_2^2 \\ H_A: S_1^2 < S_2^2 \end{array}\right\} \text{One-tailed LHS test.}$$

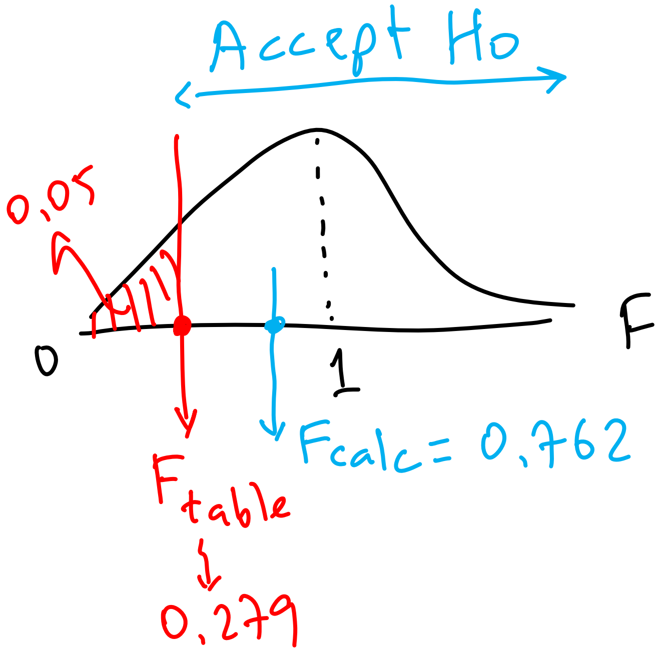
$$F_{calc} = \frac{\text{Smaller variance}}{\text{Larger variance}}$$

$$F_{calc} = \frac{90.1}{118.3} = \underline{0.762}$$

$$F_{table} = ? \quad F_{(1-\alpha)}(V_1, V_2) \equiv \frac{1}{F_\alpha(V_2, V_1)}$$

$$F_{(1-0.05)}(8,6) = F_{0.95}(8,6) = \frac{1}{F_{0.05}(6,8)}$$

$$F_{0.95}(8,6) = \frac{1}{3.58} = \underline{0.279}$$



Decision: Do not reject Ho.

Conclusion: Sample #1 is not less variable than sample #2.

Example: If $s_1 = 1.93$, $s_2 = 3.10$, $n_1 = 18$, $n_2 = 13$ and $\alpha = 0.10$. Test if two populations have the same variability.

Solution:

$$\left.\begin{array}{l} \times\ H_0:\ \sigma_1^2 = \sigma_2^2 \\ \checkmark\ H_A:\ \sigma_1^2 \neq \sigma_2^2 \end{array}\right\} \Rightarrow \text{Two tailed test}$$

$$F_{calc} = \frac{\text{Larger variance}}{\text{Smaller variance}}$$

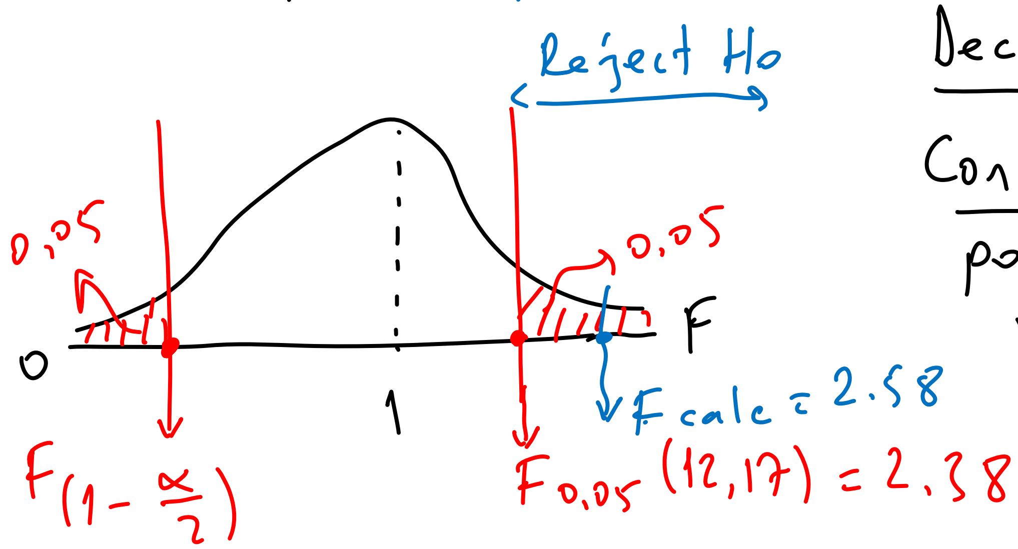
$$F_{calc} = \frac{(3.10)^2}{(1.93)^2} = \underline{2.58}$$

$$F_{table}(\nu_1, \nu_2) = ?$$

$$\left.\begin{array}{l} \nu_1 = n_2 - 1 = 13 - 1 = 12 \\ \nu_2 = n_1 - 1 = 18 - 1 = 17 \end{array}\right\} \quad \begin{array}{l} \alpha = 0{,}10 \Rightarrow \\ \alpha/2 = \frac{0{,}10}{2} = 0{,}05 \Rightarrow \end{array}$$

Use $F_{0,05}$ table.

$$F_{0,05}(12, 17) = \underline{2.38}$$



__Decision__: Reject Ho

__Conclusion__: The population variances are different.

Example: Two samples of size $n_1 = 13$, $n_2 = 16$, $s_1^2 = 45$, $s_2^2 = 33$. Use $\alpha = 0.05$, then, test whether variances of populations from which the samples were drawn are different.

Solution:

$$\left.\begin{array}{l} H_0: \sigma_1^2 = \sigma_2^2 \\ H_A: \sigma_1^2 \neq \sigma_2^2 \end{array}\right\} \text{two-tailed test}$$

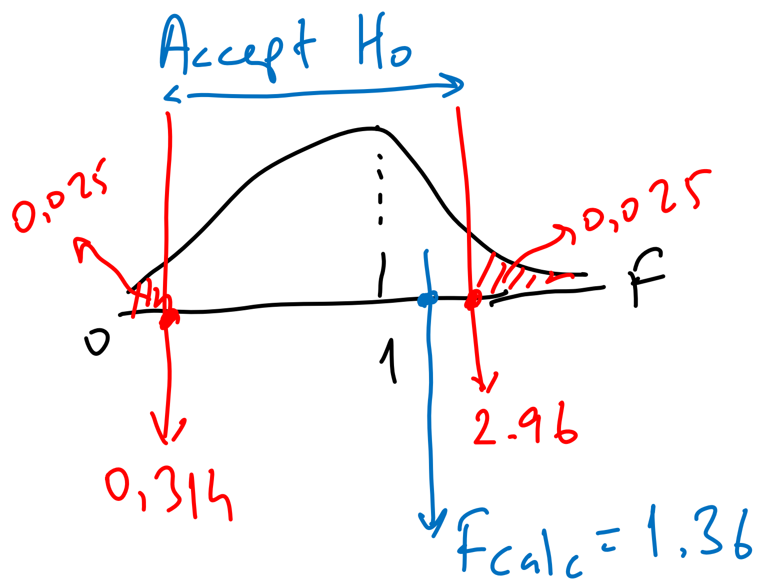
$$F_{calc} = \frac{45}{33} = 1.36$$

$$\left.\begin{array}{l} \nu_1 = 13 - 1 = 12 \\ \nu_2 = 16 - 1 = 15 \end{array}\right\} \Rightarrow F_{table} = F_{\frac{0,05}{2}}(\nu_1, \nu_2) \rightsquigarrow \text{upper critical point.}$$

$$F_{0,025}(12,15) = 2.96$$

$$F_{(1-\frac{\alpha}{2})}(\nu_1, \nu_2) = \frac{1}{F_{\alpha/2}(\nu_2, \nu_1)} \rightsquigarrow \text{lower critical point.}$$

$$= \frac{1}{F_{0,025}(15,12)} = \frac{1}{3.18} = 0,314$$



Decision: Do not reject H_0

Conclusion: The variances of populations are not different.

Example: There are two fruit juice filling machines. Does the following sample information present sufficient evidence to reject the manufacturer's claim that his modern high-speed bottle filling machine (M) fills bottles with no more variance than the company's present machine (P) ? Use α = 0.01.

	n	s^2
P	22	0.0008
M	25	0.0018

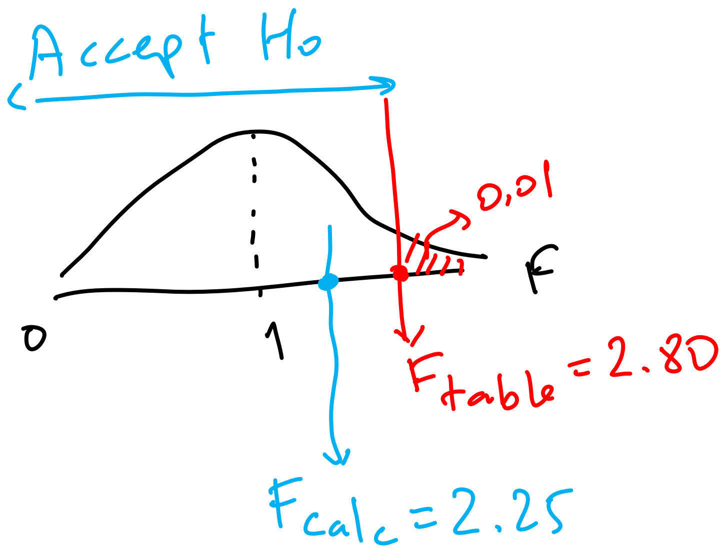
Solution:

$$\left.\begin{aligned} H_0 &: \sigma_M^2 = \sigma_P^2 \\ H_A &: \sigma_M^2 > \sigma_P^2 \end{aligned}\right\} \text{RHS test}$$

$$F_{calc} = \frac{\text{Larger variance}}{\text{Smaller variance}} = \frac{0{,}0018}{0{,}0008} = 2.25$$

$$\left.\begin{aligned} \nu_1 &= 25-1 = 24 \\ \nu_2 &= 22-1 = 21 \end{aligned}\right\} \text{Use } F_\alpha = F_{0{,}01}(24,21) \text{ table}$$

$$F_{0{,}01}(24,21) = 2.80$$



Decision: Do not reject Ho

Conclusion: The samples do not present sufficient evidence to reject the manufacturer's claim.

Homework: Investment risk is generally measured by the volatility of possible outcomes of the investment. The most common method for measuring investment volatility is by computing the variance (or standard deviation) of possible outcomes. Return over the past 10 years for the first alternative and 8 years for the second alternative produced the following data:

Investment 1: n_1 = 10, $\overline{x}_1$ = 17.8 %, s_1^2 = 3.21

Investment 2: n_2 = 8, $\overline{x}_2$ = 17.8 %, s_2^2 = 7.14

Do the data present sufficient evidence to indicate that the risks for investments 1 and 2 are unequal ? α = 0.05

Solution:

COMPARING TWO POPULATION MEANS

pop.

sample

$\mu = \mu_0$ (single mean)

(single population)

Two populations $\Rightarrow$

$\mu_1 = \mu_2$

Population 1
(Public institution salaries)

Sample 1

Compute $\bar{X}_1$

Population 2
(Private institution salaries)

Sample2

Compute $\bar{X}_2$

- Compare $\bar{X}_1$ and $\bar{X}_2$
- Make a decision

1) Test for the Difference Between Two Population Means: Given Two Large Samples (n_1 and $n_2 > 30$) and the Population Variances σ_1^2 and σ_2^2 are Known:

Suppose $\bar{x}_1$ and $\bar{x}_2$ are the means of two independent random samples of size n_1 and n_2 from populations with unknown means and known variances σ_1^2 and σ_2^2 respectively.

To test the hypothesis about the value of the difference $\mu_1 - \mu_2$, then the z value is

a) If $\sigma_1^2 \neq \sigma_2^2$ and known $\Rightarrow$

$$z_{calc} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$H_0: \mu_1 = \mu_2 \Rightarrow \mu_1 - \mu_2 = 0 \Rightarrow$

$$\boxed{z_{calc} = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}}$$

b) If $\sigma_1^2 = \sigma_2^2 = \sigma^2$ and known $\Rightarrow$

$$\boxed{z_{calc} = \frac{\bar{x}_1 - \bar{x}_2}{\sigma \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}}$$

CI for $\mu_1 - \mu_2 \Rightarrow$

$$\underbrace{(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2} \cdot \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2} \cdot \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}_{\text{for population}}$$

$$\underbrace{(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < (\mu_1 - \mu_2) < \cdots}_{\text{for sampling distribution.}}$$

Example: Two independent random samples of sizes 80 and 100 respectively are drawn from populations whose variances are 4 and 1, respectively. The means of the samples are 54.5 and 56.2. Is there evidence to suggest that there is no difference in the population means ? Assume population variances are known and not equal. Use α = 0.05 significance level.

Solution: $\bar{x}_1 = 54.5$, $\bar{x}_2 = 56.2$, $\sigma_1^2 = 4$, $\sigma_2^2 = 1$

$n_1 = 80$, $n_2 = 100$, $\alpha = 0.05$

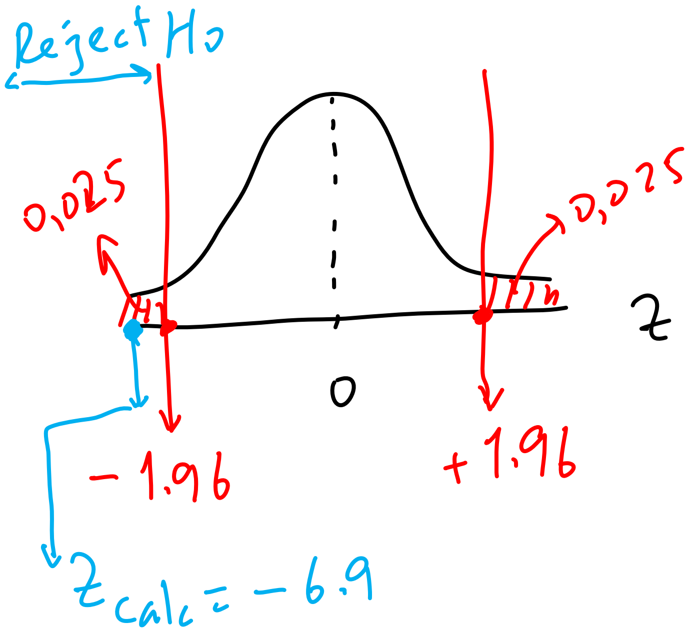
X $H_0: \mu_1 = \mu_2$

✓ $H_A: \mu_1 \neq \mu_2$

$$z_{calc} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \rightsquigarrow \text{since } \sigma_1^2 \neq \sigma_2^2$$

$$z_{calc} = \frac{54.5 - 56.2}{\sqrt{4/80 + 1/100}} \cong -6.9$$

$$z_{table} = z_{\alpha/2} = z_{0,025} = \mp 1.96$$



Decision: Reject Ho

Conclusion: There is difference between the population means.