One – Tailed (Sided) Test

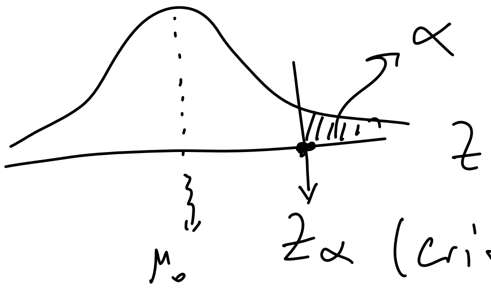
$H_0: \mu = \mu_0$

$H_A: \mu > \mu_0$

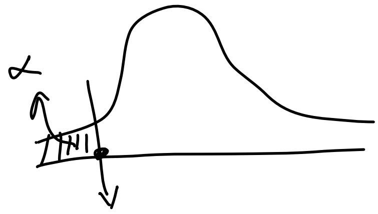
, OR

$H_0: \mu = \mu_0$

$H_A: \mu < \mu_0$



RHS



LHS.

Example: **A random sample of 100 recorded deaths in a country showed an average life span of 71.8 years with a standard deviation of 8.9 years. Does this seem to indicate that the average life span today is greater than 70 years ? Use α = 0.05.**

Solution:

$$\left.\begin{array}{l} H_0: \mu = \mu_0 = 70 \text{ years} \\ H_A: \mu > \mu_0 = 70 \end{array}\right\} \text{one – tailed RHS test.}$$

$\alpha = 0{,}05$

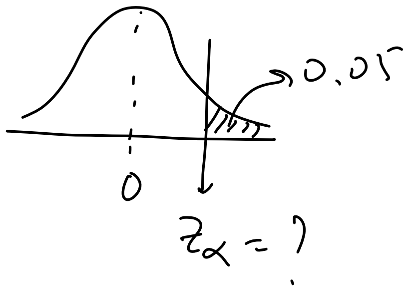
Test statistic value: z_{calc} =)

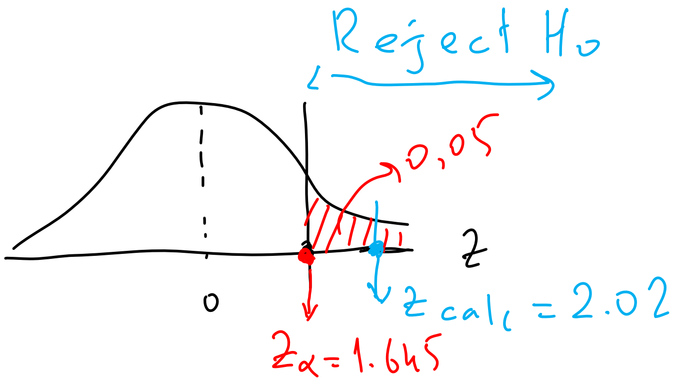
$$z_{calc} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{71.8 - 70}{8.9/\sqrt{100}} = 2.02$$

$\sigma \to s$

Table value: z_α = ?

$z_{0{,}05} \cong 1.645$





Decision: Reject Ho

Conclusion: The average life span today (71.8) is greater than 70 yrs.

Example: **Test the hypothesis that weight loss in a new diet program exceeds 20 pounds during the first month at 95 % confidency.**

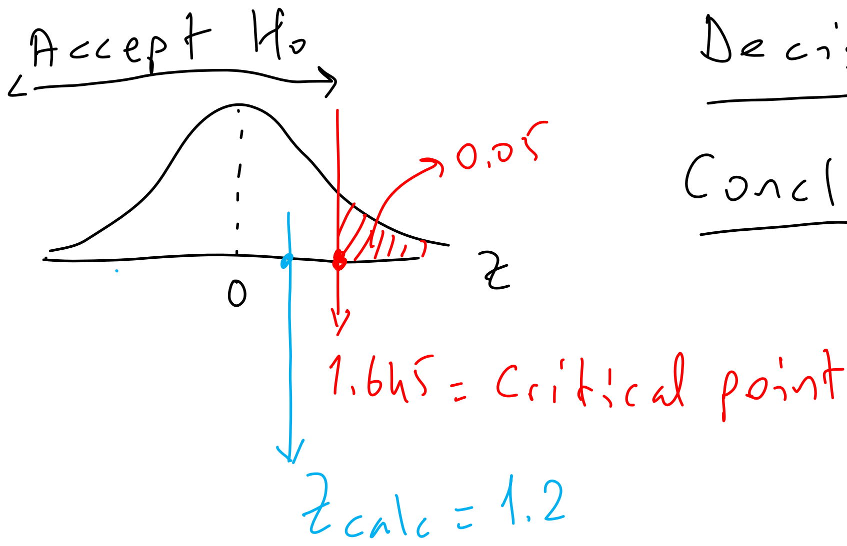
Sample data: n = 36, $\bar{x}$ = 21, s^2 = 25

Solution: 95 % CI $\Rightarrow$ $\alpha = 0{,}05$

$$\left.\begin{array}{l} H_0: \mu = \mu_0 = 20 \\ H_A: \mu > \mu_0 = 20 \end{array}\right\} \Rightarrow \text{RHS - test}$$

$$z_{calc} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{21 - 20}{5/\sqrt{36}} = 1{,}2$$

$$z_{table} = z_\alpha = z_{0{,}05} \cong 1.645$$



Decision: Do not reject Ho.

Conclusion: Weight loss in the new diet program does not exceed 20 pounds.

Example: **(Confidence interval estimation)**

The mean and Standard deviation for the quality grade-point averages of a random sample of 36 college seniors are calculated to be 2.6 and 0.3, respectively. Find the 95 % and 99 % confidence intervals for the mean of the entire senior class.

Solution:

For 95% CI $\Rightarrow$ $\alpha = 0{,}05$

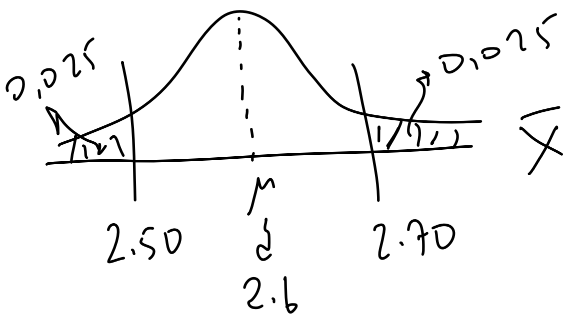
$n > 30$, $s = 0{,}3$

$$z_{\alpha/2} = z_{0{,}05/2} = z_{0{,}025} = \mp 1.96$$

$$2.6 - (1.96) \times \frac{0{,}3}{\sqrt{36}} < \mu < 2{,}6 + (1{,}96) \times \frac{0{,}3}{\sqrt{36}}$$

$$L_1 \leftarrow 2.50 < \mu < 2.70 \rightarrow L_2$$

$\therefore$ We are 95% confident (sure) that the interval from 2.50 to 2.70 includes the true grade average of 2.6.



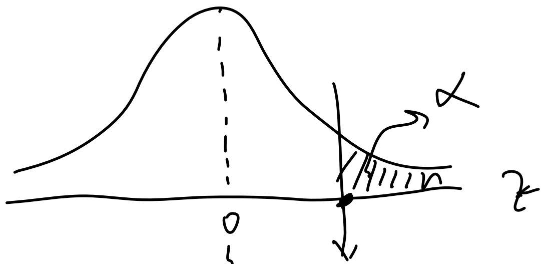
For 99% CI $\Rightarrow$ $\alpha = 0{,}01$

$$z_{\alpha/2} = z_{0{,}01/2} = z_{0{,}005} = \mp 2.575$$

$$2.6 - (2.575)\times\frac{0.3}{\sqrt{36}} < \mu < 2.6 + (2.575)\times\frac{0.3}{\sqrt{36}}$$

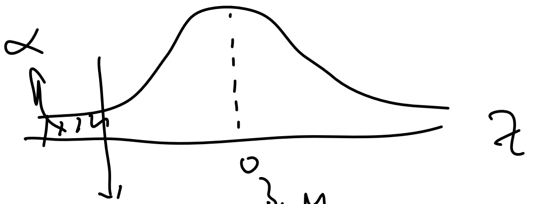
$$\underbrace{2.47}_{L_1} < \mu < \underbrace{2.73}_{L_2}$$

One-Sided Confidence Limit For μ



μ $\quad z_\alpha$ for upper limit

Upper limit: $\left[\mu \leq \bar{X} + z_\alpha \cdot \frac{\sigma}{\sqrt{n}}\right]$ (not $z_{\alpha/2}$!!!)



$-z_\alpha$

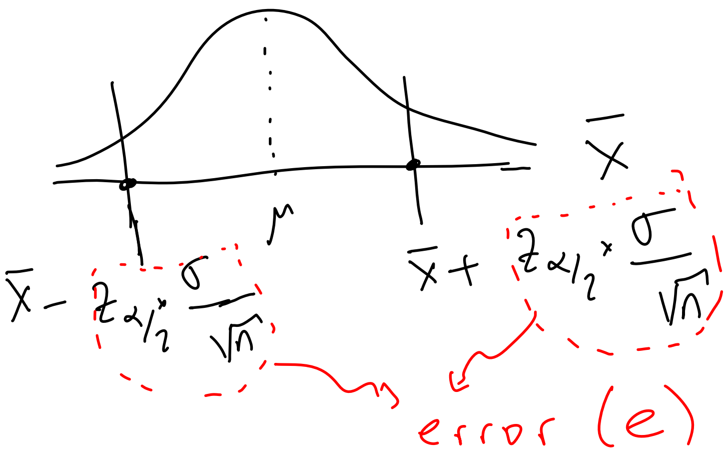
→ Lower confidence z-value

Lower confidence Limit: $\left[\mu \geq \bar{X} - z_\alpha \cdot \frac{\sigma}{\sqrt{n}}\right]$

⊗ as $n \nearrow \Rightarrow CI = (L_2 - L_1) \downarrow$ for a given α.

⊗ as $\alpha \downarrow \Rightarrow CI \nearrow$ for a given sample size.

Margin of Error in Estimating μ:



If $\bar{x}$ is used as an estimate of μ , we can be 100(1- α)% confident that the error will not exceed $z_{\alpha/2} * \frac{\sigma}{\sqrt{n}}$ **.**

$$\boxed{\text{Error } (e) = z_{\alpha/2} * \frac{\sigma}{\sqrt{n}}}$$

is used when σ is known. If σ is unknown, use S.

Sample Size for Estimating μ:

If $\bar{x}$ is used as an estimate of μ , we can be 100(1- α)% confident that the error will not exceed a specified amount "e" when the sample size is

$$n = \left(\frac{z_{\alpha/2} * \overbrace{\sigma}^{s}}{e} \right)^2$$

(for one-tailed => take z_α not $z_{\alpha/2}$).

<u>Example</u>: How large a sample size is required in previous example (remember college students) if we want to be 95 % confident that our estimate of μ is not off by more than 0.06 ?

<u>Solution</u>:

St. dev. $= S = 0{,}3$ (given) will be used for $\sigma \Rightarrow$

$z_{\alpha/2} = z_{0{,}025} = 1.96$

$$n = \left[\frac{(1.96) \times (0{,}3)}{0{,}06}\right]^2 \cong 96 \text{ samples.}$$

<u>Example</u>: What is the error in the example of college students ?

<u>Solution</u>:

$n = 36$ (in previous example of college students), $S = 0{,}3$, $\alpha = 0{,}05 \Rightarrow z_{\alpha/2} = 1.96$

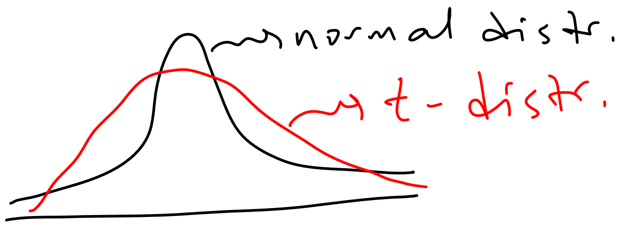
$$e = \frac{1.96 \times 0{,}3}{\sqrt{36}} = 0{,}098 \quad (n \downarrow \Rightarrow e \nearrow)$$

Small Sample Confidence Interval for μ (σ is unknown)

In most cases σ is unknown and sample size may be too small to allow the use of sample variance instead of population variance. Then, we shall describe a new distribution known as the Student's t-Distribution which permits exact inferences to be made, when s^2 is computed from a sample of any size.

t- Distribution

- t- distribution is more variable (flat) than the normal distribution.



- it is used when σ is unknown and $n<30$.
- it is very similar to the equation for z.

Test statistic value: t_{calc}

$$\boxed{t_{calc} = \frac{\bar{x} - \mu}{s/\sqrt{n}}}$$

If $\bar{X}$ and S are the mean and st. dev. of a random sample of size $n < 30$ from an approximate normal population with unknown variance σ^2, then, a CI for mean is

$$\boxed{\bar{X} - t_{\alpha/2} \cdot \frac{S}{\sqrt{n}} < \mu < \bar{X} + t_{\alpha/2} \cdot \frac{S}{\sqrt{n}}}$$

$t_{\alpha/2}$ value is the t value with $\nu = n-1$ degrees of freedom, leaving an area of $\alpha/2$ to the right, $\alpha/2$ to the left.

As $n \to \infty \Rightarrow t \to z$

Example: The content of 7 similar containers of sulfuric acid are 9.8, 10.2, 10.4, 9.8, 10.0, 10.2 and 9.6 liters. Find a 95 % confidence interval for the mean content of all such samples assuming an approximate normal distribution for container contents.

Solution: Calculate $\bar{X}$ and $S \Rightarrow$

$\bar{X} = 10.0$, $S = 0{,}283$

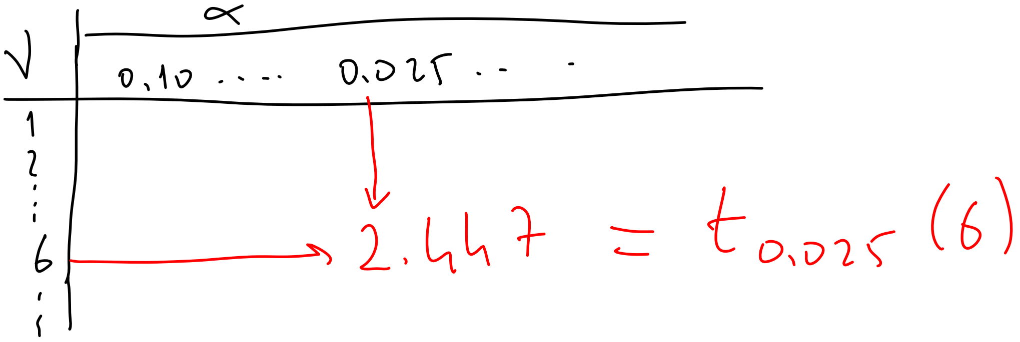
$\alpha = 0{,}05 \Rightarrow t_{\alpha/2} = t_{0{,}05/2} = t_{0{,}025}$

$t_{table} = t_{\alpha/2}(\nu) = ?$

$\nu = d.f. = n - 1 = 7 - 1 = 6$

$t_{0{,}025}(6) = ?$

t - table



$$10 - (2.447) \times \frac{0,283}{\sqrt{7}} < \mu < 10 + (2.447) \times \frac{0,283}{\sqrt{7}} \Rightarrow$$

$$\underbrace{9.74}_{L_1} < \mu < \underbrace{10,26}_{L_2}$$

<u>Example</u>: Suppose NASA wants to estimate the mean lifetime of a particular mechanical component used in the space shuttle Discovery. Due to prohibitive costs, only ten components can be tested under simulated space conditions. The lifetimes (in hr) of ten components were recorded with the following results:

$\bar{x}$ = 1173,6 hr; s = 36.3 hr

Estimate μ, the mean lifetime of the mechanical components with a 95 % confidence interval.

<u>Solution</u>:

$$n = 10, \quad \nu = n - 1 = 10 - 1 = 9, \quad \alpha = 0,05$$

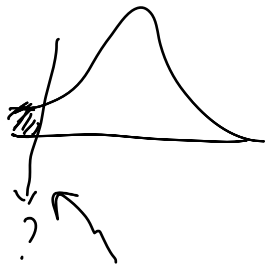
$$t_{\alpha/2}(\nu) = t_{0,025}(9) = 2.262 \quad (t_{table})$$

$$CI: 1173,6 - 2.262 \times \frac{36.3}{\sqrt{10}} < \mu < 1173.6 + 2.262 \times \frac{36.3}{\sqrt{10}}$$

$$1147.6 < \mu < 1199.56$$

We are 95 % confident that the interval from 1147.6 hr to 1199.56 hr contains the true mean lifetime of the mechanical components.

⊗ What is 95% lower limit?



$$\bar{x} - \underbrace{t_{\alpha}}_{t_{0.05}(9)\,=\,?} \times \left(\frac{s}{\sqrt{n}}\right) = 1173.6 - 1.833 \times \frac{36.3}{\sqrt{10}} = 1152.5$$

∴ We are 95% confident that true but unknown value of μ is greater than 1152.5 hr.

Hypothesis Testing by Using t-Distribution

Example: Suppose that we have s = 1.732, $\bar{x}$ = 31 and n = 7. Test the hypothesis whether μ = 30 or not. Use 99 % CI.

Solution: Two-tailed test =>

$n < 30$ and σ is unknown => apply t-test.

✓ $H_0: \mu = \mu_0 = 30$

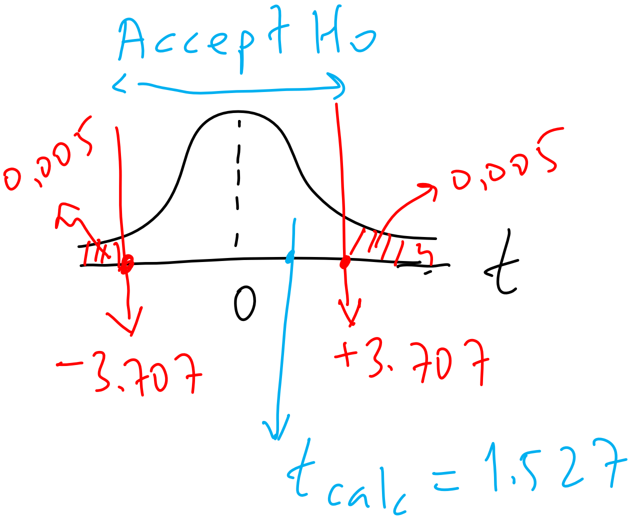
$H_A: \mu \neq \mu_0 = 30$

Calculate test statistic value = t_{calc}

$$t_{calc} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{31 - 30}{1.732/\sqrt{7}} = 1.527$$

$$\nu = n-1 = 7-1 = 6$$

$$\alpha = 0{,}01 \Rightarrow t_{\alpha/2}(\nu) = t_{0{,}005}(6) = 3.707$$



Decision: Do not reject Ho

Conclusion: μ is equal to 30.

Example: For the sample data given below, test the hypothesis that weight loss in a new diet program exceeds 20 pounds per first month.

Sample data: n = 25, $\bar{x}$ = 21.3, s^2 = 25. Use α = 0.05 significance level.

Solution: σ is unknown, $n < 30 \Rightarrow$ t-distribution.

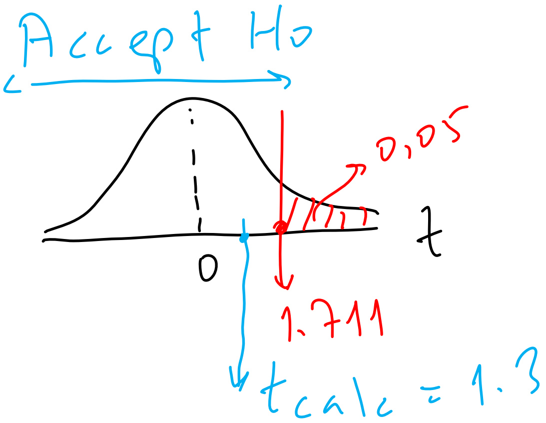
$H_0: \mu = \mu_0 = 20$

$H_A: \mu > \mu_0 = 20$

$$\nu = n-1 = 25-1 = 24$$

$t_\alpha(\nu) = t_{0{,}05}(24) = 1.711 \rightsquigarrow$ critical point

$$t_{calc} = \frac{21.3 - 20}{5/\sqrt{25}} = 1.3$$



Decision: Accept Ho

Conclusion: The weight loss doesn't exceed 20 pounds.

Homework: For the same data, but if $\bar{x} = 18$, test whether the weight loss in new diet program is less than 20 pounds per first month.

Date due: